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THE APPLICATION OF BENFORD'S LAW IN PSYCHOLOGICAL PRICING DETECTION

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Abstract

This paper presents the application of Benford's law in psychological pricing detection. Benford's law is naturally occurring law which states that digits have predictable frequencies of appearance with digit one having the highest frequency. Psychological pricing is one of the marketing pricing strategies directed on price setting which have the psychological impact on certain consumers. In order to investigate the application of Benford's law in psychological pricing detection, Benford's law is observed in the case of first and last digits. In order to inspect if the first and last digits of the observed prices are distributed according to the Benford's law distribution or discrete uniform distribution respectively, mean absolute deviation measure, chi-square tests and Kolmogorov-Smirnov Z tests are used. Results of the analysis conducted on three price datasets have shown that the most dominating first digits are 1 and 2. On the other side, the most dominating last digits are 0, 5 and 9 respectively. The chi-square tests and Kolmogorov-Smirnov Z tests have showed that, at significance level of 5%, none of the three observed price datasets does have first digit distribution that fits to the Benford's law distribution. Likewise, mean absolute deviation values have shown that there are large differences between the last digit distributions and the discrete uniform distribution implying psychological pricing in all price datasets.

Keywords: *Benford's distribution, first digit, price endings, psychological pricing*

JEL: C46, D91

1. INTRODUCTION

Benford's law or first digit law is an anomalous distribution of leading digits noted in many real-life datasets. It is named after Frank Benford in 1938 but the originator of the law was Simon Newcomb who previously stated it in 1881. Benford's law has been mainly recognised as a forensic tool for financial fraud detection but it has application in various fields of economics such as election fraud detection, macroeconomic data analysis, price digits analysis, scientific fraud detection, economic research and forecasting, stock market turnovers, on-line social networks, etc.

Goal of the paper is application of Benford's law in psychological pricing detection using statistical deviation measures from the Benford's law and uniform distribution. Hypotheses of the paper are as follows:

H_1 : *Distribution of first digits in prices fits to the Benford's law distribution.*

H_2 : *Distribution of last digits in prices differs from the uniform distribution pointing to the psychological price formation.*

Paper is structured in six chapters. After the introduction, second chapter presents history and analytics of Benford's law giving the proof of first, second and last digit occurrence according to expected frequencies and the necessary conditions for data to obey Benford's law. Third chapter elaborates the application of Benford's law in various fields of economics. Methodology and data are displayed in chapter four while results and discussion are presented and elaborated in chapter five. Final chapter exhibits concluding remarks.

2. HISTORY AND ANALYTICS OF BENFORD'S LAW

Benford's law is natural occurring law that states that digits have predictable frequencies of appearance. History of Benford's law starts with Simon Newcomb, a Canadian-American astronomer, in 1881 who noticed that first pages of logarithm tables were more worn out than the later pages implying that lower digits occur most frequently than higher digits. In his paper "*Note on the Frequency of Use of the Different Digits in Natural Numbers*" he described this phenomenon by calculating frequencies of first and second digit occurrence. Fifty years later, unaware of Newcomb's idea, Frank Benford in 1938 came to the same discovery in his paper "The Law of Anomalous Numbers". Benford analysed and tested his hypothesis on more than 20,000 observations in the various fields such as hydrology, chemistry, sport (baseball) results and daily press.

2.1. The analytics of Benford's law

According to Newcomb (1881) and Benford (1938) the Benford's law is defined by the following equation:

$$P(d) = \log_{10} \left(1 + \frac{1}{d} \right) \tag{1}$$

where d are the values of digits from 1 to 9.

Although one can think that first digits of number are uniformly distributed, it is not the case. In Benford's law distribution, probability for first digit to appear is about 30.1 percent, for digit two is 17.6 percent and so on with a decreasing probability of appearance for later digits. Digit nine has the lowest probability to appear with only 4.5 percent. Digit zero as a first digit is excluded.

Table 1. Benford's law distribution of first digit

| d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| P(d) | 0.301 | 0.176 | 0.125 | 0.097 | 0.079 | 0.067 | 0.058 | 0.051 | 0.046 |

In Table 1 is presented distribution of first digits according to Benford's law distribution. Digit 1 has the highest probability of occurrence while the digit 9 has the lowest probability of occurrence on a first digit place.

2.2. Proof of second and higher-order significant digit occurrence

Relative frequencies of second significant digit is derived as:

$$P(d) = \sum_{k=1}^9 \log_{10} \left(1 + \frac{1}{10k + d} \right) \tag{2}$$

where $d = 0, 1, 2, \dots, 9$.

Higher-order (to the last) significant digit occurrence is presented in equation 3:

$$P(d_k) = \sum_{d_1=1}^9 \sum_{d_2=0}^9 \dots \sum_{d_{k-1}=0}^9 \log_{10} \left(1 + \frac{1}{\sum_{i=1}^k d_i \cdot 10^{k-i}} \right) \tag{3}$$

where $d_k = 0, 1, 2, \dots, 9$.

In Table 2 are presented expected frequencies of digits based on Benford's law from the first to fifth place.

Table 2. Expected frequencies of digits in Benford's law distribution

| Digit | 1st place | 2nd place | 3d place | 4th place | 5th place |
|-------|-----------|-----------|----------|-----------|-----------|
| 0 | - | 0.11968 | 0.10178 | 0.10018 | 0.1000 |
| 1 | 0.30103 | 0.11389 | 0.10138 | 0.10014 | 0.1000 |
| 2 | 0.17609 | 0.10882 | 0.10097 | 0.10010 | 0.1000 |
| 3 | 0.12494 | 0.10433 | 0.10057 | 0.10006 | 0.1000 |
| 4 | 0.09691 | 0.10031 | 0.10018 | 0.10002 | 0.1000 |
| 5 | 0.07918 | 0.09668 | 0.09979 | 0.09998 | 0.1000 |
| 6 | 0.06695 | 0.09337 | 0.09940 | 0.09994 | 0.1000 |
| 7 | 0.05799 | 0.09035 | 0.09902 | 0.09990 | 0.1000 |
| 8 | 0.05115 | 0.08757 | 0.09864 | 0.09986 | 0.1000 |
| 9 | 0.04576 | 0.08500 | 0.09827 | 0.09982 | 0.1000 |

Source: Nigrini (1996).

It can be noticed that distribution of frequencies for fifth place is identical with the uniform distribution where every digit has the same probability of appearance of 0.10.

2.3. Necessary requirements for dataset to be compatible with Benford's law distribution

There are number of necessary requirements for a dataset to fulfil in order to obey Benford's law distribution. According to Goodman (2016) the requirements for a dataset to be compatible with Benford's law distribution are sufficient sample size, large span of number values, positively skewed distribution and not-human assigned numbers. Durtschi, Hillison and Pacini (2004) suggested that distributions can be expected to obey Benford's law distribution when the mean of particular dataset is greater than median and the skewness is positive, larger datasets, transactions data and numbers that result from mathematical combination of numbers. Benford's law distribution is not likely expected when dataset is comprised of assigned numbers, numbers that are influenced by human thought such as psychology formed prices and accounts with a larger of firm-specific numbers and where are no transactions recorded. Pomykacz, Olmsted and Tantanin (2017) explained that Benford's law is not applicable to non-logarithmic datasets, datasets that have normal distributions around averages, that change over time in linear way, have minimum and maximum value, have assigned numbers, etc.

3. THE APPLICATION OF BENFORD'S LAW IN ECONOMICS

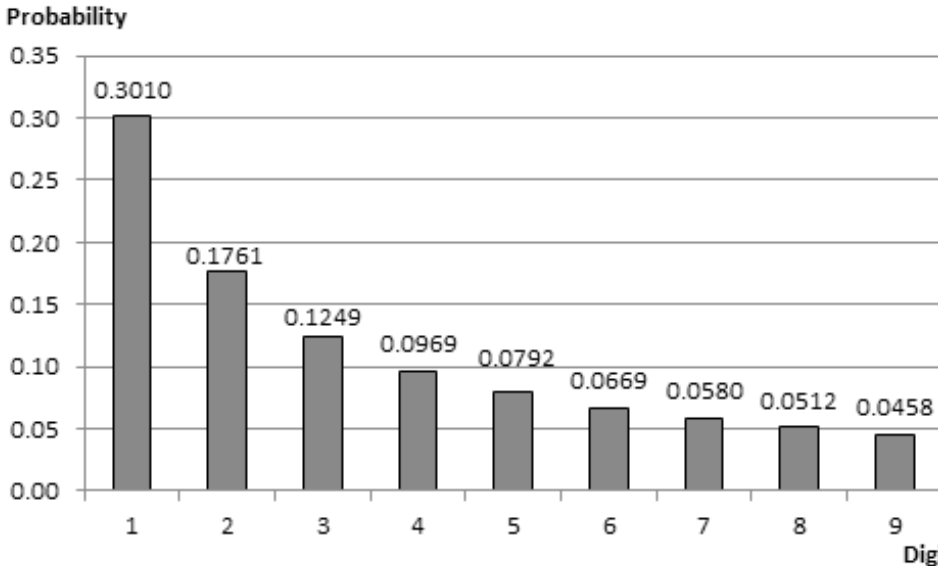
Benford's law has the application in various fields of economics. Mark Nigrini (1996) was the first to suggest using Benford's law for fraud detection in financial datasets. Since Nigrini's initial paper, the use of Benford's law in forensic accounting has been subject of investigation of many researchers. Application of Benford's law in forensic accounting can be found in following papers: *Nigrini and Mittermaier (1997)*, *Durtschi, Hillison and Pacini (2004)*, *Krakar and Žgela (2009a)*, *Nigrini (2012)*, *Slijepčević and Blašković (2014)*, *Tota, Aliaj and Lamčja (2016)*, *Papić, Vudrić and Jerin (2017)* and many others. Idea of using Benford's law in fraud detection has been also used in campaign fraud detection, *Cho and Gaines (2007)*, *Deckert, Myagkov and Ordeshook (2011)*. The application of Benford's law and possibilities for its use in international and governmental macroeconomic statistics can be found in papers *Nye and Moul (2007)* and *Hindls and Hronová (2015)*. It was found that World Bank international GDP data and purchasing power parity corrected Penn World tables for OECD countries conformed well to Benford's Law. There was not an overlap of Czech Republic national accounts data in the year 2013 to Benford's Law distribution. The differences were significant for the digits 0, 4, and 6 but any general conclusions due this phenomenon cannot be drawn. *Michalski and Stoltz (2013)* investigated if countries falsify their economic data strategically while *Gonzalez-Garcia and Pastor (2009)* applied Benford's Law in macroeconomic data quality analysis. There was an evidence that countries sometimes misreport their economic data strategically. On a dataset for 80 countries, most of the macroeconomic data conformed with Benford's law. Furthermore, it was shown that nonconformity with Benford's law cannot be interpreted as an indicator of poor quality in macroeconomic data. *Günnel and Tödter (2009)* investigated does Benford's law hold in economic research and forecasting. Relative frequencies of first digits in economic research published in journals *Empirica* and *Applied Economics Letters* were broadly consistent with Benford's law. Similarly, *Diekmann (2007)* used Benford's Law to detect fraudulent scientific data. The results have shown that first and second digit of regression coefficients were approximately Benford distributed. Another application of Benford's law was made by *Carrera (2015)* who used Benford's law in tracking exchange rate management in countries of Latin America. He found that the US dollar exchange rate does not satisfy Benford's Law. On the other hand, Benford's law holds when the euro is considered. *Giles (2007)* tested Benford's law on naturally occurring prices in certain eBay auctions while *Golbeck (2015)* investigated does Benford's Law applies to online social networks. Certain winning bids for eBay auctions obeyed Benford's Law which implied that it is unlikely for these bids to be subject to 'shilling' on the part of sellers and collusion among bidders. The investigation on social data from five major social networks (Facebook, Twitter, Google Plus, Pinterest,

and LiveJournal) have shown that the first digits distribution of user friend and follower counts followed Benford's Law. Application of Benford's law for stock prices and stock turnover was made by Krakar and Žgela (2009b). De Ceuster, Dhaene and Schatteman (1998) tested the hypothesis of psychological barriers in stock markets and Benford's law. As uniform distribution was not the right benchmark against the test, the alternative test used was cyclical permutations of the actual returns. The application of this test to the Dow Jones 30 Industrial Average, Nikkei Stock Average 225 and the Financial Times – Stock Exchange 100 has found no evidence for hypothesis of psychological barriers in stock markets. Application of Benford's law on price of medicaments was made by Novalija (2012). The data for list of medicines were provided for the period from January to March 2011 from Health Insurance Institute of Tuzla Canton. Based on the conducted research it has been established that the application of Benford's law is possible as suitable pricing technique which can result in the realization of certain savings of available funds for medicaments. Benford's law has also been successfully applied in psychological price research. Seminal paper in this field was „Price Developments after a Nominal Shock: Benford's Law and Psychological Pricing after the Euro Introduction” published by El Sehity, Hoelzl and Kirchler (2005). The authors investigated consumer price digits before and after the introduction of the euro in 2002 using Benford's law as a benchmark for price adjustments. The study showed deviations from the benchmark for the second and third digit-suggesting tendency towards psychological pricing. Wagner and Jamsawang (2012) investigated several aspects of psychological pricing such as price endings, first non-zero digit in a price, symbolic meanings of price endings, eye-catching sequences of digits in a price and framing of price cuts in percentage versus euro terms. Results have shown that Benford's law regarding first digits applied for furniture shops, the unlucky digit 4 appears less frequently in Chinese restaurants while eye-catching prices were not widely identified.

4. METHODOLOGY AND DATA

In the paper, the Benford's law will be observed on the case of first digits. Therefore, the value d is defined as $d \in \{1, 2, 3, \dots, 8, 9\}$. Accordingly, the distribution of the Benford's law is presented in the Figure 1.

Figure 1. The Benford's law distribution



Source: Authors'.

Figure 1 suggests that according to the Benford's law digit 1 has the highest probability of occurrence (0.3010) whereas the digit 9 has the lowest probability of occurrence (0.0458). As opposite, at a discrete uniform distribution each of k values have the same probability of occurrence that is equal to $1/k$. The discrete uniform distribution will be used to inspect last digits distributions. Since the last digit can be a discrete value in interval from 0 to 9, all last digits have the same probability of occurrence equal to 0.10.

In the analysis, three price datasets will be used. First dataset is Online retail dataset (Daqing Chen, Sai Liang Sain, Kun Guo, 2012). It is a transnational data set containing all the transactions occurring between December 1, 2010 and December 9, 2011. The dataset contains information about invoice number and date, stock code, description, quantity, unit price in sterling and customer ID for a UK-based and registered non-store online retail. Items that company mainly sells are unique all-occasion gifts while many customers of the company are wholesalers. Second dataset is Retail food prices in Croatia collected and modified by Faculty of Agriculture in Osijek (PFOS, 2011). This dataset contains information about products name and unit price in kunas. Third dataset is the Basic list of medicines dataset composed and published by Croatian Health Insurance Fund (CHIF, 2014). It contains information about code, label, name of the producer, price for original package in kunas, protected name of the medicine, etc. Data are available for January 2014. The reason for using three different datasets is for

assessment purpose and for making a conclusion about psychologically determined prices. It is expected that the prices of medicines will not be psychologically determined unlike prices of consumer goods.

In order to inspect if the first digits of the observed prices are distributed according to the Benford's distribution and if the last digits of the observed prices are distributed according to the discrete uniform distribution, chi-square tests and Kolmogorov-Smirnov Z tests will be conducted. In addition, mean absolute deviation measure will be calculated as well. The chi-square test statistic is defined as follows:

$$\chi^2 = \sum_{i=1}^k \frac{(f_i - e_i)^2}{e_i} \quad (4)$$

where f_i is the real number of prices in the sample with certain value of the first or the last digit i , e_i is the expected number of prices in the sample with certain value of the first digit i defined by the Benford's law or the expected number of prices in the sample with certain value of the last digit i defined by the discrete uniform distribution. If the differences between real and expected values are found out to be statistically small, the null hypothesis of the chi-square test can be rejected and it can be concluded that, at certain statistical significance level, the first digits follow the Benford's law distribution or that the last digits follow the discrete uniform distribution.

In order to get more supportive results, in addition to the chi-square tests, Kolmogorov-Smirnov Z tests will be conducted as well. The Kolmogorov-Smirnov Z test will be used to inspect the same thing as the chi-square test or it will be used to inspect whether the first digits of the observed prices follow the Benford's law distribution and whether the last digits follow the discrete uniform distribution. However, as opposite to the chi-square test, the Kolmogorov-Smirnov Z test observes and compares cumulative density functions of the sample distribution (here the distributions of prices) and the chosen distribution (here the Benford's law distribution for first digits and the discrete uniform distribution for last digits). The Kolmogorov-Smirnov Z test statistics is equal to the largest absolute difference between the cumulative density functions of the sample distribution and the chosen distribution. This value is compared to the theoretical limit that is calculated as follows:

$$K - S = \frac{\sqrt{-\frac{1}{2} \cdot \ln\left(\frac{\alpha}{2}\right)}}{\sqrt{n}} \quad (5)$$

where α is the significance level and n is the sample size. If a Kolmogorov-Smirnov Z test statistics is larger than the theoretical limit the conclusion is that, at the significance level α , the null hypothesis of the Kolmogorov-Smirnov Z test can

be rejected. In that case, the conclusion is that the cumulative density function of the sample distribution does not follow the cumulative density function of the chosen distribution.

The difference between the sample distribution of the observed prices and the Benford's law distribution for the first digit and the difference between the sample distribution of the observed prices and the discrete uniform distribution for the last digit will be also inspected by using mean absolute deviation measure. In order to calculate mean absolute deviation following equation will be applied:

$$MAD = \frac{\sum_{i=1}^k |p_i - r_i|}{k} \quad (6)$$

where p_i is the proportion of the number of prices with certain digit i in the total number of prices in the sample, r_i is the proportion of the number of prices with certain first digit i in the total number of prices at the Benford's law distribution or the proportion of the number of prices with certain last digit i in the total number of prices at the discrete uniform distribution. According to Nigrini (2012) the difference between the real and selected distribution is considered to be small if mean absolute deviation value is less than 0.0006. In case that mean absolute deviation is between 0.0006 and 0.0012 the difference is considered to be acceptable and if it is between 0.0012 and 0.0015 the difference is marginal acceptable. Furthermore, in case that mean absolute deviation is larger than 0.0015, the conclusion is that there is a large difference between the real and the selected distributions.

5. RESULTS AND DISCUSSION

In this chapter, most important results of research and discussion will be presented and elaborated.

5.1. Descriptive statistics analysis

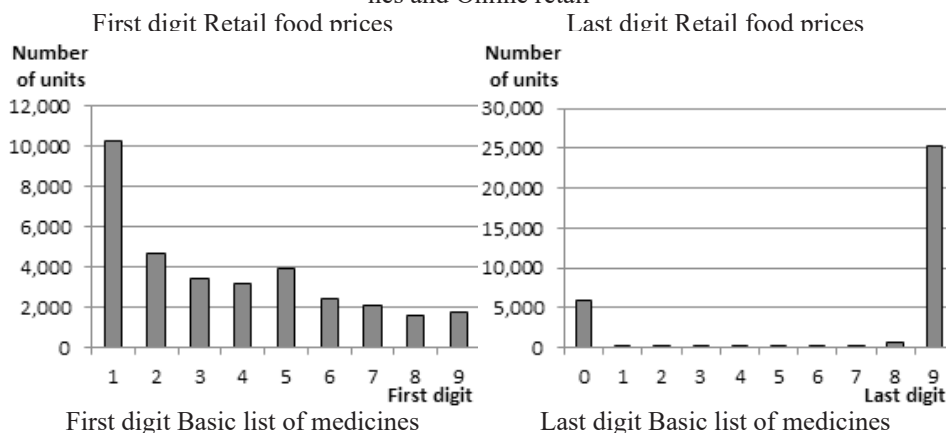
Before the first digit and the last digit analyses are going to be conducted, the descriptive statistics analysis of the observed set of prices is made. The descriptive statistics results are given in Table 1.

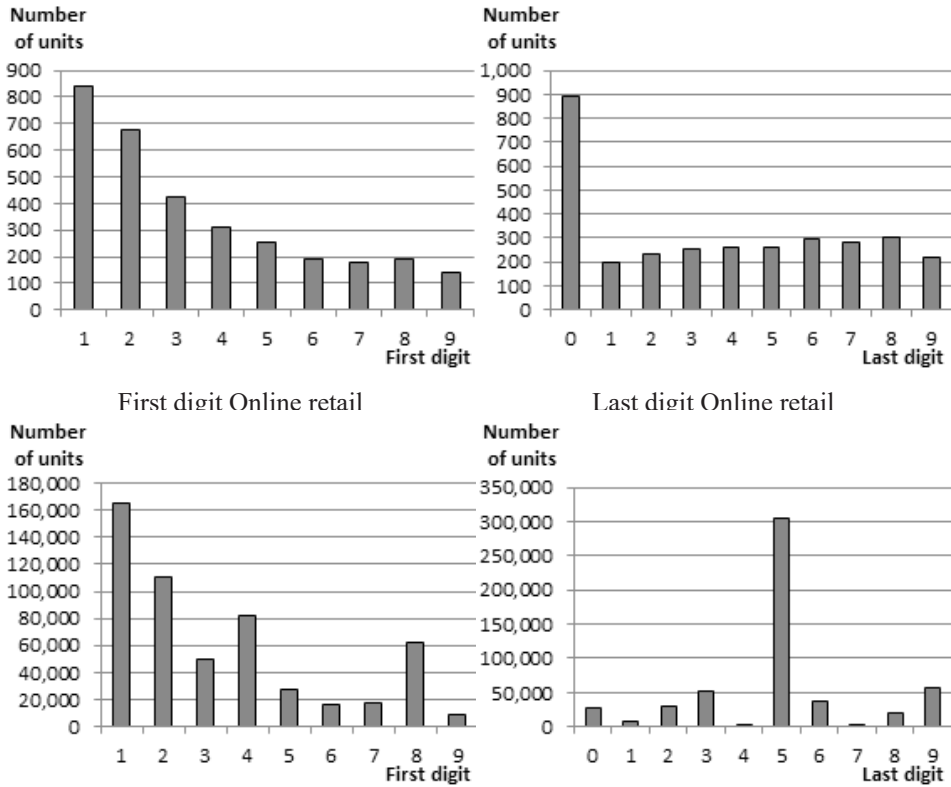
Table 3. Descriptive statistics results of Retail food prices, Basic list of medicines and Online retail

| Statistics | Retail food prices | Basic list of medicines | Online retail |
|---------------------|--------------------|-------------------------|---------------|
| Sample size | 33,503 | 3,207 | 539,388 |
| Mean | 33.05 | 699.90 | 4.67 |
| Standard deviation | 166.83 | 3,511.12 | 94.62 |
| Coeff. of variation | 505% | 502% | 2,024% |
| Skewness | 63 | 20.89 | 206 |
| Kurtosis | 5,166 | 649.34 | 64,148 |
| Mode | 1.00 | 62.28 | 1.25 |
| Minimum | 0.01 | 2.70 | 0.01 |
| 1st quartile | 5.00 | 30.30 | 1.25 |
| Median | 11.99 | 74.07 | 2.08 |
| 3rd quartile | 34.95 | 280.61 | 4.13 |
| Maximum | 15,999.00 | 130,471.82 | 38,970 |
| Range | 15,998.99 | 130,469.12 | 38,969.99 |
| Interquartile range | 34.94 | 250.31 | 2.88 |

Table 3 reveals that three observed price variables are rather different. At first, the difference in the sample sizes must be emphasized. Variable Retail food prices is consisted of 33,503 prices, variable Basic list of medicines has 3,207 prices and variable Online retail has 539,388 prices. It has to be kept on mind that variables are given in different monetary unit and consequently some results cannot be compared across the variables. However, at all three observed price variables mean statistics turned out not to be a representative measure. Furthermore, all price variables tend to be positively skewed meaning that there are some prices which are considerably higher than are the others.

Figure 2. Distributions of first and last digits of Retail food prices, Basic list of medicines and Online retail





In Figure 2 the distributions of first and last digits of the three observed price indicators are given. It can be concluded that the most dominating first digits are 1 and 2. On the other side, at variable Retail food prices the most dominating last digit is 9, at variable Basic list of medicines 0, and at variable Online retail the most prevailing last digit is 5. Holdershaw, Gendall and Garland (1997), on the analysis of 840 advertisements, revealed that odd prices clearly outnumbered all other price endings. Digits 0, 5 and 9 accounted for 97% of all price endings indicating bias in favour of odd price endings.

Table 4. Descriptive statistics results of first digits of Retail food prices, Basic list of medicines and Online retail

| Statistics | Retail food prices | Basic list of medicines | Online retail |
|---------------------|--------------------|-------------------------|---------------|
| Sample size | 33,503 | 3,207 | 539,388 |
| Mean | 3.60 | 3.48 | 3.33 |
| Standard deviation | 2.51 | 2.44 | 2.44 |
| Coeff. of variation | 70% | 70% | 73% |
| Skewness | 0.64 | 0.82 | 0.88 |
| Kurtosis | -0.78 | -0.51 | -0.48 |
| Mode | 1 | 1 | 1 |
| Minimum | 1 | 1 | 1 |
| 1st quartile | 1 | 1 | 1 |
| Median | 3 | 3 | 2 |
| 3rd quartile | 5 | 5 | 4 |
| Maximum | 9 | 9 | 9 |
| Range | 8 | 8 | 8 |
| Interquartile range | 4 | 4 | 3 |

In Table 4 descriptive statistics results of first digits for the three observed price variables are given. Despite very different sample sizes, all three observed prices variables have quite similar mean values. Furthermore, all three price variables have almost the same coefficient of variation values and are moderate positively skewed.

Table 5. Descriptive statistics results of last digits of Retail food prices, Basic list of medicines and Online retail

| Statistics | Retail food prices | Basic list of medicines | Online retail |
|---------------------|--------------------|-------------------------|---------------|
| Sample size | 33,503 | 3,207 | 539,388 |
| Mean | 7.16 | 3.73 | 4.94 |
| Standard deviation | 3.49 | 3.13 | 2.14 |
| Coeff. of variation | 49% | 84% | 43% |
| Skewness | -1.47 | 0.19 | -0.10 |
| Kurtosis | 0.27 | -1.38 | 0.51 |
| Mode | 9 | 0 | 5 |
| Minimum | 0 | 0 | 0 |
| 1st quartile | 9 | 0 | 5 |
| Median | 9 | 4 | 5 |
| 3rd quartile | 9 | 7 | 5 |
| Maximum | 9 | 9 | 9 |
| Range | 9 | 9 | 9 |
| Interquartile range | 0 | 7 | 0 |

However, if the descriptive statistics results of last digits for the three observed price variables are observed it can be concluded that the price variables are not so similar as they are according to the first variable. According to the Table 5

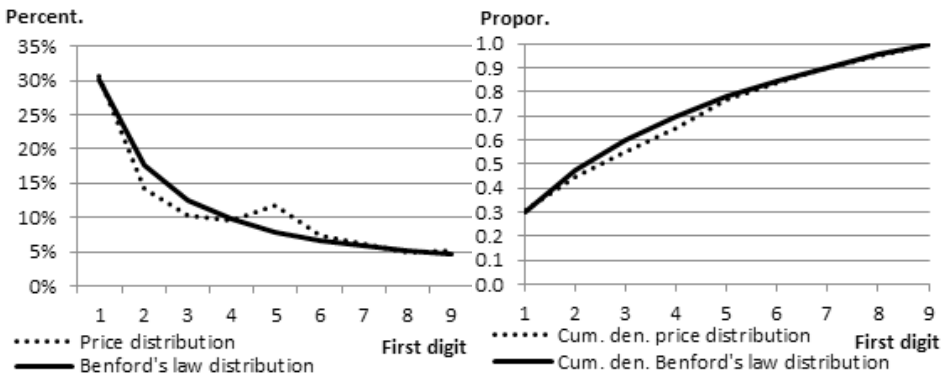
mean of the last digit at variable Retail food prices is 7.16, at variable Basic list of medicines 3.73 and the mean of last digit at variable Online retail is 4.94. Furthermore, the distributions of the price variables are quite different according to the skewness and the kurtosis statistics as well.

5.2. First digit analysis

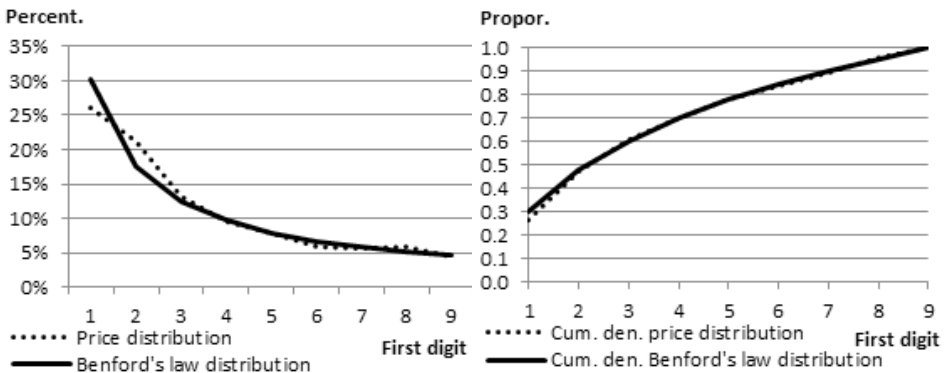
In this part of the paper it will be inspected if the first digits of the three observed price variables follow the Benford's law distribution or not. This will be made first by graphical approach and after that by using selected statistical tests.

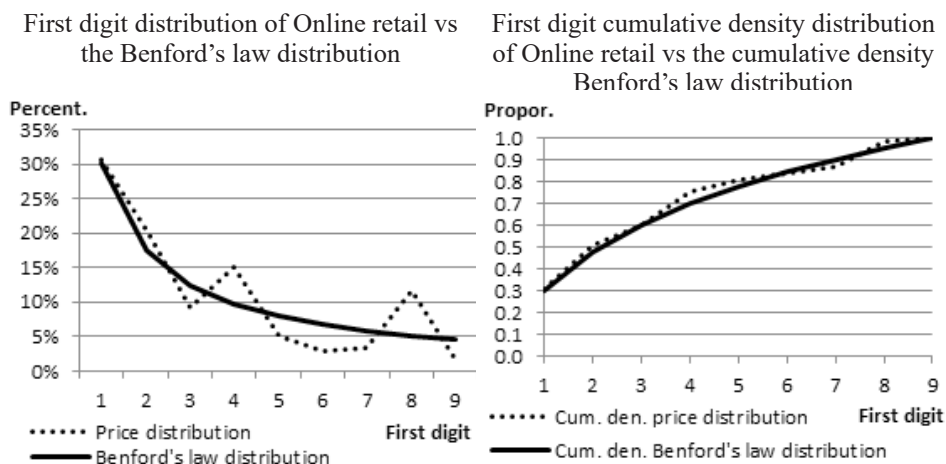
Figure 3. Comparisons of first digit distributions and cumulative density distributions of Retail food prices, Basic list of medicines and Online retail with the Benford's law distribution

First digit distribution of Retail food prices vs the Benford's law distribution First digit cumulative density distribution of Retail food prices vs the cumulative density Benford's law distribution



First digit distribution of Basic list of medicines vs the Benford's law distribution First digit cumulative density distribution of Basic list of medicines vs the cumulative density Benford's law distribution





If first digit distribution of the three observed price variables in the left column in Figure 3 are observed, it can be concluded that the first digit distribution of variable Basic list of medicines seems to be the closest to the Benford's law distribution. On the other hand, the first digit distribution of variable Online retail seems to be the worst in following the Benford's law distribution among the three observed price variables. The cumulative density distributions of the three observed price variables, which are given in the right column in Figure 3, confirmed that the first digit distribution of variable Basic list of medicines is the most similar to the Benford's law distribution in compare to the other two price variables.

Table 6. Results of chi-square tests, Kolmogorov-Smirnov Z tests and mean absolute deviations of first digits of Retail food prices, Basic list of medicines and Online retail

| Test / measure | Statistics | Retail food prices | Basic list of medicines | Online retail |
|---------------------------|-------------------------------------|--------------------|-------------------------|---------------|
| Chi-square test | Empirical value | 1,082 | 49 | 101,790 |
| | Degrees of freedom | 8 | 8 | 8 |
| | Theoretical value ($\alpha=0.05$) | 15.51 | 15.51 | 15.51 |
| | p-value | <0.0001 | <0.0001 | <0.0001 |
| Kolmogorov-Smirnov Z test | Empirical value | 0.0533 | 0.0391 | 0.0549 |
| | Theoretical value ($\alpha=0.05$) | 0.0074 | 0.0240 | 0.0018 |
| Mean absolute deviation | Empirical value | 0.0137 | 0.0115 | 0.0341 |

Despite the Figure 3, the results of conducted chi-square tests and Kolmogorov-Smirnov Z tests have shown that, at significance level of 5%, none of the three observed price variables does not have first digit distribution which follows

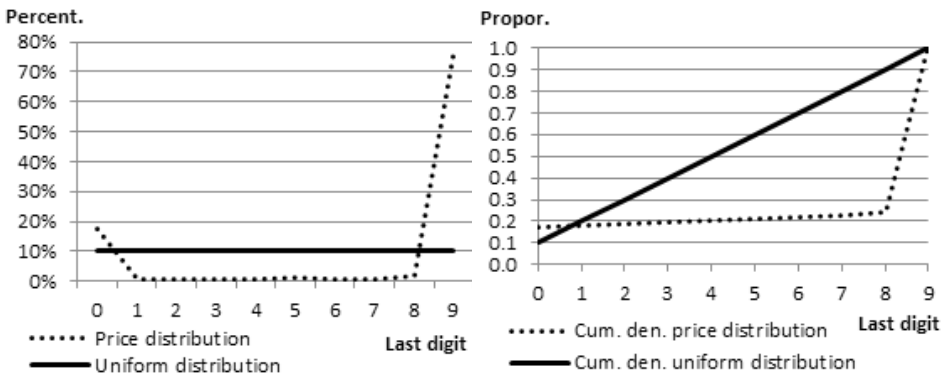
the Benford's law distribution. Furthermore, mean absolute deviation values shows that there are large differences between the first digit distributions of the three observed price variables and the Benford's law distribution.

5.3. Last digit analysis

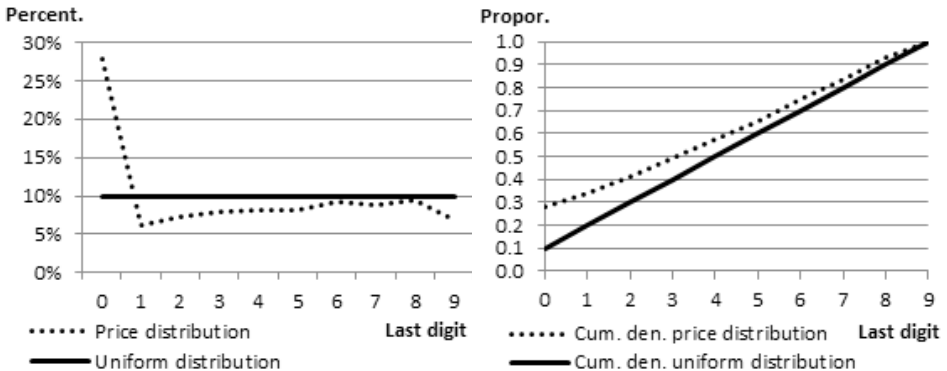
The last digit analysis will be conducted on the same way as the first digit analysis. Firstly, it will be inspected whether the last digits of the three price variables follow the discrete uniform distribution or not by using graphical approach and after that by using statistical tests.

Figure 4. Comparisons of last digit distributions and cumulative density distributions of Retail food prices, Basic list of medicines and Online retail with the discrete uniform distribution

Last digit distribution of Retail food prices vs the discrete uniform distribution Last digit cumulative density distribution of Retail food prices vs the cumulative density discrete uniform distribution



Last digit distribution of Basic list of medicines vs the discrete uniform distribution Last digit cumulative density distribution of Basic list of medicines vs the cumulative density discrete uniform distribution



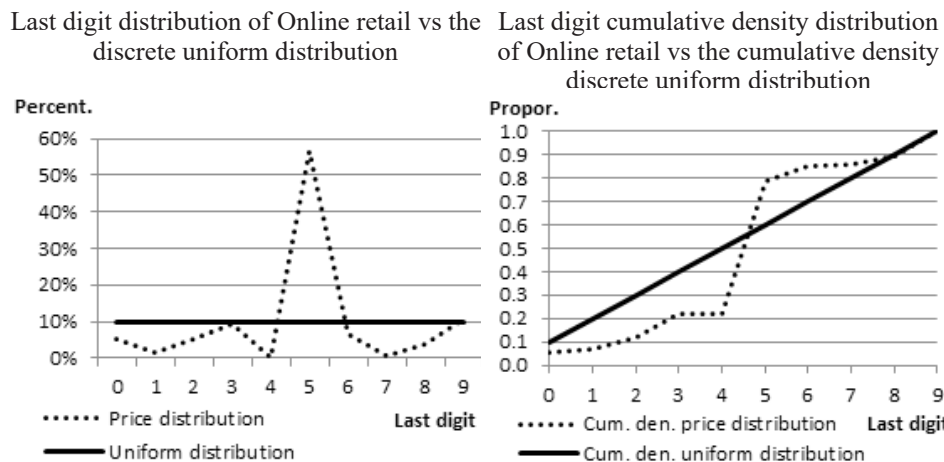


Figure 4, where the last digit distributions and cumulative density distributions of the three price variables are compared with the discrete uniform distribution, clearly shows that no one of the three price variables follows the discrete uniform distribution.

Table 7. Results of chi-square tests, Kolmogorov-Smirnov Z tests and mean absolute deviations of last digits of Retail food prices, Basic list of medicines and Online retail

| Test / measure | Statistics | Retail food prices | Basic list of medicines | Online retail |
|---------------------------|-------------------------------------|--------------------|-------------------------|---------------|
| Chi-square test | Empirical value | 168,482 | 1,160 | 1,358,549 |
| | Degrees of freedom | 9 | 9 | 9 |
| | Theoretical value ($\alpha=0.05$) | 16.92 | 16.92 | 16.92 |
| | p-value | <0.0001 | <0.0001 | <0.0001 |
| Kolmogorov-Smirnov Z test | Empirical value | 0.6559 | 0.1781 | 0.2807 |
| | Theoretical value ($\alpha=0.05$) | 0.0074 | 0.0240 | 0.0018 |
| Mean absolute deviation | Empirical value | 0.1462 | 0.0356 | 0.0946 |

Results from Table 7 confirmed the conclusions from the graphical analysis. The conducted chi-square tests and Kolmogorov-Smirnov Z tests are showing that, at significance level of 5%, none of the three observed price variables does not have last digit distribution which follows the discrete uniform distribution. In addition, mean absolute deviation values shows that there are large differences between the last digit distributions of the three observed price variables and the discrete uniform distribution.

First hypothesis of the paper which stated that distribution of first digits in prices fits to the Benford's law distribution cannot be accepted although first digit distribution of variable Basic list of medicines seems to be the closest to the Benford's law distribution. Second hypothesis of the paper which stated that distribution of last digits in prices differs from the uniform distribution pointing to the psychological price formation can be accepted. Digits with the most probability to appear as a last digit in price datasets were digits 0, 5 and 9 respectively.

6. CONCLUSIONS

Benford's law is naturally occurring law which states that digits have predictable frequencies of appearance with digit one having the highest frequency. The goal of the paper was application of Benford's law in psychological pricing detection using main statistical deviation measures. First hypothesis of the paper that was tested claimed that distribution of first digits in prices fits to the Benford's law distribution. Results of the analysis has shown that it cannot be accepted suggesting that first digit numbers that are influenced by human thought such as psychology formed prices. Second hypothesis of the paper which stated that distribution of last digits in prices differs from the uniform distribution pointing to the psychological price formation can be accepted. As expected, according to economic theory, digits with the most probability of appearance as a last digit in price datasets were digits 0, 5 and 9 respectively. Results and conclusions of this study can be useful to economic policy makers and many other economic subjects for detection of psychological price formation. The conclusions about price formations are not general and cannot be taken as generally valid. It can be observed as specific price formations in Croatia and UK. Limitation of research is related to the application of Benford's law in psychological pricing detection using first and last digit psychological pricing techniques and not all psychological pricing techniques. Recommendations for future research can go in the way to differentiate between countries and various product classes in different time periods using additional psychological-pricing techniques.

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PRIMJENA BENFORDOVA ZAKONA PRILIKOM OTKRIVANJA PSIHOLOŠKO ODREĐENIH CIJENA

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Prethodno priopćenje

Sažetak

Ovaj rad predstavlja primjenu Benfordovog zakona prilikom otkrivanja psihološko određenih cijena. Benfordov zakon je prirodni zakon koji navodi da znamenke imaju predvidljive frekvencije pojavljivanja počevši sa znamenkom jedan koja ima najveću frekvenciju. Psihološko određivanje cijena jedna je od marketinških strategija usmjerenih na postavljanje cijena koje imaju psihološki utjecaj na određene potrošače. Kako bi se istražila primjena Benfordovog zakona prilikom otkrivanja psihološki određenih cijena, Benfordov zakon se promatra u slučaju prve i zadnje znamenke. Kako bi se utvrdilo da li se prve i posljednje znamenke promatranih cijena pojavljuju prema Benfordovoj ili diskretnoj uniformnoj distribuciji, koriste se srednja vrijednost apsolutne devijacije, Hi-kvadrat i Kolmogorov-Smirnov Z test. Rezultati analize provedene na tri cjenovna seta podataka pokazali su da su najbrojnije prve znamenke 1 i 2. S druge strane, najbrojnije posljednje znamenke su 0, 5 i 9. Hi-kvadrat i Kolmogorov-Smirnov Z testovi pokazali su da, pri razini značajnosti od 5%, niti jedan od triju promatranih skupova cijena nema prvu znamenkastu distribuciju koja odgovara Benfordovoj distribuciji. Isto tako, srednja vrijednost apsolutne devijacije pokazala je da postoje velike razlike u frekvenciji pojavljivanja posljednje znamenke i diskretne uniformne distribucije što ukazuje na psihološko određivanje cijena u svim skupovima cijena.

Ključne riječi: *Benfordova distribucija, prva znamenka, završeci cijena, psihološko određivanje cijena*

JEL: C46, D91