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Introduction to the translation of A. Mohorovičić's »Earthquake of 8 October 1909«

ANDRIJA MOHOROVIČIĆ, the outstanding Croatian meteorologist and seismologist of international fame, was born in Volosko near Opatija, in Croatia, on 23 January 1857. After he finished the high school passing examinations in Rijeka with excellent grades, he enrolled in the Department of Mathematics and Physics at the University of Prague. Having completed his studies A. Mohorovičić was appointed a teacher at the high school in Zagreb and then in Osijek and Bakar. In the period 1891–1921 he was the director of the nowadays Andrija Mohorovičić Geophysical Institute, Faculty of Science, University of Zagreb. He died on 18 December 1936.

The early Mohorovičić's scientific activities include mostly meteorological problems. He is considered to be the first Croatian scientist in the fields of meteorology and climatology and was the organizer of the systematically structured meteorological service. He was mostly interested in high atmospheric layers, systematic cloud observations, unusual atmospheric phenomena, climate of the city of Zagreb and hail prevention. In 1902. he initiates the publication of the *Annual Report of the Zagreb Meteorological Observatory*, the forerunner of the todays journal *Geofizika*.

Mohorovičić's last study in the field of meteorology was published in 1901. The circumstances which caused him to stop publishing meteorological papers are not known. The fact is that after the turn of the century his scientific interests turned exclusively to seismology, always keeping in mind the goal of physical seismology »...to investigate the interior of the Earth and to take over where the geologist stops, because the modern seismographs can serve as a binocular for observing even the greatest depths.«[1] The span of Mohorovičić's interests in seismology is very wide, and some of his basic ideas are quite relevant even today. Let's take for instance his papers on how an earthquake acts upon buildings[2] in which he analytically considers the forced oscillations of building models under seismic motion load. Of course, some of his work is less relevant today, in the times of electronics and computers, then it was in the first quarter of this century – for instance the ides how to construct the new type of mechanical seismograph[3] or his method for near earthquake epicentre location[4].

A. Mohorovičić gained worldwide reputation by discovering the existence of the velocity discontinuity in the uppermost part of the Earth. Namely in 1909 he detected

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[1] Mohorovičić, A. (1913): Razvoj seizmologije posljednjih pedeset godina. Ljetopis JAZU, sv. 27, Zagreb, reprint, 1–31.

[2] Mohorovičić, A. (1911): Djelovanje potresa na zgrade. Vijesti Hrvatskog društva inžinjera i tehničara, Zagreb, XXXII, No. 2, 17–18, No. 3, 33–35, No. 4, 51–53, No. 5, 69–72, No. 6, 85–86, No. 7, 103–105, No. 8, 112–116, No. 9, 126–129, No. 10, 139–142.

[3] Mohorovičić, A. (1917): Principi konstrukcije seizmografa i prijedlog za konstukciju nova seizmografa za horizontalne komponente gibanja zemlje. Rad JAZU, knjiga 217, Zagreb, 114–150.

[4] Mohorovičić, A. (1916): Die Bestimung des Epizentrums eines Nahbebens. Gerl. Beitr. zur Geophysik, Bd. XIV, H. 3, Leipzig, 199–205.

two distinct pairs of P and S phases on seismograms of the Kupa Valley (Croatia) earthquake of 8 October 1909 and inferred the presence of a marked structural discontinuity some distance below the surface of the Earth.[5,6] It was later named after him the Mohorovičić discontinuity or abbreviated MOHO or M-discontinuity. Subsequent studies in Europe, and later over the whole globe, showed that the Mohorovičić discontinuity exists worldwide, though not always as a sharp transition and at average depth of less than 54 km (as obtained by A. Mohorovičić).

This important Mohorovičić's discovery was firstly poorly known, because it was hard to recognize the importance of the work published under such unappealing title. In the year of 1911 H. Bendorf pointed to that paper[7] not only as to one of the most important seismological papers but also to show how interesting problems seismology has to solve. Theoretical review of the paper was given by E. Rothè[8] in 1924.

Let us also mention that in this paper Mohorovičić also introduced the new method for location of near earthquakes. His conclusion about the maximal phase: »If the focus of the earthquake was in the lower layer of the Earth[9], it would ... be an earthquake without the maximum phase...« was proved in 1929 by K. Wadati in Japan. A velocity distribution given by Mohorovičić's law $v = a\,r^b$ is especially important because of the simple form of the (T, Δ) relation. It is very close approximation of the actual velocity variation over wide range of depths in the Earth.

Because of great interest the Školska knjiga Publishing Co, Zagreb, published in 1977 a reprint of Mohorovičić's seminal paper[5] which was faithful to the original in every detail. The same publishing house issued in 1982 a bilingual monograph[10] about A. Mohorovičić, in Croatian and English.

Since the original paper[5] was published in Croatian with the translation in German, the Editorial Board of *Geofizika* decided to publish this English translation on the occasion of 135th anniversary of Andrija Mohorovičić's birth.

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[5] Mohorovičić, A. (1910): Potres od 8. X. 1909. Godišnje izvješće zagrebačkog meteorološkog opservatorija za godinu 1909, Zagreb, 1–56.
[6] Bullen, K. E. and B. A. Bolt (1985): An introduction to the theory of seismology, Fourth ed, Cambridge University Press, Cambridge, 499 pp.
[7] Bendorf, H. (1912): A. Mohorovičić – Das beben vom 8. X. 1909. Gerl. Beitr. zur Geophysik, Bd. XI, Leipzig, 348–352.
[8] Rothè, E. (1924): Sur la propagation des ondes séismiques au voisinage de l'épicenre. Préliminaires continues et trajets a réfraction, UGGI, Section ed Seismologie, Série A, Trav. Sci, Fasc. 1, Paris, 17–59.
[9] e. g. in the upper mantle (*D.S.*)
[10] Skoko, D. and J. Mokrović (1982): Andrija Mohorovičić. Školska knjiga, Zagreb, 147 pp.

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Earthquake of 8 October 1909*

Andrija Mohorovičić

Foreword

On the day of 8 October 1909 at 9 h 59 m G.M.T. a strong earthquake was felt in Zagreb toppling a significant number of chimneys, but otherwise causing no extensive damage. The news which reached Zagreb by telegraph and telephone immediately after the earthquake have shown that the earthquake raged with a much greater force to the south of Zagreb, particularly in the Kupa Valley, where significant damage occurred and even people were killed.

Upon the request of the observatory, district offices, elementary schools, public offices and private persons have sent us so much macroseismic material to make us able to precisely determine the location of the epicentre and the extent of the shaken area.

All European seismic stations have readily put data read from their seismographs at my disposal, and many of them, which I addressed repeatedly with inquiries, gave me all possible explanations. Let here the warmest gratitude be extended to all of them.

A large number of seismic stations within 800 km from the epicentre and the quality of their observations awakened a hope in my mind that I may be able to have a deeper look into the mechanism of earthquake waves propagation. The work of many researchers, particularly of E. Wiechert and his disciples, provided us with very reliable travel-time curves of the specific earthquake phases at distances of 1000 to over 10 000 km from the epicentre. The curves from the epicentre to 1000 km may be corrected and altered in many instances because the present material for near earthquakes is deficient. With this paper I intend to possibly rectify this deficiency.

Due to a large quantity of macroseismic material and the impossible task of processing the macroseismic data in short time the 4th part of the 1909 Annual

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* English translation from: *Potres od 8. X. 1909,* Godišnje izvješće zagrebačkog meteorološkog opervatorija za godinu 1909, godina IX, dio IV, polovina 1, 1910, 1–56, and the German version: *Das Beben vom 8. X. 1909,* Jahrbuch des meteorologischen Observatoriums in Zagreb (Agram), Jahrgang IX, IV. Teil, Abschnitt 1, 1910, 63 pp (printed by Royal Government of Croatia, Slavonia and Dalmatia, Department of Theology and Education, C. Albrecht Print Shop and Lithography).

Report must be divided into two parts. The first of these halves contains the treatment of the macroseismic material from the earthquake on 8 October, and the following earthquakes from the same epicentre, whereas the other half shall contain the macroseismic material and data read from seismographs for the whole of the year 1909, and shall appear at the beginning of the year 1911.

1. Introduction

Modern sensitive seismographs in every station plot a diagram of ground motion in the immediate vicinity of the instrument as soon as the environment is set in motion by the earthquake waves.

A. If we compare the diagrams of an earthquake registered on two stations which are at equal distance from the earthquake focus we will notice that these diagrams are very similar but a precise comparison will show that they are not completely equal. There are two main reasons for this disparity: different capability of both instruments to faithfully record the motion of the Earth at the location where they have been placed, and a factual difference in the motion of the Earth in both locations. A third cause may be quoted too, i.e. the different shape of waves which are leaving the earthquake focus in different directions. However we do not know anything about that today, because even the equal parts of both diagrams are not completely intelligible to us. The instants in which both diagrams begin or equal parts of the diagrams begin, are also nearly equal. The time differences may have various causes, which I shall discuss here in detail.[1)]

1. Clock error. Every proper seismic station usually has three clocks, which all may be off time.

a) *The main clock* as the most precise time measuring device. Permanent correction and daily clock run are determined at the station itself by astronomical observations, or the clock is matched from time to time with some other clock at the location or in some other location. So many errors stem from that last method of matching that we are never sure to have the correct time unless our main clock is so accurate to allow the detection of the matching error. The error of such a matching may amount to 10 s and more.

b) *Contact clock,* which marks the minutes on the earthquake diagram. On some stations the main clock is the contact clock at the same time, but in general the station has a less precise clock for the contacts, being matched with the main

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1) About that refer to: H. Benndorf – Beiträge zur rationellen Seismometrie I. – Beiträge zur Geophysik B. X. H. 1.

clock every day. The more or less uneven run of the contact clock, in-between the two matching procedures, results in erroneous minute marks.

For those instruments where a special pen produces the minute marks far from the earthquake diagram, the time parallax is subject to endless variations. Any adjustment of the instrument changes the parallax.

c) *Motion of paper on the seismograph.* The mechanism holding the paper in uniform motion is usually very primitive, so that some minutes are unequal thus giving another possibility of a time error. This irregularity is particularly noticeable if the clock does not mark some minutes. If an earthquake came within that time, it may not be accurately read. If however within a single hour the marks of some minutes are missing, than an earthquake recorded within that period can not be read at all.

d) *Speed of paper flow,* i.e. the length of the diagram for a one minute time period is important for precise time determination. If the velocity is to low the small time intervals can not be read precisely enough. For too high a velocity the beginning of the earthquake can not be determined, unless it is very strong.

For near earthquakes a higher paper speed is necessary than for the distant ones. Waves with a period of 0.5–1 s may be determined well only if the velocity does not drop below 18–20 mm per minute. For strong earthquakes even this velocity is too low. The velocity of paper must agree with the instrument sensitivity i.e. according to the virtual magnification of the true Earth movements. Low sensitivity instruments do not tolerate high speeds if they are not set up for the recording of only the strong earthquakes. Very sensitive instruments must have very high speeds, in order to utilize all the details of the diagram.

2. Accidental errors. A good strong deflection may be read wrongly or an unlearned person may be allowed to read it. A good time reading may be wrongly summed with a time correction. Well determined time may be wrongly copied etc. In addition there are crude time errors which can never be detected, for example a single minute error in a single phase, various print errors etc.

3. Earthquake onset uncertainty or onset uncertainty of its respective parts. The beginning of an earthquake diagram may be either a strong jar (*impetus*), if the pen suddenly shifts from the rest position, or a slow emergence of waves from the state of rest (*emersio*). In the first case the time is easily determined, and in the second case the time of the first wave observation depends on the magnification and friction. Low sensitivity instruments and instruments with high friction, will always show the onset too late.

4. Microseismic and local noise may derange the onset of an earthquake so that it may not be observable at all, or lead to taking any stronger unrest jar as the earthquake start. Particular hindrance is the noise with periods equal or nearly equal to the period of the first earthquake waves. This happens very often in winter. Local noise of a small period is very harmful for near earthquakes.

5. Possible nonuniform spreading of waves from an earthquake focus in various directions.

Recently a lot has been written on that subject, but my opinion is that we may not yet discuss it today because even the average values are not sufficiently known to us and because the time errors are still so high, rendering data from the majority of stations unusable.

Today we have first and second class seismic stations. First class stations generally have first class clocks and instruments of the same order. Second class stations generally have lower sensitivity instruments, appropriate for the purpose intended, whereas their clocks are generally poor. The time is determined individually only on few stations. A first class station should not only have a main clock but also a first class contact clock, running always properly, so that we may at any instant have the correct time within 0.5 s. However there are many stations which allow an error of ±3 to 4 seconds.

Second class stations should have a first class clock as have the first class stations, or their data may lead to false conclusions. When I compared data from all the stations which I intended to use in this paper with average values obtained from the stations with best reputation, I could witness that the data from good stations have so small variances from average values that their errors fall within the boundaries of probable errors of ±4 s, an error to be tolerated today. All stations whose error exceeds this boundary had to be eliminated. When the best experts seek precision of ±2 s or even of ±1 s, then it is only *pium desiderium*, which is satisfied by only a few stations in Europe.

It would hence be necessary to either close the II class stations where the state is financing them, or to reorganize them so as to make their data usable. These stations are not adequate even for purely statistical purposes. A good macroseismic observer will record more earthquakes which actually occurred than a II class instrument. Only a pendulum of 1000 kg could record all the earthquakes which were felt in the epicentral region, i.e. only 40 km away from Zagreb. We could find no earthquake recorded by the instruments which had not been reported by at least one observer, without being particularly questioned about it. Poorer instruments missed many of these earthquakes.

B. If we compare two diagrams of the same earthquake, obtained with instruments of the same type and of nearly same sensitivity, but with unequal distance from the focus, we see that both of the diagrams are similar, only the diagram obtained at the more distant station lasts longer and starts later.

A diagram of a well defined earthquake is composed of two parts, the preliminary and the main earthquake. The duration of the preliminary earthquake is the longer the further away the station is from the earthquake focus. The duration of the main earthquake depends on the distance from the focus and on the magnitude of the earthquake at the focus itself. It has not yet been determined if a seismograph placed on the Earth surface exactly, or almost perpendicularly, above the earthquake focus – imagined to be at not to great a depth – will record a preliminary earthquake.

At a short distance from the focus the preliminary earthquake starts with waves of a short period. For weaker earthquakes that period amounts to only fractions of a second; for stronger earthquakes these waves are joined by waves with periods of 1–2 s, and for a very strong earthquake by even slower waves. As the distance from the focus grows so are the fast waves ever weaker and only the slower ones remain. But the stronger the earthquake the further we still find those faster waves.

The preliminary earthquake is further divided into two main parts: the first and the second preliminary earthquake. The second preliminary earthquake starts with a sudden amplitude and period increase. Usually the average amplitude of the second part is larger than the amplitude of the first one. For strong earthquakes we may observe, as is known, some similarity of the first waves for both of the preliminary earthquake parts.

Except for these main phases for stronger earthquakes a multitude of additional phases may be observed starting either with a sudden amplitude increase or a sudden period change. Some similarity of their first waves with the first waves of the earthquake onset may be sometimes observed.

At the start of the first phase the waves are the shortest. During the preliminary earthquake these waves are joined by ever longer and stronger waves, and the shorter waves become ever weaker. Every new phase brings new kinds of waves, until towards the end very long waves with periods of 30 s or more arrive.

The main earthquake starts with a more or less pronounced increase of the amplitude and decrease of the period. During the main part of the earthquake periodic alterations of the amplitude may be observed but in comparison to preliminary phases relatively small changes of period. Various maxima follow one another, the first or one of the first is the strongest, and little by little the weaker follow until finally the motion stops and the state of rest prevails.

The period of the main earthquake depends on the distance of the earthquake focus. For weak near earthquakes the main phase period amounts to 1–2 seconds. For strong earthquakes even larger periods arrive. The larger the distance from the focus the longer are the waves, so that their length increases up to 18 seconds within a distance of 90° or even more. For very strong earthquakes, shorter waves may be observed upon these long ones, like the ones arriving only in a preliminary earthquake. During the main earthquake the period alternates so as to be now longer and then shorter, until it becomes longer than at the beginning towards the end of the earthquake.

The precision of determining the start of individual phases is different. The onset of the earthquake itself can be determined most precisely. The beginning of both the preliminary phases may always be determined precisely only at those stations which posses an instrument for the vertical component too. The start of the main earthquake is very difficult to determine.

C. The problem of earthquake wave propagation has been solved in general. If we imagine the Earth as a solid body composed of concentric homogeneous spherical layers, every sudden change of equilibrium will, regardless if it occurs at the surface of the Earth or at any point in the interior, cause two systems of elastic waves which propagate with different velocities from the earthquake focus in all directions. Because the velocity of propagation increases from the surface down to a significant depth, so the path of every impact from the focus to the Earth surface will be an upward concave curve. In the same way every ray reaching the surface of the Earth will cause various new systems of waves of which some will further extend along the surface and the others will reflect towards the interior of the Earth. If in the interior of the Earth there exists a spherical surface as a dividing interface between two media with different elasticity, then new wave systems will be generated at that surface due to refraction and reflection.

The first preliminary earthquake consists of longitudinal waves, which arrive through the interior of the Earth to the instrument, partly directly and partly from one or more of the reflections. The second preliminary earthquake consists predominantly of transversal waves, whose velocity has a relation to the velocity of longitudinal waves of approximately 1:1.8 .

The main earthquake is supposed to consist, according to present day views, of surface waves which originate in the epicentre and expand along the surface in all directions. No one can deny that surface waves must be generated in every location reached by the interior wave, but the theory that the main earthquake consists of surface waves only, may be exposed to many objections:

1. Surface waves arise for every earthquake not only at the epicentre but everywhere. In the epicentre the strongest waves arise, and the further away the weaker they are. The station close to the epicentre should than receive no preliminary phases, but the amplitude should already at the first stroke be high, then rise little by little to a maximum and later decrease in the same way. But it is not so. In the Zagreb Observatory several earthquakes have been recorded with an epicentre distance of approximately 15 km. All of them show a nice preliminary earthquake, which lasts 2–3 seconds, then the amplitude suddenly rises and various maxima occur.

Fig. 1 shows a diagram of one earthquake in Stubica at a distance of 15 km from Zagreb.

Slow amplitude growth from the first waves up to the maxima can never be observed. For distant earthquakes also, the amplitude should grow slowly at first, and then ever faster up to the maxima, because the waves arriving directly through the interior of the Earth must be joined by the surface waves, which arrive from an ever smaller epicentral distance, until those waves arrive which come from the very epicentre. The amplitude regularly grows suddenly after various time spans, and at the start of the main earthquake it rises the most.

2. If the main phase consists only or mainly of surface waves, than it is difficult to explain its long duration. In the epicentre even the strongest earthquake lasts only for a short time, and its main force at most a few minutes, rarely

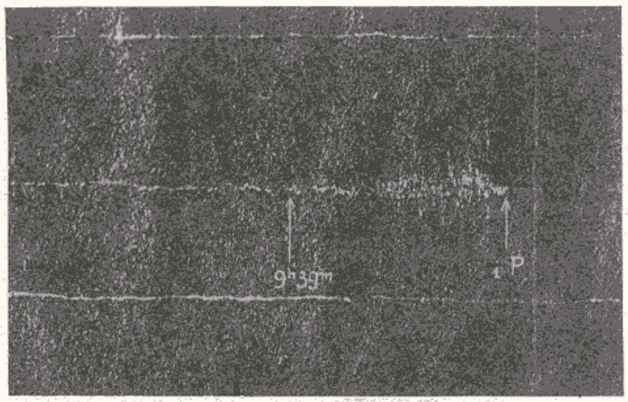


Figure 1. The earthquake of 23 May 1909, Stubica, NW component, magnified 3 times.

longer. At a distant station the main earthquake lasts often a whole hour or longer. This could only be explained by surface waves of different velocities propagating from the epicentre so that the slower ones arrive at the station ever later. I doubt very much that such a velocity differentiation is possible.

3. The further from the epicentre, the longer the period of the surface waves. This property of theirs would be difficult to explain too.

4. If the surface waves pass on their long journey along the bottom of the sea, the water should destroy them very quickly. At the European stations we should then observe a great difference between the earthquakes from east Asia, whose surface waves arrive exclusively by land and those from America or even from the Pacific Ocean, whose surface waves come exclusively along the bottom of the seas.

2. Epicentre of the earthquake on 8 October 1909

Details on the epicentral region and the propagation of the earthquake will be contained in the 2nd half of this Annual Report. Let it suffice here to submit an approximate picture of the epicentral region.

In order to determine the epicentre, main damages have been entered into a map and isoseismal lines drawn. One of the most important isoseismal lines encloses the region in which chimneys were toppling.

That region extends almost symmetrically on both sides of one axis which starts at Doberlin on the river Una (45° 09' N, 16° 29' E) and runs straight to Farkašić (45° 28' N, 16° 06' E) on the river Kupa. From here the axis runs along the river Kupa to Lasinja (45° 33' N, 15° 51' E), where it suddenly turns towards N somewhat W, and stops on west of the Zagreb Mountain. Approximately at

Figure 2. The earthquake of 8 October 1909 recorded at Zagreb. Magnification 20, NE component.

45° 44' N, 15° 48' E one branch parts towards Zagreb. This region has the largest width (30–35 km) between Petrinja at SE and Lasinja at NW for a length of some 50 km. The earthquake has caused the greatest damage in the region between 45° 20' N, 15° 50' E; 45° 30' N, 16° 17' E and 45° 31' N, 16° 03' E, where even the ground cracked. The middle of the central region is approximately at 45° 29' N and 16° 01' E, and it is the middle of a wide region where the ground cracked, at 45° 28' N and 16° 06' E. As the cracks are mostly in alluvial ground, I took as the epicentre a point in between the quoted points, at 45° 29' N and 16° 03' E. The error may amount to ±3 km at most.

It is very difficult to determine the strength of the earthquake in the epicentre, because the houses in that region are generally made of oak wood, so they were not damaged. Only churches, parish residences and public buildings are made of brickwork and stone, so they were significantly damaged. Maximum intensity has surely exceeded grade IX.

So far it may be concluded from the multitude of the macroseismic material, that for all the earthquakes which were felt in that region after the main earthquake the focus is identical to the focus of the main earthquake.

3. Seismogram data and time curves for P, $\overline{P}$ and S

Since the data from all the stations are more or less in error, for the construction of the time curves I decided not to use only the earthquake on October 8th, but also data for all the earthquakes that followed if they were appropriate for the purpose. Data on all the earthquakes characterize then some average earthquake for which all the derived results will be valid. This method would strictly be allowed only when all of these earthquakes emanated from the same epicentre and from equal depth. According to macroseismic observations all of them do come from the same epicentre, but surely not from the same depth. The earthquake on 29 January 1910 was in the epicentre and in Zagreb of almost the same strength as the main earthquake, but lasted only a short time. Its intensity fell so rapidly with distance, that already at a distance of 500 km it was recorded so weakly with the most sensitive instruments, rendering the data unusable. Therefore it should be reasoned that its focus was very shallow. Both earthquakes on 10 October, 1909 were very weak in Zagreb, but were quite well recorded in Hamburg too. Hence we should conclude of great depth. The depth of the earthquake has no great influence on $\overline{P}$, because it only changes the curve inflection point coordinates. The curve derived from all the earthquakes represents the average curve, whose points are much more precise than would be if they were derived from one single earthquake.

From the average values obtained from the specific instruments the approximate epicentral time was calculated first, i.e. the moment when the earthquake occurred in the epicentre. Later it was shown, based on strict calculation, that this time is correct so it was taken as the time origin under the condition that the time when the first strike arrived at Zagreb, is one and the same for all the earthquakes.

The main observatory clock is excellent – it is a Riefler clock – and it is used at the same time as the clock for time contacts. It's state and run are determined at the observatory by astronomic observation so that the clock, particularly during the earthquake period, guarantees a precision of ±0.5 s The errors of determining the individual phases may stem only from the unevenness of the respective minute-marks on paper and from the different shape of the first onsets for different earthquakes. For stronger earthquakes the error may at most amount to ±1 s, and for weaker ones it may be somewhat larger. The earthquake on 8 October needed 5 s to come to Zagreb, hence it started in the epicentre at 9 h 59 m 09 s ±0.5 s. This epicentral time has been taken as the time origin for all stations. For all the following earthquakes the epicentral time was deduced as the time, which is obtained by subtracting 5 seconds from the time when the earthquake arrived in Zagreb.

The values read on known instruments have been considered for the following table, partly as published by individual observatories, and partly from data communicated in writing. Where I could obtain copies of diagrams or the originals, they were read once more but a noteworthy difference was never found.

In order to save space absolute times were not entered in the tables but only differences from epicentral time, i.e. the times of earthquake travel. This method was taken from meteorological practice and will probably become familiar in seismology too, particularly in those cases when the epicentral time is precisely known.

In Table I are the travel times of the earthquake on 8 October 1909 and of subsequent earthquakes, as noted at individual stations.

Average times derived in this way have been plotted into the coordinate grid as small circles. 1 mm = 5 km has been taken as the epicentre distance length unit, and 1 mm = 1 s for the time unit, Tab. I.

During the construction of the time curve, it became apparent that the earthquake onset times – *undae primae* (P) – could not be represented by one single curve but that there are two curves: one starting in the epicentre, and running approximately up to 700 km distance, but surely not further than 800 km. The other lower curve starts positively at 400 km but it is possible that it starts already at 300 km, as the Viennese observations show. On the basis of our earthquake data this curve could be drawn if needed up to a distance of 1800 km. If it is compared to data published in the »Göttinger wöchentliche Erdbebenberichte« for the start of the first phase *(undae primae)*, it may be observed that both curves are identical.

Most of the stations at distances of 400–720 km have both phases recorded. These are Munich, Ischia, Hohenheim, Moncalieri and Strasburg. If the earthquake was somewhat stronger, the other stations would have had both phases in their records.

Tab. I. Epicentre near Pokupsko – 45° 26' N, 16° 03' EG.

Station	Day 1909	Time in epicentre h m s	Beginning m s	e M. m s	Max. m s
Zagreb	8. X.	9 59 09	0 05	0 09	0 16
epic. dist. 39 km		10 59 40	"	0 09	0 13
		11 09 39	"	0 10	0 11
Instruments:		14 39 51	"	0 09	0 11
Wiechert pendulum of		17 10 01	"	0 10	0 10
80 kg with 20-fold	10. X.	5 37 10	"	0 09	0 13
magnification		5 55 01	"	0 10	0 11
(Spindler & Hoyer)		6 08 56	"	0 09	0 12
with a paper speed of		8 56 33	"	0 09	0 11
30 mm per min. for		8 57 53	"	0 09	0 11
earthquakes on	11. X.	20 50 10	"	0 09	0 13
8 October, 9h 29 min,	14. X.	0 46 34	"	0 09	0 11
10 October, and 28	21. X.	17 03 17	"	0 10	0 13
January 1910, 23 h 28 m.	22. X.	6 36 03	"	0 10	0 14
		6 47 20	"	0 10	0 16
For other earthquakes	23. X.	4 01 07	"	0 10	0 13
pendulum of 1000 kg		8 29 57	"	0 09	?
(Bartels)	25. X.	22 45 35	"	0 09	0 11
	28. X.	5 37 30	"	0 08	0 14
	6. XI.	16 17 01	"	0 10	0 12
	2. XII.	22 59 19	"	0 09	0 13
	7. XII.	21 12 07	"	0 11	0 14
	13. XII.	0 21 41	"	0 10.5	0 12
	24. XII.	0 14 30	"	0 10	0 15
	1910				
	28. I.	23 57 45	"	0 10.5	0 11
	29. I.	0 12 02	"	0 10.5	0 11
		2 59 24	"	0 10.5	0 11
		13 29 28	"	0 10.5	0 11
Average			0 05	0 09.6	0 12.4

Earthquakes for which only traces were recorded were not taken into account. In the same manner, earthquakes after 29 January 1910 were not taken into account because except Zagreb no other station recorded them.

Rijeka ep. dist. 134 km *Instrument:* Vicentini-Konkoly	8. X.	9 59 09	0 25	0 42	0 50

Earthquakes recorded in Rijeka on 10 October 1909, as well as on 28 January and 29 January 1910 could not be taken into consideration due to large time errors.

Station	Day 1909	Time in epicentre h m s	Beginning m s	e M. m s	Max. m s
Ljubljana	8. X.	9 59 09	0 20	0 37	0 40
ep. dist. 141 km	10. X.	5 37 10	0 17	0 36	0 46
		5 55 01	0 20	0 35	0 45
Instruments:		6 08 56	0 21	0 39	0 44
various without damping	22. X.	6 36 03	0 19	–	0 43
	13. XII.	0 21 41	0 20	–	0 48
	24. XII.	0 14 30	0 23	–	0 52
	1910				
	28. I.	23 57 45	0 42?	–	1 05?
	29. I.	0 12 02	0 38?	–	1 09?
Average			0 20	0 37	0 45
Due to time error earthquakes on 28 January and 29 January 1910 could not be considered.					
Graz	8. X.	9 59 09	0 30	0 56	–
ep. dist. 184 km		11 09 39	0 33	–	–
	10. X.	5 37 10	0 26	–	–
Instrument:		5 55 01	0 29	–	–
Wiechert pendulum	13. XII.	0 21 41	0 25	–	0 53
	24. XII.	0 14 30	0 20?	–	0 38?
	1910				
	28. I.	23 57 45	0 28	–	0 58
	29. I.	0 12 02	0 27	–	1 01
Average			0 28	0 56	0 57
Earthquake on 24 December 1909 has not been considered due to probable time error.					
Trieste	8. X.	9 59 09	0 35	0 55	1 05
ep. dist. 185 km		10 59 40	0 32	0 53	0 58
	1910				
Instruments:	28. I.	23 57 45	0 32	0 51	1 01
Vicentini, Ehlert	29. I.	0 12 02	0 27	0 49	0 58
		2 59 24	0 30	0 52	0 56
Average			0 31	0 52	0 59
Pula	8. X.	9 59 09	0 36	0 57	1 01
ep. dist. 193 km	10. X.	5 37 10	0 31	0 54	0 54
		5 55 10	0 33	0 54	0 55
Instrument:	13. XII.	0 21 41	0 28	0 52	0 55
Wiechert (200 kg)	24. XII.	0 14 30	0 26	–	1 06
	1910				
	28. I.	23 57 45	0 30	0 57	1 01
	29. I.	0 12 02	0 29	0 51	1 05
		2 59 24	(0 40)	0 52	0 55
Average			0 30	0 54	1 00
Earthquake onset on 29 January 1910, 2 h 59 m was not considered.					

Station	Day 1909	Time in epicentre h m s	Beginning m s	e M. m s	Max. m s
Sarajevo					
Due to inacurate times **Sarajevo** station data could not be used.					

Station	Day 1909	Time in epicentre h m s	Beginning m s	P̄ m s	R m s	S m s	eM m s	M m s
Vienna	8. X.	9 59 09	–	0 52	–	–	1 33	1 59
ep. dist. 308 km	10. X.	5 37 10	0 42	–	–	–	1 28	1 40
		5 55 01	0 45	–	–	–	1 34	1 43
Instruments:	22. X.	6 36 03	–	–	–	–	1 33[1)]	1 45
Wiechert	25. X.	22 45 35	–	–	–	–	1 35[1)]	1 45
pendulum, Conrad	13. XII.	0 21 41	0 45	0 54	1 09	1 16	–	1 55
pendula	24. XII.	0 14 30	0 42	–	1 08	1 17	–	(1 20)
	1910							
	28. I.	23 57 45	0 42	0 50	–	–	1 27	1 42
	29. I.	0 12 02	0 48	0 58	–	–	1 27	1 47
		2 59 24	–	0 53	–	–	–	–
Average			0 44	0 53	1 09	1 17	1 31	1 47
[1)] In the Viennese »Wöchentliche Erdbebenberichte« marked as iP.								
Budapest	8. X.	9 59 09	–	1 07?	–	–	1 48	2 02
ep. dist. 316 km	10. X.	5 37 10	–	0 52	–	–	–	1 59
		5 55 01	–	0 58	–	–	–	1 43
Instrument:	1910							
Wiechert	28. I.	23 57 45	–	0 59	–	–	1 36	1 59
	29. I.	0 12 02	–	0 58	–	–	1 42	3 49
Average			–	0 58	–	–	1 39	?
For the calculation of average values the data of the earthquake on 8 October 1909 have been neglected. Data from stations O-Gyalla, Temesvar, Firenze-Ximeniano, Valle di Pompei and Pavia could not be used due to large time differences in relation to other stations.								
Padova	8. X.	9 59 09	–	0 55	–	–	1 40	–
ep. dist. 332 km		10 59 40	–	–	–	–	1 35	1 44
	10. X.	5 37 10	–	0 52	–	–	1 34	1 42
Instrument:		5 55 01	–	0 55	–	–	1 35	1 53
Vicentini	13. XII.	0 21 41	–	0 55	–	–	1 36	(3 07)?
	1910							
	28. I.	23 57 45	–	0 55	–	–	1 36	–
	29. I.	0 12 02	–	0 56	–	–	1 37	–
Average			–	0 55	–	–	1 38	1 46

Station	Day 1909	Time in epicentre h m s	Beginning m s	$\overline{P}$ m s	R m s	S m s	eM m s	M m s
Florence Quarto-Castello ep. dist. 431 km *Instrument:* Stiattesi	8. X	9 59 09	0 56	–	1 35	–	2 03 N 2 17 E	–
Munich ep. dist. 454 km *Instrument:* Wiechert pendulum	8. X.	9 59 09	0 57±[1]	1 17	–	–	2 17	–
	1910 28. I.	23 57 45	1 03	1 20	1 46	1 54	2 15±[1]	2 54
	29. I.	0 12 02	–	1 17	1 39	1 51	2 12	2 54
Average			1 03	1 19	1 43	1 53	2 15	2 54

[1] In the minute mark.

Station	Day 1909	Time in epicentre h m s	Beg.	$\overline{P}$	R	S	R	eM	M
Rocca di Papa ep. dist. 496 km *Instruments:* Agamennons pendula	8. X.	9 59 09	–	1 19	1 44[2]	–	–	–	–
		10 59 40	–	1 32	–	–	–	–	–
	10. X.	5 55 10	–	1 23	–	–	–	4 07	–
		5 37 10	–	1 20	–	–	–	–	–
Average			–	1 23	–	–	–	–	–

[2] Marked as eM.

Station	Day 1909	Time in epicentre h m s	Beg.	$\overline{P}$	R	S	R	eM	M
Ischia ep. dist. 555 km *Instrument:* Grablowitz	8. X.	9 59 09	1 20	1 34	–	2 20	–	3 08	3 29
Taranto ep. dist. 605 km *Instruments:* Wiechert (160 kg) without damping, Vicentini with damping	8. X	9 59 09	1 21	1 42	2 17	2 30	–	3 18	–
Hohenheim ep. dist. 633 km *Instrument:* Bosch-Omori	8. X.	9 59 09	1 19	1 50	2 15	–	–	3 14	–

Station	Day 1909	Time in epicentre h m s	Beg.	$\overline{P}$	R	S	R	eM	M
Moncalieri ep. dist. 644 km *Instrument:* Stiattesi	8. X.	9 59 09	1 23	1 54	–	–	2 51	3 30	–
Sofia ep. dist. 652 km *Instrument:* Bosch-Omori	8. X.	9 59 09	–	1 46	–	–	2 59	3 08	–
Jena ep. dist. 688 km	8. X.	9 59 09	1 37	–	–	–	(3 02)	–	3 32
	10. X.	5 37 10	–	–	2 02	–	–	3 30	3 40
Instrument:		5 55 01	1 41	–	–	–	–	3 30	3 41
Wiechert	13. XII.	0 21 41	1 31	–	–	2 51	3 20	3 29	–
Average			1 36	–	2 02	–	–	3 30	3 40

Station	Day 1909	Time in epicentre h m s	Beg.	R_1	R_2	S	R	eM	M
Heidelberg ep. dist. 704 km *Instrument:* Wiechert	8. X.	9 59 09	1 35	–	–	–	–	3 35	4 18
Leipzig	8. X.	9 59 09	–	1 43	2 13	–	3 08	3 31	3 41
ep. dist. 707 km	10. X.	5 37 10	–	1 48	–	–	3 05	3 18	3 31
		5 55 01	–	1 52	–	2 52	3 08	3 19	3 29
Instrument: Wiechert	13. XII.	0 21 41	–	1 53	–	2 59	–	(3 49)	–
Average			–	1 49	2 13	2 59	3 07	3 19	3 30
Strasburg ep. dist. 720 km *Instrument:* Wiechert	8. X.	9 59 09	1 43	1 57	2 25 2 46	3 04	–	3 25 3 30	–
Potsdam	8. X.	9 59 09	–	e2 07	–	–	–	i 3 52	4 (15)
ep. dist. 798 km	10. X.	5 37 10	–	–	–	3 (14)	–	3 (56)	4 (02)
		5 55 01	–	–	–	3 (11)	–	–	–
Instrument: Wiechert	13. X.	0 21 41	–	–	–	3 (07)	i 3 30	i 4 03	4 (43)
	1910								
	28. I.	23 57 45	–	–	–	–	e 3 28	–	4 (15)
	29. I.	0 12 02	–	–	–	–	e 3 31	–	4 (10)
Average				2 07	–	3 11	3 30	3 54	4 12

Station	Day 1909	Time in epicentre h m s	Beg.	R_1	R_2	S	R	eM	M
Göttingen	8. X.	9 59 09	1 49	–	–	3 20	–	3 59	4 19
ep. dist. 810 km		10 59 40	–	–	3 (08)	–	–	–	4 08
	10. X.	5 37 10	–	–	–	–	–	–	4 14
Instrument: Wiechert		5 55 01	–	–	–	–	–	–	4 15
	13. XII.	0 21 41	1 49	–	–	3 24	–	–	–
	24. XII.	0 14 30	1 43	–	–	–	–	(3 54)	–
Average			1 49	–	3 (08)	3 22	–	3 59	4 14
Catania ep. dist. 889 km *Instrument:* Big pendulum	8. X.	9 59 09	1 58	–	–	–	3 51	3 55	–
Hamburg ep. dist. 999 km	8. X.	9 59 09	2 15	–	–	–	4 25	–	–
Instrument: Wiechert	13. XII.	0 21 41	–	–	–	4 (05)	–	5 (13)	
Average			2 15	–	–	4 (05)	4 25	5 (13)	–
Uccle ep. dist. 1044 km *Instrument:* Wiechert	8. X.	9 59 09	–	2 51	–	–	–	5 09	–
Paris ep. dist. 1063 km *Instruments:* Wiechert (200 kg), Mainka	8. X.	9 59 09	–	2 54	–	–	4 41	–	–
Granada ep. dist. 1892 km *Instrument*: horizontal pendulum	8. X.	9 59 09	3 55	–	–	–	–	8 17	–
Tiflis ep. dist. 2405 km *Instruments:* various	8. X.	9 59 09	–	–	6 50	9 20	–	–	–

The last station which has both phases is Strasburg at a distance of 720 km. If the earthquake start recorded in Potsdam belongs to the upper curve or if it is only a late first phase, is difficult to decide. Göttingen at a distance of 810 has a normal beginning recorded, belonging to the lower curve, but even on the 17,000 kg pendulum no trace of a second beginning could be found, which would belong to the upper curve. The upper curve therefore reaches almost 720 km, but it is uncertain, if it reaches even further.

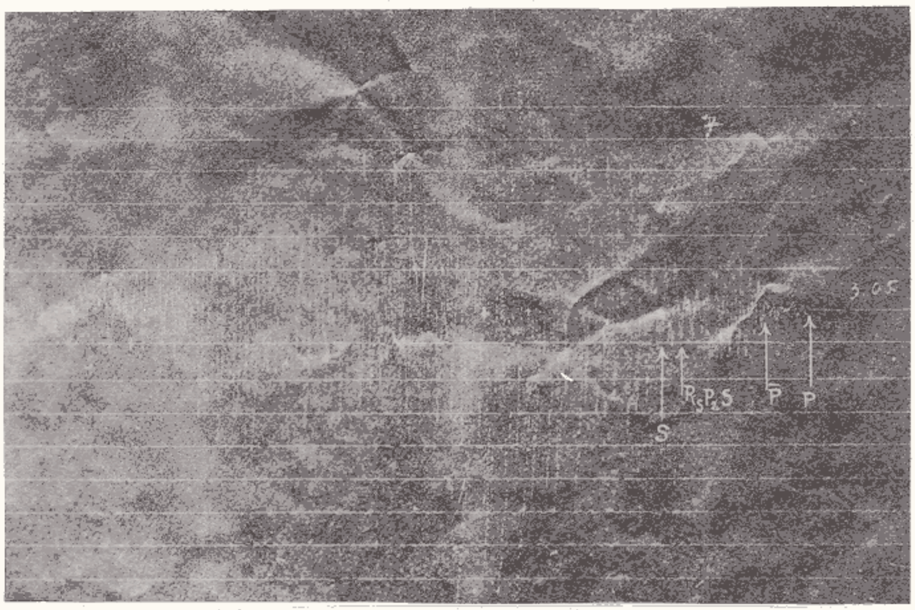


Figure 3. The earthquake of 7 June 1910, Calitri, NE component.

I will designate the lower curve as the normal primae curve (P) and the upper as the curve of individual or upper primae ($\overline{P}$).

The first phase (of normal primae) starts on all stations with very weak waves. After 20–25 s the individual primae start as a stronger or weaker strike. Our earthquake is too weak to show some diagram as an example. The earthquake on 7 June 1910 (southern Italy earthquake) has been recorded here in Zagreb at a distance of 550 km so nicely, that we are publishing it here (Fig. 3).

The run of the $\overline{P}$ curve is certain up to a distance of 600 km according to average data of individual stations at ±1 s. From 600–700 km the values are distributed so as to make the further run of the curve somewhat uncertain due to the weakness of the earthquake at that distance. I shall not be much in error if I draw a curve based on average values here, from 600 to 700 km too.

After finding both of these curves I wanted to draw as precise as possible curves for transversal waves (S) and for the beginning of the earthquake main phase (eM). Data of our earthquake were not sufficient for that because the stations particularly beyond a distance of 400 km exhibited too much difference between them. I had therefore to take other earthquakes to assist me. Up to the year 1906 and the next year no nearby earthquake has been published and it would take too much time to collect material required for that. There was nothing else left to do but to take into consideration those earthquakes which were treated in Strasburg publications for the year 1904 and 1905. First I considered the earthquake on 9 September 1905 as the best earthquake in this period and calculated for it, based on best stations and the P and $\overline{P}$ curves, the epicentral time.

Messina	Distance (km)	87	$\overline{P}$ 1h 43m 17s – 0m 11s = 1h 43m 06s
Catania	"	173	$\overline{P}$ 1h 43m 30s – 0m 28s = 1h 43m 02s
Ischia	"	290	$\overline{P}$ 1h 43m 47s – 0m 49s = 1h 42m 58s
Rocca di Papa	"	434	P 1h 44m —s – 1m 01s = 1h 42m 59s
			Mean epicentral time 1h 43m 01s ±3s

The propagation times for different distances are shown by the following table, and they are plotted into the coordinate grid (Appendix) as small crosses.

8 September 1905 – Calabrian earthquake – Epicentral time = 1h 43m 01s

Station	Epicentral distance	P m s	P m s	S m s	eM m s
Messina	87	–	0 15	–	–
Catania	173	–	0 28	–	–
Ischia	290	–	0 45	–	–
Rocca di Papa	434	0 58	1 15	–	–
Florence Ximeniano	683	1 32	–	–	3 10
Florence Q C	688	1 27	–	–	3 31
Pula	699	–	1 50	2 51[2]	–
Rijeka	734	–	1 58	3 08	3 38
Sofia	746	–	1 55	3 21	3 51
Padova	810	1 53	–	–	4 36
Salo	884	1 58	–	–	4 02
Pavia	924	1 59	–	4 02	4 59
Torino	973	2 08	–	3 29	5 28
Vienna	1069	2 16	–	–	5 21
Munich	1100	2 41[3]	–	–	5 32
Hohenheim	1233	2 40	–	5 32	7 04
Strassburg	1284	2 54[4]	–	5 08[4]	–
Heidelberg	1296	2 38	–	–	–
Tortosa	1351	2 58	–	5 02	7 04
Jena	1391	3 00	–	5 04	6 07
Leipzig	1422	3 03	–	5 48	7 26
Götingen	1491	3 20	–	5 59	7 05
Potsdam	1525	3 20	–	6 04	7 03
Uccle	1618	3 36	–	5 49	6 50
Hamburg	1725	3 53	–	–	7 37
Kiew	1898	3 58	–	–	–
Jurjev	2304	4 46	–	8 39	10 22
Akhalkalaki	2334	4 56	–	8 50	12 36
Uppsala	2360	4 48	–	8 42	12 04
Edinbourg	2382	4 58	–	8 06	11 02
Tiflis	2448	5 05	–	9 03	14 00
Moscow	2457	5 17	–	9 21	12 05

[1,2] In Strasburg publication marked as eM.
3 Approximately 15 s late time
[4] Corrected for + 1 min

Based on points derived for both earthquakes a preliminary curve of normal primae was drawn up to a distance of 2500 km.

I went up to such a great distance in order to possibly get correct propagation time values for the distance of 2000 km.

In order to even better correct this curve and to get as many values for other phases as possible, I searched among all recorded earthquakes in the years 1904 and 1905 for those earthquakes which had a precisely known epicentre and which possibly produce good values of epicentral time. For the reduction to epicentral time I took only the best stations and only those earthquakes whose epicentral time may be calculated to a precision of ±1 s. Epicentral time was calculated according to both the curves for P and $\overline{P}$, and I plotted the times derived onto the coordinate grid and corrected the curve for P where it was necessary. Based on such a corrected curve I calculated again the epicentral times and on the basis of data so corrected I adjusted the curve again. In this way the final curve was derived after some five to six attempts.

Earthquakes which I used are the following:

1. Balkan earthquake on 4 April 1904. Calculation of epicentral time:

	Dist. km		h m s	m s	h m s
Bucharest	350	$\overline{P}$	10 03 34	– 1 01	= 10 02 33
Ischia	750	$\overline{P}$	– 4 34	– 2 04	= – – 30
Messina	800	P	– 4 15	– 1 48	= – – 27
Trieste	900	P	– 4 26	– 2 00	= – – 26
Rocca di Papa	900	P	– 4 30	– 2 00	= – – 30
Padova	1050	P	– 4 49	– 2 19	= – – 30
Florence Quarto	1050	P	– 4 43	– 2 19	= – – 24
Leipzig	1350	P	– 5 28	– 2 57	= – – 31
Strassburg	1450	P	– 5 37	– 3 10	= – – 27
Göttingen	1500	P	– 5 49	– 3 16	= – – 33
Tbilisi	1800	P	– 6 20	– 3 53	= – – 27

Mean epicentral time = 10 02 29 ± 2 s

2. Indian earthquake on 4 April 1905 – Epicentral time according to Omori 0 h 49 m 48 s. Only for Tashkent and Bombay stations.

3. 1 April 1905. Calculation of epicentral time:

	Dist. km		h m s	m s	h m s
Ischia	479	P	4 43 23	– 1 07	= 4 42 16
Catania	634	P	– 43 36	– 1 26	= – – 10
Florence Quarto	688	P	– 43 40	– 1 33	= – – 07
Padova	699	P	– 43 49	– 1 35	= – – 14
Jena	1140	P	– 44 45	– 2 30	= – – 15
Strassburg	1147	P	– 44 49	– 2 31	= – – 18
Potsdam	1227	P	– 45 00	– 2 41	= – – 19

Göttingen	1267	P	– 45 01	– 2 46	= – –	15
Uccle	1491	P	– 45 22	– 3 15	= – –	07
Batum	1845	P	– 46 09	– 3 58	= – –	11
Jurjew	1878	P	– 46 10	– 4 02	= – –	08
Boržom	2100	P	– 46 22	– 4 14	= – –	08
Tbilisi	2100	P	– 46 44	– 4 26	= – –	18

Mean epicentral time = 4 42 13 ± 3 s

4. 8 November 1905. Calculation of epicentral time:

	Dist. km		h m s	m s	h m s
Athens	250	$\overline{P}$	22 06 57	– 0 41	= 22 06 16
Sofia	294	$\overline{P}$	– 07 03	– 0 50	= – – 13
Ischia	801	P	– 08 05	– 1 48	= – – 17
Catania	850	P	– 08 03	– 1 54	= – – 09
Pula	1000	P	– 08 31	– 2 13	= – – 18
Ljubljana	1028	P	– 08 32	– 2 17	= – – 15
Munich	1346	P	– 09 05	– 2 56	= – – 09
Jena	1525	P	– 09 35	– 3 19	= – – 16
Leipzig	1545	P	– 09 35	– 3 22	= – – 13
Potsdam	1606	P	– 09 45	– 3 29	= – – 16
Strassburg	1607	P	– 09 42	– 3 29	= – – 13
Götingen	1679	P	– 09 45	– 3 38	= – – 07
Tbilisi	1728	P	– 09 48	– 3 45	= – – 03
Uccle	1966	P	– 10 21	– 4 11	= – – 10

Mean epicentral time = 22 06 13 ± 3 s

In the whole series of published earthquakes I could not find any other earthquake that I could use to construct the time curves. The table overleaf shows the times for these four earthquakes.

The normal P curve drawn on the basis of all the values which could be taken into consideration is only slightly different from the curve published in »Göttinger Erdbebennachrichten«.

The curve for S could be hardly noticed up to a distance of 1000 km. At a distance of 500 km it is equal to the Göttingen one. At a distance of 1000 km it is 3 s lower than the latter, and at 1500 km and 2000 km it is 2 s lower.

Up to 1600 km the curve for P is almost a straight line, and then it turns suddenly downwards. The turn is maybe even sharper than the drawn curve shows, but the data for distances from 1900 to 2000 km did not seem reliable enough to me to place the curve lower.

The curve for S starts as the one for P approximately beyond 300 km. I found very few good values for its construction, so I suppose it will take further near earthquakes in order to determine that curve to the precision of a second. From 300 to 1000 km I drew it on the basis of data for Vienna, Munich, Ischia, Jena, Leipzig and Göttingen, and for distances from 300 to 600 km I replaced it later with the calculated curve.

Station	Earthquake	Epicent. distance	Epicent. time	P m s	$\bar{P}$ m s	S m s	eM m s
Athens	1905 XI 8	250	22 06 13	–	0 44	–	0 46
Sofia	"	294	"	–	0 50	–	–
Bucharest	1904 IV 4	350	10 02 29	–	1 05	1 25	1 43
Ischia	1905 VI 1	479	4 22 13	1 10	–	2 11	–
Catania	"	634	"	1 23	–	2 26	–
Florence Q.C.	"	688	"	1 27	–	2 14	3 46
Padova	"	699	"	1 36	–	–	3 32
Messina	1904 IV 4	800	10 02 29	1 46	–	3 38	–
Ischia	1905 XI 8	803	22 06 13	1 52	–	–	6 27
Catania	"	850	"	1 50	–	2 57	–
Trieste	1904 IV 4	900	10 02 29	1 57	–	–	–
Rocca di Papa	"	900	"	2 01	–	3 31	4 16
Pula	1905 XI 8	1000	22 06 13	2 18	–	2 53	4 59
Ljubljana	"	1028	"	2 19	–	2 59	–
Padova	1904 IV 4	1050	10 02 29	2 20	–	–	–
Florence Q.C.	"	1050	"	2 14	–	–	4 41
Jena	1905 VI 1	1140	4 22 13	2 32	–	–	–
Strassburg	"	1147	"	2 36	–	4 18	4 53
Tashkent	1905 IV 4	1200	0 49 48	2 36	–	4 38	–
Potsdam	1905 VI 1	1227	4 22 13	2 47	–	–	–
Göttingen	"	1267	"	2 48	–	–	5 32
Munich	1905 XI 8	1346	22 06 13	2 52	–	–	–
Leipzig	1904 IV 4	1350	10 02 29	2 59	–	5 19	7 05
Strassburg	"	1450	"	3 08	–	5 56	6 56
Uccle	1905 VI 1	1491	4 22 13	3 09	–	–	6 41
Göttingen	1904 IV 4	1500	10 02 29	3 20	–	5 55	–
Jena	1905 XI 8	1525	22 06 13	3 22	–	–	–
Leipzig	"	1545	"	3 22	–	6 08	8 30
Potsdam	"	1606	"	3 32	–	6 13	–
Strassburg	"	1607	"	3 29	–	6 34	–
Bombay	1905 IV 4	1610	0 49 48	3 20	–	6 51	–
Göttingen	1905 XI 8	1679	22 06 13	3 32	–	6 42	8 46
Tbilisi	"	1728	"	3 35	–	6 42	–
Tbilisi	1904 IV 4	1800	10 02 29	3 51	–	6 57	11 11
Batum	1905 VI 1	1845	4 22 13	3 56	–	7 13	10 13
Jurjev	"	1878	"	3 57	–	7 03	–
Uccle	1905 XI 8	1966	22 06 13	4 06	–	7 08	–
Boržom	1905 VI 1	1990	4 22 13	4 09	–	7 39	11 01
Tbilisi	"	2100	"	4 31	–	8 23	11 01

4. Propagation of earthquake waves in the uppermost layers of the Earth

When I was sure, based on data obtained in 3, that two kinds of first preliminary waves exist, both kinds reaching all locations from 300 to 700 km distance, and that from the epicentre to approximately 300 km distance only the first kind arrives, whereas from 700 km distance onward only the second kind arrives, I tried to explain this until now unknown fact.

Individual primae and the depth of the quake focus. As it is totally impossible that two different kinds of longitudinal waves emanate from the earthquake focus with differing velocities and further because the normal primae (P) start only at a greater distance from the epicentre, I first looked for a solution to those waves that we may follow already from the epicentre and which surely arrive by a direct path to the respective stations.

The attempt to calculate the depth of the earthquake focus on the basis of the assumption that the P waves propagate linearly, did not succeed, because I could not find any depth from which the waves could propagate at a constant velocity so that the calculated propagation times satisfy the time curve. The calculation has shown that the velocity of propagation must increase slowly with depth.

The attempt to calculate the depth of the focus according to presently known wave propagation formulas also failed because of the great complexity of these formulas. Subsequently I tried with some simple assumptions on the increase of velocity with depth, in the hope that in this way I may find simpler formulas. If I succeed to calculate the propagation times in this way so that they satisfy the time curve I will get approximate formulas which will be the closer to truth the more they satisfy the curve of times calculated by using them.

The assumption that the velocity increases proportionally to depth, leads to the integration of the differential equation of motion in a finite form, which is too complicated for practical calculations. Another assumption, which is almost identical to the first, gives a solution in a very simple form, which is very much suited for practical calculations. It is implicit that the formulas obtained will be valid only for small depth differences.

Let ρ_o be the radius vector drawn from the centre of the Earth to the earthquake focus, and let ρ and φ be the radius and angular distance of a convenient point reached by the earthquake ray during the time t, let further c_o be the origin velocity in the focus, and c the velocity at a point determined by ρ, φ and t. Similarly, let e_o and e be the angles defined by extensions of vector radii towards Earth surface and the direction, in which the earthquake wave is propagating at both points. If we assume, as customary, that the velocity is a function only of the radius vector, then the well known refraction formula is valid:

$$\frac{1}{c}\rho \sin e = \frac{1}{c_o}\rho_o \sin e_o = \mathrm{a} \qquad (1)$$

Since $\sin e = \rho \frac{\mathrm{d}\varphi}{\mathrm{d}s}$ it follows that $\rho^2 \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \mathrm{a}c \frac{\mathrm{d}s}{\mathrm{d}t} = \mathrm{a}c^2$, where $\mathrm{a} = \frac{1}{c_o}\rho_o \sin e_o$.

If in the equation:

$$c^2 = \rho^2 \left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}\rho}{\mathrm{d}t}\right)^2$$

for $\rho^2(\mathrm{d}\varphi/\mathrm{d}t)^2$ we substitute the value from the first equation, we get:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \pm \frac{c\sqrt{\rho^2 - \mathrm{a}^2c^2}}{\rho}$$

and from it by integration:

$$t + \mathrm{C} = \pm \int \frac{\rho \, \mathrm{d}\rho}{c\sqrt{\rho^2 - \mathrm{a}^2c^2}}$$

where C is the constant of integration.

The integral on the right side of the equation above may be written in the following form:

$$\int \frac{\rho \, \mathrm{d}\rho}{\mathrm{c}\sqrt{\rho^2 - \mathrm{a}^2c^2}} = \frac{1}{c} \sqrt{\rho^2 - \mathrm{a}^2c^2} + \int \frac{\rho^2 \, \mathrm{d}c/\mathrm{d}\rho \, \mathrm{d}\rho}{c^2\sqrt{\rho^2 - \mathrm{a}^2c^2}}$$

Here we may substitute for $\mathrm{d}c/\mathrm{d}\rho$ a convenient assumption giving the integral such a form which makes it easy to solve. If we assume:

$$\frac{\mathrm{d}c}{\mathrm{d}\rho} = -k\frac{c}{\rho}$$

where k is a constant which must be determined later, we get:

$$t + \mathrm{C} = \pm \frac{1}{c(k+1)} \sqrt{\rho^2 - \mathrm{a}^2c^2} \tag{2}$$

Every value of ρ associates with two equal values for t, but designated with an opposite sign, which correspond to both and opposite sides to which the ray propagates. If we consider ray propagation to one side only, we may even drop the »–« sign. From the equation:

$$\frac{\mathrm{d}c}{\mathrm{d}\rho} = -k\frac{c}{\rho}$$

we get by integration:

$$c = c_0 \left(\frac{\rho_0}{\rho}\right)^k \tag{3}$$

where c_o is the velocity which corresponds to the depth ρ_o.

If in the equation (2) we substitute for c the value from (3) and for a the value from (1), we get the arranged equation:

$$t + C = \frac{\rho}{(k+1)\, c_o\, (\rho_o/\rho)^k} \sqrt{1 - (\rho/\rho_o)^{2(k+1)} \sin^2 e_o}$$

As for time $t = 0, \rho = \rho_o$, so the value of the constant is:

$$C = \frac{\rho}{(k+1) c_o} \cos e_o$$

and finally if we substitute $(\rho_o/\rho) = r$:

$$t = \frac{\rho}{(k+1)\, c_o\, r^k} \left[\sqrt{1 - r^{2(k+1)} \sin^2 e_o} - r^{k+1} \cos e_o\right] \qquad (4)$$

If the wave of some earthquake would propagate in a layer where the velocity would be constant $c = c_o$, then also $k = 0$, hence

$$t = \frac{\rho}{c_o} \left[\sqrt{1 - r^2 \sin^2 e_o} - r \cos e_o\right]$$

If in the equation (1) we substitute for c the value from equation (3) we obtain the angle of the direction associated with the given r and e_o from the equation:

$$\sin e = r^{k+1} \sin e_o \qquad (5)$$

If in a special case the value of r is given for the surface of the Earth, then equation (5) represents the relation between the angle of the pulse e_o and the emergence angle e.

For $e_o = 0$ we also have $e = 0$ on the whole path to the surface of the Earth. For $e_o = 90°$ the maximum value is for $\sin e = r^{k+1}$, and the emerging ray closes the smallest angle with the Earth's surface at that location.

For $e_o = 0$ we obtain from (4):

$$t = \frac{\rho}{(k+1)\, c_o\, r^k} \left(1 - r^{(k+1)}\right) \qquad (6)$$

and for $e_o = 90°$:

$$t = \frac{\rho}{(k+1)\, c_o\, r^k} \sqrt{1 - r^{2(k+1)}} \tag{7}$$

In an analogous manner we get the relation between ρ and the refraction angle φ.

From equation (1) we get:

$$\rho^2 \, d\varphi = ac \, ds$$

and from there by substitution of the value for ds:

$$d\varphi = \pm \frac{ac \, d\rho}{\rho \sqrt{\rho^2 - a^2 c^2}}$$

If in this equation we substitute the value for a from (1) and the value for c from (3), and integrate, we get:

$$\varphi + C = \pm \frac{1}{k+1} \arcsin\left\{ \left(\frac{\rho_o}{\rho}\right)^{k+1} \sin e_o \right\}$$

Because for $\rho = \rho_o$ we have $\varphi = 0$, the value of the constant is:

$$C = \pm \frac{e_o}{k+1}$$

If we substitute this value in the equation above, we finally get:

$$\varphi = \pm \frac{1}{k+1} \left[e_o - \arcsin\left\{ \left(\frac{\rho_o}{\rho}\right)^{k+1} \sin e_o \right\} \right] \tag{8}$$

as the pursued relation between ρ and φ.

For $k = 0$ this equation transforms into the equation of a straight line as the connection of the points ρ_o, 0 and ρ, φ. If here also we ignore the »–« sign and substitute $\rho_o/\rho = r$, we get:

$$\varphi = \frac{1}{k+1} \left[e_o - \arcsin\left\{ r^{k+1} \sin e_o \right\} \right] \tag{9}$$

If in contrast we want to have ρ as a function of φ, then we get from the equation (8):

$$\rho^{k+1} = \frac{\rho_o^{k+1} \sin e_o}{\sin\left[e_o - (k+1)\,\varphi\right]} \tag{10}$$

ρ will be the minimum for

$$\varphi = \frac{e_o - 90^\circ}{k+1} \tag{11}$$

If $e_o > 90^\circ$, then φ is positive and if $e_o < 90^\circ$, then φ is negative. For $e_o = 90^\circ$ the focal point is at the same time the lowest point of the ray.

For the minimum value of ρ we get from (10):

$$\rho = \rho_o \sqrt[k+1]{\sin e_o} \tag{12}$$

If we look for the run of the curve from some point $(\rho_o, 0)$ to the point (ρ_o, φ) for example from some point on the surface of the Earth to some other point of the Earth surface, then we may not get to know it from equation (9), but we have to calculate $\varphi/2$ from equation (11), because the curve is symmetric for both sides of the point for which ρ is minimal. In the same way we must also calculate the time t for a half of the curve, because the equation (6) gives only a value of $t = 0$ for that case.

To calculate the depth of the earthquake focus (hypocentre) we have at our disposal only the time curve. The abscissae of this curve represent the distances from the epicentre, and are only correct as far as the epicentre has been correctly determined. The ordinatae of the curve are times calculated from a convenient time origin as zero. The shape of the curve is known only starting from the station nearest to the epicentre, and only to the degree the quality of observation of the respective stations allows. The shape of the curve from the epicentre to the nearest station is unknown. If t_o is the time when the earthquake started at the focus, t_1 the time of the first onset on some station at a distance D_1 from the epicentre, t_2 the first onset at some other station at a distance D_2, then we know only the difference t_2–t_1. This difference means that the first impact arrived t_2–t_1 time units, for example seconds, later at the second station than at the first one.

The time curve is concave upwards at small distances from the epicentre, and downwards at larger distances.

The apparent velocity calculated for the surface of the Earth becomes ever smaller as we get further from the epicentre. At the point of curve inflection this velocity is minimal and then it increases again with distance. It is known and we may easily certify that the ray must close the smallest angle with the Earth surface at the inflection point.

For our $\overline{\mathrm{P}}$ curve the inflection point is at a distance of 280±10 km. In order to determine this distance, I calculated the apparent velocities on the surface and adjusted graphically.

Based on both assumptions: knowledge of time differences and the epicentral distance of the curve inflection point, we may unambiguously determine the depth of the earthquake focus.

Three unknowns should be determined: c_o, ρ_o and k. As this calculation may be performed only through trials, we may perform them in two ways.

1. Let t_1 be the time of the first arrival of the ray $\overline{\mathrm{P}}$ at the station with a distance D_1, likewise let t_2, t_3, t_4 be the corresponding times for further three stations at distances of D_2, D_3, D_4. We shall not use factual observations for these times, but those average values that the time curve gives us. Let D_1 be some, if possible, small distance from the epicentre for which t is certain, for example approximately 50 km. In the same way let D_4 be some, if possible, large distance, let D_2 and D_3 be, if possible, at equal distances between D_1 and D_4. According to (4) we have:

$$t_1 = \frac{\rho}{(k+1)\,c_o\,r^k}\left[\sqrt{1 - r^{2(k+1)}\,\sin^2 e_1} - r^{k+1}\cos e_1\right]$$

for the first station and corresponding equations for the other three stations. The pulse angles are marked with e_1, e_2, e_3, e_4.

From the first and second equation and from the third and fourth equation we get by subtraction

$$\begin{aligned} T_1 = t_2 - t_1 = \frac{\rho}{(k+1)\,c_o\,r^k}\Big[&\sqrt{1 - r^{2(k+1)}\sin^2 e_2} - \\ &- \sqrt{1 - r^{2(k+1)}\sin^2 e_1} + r^{k+1}(\cos e_1 - \cos e_2)\Big] \\ T_2 = t_4 - t_3 = \frac{\rho}{(k+1)\,c_o\,r^k}\Big[&\sqrt{1 - r^{2(k+1)}\sin^2 e_4} - \\ &- \sqrt{1 - r^{2(k+1)}\sin^2 e_3} + r^{k+1}(\cos e_3 - \cos e_4)\Big] \end{aligned} \tag{13}$$

If in these equations we exchange c_o and T, we may solve both equations for c_o.

Now we choose convenient values for r among which we may expect r to be, – for example so that we successively take 5, 10 etc. km as the hypocentral depth – and substitute for each of these depths various values of k, and change them until we get equal values for c_o from both equations.

After a short exercise we may find in this way such values for e_1 to e_4, that the four points (stations) are uniformly distributed on the time curve.

For each group of assumed r, k and e we find from equation (9) the corresponding values for φ and t.

In this way we may calculate a table into which values of c_o and k corresponding to respective r are inserted.

From equation (10) follows, due to $e_o = 90^o$ for the inflection point:

$$r^{k+1} = \sin\left[90^o - (k + 1)\,\varphi\right] \tag{14}$$

From this equation the value of k may be easily calculated by trials for each assumed r.

If we now look for such a value of r in the table with a corresponding value of k calculated from equation (14), then the task is thus solved because the curve determined in this way falls together with the time curve in the four points, and in some fifth point it has a common inflection point.

2. The second method is just an inverted first method. First, the values of k which correspond to various assumed values of r, are determined from the equation (14), with the condition that the inflection point is at a distance of 280 km. With every pair of r and k a trial is performed until a pair of values r and k is found for which both equations (13) give the same value for c_o.

The first method is very tedious and impractical, while the second one leads to the solution very easily.

I took the round number of 6370 km as the Earth radius. The distance of 280 km corresponds in that case to an angle of 151.1' at the centre of the Earth. Because it is necessary to do square roots for the solution of the equations (13), I computed with seven digit logarithms.

The following table shows the results of solving the equation (14) for depths of 10–40 km for every 5 km:

Depth km	log r	k	log r^{k+1}	log r^k
10	0.9993176	0.625	0.9988912	0.99958
15	0.9989761	1.436	0.9975058	0.99854
20	0.9986344	2.244	0.9955701	0.99693
25	0.9982923	3.049	0.9930857	0.99479
30	0.9979499	3.850	0.9900569	0.99211
35	0.9976072	4.645	0.9864927	0.98890
40	0.9972643	5.434	0.9823980	0.98511

The times which the earthquake requires to arrive to the respective stations were plotted on millimeter grid paper and a curve for P was drawn according to the quality of the respective stations. Subsequently the curve was read for distances in steps of 20 km and the numbers obtained were adjusted to get a regular curve if possible.

The travel times for epicentral distances between 40 and 700 km in increments of 20 km are:

km	Travel time	d	km	Travel time	d	km	Travel time	d
40	4.3		280	46.2	3.8	520	89.9	3.4
60	7.1	2.8	300	50.0	3.8	540	93.2	3.3
80	10.1	3.0	320	53.8	3.8	560	96.5	3.3
100	13.3	3.2	340	57.6	3.8	580	99.7	3.2
120	16.7	3.4	360	61.4	3.8	600	102.9	3.2
140	20.2	3.5	380	65.1	3.7	620	106.1	3.2
160	23.8	3.6	400	68.8	3.7	640	109.3	3.2
180	27.4	3.6	420	72.5	3.7	660	112.5	3.2
200	31.1	3.7	440	76.1	3.6	680	115.7	3.2
220	34.8	3.7	460	79.6	3.5	700	118.9	3.2
240	38.6	3.8	480	83.1	3.5			
260	42.4	3.8	500	86.5	3.4			

Thereafter I passed over to solving the equations (13). Here is an example of the depth calculation for the depth of 25 km. The angles of 65°, 90°, 97° and 100° were taken as convenient angles of pulses.

25; $k = 3.094$; $\log r^{k+1} = 0.9930857$; $\log r^{k} = 9.99479$

Pulse angle	65°	90°	97°	100°
$\log \cos e_n$	9.62595	$\cos e_o = 0$	9.08589	9,23967
$\log \sin e_n$	9.9572757	$\log \sin e_o = 0$	9.9967507	9.9933515
$\log r^{k+1} \sin e_n$	9.9503614	9.9930857	9.9898364	9.9864372
$2\log r^{k} \sin e_n$	0.9007228	0.9861714	0.9796728	0.9728744
$r^{2(k+1)} \sin^2 e_n$	0.7956514	0.9686600	0.9541733	0.9394515
$1 - r^{2(k+1)} \sin^2 e_n$	0.2043486	0.0313400	0.0458267	0.0605485
$\sqrt{1 - r^{2(k+1)} \sin^2 e_n}$	$0.4520 = m_1$	$0.1770 = m_2$	$0.2141 = m_3$	$0.2461 = m_4$
$\log r^{k+1} \cos e_o$	0.61904	–	0.07898	0.23276
$r^{k+1} \cos e_o = n$	$0.4159 = n_1$	$0 = n_2$	$0.1199 = n_3$	$0.1670 = n_4$
$m_p \pm n_p$	0.0361	0.1770	0.3340	0.4131
$(m_p \pm n_p)\cdot(m_{p-1} \pm n_{p-1})$	$F_1 = 0.1409$		$F_2 = 0.0791$	
Emergence angle $\sin e = r^{k+1} \sin e_o$	63° 07' 30"	79° 48' 10"	77° 39' 10"	75° 45' 20"
$(e_o - e)/(k+1) = \varphi$	27.8'	151.1	286.7'	359.2
Distance of emerging ray	52	280	531	666
t according to time curve	6.0 s	46.2 s	91.8 s	113.5 s
$t_n - t_{n-1}$	$T_1 = 40.2$ s		$T_2 = 21.7$ s	

$$-\left\{\begin{aligned}\log \rho &= 3.80414\\ \log (k+1) &= 0.60735\\ \log r^k &= \underline{0.99479-1}\\ &\ 3.20200\end{aligned}\right. \quad + \Bigg\} \quad \begin{aligned}\log F_1 &= 0.14891-1\\ &\underline{+3.20200}\\ &\ 2.35091\\ -\log T_1 &= \underline{1.60423}\\ \log c_1 &= 0.74668\\ c_1 &= 5.58\end{aligned} \qquad \begin{aligned}\log F_2 &= 0.89818-2\\ &\underline{+3.20200}\\ &\ 2.10018\\ -\log T_2 &= \underline{1.33646}\\ \log c_2 &= 0.76372\\ c_2 &= 5.80\end{aligned}$$

In this way velocities were calculated for all depths from 10 to 50 km. Four assumed distances could not be precisely equal in all the calculations, because many preliminary calculations would have to be performed.

The following values were found:

km	10	15	20	25	30	35	40	45	50
c_1	5.60	5.54	5.46	5.45	5.42	5.39	5.33	5.25	5.23
c_2	6.30	6.60	6.13	5.80	6.18	6.70	6.06	5.99	6.19

With increasing depth c_1 decreases uniformly. The value of c_2 oscillates in contrast so that it gets closest to c_1 for a depth of 25 km, and for all other larger or smaller depths it departs from that value. If same distances could always be taken for the calculation there would be no such oscillations in c_2.

If indeed the equations (4) and (9) do represent the true path of the earthquake rays in the uppermost layers of the Earth, then the depth of the earthquake focus of 25 km corresponds to them as well as the initial velocity in the focus between 5.45 and 5.80 km/s. Subsequently I took a few velocities close to the arithmetic mean of both of these velocities as initial velocities and with each of these velocities I calculated for eight distances the time which it takes the rays to reach the respective locations. The work is very simple because the calculation for four points is already finished. The best result is obtained for the focal velocity of 5.60 km/s. In the following table ray travel times are calculated for some distances and adjacent to them the times obtained by observation. In the fourth column are travel times calculated from the epicentre as origin. In the fifth column are the differences between the read and the calculated times.

The average difference of both curves is smaller then the time curve construction error. If we inspect the time curve closer we will certify that the calculated curve, having smaller curvature, better approximates the times of certain stations then the constructed one. The difference between the curves is so small that we may claim that equations (4) and (9) represent the time curve quite accurately in every detail.

If we subtract from equation (4) the analogous equation for the travel time from the hypocentre to the epicentre i.e. for the time required by the first stroke to arrive from the hypocentre to the epicentre, we get the time curve equation:

Dis-tance km	Travel Time Recorded	Travel Time Calculated	Calculated – 4.5 s	Difference Rec. – Cal.	Pulse angle e_0	Emergence angle e	90 – e
	Second						
0	0	4.5	0	–	0°	0°	90°
21	–	5.9	1.4	–	40°	39° 14' 30"	50° 45' 30"
52	6.0	10.3	5.8	0.2	65°	63° 07' 30"	26° 52' 30"
84	10.7	15.8	11.3	–0.6	75°	71° 55' 40"	18° 04' 20"
117	16.2	21.1	16.6	–0.4	80°	75° 44' 30"	13° 15' 30"
147	21.5	26.8	22.3	–0.8			
174	26.3	31.5	27.0	–0.7	85°	78° 30' 20"	11° 29' 40"
280	46.2	50.3	44.8	0.4	90°	79° 48' 10"	10° 11' 50"
374	64.0	67.6	63.1	0.9	93°	79° 22' 30"	10° 37' 30"
449	77.7	80.3	75.8	1.9	95°	78° 30' 20"	11° 29' 40"
531	91.7	95.4	90.9	0.8	97°	77° 39' 10"	12° 20' 50"
666	113.5	118.0	113.5	0.0	100°	75° 45' 20"	14° 14' 40"
713	120.9	127.4	120.3	0.6	101°	75° 02' 40"	14° 57' 20"
Average difference				±0.7 s			

$$t = \frac{\rho}{(k+1)c_0 r^k}\left[\sqrt{1 - r^{2(k+1)}\sin^2 e_0} - r^{k+1}\cos e_0 + r^{k+1} - 1\right]$$

while for $e_0 = 0$ we get

$$t_0 = \frac{\rho}{(k+1)c_0 r^k}\left(1 - r^{k+1}\right)$$

Because for the surface of the Earth $c = c_0\, rk = 5.53$ km/s and $r^{k+1} \sin e_0 = \sin e$, hence equal to the sine of the emergence angle, the upper equation may then be also rearranged so that it includes only the velocity c with which the earthquake rays arrive to the surface of the Earth, and the emergence angle i.e. the angle closed by the arriving ray with a line vertical to the surface of the Earth.

Earlier it was explained that $\overline{\text{P}}$ probably reach up to 720 km distance. This is however completely uncertain, because we shall later discuss another time curve, which is in any case the extension of this first one, so it is uncertain where the first one ends. Because all the following analyses are based on the assumption that the curve $\overline{\text{P}}$ stops somewhere, we shall presently keep approximately this distance and later explain how we can arrive at a correct definition of the final distance of this curve based on further proper earthquakes.

If the curve end point is at a distance of 720 km from the epicentre, according to equation (12) the lowest point of the ray travel reaches the depth of 54 km at a distance of 3° 41.6' (411 km). If, however, the largest pulse angle in the hypocentre is only 100°, which corresponds to the end point of the curve at a distance of 666 km, then the largest depth of travel is 49 km.

In order to be able to further discuss this I had to decide on some depth, which can be at least approximately confirmed by observation. I decided for the round depth of 50 km.

As $\overline{\mathrm{P}}$ can reach only up to a depth of 50 km, at that depth there is a boundary surface of the uppermost layer of the Earth i.e. of the earths crust. At that depth there must be a sudden change of material making up the interior of the Earth, because a sudden jump in the earthquake wave velocity must occur here.

5. Normal primae (P)

The normal primae propagate in the interior of the Earth with a much higher velocity than the individual ones. In order to determine this velocity, particularly up to distances of 1600 or 1700 km, up to which distance the persistence of their travel is considerably certain, first the velocity must be determined with which these waves penetrate into the Earth's crust as well as the value k_1 of the increase of that velocity with depth.

Because in this case we have no initial assumptions to hold to while calculating the velocity change at the transition of waves from the upper crust into the interior of the Earth, I had to manage in a special manner.

The ray which emerges from the earthquake focus at a convenient angle will arrive at the boundary surface and penetrate into the interior of the Earth there. At passing through it will refract and further propagate along a curved line. The ray will then again reach the boundary surface, and as it refracts again, it will reach through the upper layer to the surface of the Earth.

If two such rays are given which reach the surface of the Earth close to each other, for example 100 km apart, we may assume that their paths in the upper layer are approximately equal. The time difference such rays require to traverse the path from the earthquake focus to the surface of the Earth depends in that case only on the time difference to traverse the lower layer.

Because here we have rays which pass from one level to another equal level, we may then employ the following equation.

For $e_o = 90°$ we have

$$\frac{\varphi}{2} = \frac{1}{k+1}\left[90° - \text{arc sin}\, r^{k+1}\right]$$

Because, however, $r^{k+1} = \sin e$ i.e. equal to the sine of the angle under which the ray penetrates into the upper layer, we have

$$\frac{\varphi}{2} = \frac{1}{k_1+1}\left[90° - e\right] = \frac{\varepsilon}{k_1+1} \tag{15}$$

The time the ray requires to traverse a half of the path is

$$\frac{t}{2} = \frac{\rho}{(k_1 + 1)\, c_o r^k} \sqrt{1 - r^{2(k+1)}}$$

or because $c_o\, r^{k_1} = c$ i.e. equal to the emergence velocity or the velocity at which the ray penetrates the upper layer, and r^{k_1+1} as before is equal to the sine of the emergence angle, then we have

$$t + \frac{2\rho}{(k_1 + 1)\, c_1} \cos e = \frac{2\rho}{(k_1 + 1)\, c_1} \sin \varepsilon$$

If one ray requires the time t_1 to pass through the second layer and the other ray takes the time t_2, then the time difference is

$$\tau_1 = t_2 - t_1 = \frac{2\rho}{(k_1 + 1)\, c_1} \left(\sin \varepsilon_2 - \sin \varepsilon_1\right) \tag{16}$$

If we initially assume that for rays up to 1700 km distance k_1 is at least an approximate constant, then we may find probable values for k_1 and c_1 from three pairs of rays.

I initially assume that the approximate projection d of the ray path onto the Earth surface is about 100 km. If the projection of the path in the lower layer is d_1, the total path is $d_1 + 100$ km with a corresponding time t_1.

In order to get correct times if possible for the purposes of this calculation I read the times from the time curves and adjusted them. The times for P so adjusted are:

Dist. km	Time s	d	Dist. km	Time s	d	Dist. km	Time s	d	Dist. km	Time s	d
500	69.4	6.4	900	120.4	6.3	1300	170.8	6.3	1700	221.1	6.2
550	75.8	6.4	950	126.7	6.3	1350	177.1	6.3	1750	227.1	6.0
600	82.2	6.4	1000	133.0	6.3	1400	183.4	6.3	1800	233.0	5.9
650	88.6	6.4	1050	139.3	6.3	1450	189.7	6.3	1850	238.7	5.7
700	95.0	6.4	1100	145.6	6.3	1500	196.0	6.3	1900	244.3	5.6
750	101.4	6.4	1150	151.9	6.3	1550	202.3	6.3	1950	249.8	5.5
800	107.8	6.4	1200	158.2	6.3	1600	208.6	6.3	2000	255.3	5.5
850	114.1	6.3	1250	164.5	6.3	1650	214.9	6.2	2050	260.8	5.5

Let us take three pairs of values for φ: 6° and 7°, 10° and 11°, 14° and 15°, further let us calculate using equation (15) for each pair of these angles the values of ε, corresponding to various conveniently assumed values of k_1. The assumed vales of φ correspond to total paths of rounded 759, 860, 1219, 1320, 1650 and 1760 km and travel times of 101.4 s, 115.4 s, 159.5 s, 173.3 s, 214.9 s and 228.3 s. The time differences for the respective pairs of φ are: 14.0 s, 13.8 s and 13.4 s. The corresponding values for c_1 are calculated using equation (16).

One example will be sufficient: For 6° and 7° the $\varphi/2$ are equal to 3° and 3° 30'. If we take $k_1 = 4$, so is $\varepsilon_1 = (k+1)\varphi/2 = 15°$, and $\varepsilon_2 = 17° 30'$, and hence according to (16) $c_1 = 7.56$ km/s.

Values of c thus obtained are the following:

$k_1 =$	4	3	2	1
6°–7°	7.56	7.68	7.77	7.85
10°–11°	7.17	7.46	7.69	7.86
14°–15°	6.64	7.20	7.64	7.95

If the obtained values of c_1 are plotted into the coordinate grid so as to put values of k_1 as abscissae and of c_1 as ordinatae, the three derived curves for c intersect approximately at $k = 1.3$ with a common velocity of 7.8 km/s.

If in this way the approximate velocity is known, then the approximate refraction angle is also known. As the velocity at the lower boundary of the upper layer equals 5.68 km/s, so $\log n = 0.12804$.

If now we take as the first approximation $k = 1.3$, and compute using (15) for example for 6°, 10° and 14° the corresponding ε, and by multiplying their sines by n we compute the angles at which these rays emerge into the upper layer, from known pulse angles we can easily calculate the paths from the boundary surface to the earthquake focus on one side, and to the Earth surface on the other. Because I expected to have to perform a number of trials, I calculated the distances (in km) and times (in s) for pulse angles of 30°, 40°, 50° and 60°.

	From 50 to 25 km		From 50 to 0 km	
Pulse angle	Distance, km	Time	Distance, km	Time
30°	14	5.0	28	10.3
40°	21	5.8	41	11.5
50°	29	6.9	58	13.6
60°	42	8.7	82	22.0

All these values were plotted into the coordinate grid and path and time curves were drawn. From this graphic diagram the total path and time may be read with adequate precision, for every pulse angle from 30° to 60° for the upper layer.

The first test with calculated approximate values has shown that k is too large. Thereafter k was reduced consecutively to 1.0 and then to 0.75, and with this last value and a velocity $c_1 = 7.747$ km/s an excellent match of the calculated values with recorded ones was obtained.

The result of the calculation is the following:

$c_1 = 7.747$ *km/s*, $k = 0.75$, $\log n = 0.13479$

φ	Total path in upper layer		In lower layer		Sum		−4.5	Time curve	Difference
	km	t	km	t	km	t			
2°	79	19.3	222	28.5	301	43.3	38.8	–	–
4°	79	19.3	445	56.9	524	76.2	71.7	72.5	0.8
6°	78	19.3	667	85.3	745	104.6	100.1	100.1	0.0
8°	78	19.2	889	113.6	967	132.8	128.3	128.8	0.5
10°	77	19.2	1112	141.8	1189	161.0	156.5	156.8	0.3
12°	77	19.1	1334	169.9	1411	189.0	184.5	184.7	0.2
14°	75	19.0	1556	197.8	1631	216.8	212.3	212.5	0.2
							Average difference: s = 0.3		

The computed values, as may be seen from the numbers above, agree completely with the time curve. The emergence angles on the Earth surface are:

Epicentral distance, km	Emergence angle		Epicentral distance, km	Emergence angle	
	e	$90°-e$		e	$90°-e$
301	45° 20'	44° 40'	1189	44° 35'	45° 25'
524	45° 08'	44° 52'	1411	44° 17'	45° 43'
745	45° 00'	45° 00'	1631	43° 57'	46° 03'
967	44° 49'	45° 11'			

The ray which reaches a distance of 1631 km has its lowest point at a depth of 132.7 km.

The ray, which emerges from the earthquake focus at an angle of 100°, touches the lower surface at a depth of 50 km, and arrives at the surface of the Earth. The rays which emerge at a larger angle are totally reflected at the lower surface. The limiting ray between the last ray which is totally reflected and the first ray which penetrates the lower layer emerges from the earthquake focus at an angle for which the refraction angle is equal to 90°, and the sine of the incoming angle $\sin\beta = 1/n$, in our case 47° 09'. This angle corresponds to a pulse angle at the earthquake focus of 133° 49'.

All rays which emerge from the earthquake focus between the angles of 0° and 133° 49' remain in the upper layer, and only 14% of the energy penetrates into the interior of the Earth. The ray which emerges from the earthquake focus at the angle of 133° 49' and reflects at the lower surface emerges at the Earth surface at a distance of 79 km. This ray, however, does not yet penetrate into the interior of the Earth; only a ray which emerges from the earthquake focus at a greater angle can refract and reach the surface of the Earth again. This explains why normal P start only at a larger distance from the epicentre – they must come in at a specific angle onto the refraction surface in order to have a sufficient part of them refracted.

6. Transversal waves

6. 1. Transversal waves (secundae) of individual primae

Because both the individual and normal primae are only one wave type and differ between themselves only in that they arrive to the surface of the Earth by different paths, both of them are longitudinal waves. Transversal waves which belong to normal primae are known for a long time now. However, because hitherto no one differentiated between the individual and the normal primae, the transversal waves belonging to the individual primae were therefore also unknown.

It is highly probable that the maximum waves are transversal waves belonging to the normal primae. This assumption is based on the fact, that the ratio of travel duration of eM to the travel duration of $\overline{P}$, is a number almost equal to the ratio between the travel times of normal S and corresponding P.

So we get for

	km	eM/P		km	eM/P
Zagreb	39	1.82	Budapest	318	1.66
Rijeka	134	1.58	Padova	332	1.72
Ljubljana	141	1.69	Munich	454	1.67
Graz	184	1.84	Hohenheim	633	1.73
Trieste	185	1.59	Moncalieri	644	1.81
Pula	193	1.70	Sofia	652	1.61
Vienna	308	1.66			
				Average	1.70

This number is still quite uncertain, and may be in error for some unit in the last digit because it is very difficult to precisely read the beginning of the maximum phase.

If we look for the ratio of travel duration of normal primae and secundae starting at 2000 km backwards, we get

at 2000 km 1.79	at 1000 km 1.81
at 1500 km 1.80	at 500 km 1.76

From 1000 km backwards the ratio decreases. This may be explained by the fact that the path of the waves in the upper layer for these small distances amounts to a high proportion of the total path. Consequently, it may also be seen that in upper layers the ratio must be smaller then in the lower ones.

As the base of further calculation I took as the ratio in the upper layer the value of 1.71. This value is about 0.5% higher than the average value for all stations, but considering the quality of the stations it seemed better.

The earthquake ray, which comes from the earthquake focus directly to the epicentre as a longitudinal wave, requires 4.5 seconds for that path. The corresponding transverse wave then requires 7.7 seconds for that same path. In this way we get at the epicentre the preliminary phase, which lasts 3.2 seconds. This preliminary phase will be the shorter, the closer the earthquake focus is to the Earth surface.

In order to determine the path of $\overline{S}$ we must know how their velocity changes with depth. At the time we can not learn that from observations. I shall therefore take for now that in the upper layer of the Earth both types of waves do not part from each other. For short paths this assumption will not introduce a big error, but later, when correct data are known for larger distances, the calculations may be more strict.

If we multiply the $\overline{P}$ travel time with 1.71, we get the following times for $\overline{S}$:

km	Travel time	T. t. – –4.5s	km	Travel time	T. t. – –4.5s	km	Travel time	T. t. – –4.5s
0	7.7	3.2	200	60.9	56.4	500	155.6	151.1
50	17.4	12.9	300	93.2	88.7	600	183.7	179.2
100	30.4	25.9	400	125.3	120.8	700	211.0	206.5

The velocity of transversal waves ($\overline{S}$) would be therefore 3.27 km/s in the earthquake focus. If the times above are taken as the travel durations of the surface waves, we get a velocities of 3.31 and 3.39 km/s for distances of 500 and 700 km respectively, i.e. the velocity which completely agrees with the velocity hitherto known for surface waves. If the velocity is 3.27 km/s in the focus, then the emergence velocity is 3.27 km/s. If the ratio of 1.71 is correct, then the velocity of surface waves which are induced by the transversal waves would amount to 3.23 km/s. If the ratio above were in error by five units of the last decimal, then the corresponding velocities calculated along the surface would amount to 3.23 or 3.14 km/s. The true velocity of surface waves must fall between these two velocities.

6. 2. Transversal waves (S) of normal primae

The calculation of velocity and the measure (k_2) of its change with depth was performed in the same way as for P, with the approximate assumption that within the Earth's crust the transversal waves do not part from the longitudinal ones.

The following travel times for S were taken as the basis of the calculation:

km	Travel time, s	km	Travel time, s	km	Travel time, s	km	Travel time, s
600	150.2	950	233.0	1300	312.3	1650	388.9
650	162.2	1000	244.5	1350	323.4	1700	399.6
700	174.2	1050	256.0	1400	334.5	1750	410.2
750	186.1	1100	267.4	1450	345.5	1800	420.7
800	197.9	1150	278.7	1500	356.5	1850	431.2
850	209.7	1200	290.0	1550	367.4	1900	441.5
900	221.4	1250	301.2	1600	378.2	1950	451.7
						2000	461.8

If we assume certain values for k_2, we get the following values for c_2:

k =	4	3	2	1
ϕ = 6° – 7°	4.46	4.52	4.54	4.60
ϕ = 10° – 11°	4.38	4.56	4.70	5.04
ϕ = 14° – 15°	4.16	4.48	4.76	–

From these velocities we get, as for P, the approximate value of $k = 2.9$, and for $c_2 = 4.55$ km/s. A trial calculation with these values shows that this velocity is too high. Because the velocity enters the formula as a denominator it is easy to find the right velocity. The second trial for $c_2 = 4.172$ shows that k is somewhat too large. Finally a suitable result was achieved with $k_2 = 2.7$. and $c_2 = 4.182$ km/s. As the velocity of $\bar{S}$ at the lower boundary of the upper layer is 3.32 km/s, $\log n = 0.10000$.

With these values we get the following travel times:

Distance, km	Calculated travel duration, s	–4.5 s	Travel duration according to time curve	Difference
319	89.5	85.0	–	–
538	141.9	137.4	–	–
758	193.2	188.7	188.0	–0.7
977	243.9	239.7	239.2	–0.5
1196	293.3	288.8	289.1	0.3
1414	341.9	337.4	337.6	0.2
1631	389.1	384.6	384.8	0.2

This calculation agrees exactly with the time curve.

Because the time curve for S is not so accurately known as the one for P, repeated calculation will be necessary as soon as more material is at hand.

The ratio between the initial velocities of P and S at the boundary surface is 1.852, i.e. significantly larger than in the upper layer. Since, however, k is different for both types of waves, so is that ratio variable with depth.

The ray which reaches the surface of the Earth at a distance of 1631 km reaches the maximum depth of 216 km. When passing from the upper layer into the lower one, the transversal waves part from the longitudinal ones and reach a much greater depth (216 km in relation to 132 km).

If we calculate the travel duration of S for a distance of 2067 km, which corresponds to a path of 18° in the lower layer, we get already a difference of 2.4 seconds between observation and calculation.

When in time, based on a larger number of observations, it becomes possible to accurately determine the thickness of the uppermost layer of the Earth, the velocities with which the rays enter from the upper layer into the lower one will change slightly too. The larger the thickness of the upper layer, the higher these velocities would be.

7. Reflections of waves in the uppermost layer of the Earth

The considerations and calculations above proved that in a shallow depth of some 50 km there is a spherical surface, where a strong ray refraction occurs. It follows that on that surface as well as on the surface of the Earth, various reflections of waves must occur. It appears at first glance that a calculation of these reflections is futile work as long as the depth of the reflecting surface is not accurately known. I consider, however, that just the precise evaluation of these reflections by observation will lead us to accurate assessment of this surface depth. If the observers are cautioned of the approximate position of these reflections in the diagram of an earthquake, then they will look for these reflections and read them. When a large quantity of these reflections is determined, in time their curves could be drawn, and an evaluation made how much they differ from the previous approximately evaluated reflections.

Two types of reflections may occur:

a) one lower reflection (R_i), where every ray comes to the surface of the Earth, after it was reflected at the lower interface,

b) one upper reflection (R_s), where the ray comes to the surface of the Earth, after it has been firstly reflected at the upper surface and then at the lower one.

These two reflections may repeat until the whole energy is used up.

As various transformations of longitudinal waves into transversal and *vice versa* may occur for reflections, we will get a multitude of time curves for each reflection instead of two.

a) For R_i α) the longitudinal wave may come unchanged to the surface of the Earth ($R_i\overline{P}$); β) transversal waves may be created at the lower surface and reach the surface of the Earth ($R_i\overline{PS}$); γ) the transversal waves, emanating from the earthquake focus, may reflect at the lower surface ($R_i\overline{S}$).

b) For R_s an even larger number of reflections may precipitate: α) longitudinal wave reflection ($R_s\overline{P}$); β) transversal waves evolve from longitudinal waves only after the second reflection ($R_s\overline{P}_2\overline{S}$); γ) the transversal waves develop at the first reflection already ($R_s\overline{PS}_2$); δ) the transversal waves, emanating from the earthquake focus, reach the surface of the Earth after a double reflection ($R_s\overline{S}$).

If the reflections repeat themslves, the number of possible time curves increases.

If another surface where the waves refract exists, then there is a chance that standing waves be created, to which Wiechert already draw attention.

7. 1. Lower reflection (R_l)

The longitudinal or transversal ray which is reflected at the lower surface reaches the surface of the Earth at a larger or smaller distance from the focus depending on the pulse angle in the focus, i.e. the focal angle being smaller or larger. If the pulse angle is equal to 180°, then the ray which leaves the focus directly downwards will after reflection reach the surface of the Earth in the epicentre. As the pulse angle is ever smaller, the angle at which the ray arrives at the lower surface becomes ever larger. If finally the pulse angle reduces to 100°, the angle at the lower surface grows to 90°. The reflected ray then unites with the direct one.

The time curves of lower reflections may hence reach only up to such an epicentral distance, which may be reached by $\overline{P}$ too. In our case only up to approximately 700 km. At this distance the time curves of lower reflections must fall on the time curve for $\overline{P}$ and $\overline{S}$. As the ray path must be calculated with earlier mentioned formulas always from below upwards, so the ray paths have been calculated emanating from the lower surface at angles of 0°, 10°, 20° etc, and run on one side towards the focus, and on the other side towards the surface of the Earth. The sums of both paths will make up the total path and the total time of the reflections.

1. Paths and times (s)

Reflection angle	From 50 to 25 km			From 50 to 0 km		
	km	Long.	Transv.	km	Long.	Transv.
0°	0	4.4	7.5	0	8.9	15.2
30°	14	5.0	8.6	28	10.3	17.6
40°	21	5.8	9.9	41	11.5	19.7
50°	29	6.8	11.6	58	13.6	23.3
60°	42	8.7	14.9	82	17.4	29.2
70°	65	12.3	21.0	123	23.6	40.4
80°	117	21.1	36.1	205	37.6	64.3
85°	175	31.0	53.0	281	50.6	86.5
90°	281	49.6	84.8	395	70.8	121.1

2. Total paths and times relative to the Earth's surface

Path	$R_i\overline{P}$	$R_i\overline{PS}$	$R_i\overline{S}$
0	8.8	15.1	18.2
42	10.8	18.1	21.7
62	12.8	21.0	25.1
87	15.9	25.6	30.4
124	21.3	33.4	39.6
188	31.4	48.2	56.9
322	54.2	80.9	95.9
456	77.1	113.0	135.0
676	115.9	166.2	201.4

In the lower table the times are relative to the surface of the Earth i.e. relative to the time in the epicentre obtained by subtracting 4.5 from each sum.

Reflected alternating waves (PS) fall almost exactly together with the normal S. That is why the estimate of S up to a distance of 700 km is very uncertain.

Repeated reflection at the lower surface may give five new time curves. $R_i^2\overline{P}$ is somewhat above R_iP up to 600 km, and from 700 to 1650 km causes the extension of this curve. In the same way $R_i^2\overline{S}$ is an extension of $\overline{S}$ and $R_i\overline{S}$.

7. 2. Upper reflection (R_s)

As this reflection consists of two reflections, one at the surface of the Earth and the other at the lower interface, it will occur that up to a pulse angle of 79° 48', corresponding to an emergence angle of 75° 36' at a distance of 115 km, an angle less than 90° on the lower surface will correspond to it. If the pulse angle is 79° 48', then the ray reflected at the upper surface will just touch the lower one, and will traverse a path of 790 km from one surface to another so that it will reach the surface of the Earth at a distance of 905 km. If the pulse angle becomes larger then the emergence angle will be larger too, and a ray reflected at the surface will not reach the lower interface, hence it will not be reflected downwards. The largest emergence angle of 79° 48' corresponds to a pulse angle of 90° at a distance of 280 km. The ray, which is reflected from the surface of the Earth at an angle of 180° – 79° 48' = 100° 12', reaches the surface of the Earth at a distance of 560 km. Hence, the ray which exits the focus at an angle of 90° and is reflected at the surface of the Earth comes again to the surface at a distance of 3×280 km = 840 km. The time curve returns therefore from 905 back to 840 km. If the pulse angle is 100° 12', than the same emergence angle corresponds to it as for a pulse angle of 79° 48', i.e. 75° 36', and the reflected ray will touch the lower interface again. The pulse angle of 100° 12' corresponds to a path of 675 km from the surface of the Earth back to it. The total path of the ray will hence be 790 + 675 = 1465 km. The time curve of the upper reflection hence makes a turn between approximately 790 km and 905 km.

The following table shows the calculated paths and times:

1. Paths and times (s)

From 25 to 0 km			From 0 km downwards		
Path	Long. time	Trans. time	Path	Long. time	Trans. time
0	4.5	7.7	0	8.9	15.2
14	5.3	9.1	28	10.3	17.6
20	5.7	9.7	41	11.5	19.7
29	6.8	11.6	58	13.6	23.3
40	8.7	14.9	82	17.1	29.2
58	11.3	19.3	123	23.6	40.4
88	16.5	28.2	205	37.6	64.3
106	19.6	33.5	280	50.6	86.5
114	21.2	36.3	395	70.8	121.1
174	31.5	53.9	312	55.9	95.6
280	50.6	86.5	280	50.6	86.5
449	80.3	137.3	312	55.9	95.6
675	120.2	205.5	395	70.8	121.1

2. Total paths and times relative to the Earth's surface

Path	$R_s\overline{P}$	$R_s\overline{P}_2\overline{S}$	$R_s\overline{PS}_2$	$R_s\overline{S}$
0	17.8	24.1	30.4	33.6
70	21.4	28.7	36.0	39.8
102	24.2	32.4	40.6	44.6
145	29.5	39.2	48.9	53.7
204	38.4	50.5	62.6	68.8
304	54.0	70.8	87.6	95.6
498	87.2	113.9	140.6	152.3
668	116.3	152.2	188.1	202.0
904	158.3	208.6	258.9	274.0
798	138.8	178.5	218.2	240.6
840	147.3	183.2	219.1	255.0
1073	187.6	227.3	267.0	324.0
1465	257.3	307.6	357.9	443.2

The upper reflection of $\overline{P}$ is the extension of $\overline{P}$ up to 1465 km. This extension may be seen indeed only for strong earthquakes. When earlier I took some stronger earthquakes into the account, I thought that $\overline{P}$ extend up to 1400 km or 1600 km distance. When later I had to drop these earthquakes, the respective data were rejected too.

In the same way the upper reflection of $\overline{S}$ is simply their extension over 700 km. If we place a rule on this extension we see almost a perfect straight line, seeming to come from the epicentre. As further reflections of $\overline{S}$ give the extension of this line over 1460 km, so $\overline{S}$ with their extensions make a straight line, which has an apparent velocity of 3.33 km/s on the Earth's surface. This tells us why these waves are considered to be surface waves.

These waves induce surface waves as transversal waves, and because these surface waves have also a velocity of 3.33 km/s, both types of waves must fall together.

8. Some further evidence of theory validity

If in the diagrams of particular earthquakes along with the main phases we find some subphases, we will thus prove that the calculated reflections really exist, that therefore the reflective surface also exists, and that the main earthquake phase should be assigned to transversal waves and to their reflections in the Earth's crust.

In the publications of seismic institutes, only three main phases have been recorded for near earthquakes: P, S and eM. P are, according to the strength of the earthquake, the distance and to the quality of the instruments, either normal or individual. S are either true S or one of close reflections; particularly often are R_sP_2S taken as S. For vertical pendula, particularly Vicentini ones, S are often mistaken for eM.

When studying diagrams which some stations have placed at my disposal I read the reflections myself while some other stations reported to me their own readings.

In the diagram of the earthquake on 13 December 1909 from the Vienna station, almost all the phases may be found. Also, in the diagrams of the Munich station, particularly for the earthquake on 28 January 1910, all the phases are observable. In the diagram of the Taranto station we may also find all the phases; equally so in the diagrams: Jena, Leipzig and Strasburg. Stations further away may not be taken into consideration because of the earthquake's weakness.

In order to collect more material, from the beginning of this year I had all the visible emergencies meticulously read for all the near earthquakes recorded on our Wiechert pendulum of 1000 kg. Regrettably, only a single of these earthquakes had a well known epicentre, and for some others only the approximate location of the epicentre according to newspaper reports was known. The reflections could be confirmed only by reducing all the phases to epicentral time according to time curves. Depending on how the respective times coincided I could judge the distance of the epicentre on one hand, and if the reflections truly exist on the other.

I could therefore use the following earthquakes:

1. Earthquake on 18 February 1910. According to the newspaper reports it was felt on Crete. According to data of stations in Hamburg, Vienna, Graz, Zagreb, Moncalieri and Valle di Pompei the epicentre could be approximately at 36° 55' N and 23° 45' E in the Aegean Sea at a distance of 1265 km from Zagreb.

The read phases reduced to the epicentre give:

	h m s	m s		h m s
P	5 11 59	– 2 47	=	5 09 12
R_sP	– 12 57	– 3 41	=	– – 16
R_sP_2S	– 13 28	– 4 26	=	– – 02
S	– 14 02	– 5 05	=	– 08 57
R_sS = eM	– 15 10	– 6 22	=	– 08 48

The P and R_sP phases correspond on average to the epicentral time, derived from all the other stations, i.e. 5 h 09 m 13 s. All other phases are too early, which would contribute to the conclusion that the epicentre is a little too far.

2. Earthquake on 23 February 1910. Epicentre unknown, probably in Bulgaria. Distance determined according to time curves is 700 km.

	h m s	m s		h m s
P	7 53 46	– 1 35	=	7 52 11 (uncertain)
	55			– – 20 (certain onset)
$\overline{P}$ or $R_i\overline{P}$	– 54 27	– 2 02	=	– – 25
e	– 54 56	–		– (unknown)
$R_s\overline{P}_2\overline{S}$	– 55 01	– 2 39	=	– – 22
S	– 55 17	– 2 54	=	– – 24
$R_s\overline{PS}_2$?	– 55 28	– 3 17	=	– – 11
eM = $\overline{S}$	– 55 53	– 3 31	=	– – 22

The phases read coincide between themselves very well, if we eliminate the uncertainty in their onsets.

3. Earthquake on 17 March 1910. Epicentre unknown. From the notes of Sarajevo and Zagreb it seems that it should be in Dalmatia. Distance from Zagreb 333 km.

	h m s	m s		h m s
P	7 33 15	– 0 47	=	7 32 28
$\overline{P}$	– – 28	– 0 56	=	– – 32
R_sP_2S	– – 43	– 1 16	=	– – 27
eM = $\overline{S}$	– 34 12	– 1 42	=	– – 30

All phases coincide very well.

4. Earthquake on 18 March 1910. According to Zagreb, Sarajevo, Pula and Ljubljana the epicentre is in Dalmatia approximately at 42° 54' N and 16° 50' E. The distance from Zagreb about 330 km, hence probably the same epicentre as the previous earthquakes.

	h m s	m s		h m s
P	20 22 23	– 0 47	=	20 21 36
$\overline{P}$	– – 33	– 0 56	=	– – 37
S	– 23 05	– 1 27	=	– – 38
eM = $\overline{S}$	– – 16	– 1 38	=	– – 38

All phases coincide very well.

5. Earthquake on 6 April 1910. Epicentre unknown, approximate distance 650 km.

	h m s	m s		h m s	
P	1 38 48	– 1 29	=	1 37 19	
$\overline{P}$	– 39 02	– 1 51	=	– – 13	
e	– – 28	–		–	(unknown)
$R_s\overline{P}_2\overline{S}$	– – 47	– 2 28	=	– – 19	
S	– 40 01	– 2 42	=	– – 19	
eM = $\overline{S}$	– – 40	– 3 16	=	– – 24	

$\overline{P}$ is a little too early, and $\overline{S}$ a little too late; other phases coincide completely.

6. Earthquake on 2 May 1910. Epicentre unknown, approximate distance 850 km.

	h m s	m s		h m s
P	21 22 56	– 1 54	=	21 21 02 ±
$R_s\overline{P}$	– 23 22	– 2 29	=	– 20 53
$R_s\overline{P}_2\overline{S}$	– 24 03	– 3 05	=	– – 58
R_sP_2S	– 24 18	– 3 15	=	– 21 03
S	– – 32	– 3 30	=	– 21 02
eM = $R_s\overline{S}$	– 25 14	– 4 17	=	– 20 57

The first stroke falls into the minute mark. All phases coincide well. Both branches of R_sP_2S are visible.

7. Earthquake on 7 June 1910. Epicentre according to report obtained from »R. uffizio centrale di meteorologia« in Rome approximately at 40° 55' N and 15° 27' E. Distance from Zagreb 550 km.

	h m s	m s		h m s	
P	2 05 14	– 1 16	=	2 03 58	
$\overline{P}$	– – 34 35	– 1 35	=	– 03 59 04 00	NE comp. NW comp.
$R_s\overline{P}_2\overline{S}$	– 06 10 11	– 2 05	=	– 04 05 06	
S	– – 19	– 2 20	=	– 03 59	
iM = R_sS	– – 56	– 2 45	=	– 04 11	

From P, $\overline{P}$ and S follows an epicentral time of 2 h 03 m 59 s. $R_s\overline{P}_2\overline{S}$ and iM are somewhat late. If we took a distance of 560 km it would all fit even better.

I investigated also the publications of those institutions, which have up to now sent their reports, so I found as the epicentral time (ET) from

Sarajevo	– Distance	445 km	– ET	2 03 57
Pula	– Distance	460 km	– ET	3 59
Trieste	– Distance	550 km	– ET	3 57
Ljubljana	– Distance	580 km	– ET	3 57
Graz	– Distance	675 km	– ET	4 04
Vienna	– Distance	800 km	– ET	3 58
Hamburg	– Distance	1470 km	– ET	4 08

The epicentral time determined from Zagreb data should be correct to ±1 s. In Figure 3 the respective phases of the Zagreb diagram are marked.

8. Earthquake on 13 July 1910. Approximate distance 380 km, epicentre in Tyrol, probably somewhat west of Innsbruck.

	h m s		m s		h m s
P	8 33 23	–	0 54	=	8 32 29
$\overline{P}$	– – 33	–	1 04	=	– – 29
R_sP_2S	– – 42 45	–	1 27	=	– – 15 18
S	– 34 07	–	1 40	=	– – 27
iM $=\overline{S}$	– – 24	–	1 54	=	– – 30

Except $R_s\overline{P}_2\overline{S}$ the other phases coincide completely.

9. Earthquake on 1 August 1910. Epicentre unknown, approximate distance 475 km.

	h m s		m s		h m s
P	10 42 03	–	1 06	=	10 40 57
$\overline{P}$	– – 23	–	1 22	=	– 41 01
S	– – 57	–	2 01	=	– 40 56 ±
eM$=\overline{S}$	– 43 19	–	2 23	=	– – 56

S fall in the minute mark. All phases coincide very well between themselves.

10. Earthquake on 2 August 1910. Epicentre unknown, approximate distance 1275 km.

	h m s		m s		h m s	
P	2 35 53	–	2 48	=	2 33 05	
R_sP_2S	– 37 30	–	4 28	=	– – 02	
e	– – 45		–		–	(unknown)
e	– – 54		–		–	(unknown)
S	– 38 10	–	5 07	=	– – 03	
eM$=R_s\overline{S}$	– 39 32	–	6 26	=	– – 06	

All phases coincide. Both unknown phases are probably higher reflections.

11. Earthquake on 30 August 1910. According to newspaper data it was felt in Reggio di Calabria. Approximate distance 900 km.

	h m s		m s		h m s
P	2 12 08	–	2 01	=	2 10 07
R_sP	– – 48	–	2 38	=	– – 10
S	– 13 52 51	–	3 41	=	– 10 11 10
? e	– 14 24				
eM=$R_s\overline{S}$	– 14 41	–	4 32	=	– 10 09

All phases coincide very well.

12. Earthquake on 31 August 1910. Epicentre unknown, approximate distance 375 km.

	h m s		m s		h m s
P	18 58 23	–	0 53	=	18 57 30
$\overline{P}$	– – 35	–	1 04	=	– – 31
R_sP_2S	– – 52	–	1 26	=	– – 26
R_iPS	– 59 05	–	1 33	=	– – 32
R_sPS_2	– – 19 16	–	1 47	=	– – 32 29
eM=$\overline{S}$	– – 22 24	–	1 53	=	– – 29 31

All phases coincide very well, S can not be found in the diagram.

13. Earthquake on 11 October 1910. According to newspaper data felt in the southwest Hungary (Deva, Orszova). Approximate distance 550 km.

	h m s		m s		h m s
P	11 53 25	–	1 16	=	11 52 09
$\overline{P}$	– – 48	–	1 35	=	– – 13
? e	– 54 02		–		–
R_sP_2S	– – 11	–	2 04	=	– – 06
S	– – 32	–	2 20	=	– – 12
eM=$\overline{S}$	– – 53	–	2 45	=	– – 08

Mean epicentral time is 11 h 52 m 10 s ±2 s. Phase after $\overline{P}$ is probably some higher reflection.

14. Earthquake on 25 August 1909. Epicentre by Siena. Distance from Zagreb 470 km (Figure 4 a, b).

	h m s		m s		h m s	
P	0 23 08±	–	1 05	=	0 22 35	±(minute mark)
$\overline{P}$	– – 24	–	1 21	=	– – 03	
$\overline{S}$	– 24 26	–	2 21	=	– – 05	

In addition to these earthquakes I add the reading of the Vogtland earthquake on 3 November 1908 according to the diagram published in »Seismische Registrierungen in Göttingen im Jahre 1907. (Nachrichten der k. Gesellschaft der Wissenschaften zu Göttingen. 1909) Fig III.«. Epicentral distance marked as 250 km.

	h m s	m s		h m s
P	13 11 29	– 0 40	=	13 10 49
R_sP_2S	– – 44	– – 59	=	– – 45
R_iPS	– – 52	– 1 03	=	– – 49
S	– – 55	– 1 08	=	– – 47
$\overline{S}$	– – 57	– 1 12	=	– – 45
$eM=R_i\overline{S}$	– – 12 00	1 14	=	– – 46

Mean epicentral time 13 10 47±1

This is the only earthquake known to me for which S comes already at a distance of 250 km. The time of 1 m 08 s lies exactly on the extension of the curve for S to 250 km.

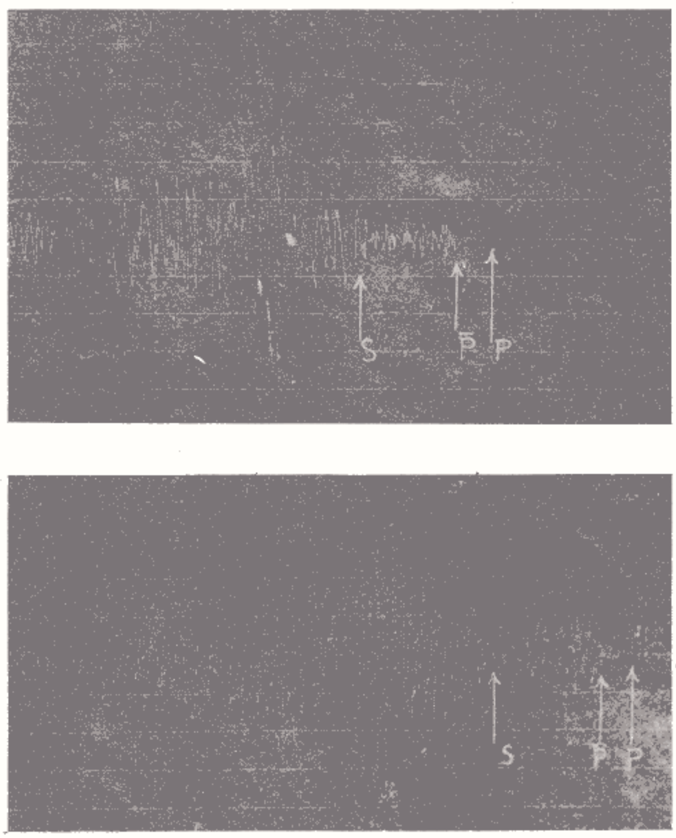


Figure 4. The earthquake of 25 August 1909. Sienna, a) Ne component, b) NW component.

From the examples presented it may be seen that reflections indeed exist, and that they may be found in good earthquake diagrams. Between 300 and 700 km we find it in all earthquake diagrams.

It is much easier to find reflections in weaker earthquakes then in the strong ones, because for weak earthquakes only the first reflections are prominent.

9. Assessment of the earthquake focal distance

In order to determine the distance of the earthquake focus for near earthquakes, various empirical formulas have been proposed in the last decades. If t is the time when the first stroke was recorded at some station, and t' the time when the start of the maximum phase was recorded, and a and b two constants determined from experience, the two empirical formulas state:

$$D = \mathrm{a}(t' - t) + \mathrm{b}$$

$$D = \mathrm{a}(t' - t)$$

First of these formulas requires that the earthquake diagram has no preliminary phase up to the distance D = b, which is doubtful, at least. The second formula, in contrast, is based on the assumption that the ratio of path times of $\overline{\mathrm{P}}$ and $\overline{\mathrm{S}}$ is constant according to 6. 1.

If τ_o is the time, when the earthquake occurred in the focus, t'_1 and t'_2 the onset times of the earthquake at the two stations at distances D_1 and D_2, and equally t''_1 and t''_2 the onset times of the maximum phases, and if it is allowed that $\overline{\mathrm{S}}$ are the onsets of maximum phases, then the following ratios are valid

$$\frac{t''_2 - \tau_o}{t'_2 - \tau_o} = \frac{t''_1 - \tau_o}{t'_1 - \tau_o} = \mathrm{m}$$

From this equation it follows:

$$\tau_o = \frac{t'_1 t''_2 - t''_1 t'_2}{(t''_2 - t'_2) - (t''_1 - t'_1)}$$

or

$$\tau_o = \frac{\mathrm{m}t'_1 - t''_1}{\mathrm{m} - 1} = \frac{\mathrm{m}t'_2 - t''_2}{\mathrm{m} - 1}$$

If this assumption is valid and if the times for $\overline{\mathrm{P}}$ and $\overline{\mathrm{S}}$ from one or two stations are known, then for distances up to 700 km we may use these formulas to find the earthquake origin time.

If we further take some mean velocities for $\overline{\text{P}}$ and $\overline{\text{S}}$, as is always allowed for an approximate calculation, we get:

$$D = v_1(t'_1 - \tau_o) \text{ and } D = v_2\,(t''_1 - \tau_o)$$

where v_1 and v_2 are the velocities of $\overline{\text{P}}$ and $\overline{\text{S}}$. If into each of these equations we substitute for τ_o the value calculated earlier, we get:

$$D = \frac{v_1}{\text{m} - 1}\,(t''_1 - t'_1)$$

or

$$D = \frac{\text{m}v_2}{\text{m} - 1}\,(t''_1 - t'_1)$$

For $\text{m} = 1.71$ and $v_1 = 5.60$ it follows

$$D = 7.9\,(t''_1 - t'_1)$$

which value is valid for distances up to 700 km.

D is the distance of the focus, and not of the epicentre. However, because for distant stations both distances are almost equal, this formula may be used also to calculate the epicentral distance. From time curves for $\overline{\text{P}}$ and $\overline{\text{S}}$ we get 13 seconds for $t'' - t'$ at a distance of 100 km, 38 s for 300 km, and 78 s for 600 km. The calculated values are 103 km, 300 km and 600 km. In this same way normal P and S could be used for the same purpose. Because, however, m is not constant for them, the calculated values will not be so accurate. If $t'' - t'$ is the time difference which it takes for S and P to reach some station, on average we have

$$D = 9.0\,(t'' - t')$$

For a distance of 300 km this calculation gives a distance 40 km too large, and a distance 60 km too short for a 1500 km distance.

Both equations for τ_o as well as for D are independent of the time origin i.e. of that moment when the time count begins. Time and distance error obtained from these formulas depends on the higher or lower accuracy with which the times for both phases were determined. A value too low for t' gives a value too low for τ_o, and too large a distance. The opposite is true for t''.

Because until now the difference between P and $\overline{\text{P}}$ was unknown, the one or the other phase was taken arbitrarily, and in this way a completely different distance was obtained. In the same way it is difficult to determine eM, because this phase is very readily misread as any of the major or additional phases which accidentally shows a particularly large deflection on the seismogram, and that is a further reason to calculate distance erroneously.

The best method of distance estimation consists of comparing all the read phases with the time curves. If we read all the noticeable stronger deflections for near earthquakes and plot the time differences between these deflections and the first deflection onto a ruler divided into millimeters, for example with chalk, and move the ruler over the time curves until all or at least most of the read phases coincide with the time curves, then we get the most probable distance. By comparing the specific phases read with the ones drawn, we may often notice that we have marked one or more of the phases wrong.

10. Influence of the focus depth on specific phases

10. 1. Individual primae

The depth of the focus changes the coordinates φ and t of the point where the time curve has its inflection, and the distance reached by the last ray which touches the lower surface.

If the earthquake focus is at the surface of the Earth, than the last ray may arrive within 141.6 s to a distance of 790 km.

If we compare this time to the one we get from the curve for 25 km depth, extended to 790 km, then we see that the ray originating at 0 km depth reaches the surface of the Earth approximately 6 s later than the one coming from 25 km. The whole time curve for $\overline{\mathrm{P}}$ for a depth of 0 km is higher than the one for a depth of 25 km.

If the focus is at a depth of 50 km, the last ray reaches only a distance of 395 km in 70.8 s. Because the ray that travels from that depth directly upwards to the epicentre requires 8.9 s for that travel, the time a ray requires for a distance of 395 km calculated from the epicentre would only be 61.9 s. For a depth of 25 km a ray requires 68 s for the same travel. The time curve for the depth of 50 km is hence lower than the one for the depth of 25 km.

If the earthquake focus were in the lower Earth layer, the earthquake would have no $\overline{\mathrm{P}}$ but also no $\overline{\mathrm{S}}$, the earthquake would hence be without a maximum phase. I do not know if such an earthquake has ever been observed. Presently I must assume that there is no seismic activity in the lower layer of the Earth.

The epicentral distances of the $\overline{\mathrm{P}}$ curve inflection point for some focal depths are the following:

Focal depth, km	km	Focal depth, km	km	Focal depth, km	km
0	0	20	251	40	354
5	125	25	280	45	375
10	174	30	307	50	395
15	217	35	331		

For small depths the distance of the inflection point changes approximately by 25 km for every km of depth change. For large depths this change is comparatively small. For the depth of 25 km, a change in the distance of the inflection point of 6 km corresponds to every km of depth change. Since for our earthquake we have an uncertainty of approximately 10 km in the distance of the inflection point, it follows that the calculated depth of the earthquake focus is uncertain approximately by ±2 km. The uncertainty of depth leads to an uncertainty of the value by which the velocity increases with depth for approximately ±0.3.

10. 2. Normal primae

For a depth of focus of 0 km, the path of the ray in the upper layer to a distance of 1600 km will be some 20 km longer and the time some 3.5 s longer, then for a depth of 25 km. The total path for the same path in the lower layer will be 20 km longer. The length of 20 km corresponds to a time of 3 s according to the time curve. The ray will hence arrive about 0.5 s later to the surface of the Earth. If we add to that the epicentral time of 4.5 s, then the time curve of P will be 5 s higher in total. For an earthquake focal depth of 50 km the time curve would be lower by approximately the same amount.

If we draw a common time curve using different earthquakes, then we reduce all the earthquakes to a common average depth of focus.

As the time curves for P and $\overline{\mathrm{P}}$ change with depth, so do the curves for S and $\overline{\mathrm{S}}$. Their change is 1.7 times larger than the change of the main phases. This is also one of the reasons why times for S agree less with the average time curve then do the times for P.

11. Final comment

From the said above it follows that equations (4) and (9) satisfy all the times within the limit of precision which may be achieved today, and that the equation $c = c_0 r^k$ represents a fairly good first approximation of the change of earthquake waves' velocity with depth. This equation is indeed valid only for small changes in depth, because we can penetrate into the interior of the Earth only gradually anyway.

In order to finally solve the problem of wave propagation, many earthquakes will have to be processed. These analyses will, however, have no greater value if all the stations do not take better care of time precision then it is still the case today.

I feel free to ask all of the stations:

1. To keep paper at a speed of at least 15 mm per minute, so that all the stronger deflections may be easily recognized.

2. To read for near earthquakes – up to 2000 km epicentral distance – not only the main phases but also all other visible stronger deflections, and to be aware of a possible period change.

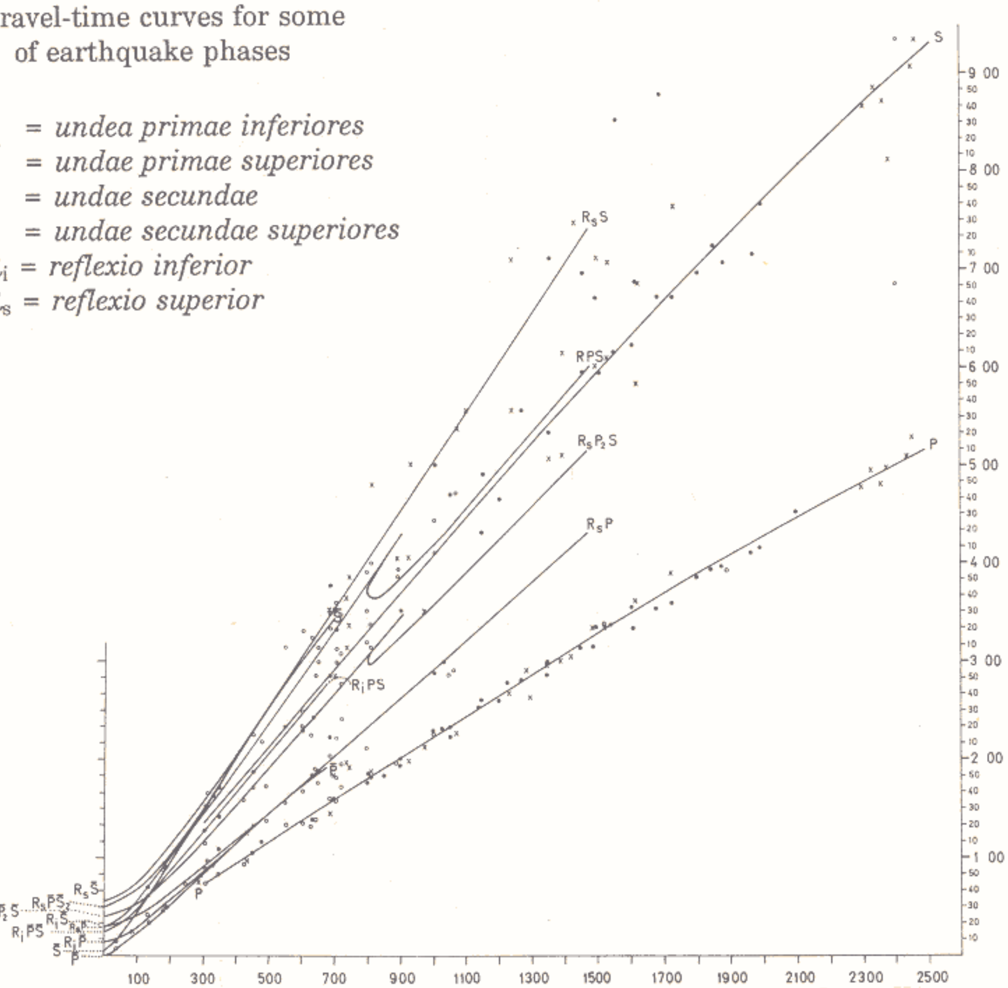
In this way it will be possible to construct time curves not only for the main phases but also for the stronger secondary ones, and from there it will be possible to conclude on the way the waves propagate in the upper layer of the Earth.

This paper should be taken as the first attempt and let it be judged as such.

Appendix

Travel-time curves for some of earthquake phases

P = *undea primae inferiores*
$\overline{\mathrm{P}}$ = *undae primae superiores*
S = *undae secundae*
$\overline{\mathrm{S}}$ = *undae secundae superiores*
R_i = *reflexio inferior*
R_s = *reflexio superior*



(Redrawn after original)