Economic Research-Ekonomska Istraživanja

ISSN: 1331-677X (Print) 1848-9664 (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

International labour movement, public intermediate input and wage inequality: a dynamic approach

Dianshuang Wang

To cite this article: Dianshuang Wang (2019) International labour movement, public intermediate input and wage inequality: a dynamic approach, Economic Research-Ekonomska Istraživanja, 32:1, 1-16

To link to this article: https://doi.org/10.1080/1331677X.2018.1546123

© 2019 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.

Published online: 22 Jan 2019.

Submit your article to this journal

Article views: 7

View Crossmark data

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=rero20

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2019, VOL. 32, NO. 1, 1–16
https://doi.org/10.1080/1331677X.2018.1546123

Routledge
Taylor & Francis Group

OPEN ACCESS Check for updates

International labour movement, public intermediate input and wage inequality: a dynamic approach

Dianshuang Wang

Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Japan

ABSTRACT

This article incorporates the public intermediate input in a dynamic model with two final private sectors and a public sector and investigates impacts of an inflow of skilled and unskilled labour on wage inequality. The public intermediate input can be accumulated and its accumulated stock serves as a public input for private production. From the analysis, in the steady state equilibrium, an increase in the skilled and unskilled labour endowment raise the stock of public intermediate input. Also, an inflow of skilled labour reduces the wage of skilled labour and raises the wage of unskilled labour, and an inflow of unskilled labour increases both the wages of skilled and unskilled labour. Concerning their impacts on the wage inequality, an inflow of skilled labour decreases the wage inequality, while the result of an inflow of unskilled labour on wage inequality is ambiguous. If the production elasticity of the public intermediate input stock in the skill-using sector is small enough, an inflow of unskilled labour narrows down wage inequality.

ARTICLE HISTORY
Received 30 January 2018
Accepted 1 August 2018

KEYWORDS
International labour movement; wage inequality; public intermediate input; dynamic analysis

JEL
F2; H41; J31

1. Introduction

The rising wage inequality between skilled and unskilled labour is a concern for both developed and developing countries. Many scholars believe that trade liberalisation and international factor movement have contributed to the widening inequality, such as Leamer (1996), Feenstra and Hanson (1996), Beyer, Rokjas and Vergara (1999), Reenen (2011) and Afonso (2012). Meanwhile, their effects on wage gap have been analysed extensively among theoretical papers. Marjit and Kar (2005) analysed how an outflow of skilled and unskilled labour affect wage inequality in a dual economy and the results depend on the capital intensities in the skilled labour-using sector and unskilled labour-using sector. Marjit and Kar (2005) analysis has been extended by incorporating domestic labour migration from various perspectives, as in Beladi, Chaudhuri and Yabuuchi (2008), Chaudhuri (2008), Gupta and Dutta (2010), Pan and Zhou (2013) and Li and Xu (2016). Beladi et al. (2008) considered a model with

CONTACT Dianshuang Wang wangds07@126.com

© 2019 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group.
This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

unemployment and impacts of international factor movement on wage inequality crucially depend on the difference in intersectoral factor intensities. Chaudhuri (2008) included unemployment and unionised wage rate of unskilled labour and showed that the results of international factor movement on wage inequality may not necessarily depend on the difference in the factor intensity. Gupta and Dutta (2010) introduced a non-traded final good sector and endogenous formation of skilled labour in a general model and found that the international factor movement on wage inequality depends on factor intensity ranking between two skilled labour sectors. Pan and Zhou (2013) established a model by accommodating environmental pollution which affects agricultural production and impacts of factors movement on wage inequality depend on the negative impact of pollution on agricultural production. Li and Xu (2016) investigated how international factor movements affect wage inequality with the existence of a modern agricultural sector and found that a decrease in the endowment of unskilled labour certainly decreases the wage inequality and the result that skilled labour movement has on wage inequality is dependent on the factor intensity between the urban and modern agricultural sector.

However, previous studies seldom consider the role of public infrastructure in the wage inequality. Nowadays, public infrastructure, such as legal and economic institutions, transportation systems and communications is playing an increasing role in a modern society, without which economic growth development will be greatly affected. The importance of public infrastructure for economic growth stems from its effect on private production. Following Meade's (1952) classification of public infrastructure, there are two types: 'creation of atmosphere' and 'unpaid factors'. In the 'creation of atmosphere' type, public infrastructure is fully available to every firm, such as free information about technology. In the 'unpaid factors' type, public infrastructure, such as highways, bridges and communication facilities, can be viewed as public intermediate input in the production process of private industry. The private production function exhibits constant returns to scale with respect to public intermediate input and primary inputs (labour and capital). However, unlike private inputs, the public intermediate input needs enormous funds for construction, maintenance, operations and overall development and usually is provided by the government and financed by taxation. Although private industries pay the cost ultimately, their payments do not affect the quantity of the public intermediate input directly and such input is essentially an unpaid input from the private industries' perspective.

The importance of public infrastructure arouses the great interest of many economic theorists, especially in international trade theory. Such studies include McMillan (1978), Manning and McMillan (1979), Tawada and Abe (1984), Tawada and Okamoto (1983), Abe (1990), Suga and Tawada (2007) and Yanase and Tawada (2012). Previous papers dealing with public infrastructure are confined mostly to a static framework or 'creation of atmosphere' type. An exceptional paper is Yanase and Tawada (2017), which considers the stock effects of an 'unpaid factors' type public intermediate good in a dynamic open economy with two consumption final goods, one public intermediate good and one primary factor (labour) and shows a country's trade pattern and whether a country gains or loses after the opening. However, existing studies dealing with the public intermediate input seldom consider wage inequality issue. Since the public intermediate input affects productivity and wages of private sectors at different levels, therefore, it is necessary to investigate how international factor movement influences the wage inequality with the presence of the public intermediate input.

In order to fill the theoretical research gap, this paper makes an analysis of how an inflow of skilled and unskilled labour affects wage inequality by a dynamic treatment of the public intermediate input. When incorporating the public intermediate input into the model, the paper obtains new conclusions. From the analysis, in the steady state equilibrium, an inflow of skilled labour reduces the wage of skilled labour and raises the wage of unskilled labour and an inflow of unskilled labour increases both the wages of skilled and unskilled labour. With regards to their impact on wage inequality, an inflow of skilled labour decreases the wage inequality, while the result of an inflow of unskilled labour on wage inequality is ambiguous. If the production elasticity of the public intermediate input stock in the skill-using sector is small enough, an inflow of unskilled labour narrows down wage inequality.

It is worth mentioning that Pi and Zhou (2012, 2014) also analyse the impacts generated by a movement of international factors on the wage inequality with the consideration of public infrastructure. The main differences between this paper and Pi and Zhou (2012, 2014) are reflected mainly in the treatment of public intermediate input. The paper uses a dynamic analysis and incorporates the stock effect of public intermediate input, while Pi and Zhou (2012, 2014) considered the static framework. In reality, however, many public intermediate inputs have the characteristics of durability or capital and dynamic analysis is closer to the reality of the economy. To my knowledge, there is no dynamic theoretical model analysis focusing on wage inequality with the stock effect of public intermediate input of an 'unpaid factor' type. In addition, the results of international factor movement on the wage of skilled and unskilled labour in Pi and Zhou (2012, 2014) also depend on the factor intensities. However, this paper shows that an inflow of skilled labour reduces the wage of skilled labour and raises the wage of unskilled labour, and an inflow of unskilled labour increases both wages of skilled and unskilled labour. These results are new in this field.

Another point worthy of mention is that the paper employs the framework of Yanase and Tawada (2017) to embed the public intermediate input. Yanase and Tawada (2017) consider an economy with two private sectors and one public sector to address international trade issues. Both private and public sectors utilize one primary factor labour, which is homogeneous. With homogeneous labour input and public intermediate input, Yanase and Tawada (2017) propose that the production frontier of the economy is strictly concave to the origin and a smaller (larger) labour endowment country tends to become an exporter of a good whose productivity is more (less) sensitive to the public intermediate input. In our case, where labour is heterogeneous, the skill-using sector uses skilled labour and the unskill-using sector and public sector only use unskilled labour for production, the paper investigates impacts of an inflow of skilled and unskilled labour factors on wage inequality. Since the skilled labour is specific to the skill-using sector, an inflow of skilled and unskilled labour brings the unskilled labour transfer between the unskilled and public sector, which contributes to changing wages of skilled and unskilled labour consequently. For example, an inflow of skilled labour leads to an increase in the demand for public intermediate input, which expands the public sector and attracts unskilled labour from the unskill-using sector. Both an inflow of skilled labour and movement of unskilled labour affect the skilled wage. As for wages of unskilled labour, besides the movement of unskilled labour between unskilled and public sectors, the more stock of public intermediate input also brings a change of unskilled wage. Such a movement of unskilled labour is crucial to the analysis; however, in Yanase and Tawada (2017), because labour is homogeneous and labour could move freely among three sectors, such a mechanism does not exist.

The remainder of this paper is organized as follows. The model is described in Section 2 and a static analysis is conducted in Section 3. In Section 4, the dynamic analysis is considered. Concluding remarks are made in Section 5.

2. The model

The article considers a small, open economy with three sectors: two private sectors, skill-using and unskill-using sector, and a public sector. The private sectors produce private final goods. While the public sector produces a public intermediate non-tradable good in Meade's (1952) 'unpaid factor' type, which can be accumulated, its accumulated stock serves in the private production. Assume two final goods are tradeable and, hence, their prices are given internationally.

As for the two private sectors, we adopt a specific-factor model related to Marjit and Kar (2005) where the skill-using sector employs skilled labour and the public intermediate input to produce the high-skill product while the unskill-using sector uses unskilled labour and the public intermediate input to produce low-skill goods. Skilled labour is specific to the skill-using sector and unskilled labour cannot enter into it. The production functions for two private sectors are assumed to be linearly homogeneous, with two respective factors, and to take the Cobb-Douglas form:

$$Y_S = R^{\alpha_S} L_S^{1-\alpha_S}, \quad 0<\alpha_S<1 \tag{1}$$

and

$$Y_U = R^{\alpha_U} L_U^{1-\alpha_U}, \quad 0<\alpha_U<1 \tag{2}$$

where Y_S (Y_U) is the output of the skill-using sector (unskill-using sector), R is the stock of the public intermediate input, and L_S (L_U) is the employment of skilled (unskilled) labour in the skill-using (unskill-using) sector. From two production functions, α_S and α_U are the production elasticity of the public intermediate input stock in the skill-using and unskill-using sector, respectively. Assume the public intermediate input stock serves more significantly in the skill-using sector than in the unskill-using sector, and impose the following assumption:

Assumption : For all $R > 0$, $\alpha_S > \alpha_U$.

According to Pi and Zhou (2014), assume the public intermediate input is produced by the aid of unskilled labour only and its production function is a linear function for simplicity's sake and expressed as $Y_R = L_{UR}$, where L_{UR} is the employment of the public sector. Following Yanase and Tawada (2012, 2017), the accumulation of the public intermediate good is described in the following dynamic equation:

$$\dot{R} = Y_R - \beta R \tag{3}$$

where β is the depreciation rate of the stock of the public intermediate input.

At each moment in time, the market-clearing conditions of the skilled and unskilled labour could be shown as follows:

$$L_S = \bar{L}_S \tag{4}$$

and

$$L_U + L_{UR} = \bar{L}_U \tag{5}$$

where $\bar{L}_S$ and $\bar{L}_U$ are the skilled and unskilled labour endowment, respectively.

Nest consider the behaviour of a representative household whose lifetime utility is

$$U = \int_0^\infty e^{-\rho t}[\alpha \ln C_S + (1-\alpha)\ln C_U]dt \tag{6}$$

where C_S and C_U are the consumption good of the skill-using and unskill-using sector, ρ is the rate of time preference, and $0<\alpha<1$ is a parameter.

With no international borrowing or lending, balance of payment implies

$$pY_S + Y_U = pC_S + C_U \tag{7}$$

where p is the world price good of the skill-using sector relative to that of the unskill-using sector and is assumed to be given and constant over time under a small open economy assumption.

The social planner determines $\{L_S, L_U, L_{UR}, C_S, C_U\}_0^\infty$ to maximize a representative household's lifetime utility (6) subject to the constraints (1), (2), (3), (4), (5) and (7).

3. Static analysis

In this section, we consider the stock of the public intermediate input as constant. Given the stock of the public intermediate input R and its shadow price θ, the static results can be obtained by solving a social planner's dynamic optimisation problem. The current-value Hamiltonian function is described as

$$H = \alpha \ln C_S + (1-\alpha)\ln C_U + \theta(L_{UR} - \beta R) \\ + \pi\left(pR^{\alpha_S}L_S^{1-\alpha_S} + R^{\alpha_U}L_U^{1-\alpha_U} - pC_S - C_U\right) + \lambda_S(\bar{L}_S - L_S) + \lambda_U(\bar{L}_U - L_U - L_{UR})$$

where π is the multiplier associated with income constraint and λ_S and λ_U are the multipliers associated with the full employment constraint of skilled and unskilled labour, respectively.

Solving the current-value Hamiltonian function,

$$yC_S = \alpha Y_S, (1-y)C_U = (1-\alpha)Y_U \tag{8}$$

$$\frac{(1-\alpha_S)y}{L_S} = \lambda_S \tag{9}$$

$$\frac{(1-\alpha_U)(1-y)}{L_U} = \theta = \lambda_U \tag{10}$$

where $y = pY_S/(pY_S + Y_U)$ is the share of the skill-using sector in national income. Equation (8) describes the consumption of good of skill-using and unskill-using sector. Equation (10) indicates the optimal allocation of unskilled labour between the unskill-using sector and public sector.

The static model, thus, consists of six equations: (1), (2), (4), (5), (10) and $y = pY_S/(pY_S + Y_U)$. Six endogenous variables, Y_S, Y_U, L_U, L_S, L_{UR} and y, are determined as a function of the state variable R, co-state variable θ and parameters $\bar{L}_S$, $\bar{L}_U$ and p. Following Yanase and Tawada (2017), these equilibrium solutions are denoted as temporary equilibrium ones.[1] Once Y_S, Y_U and y are determined, C_S and C_U are determined from equation (8). Totally differentiating the system, we have the following Lemma 1.

Lemma 1: The equilibrium solutions of $y(R, \theta; \bar{L}_S, \bar{L}_U, p)$, $Y_S(R, \theta; \bar{L}_S, \bar{L}_U, p)$, $Y_U(R, \theta; \bar{L}_S, \bar{L}_U, p)$ and $L_{UR}(R, \theta; \bar{L}_S, \bar{L}_U, p)$ have the following properties:

a. Under Assumption 1, $\partial y/\partial R > 0$, $\partial L_{UR}/\partial R > 0$. As for output, $\partial Y_S/\partial R > 0$, while the sign of $\partial Y_U/\partial R$ is ambiguous, depending on the values of α_S and α_U.
b. $\partial y/\partial \theta > 0$, $\partial Y_U/\partial \theta < 0$, $\partial L_{UR}/\partial \theta > 0$
c. $\partial y/\partial \bar{L}_S > 0$, $\partial Y_S/\partial \bar{L}_S > 0$, $\partial Y_U/\partial \bar{L}_S < 0$, $\partial L_{UR}/\partial \bar{L}_S > 0$
d. $\partial y/\partial \bar{L}_U = 0$, $\partial Y_U/\partial \bar{L}_U = 0$, $\partial L_{UR}/\partial \bar{L}_U = 1$

Proof. See Appendix A1.

The economic intuition behind Lemma 1 is as follows. Concerning (a), an increase in the stock of public intermediate input has a positive impact on the outputs of both two private sectors; meanwhile, unskilled labour flows from the unskill-using sector to the public sector to increase the stock of public intermediate input, which has a negative impact on the production of the unskill-using sector. If the difference in production elasticity of the public intermediate input stock between the two sectors is not very large, the positive effect outweighs the negative effect on output and $\partial Y_U/\partial R > 0$. However, if α_S is much larger than α_U, which means the unskill-using sector benefits little from more provision of public intermediate input, the negative effect dominates the change and $\partial Y_U/\partial R < 0$. Although an increase in R has a positive effect on output of private sectors, the skill-using sector gains more than the unskill-

[1] Here, the equilibrium is the one derived for a given level of the stock of the public infrastructure R, which is a given constant at each moment in time.

using sector under the Assumption, $\partial y/\partial R>0$. Given the production function of public intermediate input, one unit of unskilled labour could produce one unit of public intermediate input, and an increase in R needs more employment of the public sector and $\partial L_{UR}/\partial R>0$. As for (b), a rise in θ promotes the production of public intermediate input, thus $\partial L_{UR}/\partial \theta>0$. Meanwhile, unskilled labour flows into the public sector and the unskill-using sector experiences a loss in output because of $0<\alpha_U<1$ and drops the share of the unskill-using sector in national income, and $\partial y/\partial \theta>0$. Regarding (c), adding the endowment of skilled labour will expand the skill-using sector and increase the demand of public intermediate input. The unskill-using sector drops its output because of the outflow of unskilled labour. Finally, (d) describes the effects of an increase in the endowment of unskilled labour. Since the contribution of one additional unskilled labourer employed in the public sector is greater than that allocated in the unskill-using sector, increased unskilled labour is wholly absorbed by the public sector and has no impact on private output.

It should be noted that, even though the paper employs the framework of Yanase and Tawada (2017), the main results are different. Since the labour is homogenous in the Yanase and Tawada (2017) study, there is no distinction between skilled and unskilled labour. An increase in the labour endowment enlarges the output of both private sectors, as shown in Lemma 4 of Yanase and Tawada (2017). However, in the established model, an increase in skilled labour enlarges the output of the skill-using sector and shrinks the output of the unskill-using sector; an inflow of unskilled labour has no impact on output of private sectors in the short-term.

Next, consider the wage inequality between skilled and unskilled labour. The wage of skilled labour$w_S$and the wage of unskilled labour w are expressed as $w_S = (1-\alpha_S)pY_S/L_S$ and $w = (1-\alpha_U)Y_U/L_U$, respectively.

Proposition 1: In the temporary equilibrium, an inflow of skilled labour decreases the wage inequality, while an inflow of unskilled labour has no effect on it.

Proof. Using the results in Appendix A1, we can get

$$\frac{dw_S}{d\bar{L}_S} = (1-\alpha_S)p\frac{\bar{L}_S\partial Y_S/\partial \bar{L}_S - Y_S}{\bar{L}_S{}^2} = -\frac{\alpha_S(1-\alpha_S)pY_S}{\bar{L}_S{}^2}<0$$

and

$$\frac{dw}{d\bar{L}_S} = (1-\alpha_U)\frac{L_U\partial Y_U/\partial \bar{L}_S - Y_U\partial L_U/\partial \bar{L}_S}{L_U{}^2} = (1-\alpha_U)\frac{L_U\partial Y_U/\partial \bar{L}_S + Y_U\partial L_{UR}/\partial \bar{L}_S}{L_U{}^2}>0$$

The impact of a change of skilled labour endowment on the skilled–unskilled wage inequality can be expressed as

$$\frac{d(w_S-w)}{d\bar{L}_S}<0$$

Similarly, we have $dw_S/d\bar{L}_U = 0$, $dw/d\bar{L}_U = 0$ and $d(w_S-w)/d\bar{L}_U = 0$.

The economic intuition behind Proposition 1 can be explained as follows. An inflow of skilled labour will raise the supply of skilled labour in the economic system, and, as a result, its wages will fall. However, from Lemma 1, a larger endowment of skilled labour augments the public sector and moves unskilled labour from the unskill-using sector to the public sector, which decreases the employment of unskilled labour in the unskill-using sector and increases its marginal product and its wage. Thus, an inflow of skilled labour reduces wage inequality. An inflow of unskilled labour has no impact on skilled wage, since the stock of public intermediate input is constant. An inflow of unskilled labour increases unskilled labour supply; however, the increased unskilled labour wholly locates in the public sector to produce the intermediate input and has no impacts on private output in the short-term. Therefore, the wages of skilled labour and unskilled labour stay the same.

4. Dynamic analysis

In this section, the public intermediate input be accumulated by the production of public sector and its stock changes. The dynamic results are characterized by the following adjoint equation and the transversality condition:

$$\dot{\theta} = (\rho + \beta)\theta - \frac{\alpha_S y + \alpha_U(1-y)}{R} \tag{11}$$

$$\lim_{t\to\infty} e^{-\rho t}\theta(t)R(t) = 0 \tag{12}$$

In equation (11), the first term of the right-hand side, $(\rho+\beta)\theta$, is the sum of the intertemporal cost ($\rho\theta$) and the replacement cost of depreciated public intermediate input ($\beta\theta$). The second term, $[\alpha_S y + \alpha_U(1-y)]/R$, is the sum of two private sectors' marginal revenue product (in GDP term) of public intermediate input. And the left-hand side, $\dot{\theta}$, is the gain (or loss if negative) of the public intermediate input. Rewrite equation (11) as $\dot{\theta} + [\alpha_S y + \alpha_U(1-y)]/R = (\rho+\beta)\theta$, which states that the optimal allocation of public intermediate input balances benefits and cost. The dynamic equilibrium (11) can be satisfied under the competitive decentralized economy by incorporating the Lindahl pricing rule for financing the cost of public goods (see Appendix A2).

Using the results in section 3 and substituting them into equations (3) and (11),

$$\dot{R} = Y_R\left(R, \theta; \bar{L}_S, \bar{L}_U, p\right) - \beta R \tag{13}$$

and

$$\dot{\theta} = (\rho+\beta)\theta - \frac{\alpha_S y\left(R, \theta; \bar{L}_S, \bar{L}_U, p\right) + \alpha_U\left(1 - y\left(R, \theta; \bar{L}_S, \bar{L}_U, p\right)\right)}{R} \tag{14}$$

The dynamic path is characterized by $\{R(t), \theta(t)\}_0^\infty$, satisfying equations (11) and (12). Denote $\bar{z}$ as a steady state solution for a variable z, and $(\bar{R}, \bar{\theta})$ is a solution for

$\dot{R} = \dot{\theta} = 0$. In light of equations (13) and (14), conditions for steady-state equilibrium are given by

$$L_{UR} \equiv Y_R\left(R, \theta; \bar{L}_S, \bar{L}_U, p\right) = \beta R \tag{15}$$

and

$$(\rho + \beta)\theta = \frac{\alpha_S y\left(R, \theta; \bar{L}_S, \bar{L}_U, p\right) + \alpha_U\left(1 - y\left(R, \theta; \bar{L}_S, \bar{L}_U, p\right)\right)}{R} \tag{16}$$

The solutions of the dynamic model can be arrived at by solving equations (1), (2), (4), (5), (10), $y = pY_S/(pY_S + Y_U)$, (15) and (16). Using equations (4), (5) and (15), the production function of skill-using and unskill-using sectors can be written as

$$Y_S = \left(\frac{L_{UR}}{\beta}\right)^{\alpha_S} {\bar{L}_S}^{1-\alpha_S}, \; Y_U = \left(\frac{L_{UR}}{\beta}\right)^{\alpha_U} \left(\bar{L}_U - L_{UR}\right)^{1-\alpha_U}. \tag{17}$$

Substituting equation (17) into $y = pY_S/(pY_S + Y_U)$,

$$\frac{1-y}{y} = \left(\frac{L_{UR}}{\beta}\right)^{\alpha_U - \alpha_S} \frac{\left(\bar{L}_U - L_{UR}\right)^{1-\alpha_U}}{p\bar{L}_S^{1-\alpha_S}} \tag{18}$$

which implicitly determines y as a function of L_{UR}. Denote this as $y = \xi(L_{UR})$, which has $\lim_{L_{UR}\to 0} \xi(L_{UR}) = 0$ and $\lim_{L_{UR}\to \bar{L}_U} \xi(L_{UR}) = 1$ and $\xi'(L_{UR}) > 0$ under the Assumption. Using equations (5), (10) and (16), we can get

$$y = 1 - \frac{\beta\alpha_S(\bar{L}_U - L_{UR})}{(\rho + \beta)(1 - \alpha_U)L_{UR} + \beta(\alpha_S - \alpha_U)(\bar{L}_U - L_{UR})}. \tag{19}$$

From equation (19), we get $y = \varsigma(L_{UR})$, and $\lim_{L_{UR}\to 0} \varsigma(L_{UR}) = -\alpha_U/(\alpha_S - \alpha_U) < 0$, $\lim_{L_{UR}\to \bar{L}_U} \varsigma(L_{UR}) = 1$ and $\varsigma'(L_{UR}) > 0$. Thus, the system can be reduced to equations (18) and (19). Moreover, we can get $\xi'(L_{UR}) < \varsigma'(L_{UR})$. Therefore, there exists a unique pair of steady-state solutions $(L_{UR}, y) \in (0, \bar{L}_U) \times (0, 1)$. C_S and C_U can be obtained once we get these solutions through equation (8).

Next, we consider the stability of the steady state. Linearizing the dynamic system of equations (13) and (14) around the steady state, we have

$$\begin{pmatrix} \dot{R} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \partial L_{UR}/\partial R - \beta & \partial L_{UR}/\partial \theta \\ \dfrac{\alpha_S y + (1-y)\alpha_U}{R^2} - (\alpha_S - \alpha_U)\dfrac{\partial y}{\partial R} & \rho + \beta - \dfrac{(\alpha_S - \alpha_U)}{R}\dfrac{\partial y}{\partial \theta} \end{pmatrix} \begin{pmatrix} R - \bar{R} \\ \theta - \bar{\theta} \end{pmatrix} \tag{20}$$

The determinant of the Jacobian matrix in equation (20) is denoted as J, and substituting the results in Appendix A1, we get

$$J = -\frac{y(1-\alpha_S)L_U(\alpha_S - \alpha_U) + \alpha_U L_{UR}[1 - y(1-\alpha_S)] + \alpha_U L_U}{\theta R^2[1 - y(1-\alpha_U)]} < 0$$

which indicates that the characteristic roots are of opposite signs. Therefore, the steady state is a local saddle point.

The steady-state solution depends on the endowment of skilled and unskilled labour. Next, we examine how a change of skilled and unskilled labour endowment affect the steady-state stock of public intermediate input and its shadow price. We use Lemma 2 to express the impacts.

Lemma 2: *An inflow of skilled and unskilled labour raises the stock of public intermediate input and drops its shadow price in the steady state equilibrium.*

Proof. See Appendix A3.

A larger endowment of unskilled labour brings more unskilled labour to produce the public intermediate input and results in a greater stock of public intermediate input. Here, it should be noted that, even though skilled labour is not an input in the production of public intermediate input, a higher skilled labour endowment also contributes to more stock of public intermediate input. From Lemma 1, an inflow of skilled labour moves unskilled labour from the unskill-using sector to public sector, which leads to more production of the public sector and a larger amount of stock of public intermediate input.

Now we proceed to the analysis of wage inequality in the steady state equilibrium. Different with the static case where the stock of public intermediate input is constant, in the steady state equilibrium, an increase in the endowment of skilled and unskilled labour can affect the stock of public intermediate input and its shadow price, which impact the wages indirectly. First, examine the impact of an inflow of skilled labour,

$$\frac{dw_S}{d\bar{L}_S} = \frac{\alpha_S(1-\alpha_S)p}{\bar{L}_S{}^2}\left(\frac{\bar{L}_S}{R}\frac{\partial\bar{R}}{\partial\bar{L}_S} - 1\right) = \frac{\alpha_S(1-\alpha_S)p}{\bar{L}_S{}^2}\frac{\alpha_U\bar{L}_U[1-y(1-\alpha_S)]}{\theta R^2 J[1-y(1-\alpha_U)]}<0$$

and

$$\frac{dw}{d\bar{L}_S} = \frac{\alpha_U(1-\alpha_U)Y_U}{L_U[1-y(1-\alpha_U)]}\left[\frac{y(1-\alpha_S)}{\bar{L}_S} + \frac{1-y(1-\alpha_S)}{R}\frac{\partial\bar{R}}{\partial\bar{L}_S} + \frac{1}{\theta}\frac{\partial\bar{\theta}}{\partial\bar{L}_S}\right]>0$$

Thus, we can get $d(w_S - w)/d\bar{L}_S<0$. Considering the impact of an inflow of unskilled labour,

$$\frac{dw_S}{d\bar{L}_U} = \frac{\alpha_S(1-\alpha_S)pY_S}{\bar{L}_S R}\frac{\partial\bar{R}}{\partial\bar{L}_U}$$

$$= \frac{\alpha_S(1-\alpha_S)pY_S}{\bar{L}_S}\frac{\alpha_U[1-y(1-\alpha_S)]}{y(1-\alpha_S)L_U(\alpha_S-\alpha_U)+\alpha_U L_{UR}[1-y(1-\alpha_S)]+\alpha_U L_U}>0$$

$$\frac{dw}{d\bar{L}_U} = \frac{(1-\alpha_U)Y_U}{L_U^2}\left[1+\frac{\alpha_U L_U[1-y(1-\alpha_S)]}{R[1-y(1-\alpha_U)]}\frac{\partial\bar{R}}{\partial\bar{L}_U} + \frac{\alpha_U L_U}{\theta[1-y(1-\alpha_U)]}\frac{\partial\bar{\theta}}{\partial\bar{L}_U}\right]$$

$$= \frac{(1-\alpha_U)Y_U}{L_U^2}\frac{(1-\alpha_S)yL_U(\alpha_S-\alpha_U)+\alpha_U L_{UR}[1-y(1-\alpha_S)]+\alpha_U L_U[1-\alpha_S y(1-\alpha_S)]}{y(1-\alpha_S)L_U(\alpha_S-\alpha_U)+\alpha_U L_{UR}[1-y(1-\alpha_S)]+\alpha_U L_U}>0$$

and

$$\frac{d(w_S-w)}{d\bar{L}_U}=\frac{\alpha_U(\alpha_S w_S-w)-(w_S-w)\alpha_S\alpha_U y(1-\alpha_S)-yw(1-\alpha_S)(\alpha_S-\alpha_U)-w\alpha_U(1-y+y\alpha_S)L_{UR}/L_U}{y(1-\alpha_S)L_U(\alpha_S-\alpha_U)+\alpha_U L_{UR}[1-y(1-\alpha_S)]+\alpha_U L_U}$$

the sign of which is ambiguous, depending on the value of α_S. Note that α_S is the production elasticity of the public intermediate input stock in the skill-using sector and, if α_S is small enough, $d(w_S-w)/d\bar{L}_U<0$.

Proposition 2: In the steady state, an inflow of skilled labour decreases the wage inequality, while the impact of an inflow of unskilled labour on wage inequality is ambiguous. If the production elasticity of the public intermediate input stock in the skill-using sector is small enough, an inflow of unskilled labour narrows down wage inequality.

We will explain the economic mechanism behind Proposition 2. An increase in skilled labour endowment raises the stock of public intermediate input, which will generate the positive effect for wages of skilled and unskilled labour, as well as the negative effect for skilled labour wage. Under the model, the negative effect dominates the interaction and the wage of skilled labour drops as a result of an increase in the supply of skilled labour. As for the wage of unskilled labour, from Lemma 1, an inflow of skilled labour raises the demand for unskilled labour. Because of the positive effect of the stock of public intermediate input and increased demand, the wage of unskilled labour rises. Thus, the wage inequality reduces as a result of an inflow of skilled labour. An inflow of unskilled labour increases skilled wage because of the higher stock of public intermediate input from Lemma 2. Concerning the wage of unskilled labour, a larger endowment of unskilled labour has two effects on its wage: a productivity effect due to the stock of public intermediate input and a supply effect. However, the latter effect exerts no impact on its wage. According to Lemma 1, the increased unskilled labour wholly absorbed by the public sector and the wage of unskilled labour increases as a result of an inflow of unskilled labour. As for its impact on the wage inequality, the direction is ambiguous since an inflow of unskilled labour raises the wages of both skilled and unskilled labour. If the production elasticity of the public intermediate input stock in the skill-using sector is small enough, implying that the public intermediate input stock serves little significantly in the skill-using sector, the impact of an inflow of unskilled labour on skilled wage will not be too large compared to that of unskilled labour, and wage inequality will narrow down.

5. Concluding remarks

Traditionally, an inflow of skilled labour will reduce the wage inequality, while an inflow of unskilled will widen it. Previous papers largely ignore the role of public intermediate input in wage inequality. This paper incorporates the public intermediate input in 'unpaid factor' type and investigates the impact of an inflow of skilled and unskilled labour on wage inequality by establishing a model with two private sectors and one public sector. The public intermediate input can be accumulated and its accumulated stock serves as a public good for private production. From the analysis, in the steady state equilibrium, an increase in the skilled and unskilled labour endowment raises the stock of public intermediate input. And an inflow of skilled labour reduces the wage of skilled labour and raises the wage of unskilled labour, and an inflow of unskilled labour increases both wages of skilled and unskilled labour. Concerning their impacts on the wage inequality, an inflow of skilled labour decreases the wage inequality, while the result of an inflow of unskilled labour on the wage inequality is ambiguous. If the production elasticity of the public intermediate input stock in the skill-using sector is small enough, an inflow of unskilled labour narrows down wage inequality. Since the public intermediate input is playing an increasing role in both developing and developed countries, the findings of this paper have revealed a possibility that the conventional emigration policy of unskilled labour may not succeed in altering the wage inequality, especially for developed countries with a well-developed public infrastructure. According to the results, the governments of such countries should be indifferent to such an influx of unskilled labour. However, for developing countries with deficient public infrastructure, governments should not only pay attention to brain drain, but also take appropriate measures to retain unskilled labour.

To our knowledge, this paper is the first to analyse in an integrated framework the role of international labour movement on the wage of skilled and unskilled labour and wage inequality by a dynamic treatment of the public intermediate input. Admittedly, our model has some limitations, especially in some of its assumptions. For example, the paper assumes that the production elasticity of the public intermediate input stock in the skill-using sector is larger than that in the unskill-using sector, which is an empirical question. However, few empirical studies have tested the elasticity of public intermediate input stock in skill-using and unskill-using sectors. Another limitation is the production function of the public sector, which assumes its production function is a linear function. In general, the production function is assumed to be strictly concave and satisfies Inada conditions. However, the general production function makes the analysis much more complicated, especially in the dynamic analysis. Using the linear function, we can clarify some economic mechanisms that explain why an inflow of skilled and unskilled labour can bring the movement of unskilled labour between the unskill-using sector and public sector and change the wages of skilled and unskilled labour.

We can possibly extend our analysis in the following three respects. First, this paper considers the impacts of international factor movement on wage inequality with the presence of a public intermediate input. As mentioned in the introduction, many scholars hold that international goods trade may also contribute to the rising wage inequality. Then, how international trade affects wage inequality is one direction for future research. Second, in the model, all private firms can use the stock of public intermediate input commonly for production without any arising congestion issue. In reality, the contribution of stock of public intermediate input (such as transportation and communication system, water supply and irrigation, etc.) to private sectors is subject to congestion. We can introduce the congestion issue by assuming that the contribution of public intermediate input is decreasing in the use of private factors. Third, existing theoretical papers on public intermediate input have adopted the full-employment framework and ignored the problem of unemployment, particularly that of unskilled labour. An inflow of unskilled labour may significantly affect the overall employment, wages, and skilled–unskilled wage inequality. Therefore, such an embeddedness may bring about some new insights that are different from the traditional literature.

Disclosure statement

The author reports no conflicts of interest.

References

Abe, K. (1990). A public input as a determinant of trade. *Canadian Journal of Economics*, 23(2), 400–407. doi:10.2307/135613

Afonso, O. (2012). Scale-independent North-South trade effects on the technological-knowledge bias and wage inequality. *Review of World Economics*, 148 (1), 181–207. doi:10.1007/s10290-011-0109-7

Beladi, H., Chaudhuri, S., & Yabuuchi, S. (2008). Can International Factor Mobility Reduce Wage Inequality in a Dual Economy? *Review of International Economics*, 16(5), 893–903.

Beyer, H., Rojas, P., & Vergara, R. (1999). Trade liberalization and wage inequality. *Journal of Development Economics*, 59(1), 103–123. doi:10.1016/S0304-3878(99)00007-3

Chaudhuri, S. (2008). Wage inequality in a dual economy and international mobility of factors: do factor intensities always matter? *Economic Modelling*, 25(6), 1155–1164. doi:10.1016/j.econmod.2008.02.007

Feenstra, R. C., & Hanson, G. H. (1996). Globalization, outsourcing and wage inequality. *American Economic Review*, 86, 240–245.

Gupta, M. R., & Dutta, P. B. (2010). Skilled–unskilled wage inequality, nontraded good and endogenous supply of skilled labour: a theoretical analysis. *Economic Modelling*, 27 (5), 923–934. doi:10.1016/j.econmod.2010.05.013

Leamer, E. (1996). Wage inequality from international competition and technological change: theory and country experience. *American Economic Review*, 86 (2), 309–314.

Li, X., & Xu, Y. (2016). Unemployment, Wage Inequality and International Factor Movement in the Presence of Agricultural Dualism. *Review of Development Economics*, 20(2), 415–425. doi:10.1111/rode.12235

Manning, R., & McMillan, J. (1979). Public Intermediate Goods, Production Possibilities, and International Trade. *Canadian Journal of Economics*, 12(2), 243–257. doi:10.2307/134599

Marjit, S., & Kar, S. (2005). Emigration and wage inequality. *Economics Letters*, 88(1), 141–145. doi:10.1016/j.econlet.2005.02.003

McMillan, J. (1978). A Dynamic Analysis of Public Intermediate Goods Supply in Open Economy. *International Economic Review*, 19(3), 665–678. doi:10.2307/2526331

Meade, J. E. (1952). External economies and diseconomies in a competitive situation. *Economic Journal*, 62(245), 54–67. doi:10.2307/2227173

Pan, L., & Zhou, Y. (2013). International factor mobility, environmental pollution and skilled-unskilled wage inequality in developing countries. *Economic Modelling*, 33, 826–831. doi:10.1016/j.econmod.2013.06.013

Pi, J., & Zhou, Y. (2012). Public infrastructure provision and skilled-unskilled wage inequality in developing countries. *Labor Economics*, 19(6), 881–887. doi:10.1016/j.labeco.2012.08.007

Pi, J., & Zhou, Y. (2014). Foreign capital, public infrastructure, and wage inequality in developing countries. *International Review of Economics and Finance*, 29, 195–207. doi:10.1016/j.iref.2013.05.012
Reenen, J. V. (2011). Wage inequality, technology and trade: 21st century evidence. *Labor Economics*, 18, 730–741.
Suga, N., & Tawada, M. (2007). International Trade with a Public Intermediate Good and the Gains from Trade. *Review of International Economics*, 15(2), 284–293. doi:10.1111/j.1467-9396.2007.00648.x
Tawada, M., & Abe, K. (1984). Production Possibilities and International Trade with a Public Intermediate Good. *Canadian Journal of Economics*, 17(2), 232–248. doi:10.2307/134955
Tawada, M., & Okamoto, H. (1983). International Trade with a Pubic Intermediate Good. *Journal of International Economics*, 15(1-2), 101–115. doi:10.1016/0022-1996(83)90043-0
Yanase, A., & Tawada, M. (2012). History-dependent paths and trade gains in a small open economy with a public intermediate good. *International Economic Review*, 53(1), 303–314. doi:10.1111/j.1468-2354.2011.00681.x
Yanase, A., & Tawada, M. (2017). Public infrastructure for production and international trade in a small open economy: a dynamic analysis. *Journal of Economics*, 121(1), 51–73. doi:10.1007/s00712-016-0519-z

Appendix

A1: Proof of Lemma 1

Totally differentiating the production function of skill-using sectors,

$$dY_S = \frac{(1-\alpha_S)Y_S}{L_S} d\bar{L}_S + \frac{\alpha_S Y_S}{R} dR \tag{A-1}$$

Totally differentiating $y = pY_S/(pY_S + Y_U)$ and substituting (A-1), we obtain

$$\frac{1}{Y_U} dY_U + \frac{1}{y(1-y)} dy = \frac{(1-\alpha_S)}{L_S} d\bar{L}_S + \frac{\alpha_S}{R} dR \tag{A-2}$$

Using equation (10), equation (5) can be written as $(1-\alpha_U)(1-y) + \theta L_{UR} = \theta \bar{L}_U$. Totally differentiating this equation, we get

$$-\frac{(1-\alpha_U)}{\theta} dy + dL_{UR} = \frac{\bar{L}_U - L_{UR}}{\theta} d\theta + d\bar{L}_U \tag{A-3}$$

Using equation (10), equation (2) can be written as $Y_U = R^{\alpha_U}[(1-\alpha_U)(1-y)\theta^{-1}]^{1-\alpha_U}$. Totally differentiating this equation, we get

$$\frac{1}{Y_U} dY_U + \frac{1-\alpha_U}{1-y} dy = \frac{\alpha_U}{R} dR - \frac{1-\alpha_U}{\theta} d\theta \tag{A-4}$$

From (A-2) and (A-4), we get

$$dY_U = -\frac{yY_U(1-\alpha_S)(1-\alpha_U)}{[1-y(1-\alpha_U)]\bar{L}_S} d\bar{L}_S - \frac{Y_U[y\alpha_S(1-\alpha_U)-\alpha_U]}{[1-y(1-\alpha_U)]R} dR - \frac{Y_U(1-\alpha_U)}{[1-y(1-\alpha_U)]\theta} d\theta \tag{A-5}$$

$$dy = \frac{y(1-\alpha_S)(1-y)}{[1-y(1-\alpha_U)]\bar{L}_S} d\bar{L}_S + \frac{y(\alpha_S-\alpha_U)(1-y)}{[1-y(1-\alpha_U)]R} dR + \frac{y(1-y)(1-\alpha_U)}{[1-y(1-\alpha_U)]\theta} d\theta \tag{A-6}$$

$$dL_{UR} = \frac{yL_U(1-\alpha_S)}{[1-y(1-\alpha_U)]\bar{L}_S}d\bar{L}_S + \frac{yL_U(\alpha_S-\alpha_U)}{[1-y(1-\alpha_U)]R}dR + \frac{L_U}{[1-y(1-\alpha_U)]\theta}d\theta + d\bar{L}_U \quad \text{(A-7)}$$

From (A-5), the sign of $\partial Y_U/\partial R$ is ambiguous, depending the value of α_S and α_U. If the difference in production elasticity of the public intermediate input stock between the two sectors is not very large, $\partial Y_U/\partial R>0$.

A2: Decentralized equilibrium result

Consider a competitive economy with two private sectors and one public sector. Assume the government finances the cost of public intermediate good, wL_{UR}, by Lindahl pricing rule $t_S pY_S + t_U Y_U$, where $t_i(i=S,U)$ is the production tax rate imposed in sector i. Assume that the (instantaneous) utility function of a representative household is in Cobb-Douglas type $U(C_S, C_U) = \gamma \ln C_S + (1-\gamma)\ln C_U$. The household's income, I, consists of profits, $\Pi_S = (1-t_S)pY_S - w_S L_S$ and $\Pi_U = (1-t_U)Y_U - wL_U$, and wage reward $w_S L_S + w(L_U + L_{UR})$. Thus, $I = pY_S + Y_U$. Under the budget constraint, the optimal consumption amount is obtained as $C_S = \gamma I/p$ and $C_U = (1-\gamma)I$. Profit maximisation conditions for two private sectors are $(1-t_S)(1-\alpha_S)pY_S/L_S = w_S$ and $(1-t_U)(1-\alpha_U)Y_U/L_U = w$.

So far, the public sector's budget constraint $wL_{UR} = t_S pY_S + t_U Y_U$, two private sectors' profit maximization conditions and equations (1), (2), (4) and (5) jointly determine the equilibrium values for w, w_S, Y_S, Y_U, L_S, L_U and L_{UR} for given R in a temporary equilibrium model, t_S and t_U are policy variables. After solving the model and obtaining the results of Y_S and Y_U, C_S and C_U could be arrived at correspondingly. From C_S and C_U, the indirect utility is $v(p, I) = \ln I - \gamma \ln p + \Omega$, where $\Omega = \gamma \ln \gamma + (1-\gamma)\ln(1-\gamma)$.

The government chooses the tax time path t_S and t_U in order to maximise the discounted sum of the utility $\int_0^\infty e^{-\rho t}[\ln(pY_S + Y_U) - \gamma \ln p + \Omega]dt$ subject to equation (3). The current-value Hamiltonian function is described as:

$$\begin{aligned} H^A = &\ \ln(pY_S + Y_U) - \gamma \ln p + \Omega + \Gamma(L_{UR} - \beta R) \\ &+ w\left[\bar{L}_U - \left(\frac{Y_U}{R^{\alpha_U}}\right)^{\frac{1}{1-\alpha_U}} - L_{UR}\right] + w_S\left[\bar{L}_S - \left(\frac{Y_S}{R^{\alpha_S}}\right)^{\frac{1}{1-\alpha_S}}\right] \end{aligned}$$

where Γ is the shadow price of public intermediate input. Solving the H^A, the optimal tax should satisfy $t_S = t_U = 1 - 1/(pY_S + Y_U)$. The adjoint equation is $\dot{\Gamma} = (\rho+\beta)\Gamma - [y\alpha_S + (1-y)\alpha_U]/R$. Setting $\Gamma = \theta$, the adjoint equation is identical to equation (11) and the centralized equilibrium results can be satisfied under decentralised competitive economy with the Lindahl rule for the provision of public intermediate input.

A3: Proof of Lemma 2

Denote the right-hand side of equation (13) as $\Phi(R, \theta; \bar{L}_S, \bar{L}_U, p) = Y_R - \beta R$ and that of equation (14) as $\Psi(R, \theta; \bar{L}_S, \bar{L}_U, p) = (\rho+\beta)\theta - [\alpha_S y + \alpha_U(1-y)]/R$. Totally differentiating the steady state conditions,

$$\begin{pmatrix} \partial L_{UR}/\partial R - \beta & \partial L_{UR}/\partial\theta \\ \dfrac{\alpha_S y + (1-y)\alpha_U}{R^2} - \dfrac{(\alpha_S-\alpha_U)}{R}\dfrac{\partial y}{\partial R} & \rho+\beta - \dfrac{(\alpha_S-\alpha_U)}{R}\dfrac{\partial y}{\partial\theta} \end{pmatrix}\begin{pmatrix} d\bar{R} \\ d\bar{\theta} \end{pmatrix} = -\begin{pmatrix} \partial L_{UR}/\partial\bar{L}_S \\ -\dfrac{(\alpha_S-\alpha_U)}{R}\dfrac{\partial y}{\partial\bar{L}_S} \end{pmatrix} d\bar{L}_S - \begin{pmatrix} 1 \\ 0 \end{pmatrix} d\bar{L}_U \quad \text{(A-8)}$$

Solving equation (A-8), we have

$$d\bar{R} = -\frac{\left(\rho + \beta - \frac{\alpha_S - \alpha_U}{R}\frac{\partial y}{\partial \theta}\right)\frac{\partial L_{UR}}{\partial \bar{L}_S} + \frac{(\alpha_S - \alpha_U)}{R}\frac{\partial L_{UR}}{\partial \theta}\frac{\partial y}{\partial \bar{L}_S}}{J} d\bar{L}_S - \frac{\left(\rho + \beta - \frac{\alpha_S - \alpha_U}{R}\frac{\partial y}{\partial \theta}\right)}{J} d\bar{L}_U \tag{A-9}$$

and

$$d\bar{\theta} = \frac{\left(\frac{\partial L_{UR}}{\partial R} - \beta\right)\frac{(\alpha_S - \alpha_U)}{R}\frac{\partial y}{\partial \bar{L}_S} + \frac{\partial L_{UR}}{\partial \bar{L}_S}\left[\frac{\alpha_S y + (1-y)\alpha_U}{R^2} - \frac{(\alpha_S - \alpha_U)}{R}\frac{\partial y}{\partial R}\right]}{J} d\bar{L}_S + \frac{\frac{\alpha_S y + (1-y)\alpha_U}{R^2} - \frac{(\alpha_S - \alpha_U)}{R}\frac{\partial y}{\partial R}}{J} d\bar{L}_U \tag{A-10}$$

From equation (A-9), we get $d\bar{R}/d\bar{L}_S > 0$ and $d\bar{R}/d\bar{L}_U > 0$, $d\bar{\theta}/d\bar{L}_S < 0$ and $d\bar{\theta}/d\bar{L}_U < 0$.