Abstract—This paper investigates cooperative non-orthogonal multiple access (NOMA) with simultaneous wireless information and power transfer (SWIPT) radio networks. A decode-and-forward relay deserves a base station to transmit information to two users. Two access schemes are addressed: direct and relay assisted transmission (DRAT) where a line-of-sight exists between the source and destination, and non-direct and relay assisted transmission (nDRAT) where the only access to the final users is through the relay. New closed-form expressions of outage probability are derived at these schemes. A generalization using Nakagami-m fading channels in considered, in order to present a complete cover of relayed NOMA systems with energy harvesting behavior in small scale fading. We consider the impact of time splitting fraction, power allocation and channel parameters on system maintainability and evaluate its maximum data rate transmission with full autonomy. By comparing the two schemes, cooperative NOMA with energy harvesting (EH) in nDRAT scenario outperforms transmission with direct link in terms of outage probability and transmission data rate.

Index Terms—Nakagami-m fading, simultaneous wireless information and power transfer, decode-and-forward, non-orthogonal multiple access, energy harvesting, outage probability.

I. INTRODUCTION

Non Orthogonal Multiple Access technique has shown remarkable performance gain in fifth generation wireless networks. With higher spectral efficiency, NOMA serves multiple users using superposition coding despite their channel conditions which may be different according to their position in the cell or even the environment quality [1–2]. NOMA design has been also applied in cooperative networks in order to enhance spectral efficiency [1–3].

The impact of cooperative NOMA in small-fading systems with SWIPT has not been well investigated. Many network configurations were modeled with different performance parameters. In [4] outage probability of power splitting SWIPT two-way relay networks was studied. Outage and ergodic sum rate of amplify-and-forward (AF) relaying system were studied in [5] in comparison with orthogonal multiple access (OMA) systems. Throughput of fixed gain AF relaying with NOMA is considered in [6]. However it is difficult to find a complete case study that addresses small-fading networks, the majority of papers are limited to Rayleigh channels. In this paper, we cover all small scale fading systems for dense signal scatters with several network configurations. We present a complete and generalized model of NOMA system with energy harvesting for 5G in Nakagami-m fading as an extension of our work [7], which is a solid foundation for multiple channel types. We compare two transmission schemes: Direct and Relay Assisted Transmission (DRAT), studied in [7], where a decode-and-forward (DF) relay helps the transmission from a source node to the destination, and no DRAT (nDRAT) where no direct link is present from the source to final users. To the best of our knowledge, the impact of time splitting fraction, power allocation and channel parameters on NOMA with EH relaying performance has not been done yet. The present work investigated the system performance taking into account these parameters.

This paper is organized as follows. Section II introduces the considered system and channel model in multiple access and broadcast phases. Section III derives performance analytical formulation for outage probability (OP) under energy harvesting gain. Section IV, provides numerical results and simulations. At the end, section V is reserved for the conclusion of the whole work.

II. SYSTEM AND CHANNEL MODEL

In this paper we are interested to down-link communication between a base station (BS) and two users $U_1$ and $U_2$ through a decode and forward (DF) relay $R$. The nodes are equipped with single antennas. And, all links experience independent and identically Nakagami-m fading.

As illustrated in The Fig. 1, the channel coefficients between the BS and other nodes are denoted $h_i$. The links between the $R$ and $U_i$ are described as $g_i$. The additive white Gaussian noise (AWGN) has the same ratio for all the links $w \approx (0, \sigma^2)$. The transmission follows the harvest-then-cooperate model [8].

In the first time-slot, the BS broadcasts during $\frac{1-\tau}{T}$ a
superposition of two messages \((\alpha_1 x_1 + \alpha_2 x_2)\) where \(\alpha_i\) is the power allocation for each symbol \(x_i\). The message of \(U_1\) is prior to that of \(U_2\), and therefore \(\alpha_1 > \alpha_2\).

### A. Case DRAT

Following signal interference cancellation (SIC) technique, the relay and \(U_1\) will first decode \(x_1\) by treating \(x_2\) as interference, then \(R\) decodes \(x_2\). According to the targeted data rate for each user \(R_i\), the conditions for \(R\) and \(U_1\) to decode both \(x_1\) and \(x_2\) are [7] \(C_{R,x_1} > R_1\), \(C_{R,x_2} > R_2\), and \(C_{U_1,x_1} > R_1\), where

\[
C_{i,j} = \frac{1}{2} \log_2(1 + \gamma_{i,j})
\]

is the capacity gain for \(i \in U_1, R\) and signal \(j\), with:

\[
\gamma_{i,x_1} = ||h_{Si}||^2 \alpha_1 \gamma; (i \in R, U1).
\]

\[
\gamma_{i,x_2} = \frac{||h_{Si}||^2 \alpha_2 \gamma}{||h_{Si}||^2 \alpha_1 \gamma + 1}; (i \in R, U1)
\]

where \(\gamma = \frac{P}{\sigma^2}\). The relay uses the received signal to collect energy during \(\tau T\) in order to ensure his autonomy during the second time-slot.
The harvested energy is expressed as $E_h = \rho (|h_{SR}|^2) \tau T$ where $\rho$ is the energy conversion efficiency with $0 \leq \rho \leq 1$.

In the second time slot, R sends $x_2$ to $U_2$ with an SNR:

$$\gamma_{U_2,x_2} = |g_2|^2 P_R \sigma^2.$$  \hspace{1cm} (3)

where $P_R = \frac{E_h}{(1+\gamma_{U_2,x_2})}$. The condition for $U_2$ to decode its own message is $C_{U_2,x_2} > R_2$.

B. Case of nDRAT

In this case, the direct links between the base station and mobile users are assumed to be absent. After decoding the two signals, the relay forwards $(\alpha_1 x_1 + \alpha_2 x_2)$ in the second time-slot, with the transmission power $P_R$. $U_2$ decodes $x_1$ then deduces $x_2$ [9]. Hence $C_{U_2,x_1} > R_1$ where:

$$\gamma_{U_2,x_1} = \frac{|g_2|^2 \alpha_1 \gamma_R}{|g_2|^2 \alpha_2 \gamma_R + \sigma^2}.$$ \hspace{1cm} (4)

where $\gamma_R = \frac{P_R}{\sigma^2}$

![Figure 4: System outage probability vs. transmission SNR of the BS, at Nakagami parameter $\mu$ as parameter](image)

![Figure 5: System outage probability vs. users data rate in bps/Hz, at Nakagami parameter $\mu$ as parameter](image)

III. OUTAGE PROBABILITY ANALYSIS

In this paper we are interested to investigate the outage behavior of our system. In fact, a successful end-to-end transmission is considered when the achievable data rate is greater than a fixed data rate threshold. This metric is conditioned by the successful message decoding during the end-to-end transmission.

In this section, the outage probability for each user $i$ denoted by $P_{out,i}$ is studied for both DRAT and nDRAT schemes.

A. Case of DRAT

As explained above, the outage probabilities at $U_1$ and $U_2$ are expressed respectively as:

$$P_{out_1} = P(C_{U_1,x_1} < R_1)$$

$$P_{out_2} = P(C_{U_2,x_2} < R_2, C_{U_2,x_2} < R_2)$$ \hspace{1cm} (5)

The probability density function (PDF) of the channel gains $\lambda = |q|^2$, $q \in \{h_1, g_1, g_2\}$ is:

$$f(\lambda) = \frac{\mu^\mu \lambda^{\mu-1}}{\omega_0 \Gamma(\mu)} e^{-\frac{\lambda}{\omega_0}}, \text{ for } \lambda \geq 0$$ \hspace{1cm} (6)
where $\mu$ is Nakagami-m multipath fading parameter, $\Gamma()$ is the Gamma function, $\omega_0 = E(\lambda)$ is the average gain for each channel. According to [4], the cumulative density function (CDF) can be obtained using:

$$F(\lambda \leq x) = \int_{0}^{x} f(\lambda) d\lambda = 1 - e^{-\frac{\mu x}{\omega_0}} \sum_{k=0}^{\mu-1} \frac{1}{k!} \left( \frac{\mu x}{\omega_0} \right)^k$$  

(7)

**Theorem 1.** The outage probabilities for $U_1$ and $U_2$ in cooperative NOMA DRAT scheme with energy harvesting over $m$-Nakagami are defined as follows:

$$P_{out_1} = 1 - e^{-\frac{z_1}{\omega_{hs}} z_1} \sum_{k=0}^{\mu_{12}-1} \frac{1}{k!} \left( \frac{\mu_{12} z_1}{\omega_{hs}} \right)^k$$

$$P_{out_2} = 1 - e^{-\frac{z_2}{\omega_{hs}} z_2} \sum_{k=0}^{\mu_{12}-1} \frac{1}{k!} \left( \frac{\mu_{12} z_2}{\omega_{hs}} \right)^k \times e^{-\frac{z_1}{\omega_{hs}} z_1} \sum_{k=0}^{\mu_{2}-1} \frac{1}{k!} \left( \frac{\mu_{2} z_1}{\omega_{hs}} \right)^k$$

where, $z_1 = 2^{R_2} - 1$ , $z_2 = 2^{R_1} - 1$; $\gamma = \frac{P}{\sigma^2}$, $\gamma R = \frac{P_R}{\sigma^2}$; $\xi_1 = \frac{\tau}{\alpha(\alpha_1 - \alpha_2)}$, $\xi_2 = \frac{\tau}{\alpha_2}$; $\lambda_{1} = \frac{\xi_1}{\gamma R(\alpha_1 - \alpha_2 z_1)}$, $\lambda_{2} = \frac{\xi_2}{\alpha_2}$. 

$\xi = max(\xi_1, \xi_2)$.

**Proof.** See Appendix A.

**B. Case of nDRAT**

The outage probability at $U_1$ and $U_2$ are expressed respectively as:

$$P_{out_1} = 1 - P(C_{R,x} > R_1, C_{u_1} > R_1, C_{u_2,x} > R_1);$$

$$P_{out_2} = 1 - P(C_{R,x} > R_1, C_{R,x} > R_2, C_{u_2} > R_2);$$

**Theorem 2.** The outage probabilities for $U_1$ and $U_2$ in cooperative NOMA nDRAT scheme with energy harvesting over $m$-Nakagami are defined as follows:

$$P_{out_1} = 1 - e^{-\frac{z_1}{\omega_{hs}} z_1} \sum_{k=0}^{\mu_{12}-1} \frac{1}{k!} \left( \frac{\mu_{12} z_1}{\omega_{hs}} \right)^k \times e^{-\frac{z_1}{\omega_{hs}} z_1} \sum_{k=0}^{\mu_{2}-1} \frac{1}{k!} \left( \frac{\mu_{2} z_1}{\omega_{hs}} \right)^k$$

$$P_{out_2} = 1 - e^{-\frac{z_2}{\omega_{hs}} z_2} \sum_{k=0}^{\mu_{12}-1} \frac{1}{k!} \left( \frac{\mu_{12} z_2}{\omega_{hs}} \right)^k \times e^{-\frac{z_1}{\omega_{hs}} z_1} \sum_{k=0}^{\mu_{2}-1} \frac{1}{k!} \left( \frac{\mu_{2} z_2}{\omega_{hs}} \right)^k$$

where, $z_1 = 2^{R_1} - 1$ , $z_2 = 2^{R_2} - 1$; $\gamma = \frac{P}{\sigma^2}$, $\gamma R = \frac{P_R}{\sigma^2}$; $\xi_1 = \frac{\tau}{\alpha(\alpha_1 - \alpha_2)}$, $\xi_2 = \frac{\tau}{\alpha_2}$; $\lambda_{1} = \frac{\xi_1}{\gamma R(\alpha_1 - \alpha_2 z_1)}$, $\lambda_{2} = \frac{\xi_2}{\alpha_2}$. 

$\xi = max(\xi_1, \xi_2)$.

**Proof.** See Appendix B.

**IV. NUMERICAL RESULTS**

In this section, the performance of the NOMA with EH DF relaying network is evaluated by numerical results over Nakagami-m fading channels for DRAT and nDRAT schemes.

In what follows, we set the average gain $\omega_1 = 1$ and noise power $\sigma^2 = 0.1$ for all links. The power allocation coefficients $\alpha_1 = 0.9, \alpha_2 = 0.1$ and the targeted data rate $R_1 = R_2 = 0.5$ bps/Hz.

Fig. 2 illustrates the simulation of outage probability versus the transmission power at the BS for different values of $\omega$. The simulation is performed using the Matlab 'makedist' function to create Nakagami probability distribution object. It is seen how the simulation results perfectly match the analytical formulation and the impact of the channel quality to enhance the outage probability. To lighten the following illustrations, the simulation results will be omitted.

In Fig. 3, we show the variation of the outage probability in function of the energy harvesting period $\tau$. Receiving a copy of the source signal at US1 seems to not help performance improvement at US1, as a slight enhancement in $P_{out_1}$ is noted for nDRAT scheme for $\tau = 0.3$ which corresponds to the optimal EH period [9].

In Fig. 4, we vary the Nakagami parameter $\mu$ to see its effect on the system outage for $\tau = 0.3$. Increasing $\mu$ enhances the spread of the distribution. It can be seen that this ensures better system maintainability due to the amount
of the collected energy.

Fig. 5 represents the outage probability for different data rates and under variant channel conditions. It is shown that data rate can reach 2bps/Hz and even 2.5bps/Hz for nDRAT system under low $\mu$ values.

Fig. 6 is dedicated to illustrate the variation of power allocation coefficients. It is shown that a balanced choice for $\alpha_1$ allows to ensure the communication for both users with close quality. It corresponds to $0.5 < \alpha_1 \leq 0.7$. For nDRAT scheme, Fig. 6(b), the difference is more visible between outage probabilities. The $U_2$ outage is more sensitive to $U_2$ prioritization.

V. CONCLUSION

In this paper we presented a complete generalization of NOMA systems with SWIPT technique in two schemes DRAT and nDRAT. The outage probability has been derived to analyze the system performance.

Simulation results are conducted to demonstrate our analytical results. It is the first occasion where the effect of power allocation coefficients and channel parameters were illustrated. It was shown that system maintainability was ensured in the two schemes with up to 2 bps/Hz with fully autonomous relay. Furthermore, the absence of direct link from source to the nearer user has a great influence on the outage probability. An enhancement were noted in all simulation conditions especially for the farther user.

Additionally, these results clarified the outage performance for NOMA with EH cooperative scheme over more general fading channels. It would be interesting in future works to generalize the proposed theorems with N users and multiple antennas use.

APPENDIX A

PROOF OF THEOREM 1

Referring to [7] the outage probability for $U_1$ is:

$$P_{out_1} = P(|h_{su1}|^2 < \xi_1)$$

assuming $z_1 = 2^\frac{2\mu_1}{2R} - 1$ and $\xi_1 = \frac{z_1}{\alpha_1 2R}$

Using the cumulative distribution function (CDF) of Nakagami distribution [4]

$$P_{out_1} = 1 - e^{-\frac{\mu_1 z_1}{\omega_{h_{su1}}}} \sum_{k=0}^{\mu_1 - 1} \frac{1}{k!} \left( \frac{\mu_1 z_1}{\omega_{h_{su1}}} \right)^k$$

$$P_{out_2} = 1 - P\left( |h_{sr}|^2 \geq \xi_2, \text{for} z_2 < \frac{2\mu_2}{\gamma_1} \right)$$

$$P_{out_2} = 1 - e^{-\frac{\mu_2 z_2}{\omega_{h_{sr}}}} \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{h_{sr}}} \right)^k$$

$$P_{out_2} = 1 - e^{-\frac{\mu_2 z_2}{\omega_{h_{sr}}}} \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{h_{sr}}} \right)^k$$

The outage probability for $U_2$ is:

$$P_{out_2} = P\left( |h|^2 \geq \xi, \text{and} |g|^2 \geq \xi R \right)$$

Similarly,

$$P(|h|^2 \geq \xi) = e^{-\frac{\mu_1 z_1}{\omega_{h}}} \sum_{k=0}^{\mu_1 - 1} \frac{1}{k!} \left( \frac{\mu_1 z_1}{\omega_{h}} \right)^k$$

$$P(|g|^2 \geq \xi R) = e^{-\frac{\mu_2 z_2}{\omega_{g}}} \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{g}} \right)^k$$

APPENDIX B

PROOF OF THEOREM 2

Referring to [9] the outage probability for $U_1$ is:

$$P_{out_1} = 1 - P(|h|^2 \geq \xi_1) * P(|g|^2 \geq \xi_1 R) * P(|g|^2 \geq \xi_1 R)$$

Using the cumulative distribution function (CDF) of Nakagami distribution [4]:

$$P(|h|^2 \geq \xi_1) = 1 - \int_0^{\xi_1} f(\lambda) d\lambda = e^{-\frac{\mu_1 z_1}{\omega_{h}}} \sum_{k=0}^{\mu_1 - 1} \frac{1}{k!} \left( \frac{\mu_1 z_1}{\omega_{h}} \right)^k$$

Similarly,

$$P(|g|^2 \geq \xi_1 R) = e^{-\frac{\mu_2 z_2}{\omega_{g}}} \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{g}} \right)^k$$

Hence:

$$P_{out_1} = 1 - e^{-\frac{\mu_1 z_1}{\omega_{h}}} \sum_{k=0}^{\mu_1 - 1} \frac{1}{k!} \left( \frac{\mu_1 z_1}{\omega_{h}} \right)^k \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{g}} \right)^k \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{g}} \right)^k$$

The outage probability for $U_2$ is:

$$P_{out_2} = P\left( |h|^2 \geq \xi, \text{and} |g|^2 \geq \xi R \right)$$

Similarly,

$$P(|h|^2 \geq \xi) = e^{-\frac{\mu_1 z_1}{\omega_{h}}} \sum_{k=0}^{\mu_1 - 1} \frac{1}{k!} \left( \frac{\mu_1 z_1}{\omega_{h}} \right)^k$$

$$P(|g|^2 \geq \xi R) = e^{-\frac{\mu_2 z_2}{\omega_{g}}} \sum_{k=0}^{\mu_2 - 1} \frac{1}{k!} \left( \frac{\mu_2 z_2}{\omega_{g}} \right)^k$$
\[
P((|g_2|^2 \geq \xi_2 R)) = e^{-\frac{\xi_2 R}{\mu}} \sum_{k=0}^{\mu-1} \frac{1}{k!} \left(\frac{\mu \xi R}{\omega g^2} \right)^k\]

(22)

\[
P_{\text{out}_2} = 1 - \left[ e^{-\frac{\mu}{\omega_h}} \sum_{k=0}^{\mu-1} \frac{1}{k!} \left(\frac{\mu \xi R}{\omega_h} \right)^k \right] \times \left[ e^{-\frac{\mu \xi R}{\omega g^2}} \sum_{k=0}^{\mu-1} \frac{1}{k!} \left(\frac{\mu \xi R}{\omega g^2} \right)^k \right]
\]

(23)

REFERENCES


Nouha Dahi is a Ph.D student at the National School of Engineers of Tunis (ENTI), SysCOM Laboratory. In 2012, she obtained the Masters degree in Electronic Systems and communication networks from Polytechnic Shool of Tunisia (EPT). Her research interests focus on physical communications. Particularly, she works on wireless radio networks, cooperative diversity, OFDMA - NOMA systems, and energy harvesting.

Noureddine Hamdi is currently an Associate Professor at The Institute of Applied Sciences and Technologies of Tunis (INSAT) since 2001. He received his Ph.D and the Habilitation in communication technology from the National School of Engineers of Tunis (ENTI), respectively in 1999 and 2007. His research interests focus on wireless communication networks. Particularly, he works on radio resource management, spatial diversity, cooperative diversity, MIMO OFDMA systems, cross layer optimization design, cognitive radio, wireless sensor networks, network coding and joint source channel coding. He is the author of more than 12 journal papers and 30 international conference communications. He has been involved in CMCU 2008 – 2009 cooperation project with University de Poitiers and Laboratory SysCOM.