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# Residential market ratings using fuzzy logic decision-making procedures

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**ABSTRACT**

The recent global financial crisis has highlighted the important role that the real estate market plays in the global economy. The specific character of the real estate market, the availability of market information and the sudden, unpredictable changes that often occur in that market, as well as investments, are affected by considerable risks and uncertainties. Objective monitoring of the real estate market is a requirement to maintain balance, increase security and minimise the risk of crises in urban spaces. One solution is to analyse and monitor the markets continuously, using comprehensive classification. In this paper, the authors propose the creation of a decision-making support system based on an analysis of the condition of real estate markets using ratings. The proposed procedure employs decision-making theory, data mining technology (Rough Set Theory and Value Tolerance Relation fuzzy theory) and rating scoring analysis.

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**JEL CLASSIFICATION**
B41; C10; D80; R31

## 1. Introduction

Making decisions is an integral element of human life and is the most frequent activity performed on a micro- and macro-scale. Making optimum decisions should rely on reliable data describing reality, in line with the decision-maker's preferences (Saaty, 2008). However, access to reliable data or information is difficult nowadays, in some cases because of a lack of access; in other cases due to excessive amounts of such data (so-called information noise) and difficulties in the proper selection of the right type of data.

Currently, analysis of a given market's condition, structure and characteristics is crucial to identify attractive prospects and the potential growth of an area as well as potential investment locations. Furthermore, the residential market cannot be

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analysed without considering its relationship with the quality of life or the surrounding urban areas. The link between real estate markets and urban growth potential has been highlighted in several studies (e.g., D'Arcy and Keogh, 1999; Leung, 2004).

Due to globalisation, the implementation of information technology (IT) solutions and the increasing mobility of people, making decisions in the real estate market is no longer limited to an analysis of local and technical factors of properties, i.e., so-called endogenous factors, and now extends to exogenous factors (e.g., labour market absorption, economic potential of the area), which influence the long-term efficiency of investments. Residential properties not only constitute a common element for securing basic existential needs and capital location, but they are also an important factor determining the conditions, development and investment potential of a given region. Cities and regions wishing to achieve a dominant position via their policies seek to attract as many entities and types of activity as possible. An accurate prediction of the real estate market potential is essential to prospective homeowners, developers, investors, appraisers, tax assessors, local authorities and other real estate market participants, such as mortgage lenders and insurers (Ball & Wood, 1999; Case, 2000; Frew & Jud, 2003; Irwin, 1993; Jaffe & Sirmans, 1989; Janowski et al., 2014; McCue & Belskym, 2007; Źróbek & Grzesik, 2013). Moreover, learning the lessons from the last outbreak of the Global Financial Crisis (2007–2008), primarily initiated by the insolvency of mortgage borrowers, current and objective monitoring of the real estate market is necessary to maintain balance, increase security and minimise the risk of crisis in many aspects of human existence in urban spaces.

Various types of classification and segmentation are used to organise the information across the wide range of phenomena characterising a real estate market. Real estate markets are usually classified on the basis of property type, location, income-producing, potential, typical investor characteristics, typical tenant characteristics and other attributes recognised by those participating in the exchange of property (Bernat et al., 2014; Razzak, 2015). For instance, Dubin and Goodman (1982) proposed methods for analysing non-nested submarkets. The idea of identifying housing submarket boundaries by developing and estimating the parameters of a hierarchical model for house prices was proposed by Goodman and Thibodeau (2003). Additionally, Kulesza and Bełej (2015) proposed the segmentation of real estate markets due to time delays, relaxation time and long-term equilibrium levels of time series in residential local markets.

Significant classification of real estate markets allowing for a mutual review of individual markets and comparison in terms of a hyper-local and/or global approach are distinguished in the following classifications: ranking classification (excluding the elements of the comparative assessment of the market) and rating classification (including elements of comparative assessment and market condition diagnosis).

The need to classify the real estate market in a rating form was expressed in several documents and standards outlined by European Property and Market Rating (2003), Research on Property and Market Rating in China based on Basel II (2008), Kalberer (2012), and Kaklauskas et al. (2015). The European Property and Market Rating (2003) and Kalberer (2012) define "Property and Market Rating" as a versatile instrument for assessing the quality of property. Other authors consider real estate market ratings to be useful tools for developing portfolio investment strategies (Anglin & Yanmin, 2011) or formulating long-short portfolio strategies on housing indices for more-risky and less-risky assets characterised by low liquidity (Beracha & Skiba, 2011).

Therefore, the authors propose a procedure for developing a decision-making system based on an analysis of the condition of real estate markets in a rating form. The proposed procedure uses decision-making theory, data mining technology – Rough Set Theory (RST) and Value Tolerance Relation fuzzy theory (VTR) – and a rating scoring analysis. This allowed decision rules to be established in a system form for comparative assessment and market condition diagnosis. The study was conducted based on the largest Polish and Italian markets in 2014. The reason for this choice was to test the efficiency of the proposed procedure by comparing two different markets with approximately the same level of real estate market information. In this particular case, the stimulation of the supply and demand and the geo-localisation aspect of these countries were taken into account.

The paper is structured by sections. Section 1 provides an explanation of the choice of rating classification. Section 2 presents the methodology of the research. Section 3 presents the calculation of the decision rules based on Polish and Italian cases and provides a discussion of the achieved results. Section 4 presents the conclusions and future directions of research. The study was prepared as a result of implementation of research project No. UMO-2014/13/B/HS4/00171 financed from the funds of the National Science Centre.

## 2. Methodology of the research

### 2.1. Proposed solutions for decision-making in a real estate market

Decision-making in a real estate market is complicated because of the needs that the property must satisfy. The difficulty also lies in the diversity and imprecision of spatial attributes, the large, multidimensional scope of data to be analysed, the sensitivity of properties to environmental and economic changes and fashion, as well as heterogeneity with respect to the nature and type of individual objects. Therefore, decision-making in a real estate market is difficult and may result in great risk and uncertainty. In order to minimise risk and facilitate the process, various kinds of systems supporting decision-making processes are recommended.

The most important actions for creating effective support decision-making systems is by determining the scope of information, estimation of databases and data learning and data extraction. The typical components of systems supporting decision-making using data-mining techniques (Hand et al. 2005; Słowiński, 1992, Zavadskas & Turskis, 2011; Kaklauskas et al. 2011) consist in: the model, the structural formula of a database, the scoring functions adjusting the model to reality, the choice and optimisation of an analytical method for data exploration and a strategy of data management, access and updating.

Considering these assumptions, the authors propose the development of a scoring system to assess the condition of residential real estate markets using a component decision-supporting system. In the analytical part, entropy and the assumptions of

Rough Set Theory (RST) were used. One of the main problems was the selection of characteristics for the database. Information in a real estate market is asymmetric, in some cases it is not available and the level of aggregation information about real estate markets may vary. Due to the specificity of the information on the real estate markets, a measure of the diffusion and characteristics of market information was proposed using entropy. Therefore, the diversity, merit and usefulness of the market information was calculated on the basis of entropy weight, which, according to Shannon and Weaver (1963), is a measure of 'disorder, chaos and randomness of certain information', which is a common characteristic of information connected with real estate markets. Furthermore, this method allows data analysis without identifying a dependent variable. Our dependent variable is a rating, which is the final stage in this procedure.

On the other hand, due to the small number of observations (16 cases for Polish markets and 20 for Italian markets – statistical techniques require the greatest number of rows (observation) to validate the model), the decision rules were established based on *Boolean Algebras* (if [conditions], then [decision]), which is the main assumption of the RST formulated by Polish mathematician, Professor Zdzisław Pawlak. The classical RST was developed (Pawlakm, 1982, 21997) to analyse imprecise and vague data which are commonly found in the real estate market and accompanies decision-making (fuzzy decision-making) in that market. In this theory, an analysed phenomenon is considered as an object characterised by features related to a specified piece of information. RST with a valued tolerance relation extension is used in many sciences and it is often applied as the main support tool in decision-making systems (e.g., Bello & Verdegay, 2012; Biłozor & Renigier-Biłozor, 2014; Chi, Yeh, &, Lai, 2011; Chung & Tseng, 2012; Guoyin & Lihe, 2012; Polkowskim, 2010; Renigier-Biłozor, 2011; Zavadskas & Turskis, 2011; Zhang, 2012). Moreover, the assumptions of this theory are relatively simple, clear and repetitive in subsequent rating years without changing. RST is used to analyse data that are qualitatively and quantitatively ambiguous, imprecise and varied, commonly existing in the real estate domain. The complexity and specificity of information and the real estate market is caused by: significant variations in the quantity of available information, complex methods of data description (differences in the scale of attribute description), significant differences between real estates (no two real estates are identical), various criteria for using real estate (every real estate can be used and managed in a variety of ways), a lack of comprehensive information (due to the lack of homogeneous systems for collecting real estate data), the inaccurate and 'fuzzy' character of real estate data and the absence of homogeneous functional dependencies between real estate attributes, decision-making strategies represented by the value, function and method of real estate management.

Data mining based on statistical techniques, such as clustering technique, neural networks and decision trees, present problems in this application. In clustering techniques, there are several possible algorithms that can be used, several different dissimilarity measures and even different classification methods (hierarchical and non-hierarchical). The application of a neural network determines a final result depending on the architecture of the nodes. It is well-known that different software applying neural networks offer different answers to the same data set (Worzala et al., 1995). This problem also occurs in the application of decision trees. This explains the reason to use RST, which allows a procedure to be applied without any inference while analysing the data and provides a single solution for a data sample, which is essential for the reliability of rating the results.

### 2.2. Data description

The efficiency of a real estate market depends on effective databases and decision-making systems. The disclosure, description and availability of the information structure are essential to select and verify the necessary data. The division of information should take into account the macro-, meso- and micro-scale of the economy, along with a division into specific categories of data/information which are connected with the analysed market. The existing knowledge was compiled to develop a set of indicators for overall evaluation of real estate markets. An attempt was made to develop features with the greatest influence on market decision-making based on literature analysis and observations of participants in a real estate market. These included categories of information strictly related to the residential, economic, political, social, spatial and location realms. Each of these realms represents a different range of information that affects quality of life to various degrees. Thus, in the long term, it has an influence on decisions concerning buying, renting or selling residential real estate. In the study, in order to develop the rating decision rules for a real estate market, the main categories of information were analysed as follows: social (describing the real estate market indirectly and shows the quality of life of residents), economic and political (describing not only the current economic situation of the cities, but also the quality of the activities of local and national governments), spatial and location (giving information about the characteristics of an area, including information about facilities and planning regulations), residential (describing in detail the real estate market characteristics and information on properties, transactions and offers).

Within the range of this study, a database was developed based on the largest residential markets in Poland (16 markets) and Italy (20 markets) (Figure 1). All of the proposed markets have a major spatial impact on other regions and are the best point of reference (representative of their region) and provide access to comprehensive data. The database was developed for 2014 and the data were divided into four categories of information and designated as determinant or destimulant in relation to the residential market (Appendix A). The condition of comparability, regarding the duplication of every indicator for both markets, was taken into consideration to develop the database. The study contains 60 indicators that were collected using many sources of information (see Appendix A).

### 2.3. Procedure for elaborating the decision rules in the form of a supportive decision system

To determine the decision rules for real estate market ratings, a decision-making procedure was developed as illustrated in Figure 2.

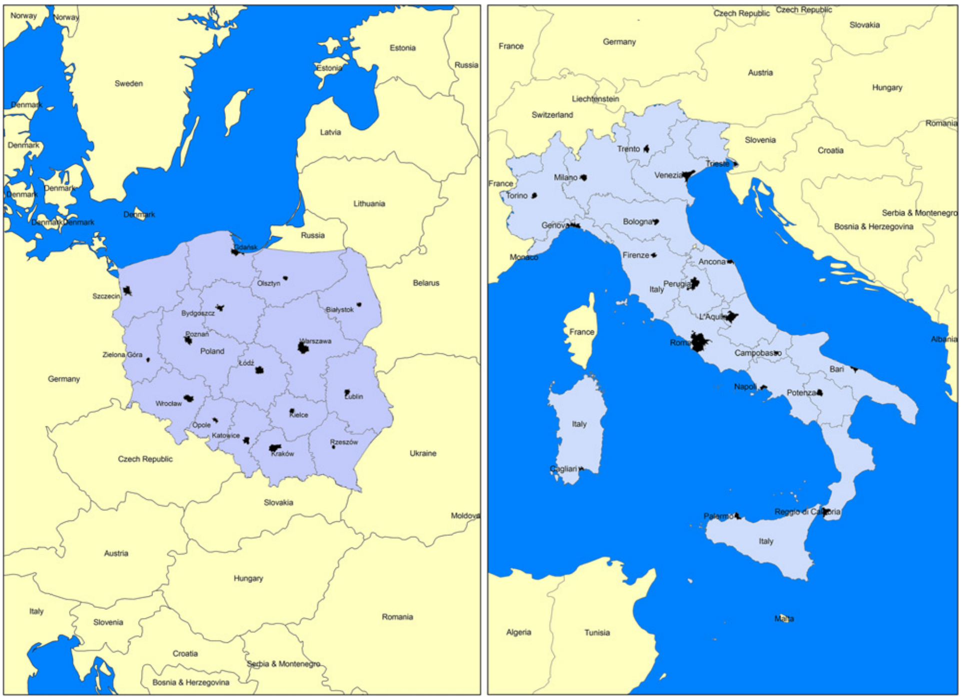


**Figure 1.** The area of real estate markets analysis for Poland and Italy. Source: Own elaboration with ArcGis utilisation.

The presented procedure was elaborated, taking into account the typical components of data-mining techniques, the specificity of market information and the phases of support decision-making systems. The stages are presented and explained in detail in the next Section.

## 3. Development of the results and implementation of the assumption based on the Polish and Italian cases

### 3.1. Preliminary preparation of the database

In the presented procedure, after the development of the database (described above), the data in the database were normalised. The aim of the normalisation was to transform the multidimensional space of the collected diagnosing variables into a one-dimensional space (objectively comparable). In the presented procedure, the normalisation of data was completed using the following formulas:

$$\text{– for determinant} \quad Z_j = \frac{X_j - X_j^{\min}}{X_j^{\max} - X_j^{\min}} \tag{1}$$

$$\text{– for destimulant} \quad Z_j = \frac{X_J^{\max} - X_j}{X_j^{\max} - X_j^{\min}} \tag{2}$$

where $Z_j$ = value of indicator after normalization, $X_j$ = value of indicator before normalization, $X_j^{max}$; $X_j^{min}$ = minimum and maximum of indicator value.

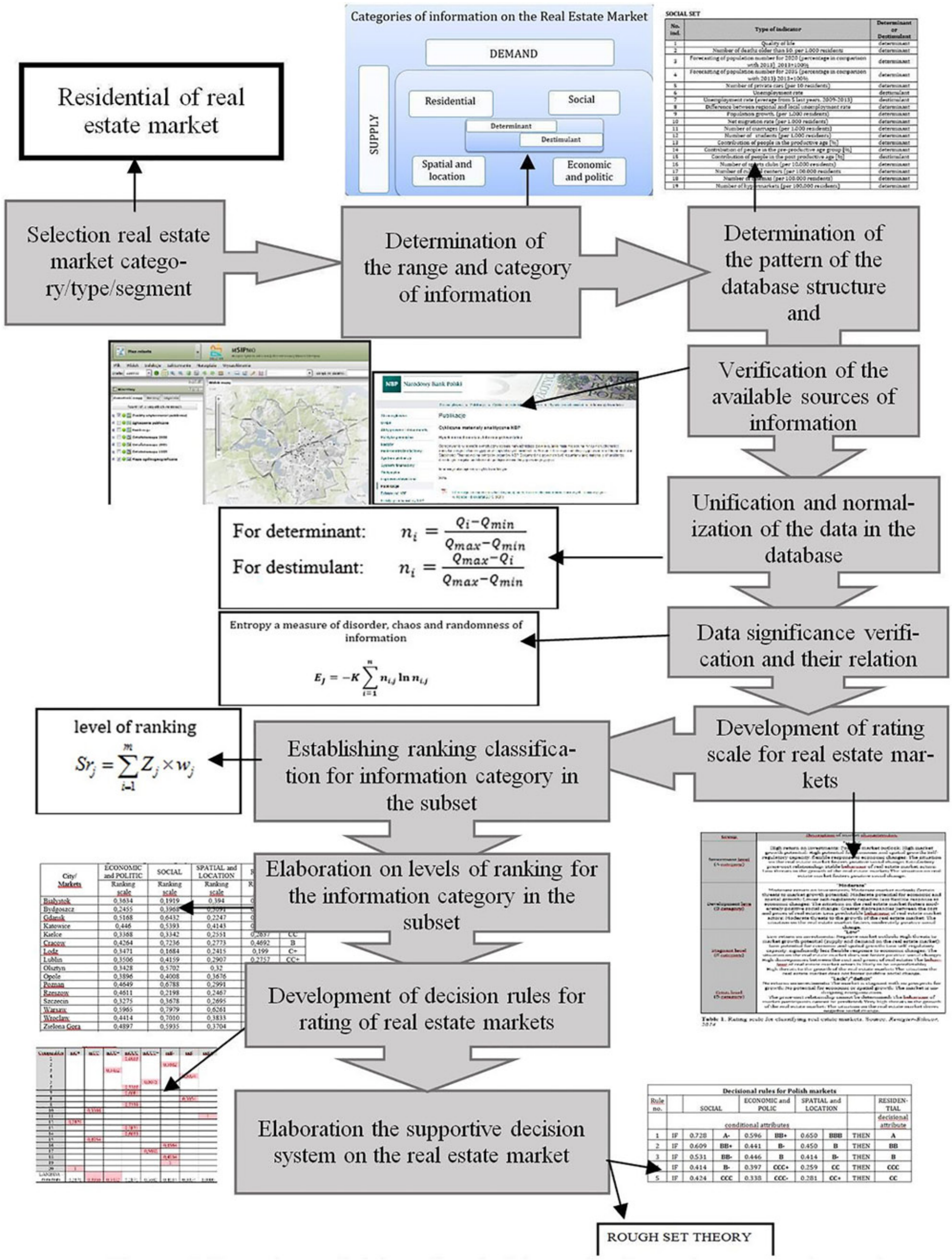


**Figure 2.** Procedure of elaborating decision rules for real estate market rating. *Source:* Own elaboration.

The determinants positively influence the features that shape the real estate market condition, while destimulants have a negative influence on them.

The next stage of the procedure consisted of the verification of the significance and diversification of data in terms of their relevance and importance concerning the purpose of the analysis. Entropy is perceived differently in numerous theories. In this simulation, a measure of entropy (weight vector determined by entropy proposed by

**Table 1.** The weight vector for social subcategories.

| No. ind. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $E_j$ | 0.9976 | 0.9962 | 0.9997 | 0.9988 | 0.9969 | 0.9784 | 0.9773 | 0.9411 | 0.9655 | 0.8689 |
| $W_j$ | 0.0062 | 0.0097 | 0.0007 | 0.0030 | 0.0081 | 0.0560 | 0.0589 | 0.1526 | 0.0895 | 0.3398 |

| No.ind | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| $E_j$ | 0.9996 | 0.9877 | 0.9967 | 0.9994 | 0.9990 | 0.9879 | 0.9879 | 0.9510 | 0.9846 |
| $W_j$ | 0.0011 | 0.0319 | 0.0084 | 0.0015 | 0.0027 | 0.0314 | 0.0314 | 0.1271 | 0.0400 |

*Source*: Own calculation.

Deng et al. 2000 and Ignasiak, 2001) was calculated for indicators in the individual categories of rating information according to the formula:

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \tag{3}$$

where $w_j$ = weight vector for particular criteria, $d_j$ = degree of internal of rating variance,

$$d_j = 1 - E_j, \quad E_j = \text{ entropy} \qquad (4),$$

$$E_j = -K \sum_{i=1}^{m} n_{i,j} \ln n_{i,j} \tag{5}$$

where $K = 1/\ln m$ (constant that guarantees $0 \le E_j \le 1$); $i = 1, \ldots, m$; $j = 1, \ldots, n$, $m$ = number of states in a particular criterion; $n$ = weight of the probability of individual information (indicator).

It must be stressed that higher entropy results in a lower weight being generated. Lower entropy indicates less uncertainty in the information flow and, consequently, the more useful the information is for the system. Table 1 presents an example of a result obtained for indicators in a social category for Polish markets.

For instance, the entropy result for indicator No. 1 (Table 1) was calculated as follows:

$$\begin{aligned} E_1 = {} & -0.3607((-0.1912) + (-0.1708) + (-0.1620) + (-0.1258) + (-0.1810) \\ & + (-0.1595) + (-0.1810) + (-0.1517) + (-0.2151) + (-0.1810) + (-0.1651) \\ & + (-0.19250) + (-0.1190) + (-0.1537) + (-0.2141) + (-0.2026)) = 0.9976 \end{aligned}$$

### *3.2. Development of the rating scale for real estate markets*

Rating, as the methodology of the comparative assessment, requires the use of rating scales. These scales must permit an assessment of the condition of real estate markets and allow for a comparable market assessment in the descriptive and numerical form. The proposed scales are shown in Table 2 and were elaborated by the authors based on the original credit rating scales.

**Table 2.** Description of the rating scale for classifying real estate markets.

| Group | Description of market characteristics | Rating scale | Numerical classification |
|---|---|---|---|
| Investment level (A category) | 'High' High return on investments; positive market outlook; high market growth potential; high potential for economic and spatial growth; self-regulatory capacity, flexible response to economic changes; the situation on the real estate market fosters positive social change; satisfactory price–cost relationship; stable behaviour of real estate market actors; low threats to the growth of the real estate market; the situation on real estate market fosters positive social change | AAA + | >0.990 |
| | | AAA | 0.956–0.989 |
| | | AAA– | 0.922–0.955 |
| | | AA+ | 0.888–0.921 |
| | | AA | 0.854–0.887 |
| | | AA– | 0.820–0.853 |
| | | A+ | 0.786–0.819 |
| | | A | 0.751–0.785 |
| | | A– | 0.718–0.751 |
| Devlopment level (B category) | 'Moderate' Moderate return on investments; moderate market outlook; certain threats to market growth potential; moderate potential for economic and spatial growth; lower self-regulatory capacity, less flexible response to economic changes; the situation on the real estate market fosters moderately positive social change; greater discrepancies between the cost and prices of real estate; less predictable behaviour of real estate market actors; moderate threats to the growth of the real estate market; the situation on the real estate market fosters moderately positive social change | BBB+ | 0.684–0.717 |
| | | BBB | 0.650–0.683 |
| | | BBB– | 0.612–0.649 |
| | | BB+ | 0.578–0.611 |
| | | BB | 0.544–0.577 |
| | | BB– | 0.510–0.543 |
| | | B+ | 0.476–0.509 |
| | | B | 0.442–0.475 |
| | | B– | 0.408–0.441 |
| Stagnant level (C category) | 'Low' Low return on investments; negative market outlook; high threats to market growth potential (supply and demand on the real estate market); low potential for economic and spatial growth; low self-regulatory capacity, significantly less flexible response to economic changes; the situation on the real estate market does not foster positive social change; high discrepancies between the cost and prices of real estate; the behaviour of real estate market actors is likely to be unpredictable; high threats to the growth of the real estate market; the situation the real estate market does not foster positive social change. | CCC+ | 0.374–0.407 |
| | | CCC | 0.340–0.373 |
| | | CCC– | 0.306–0.339 |
| | | CC+ | 0.272–0.305 |
| | | CC | 0.238–0.271 |
| | | CC– | 0.204–0.237 |
| | | C+ | 0.170–0.203 |
| | | C | 0.136–0.169 |
| | | C– | 0.102–0.135 |
| Crisis level (D category) | 'Lack'/'deficit' No returns on investments; the market is stagnant with no prospects for growth; no potential for economic or spatial growth; the market is undergoing reorganisation. The price–cost relationship cannot be determined; the behaviour of market participants cannot be predicted; very high threats to the growth of the real estate market; the situation on the real estate market drives negative social change. | D+ | 0.068–0.101 |
| | | D | 0.034–0.067 |
| | | D– | <0.033 |

*Source:* Own elaboration on the basis of Renigier-Biłozor et al. (2014).

There are three scores per group: AAA/BBB/CCC, AA/BB/CC and A/B/C/D, where scores AAA/BBB/CCC represent the highest rating and A/B/C/D the lowest. The numerical classification assumed the quantitative comparison conducting on the basis of proportional sliding scale. It involves the assumption that measure of the ranking category depends on mutual relation between the analysed objects, assuming absolute 0 as the worst measure and 1 as the best.

### 3.3. Development of rankings and ratings for real estate markets

In the next step, the level of ranking was specified using the following formula:

$$Sr_j = \sum_{i=1}^{m} Z_j \times w_j \tag{6}$$

where $Sr_j$ = level of ranking; $Z_j$ = normalised indicator; $w_j$ = weight vector.

The developed level of rankings for every category in the subset is shown in Tables 3 (Polish markets) and 3b (Italian markets). For instance, for Bialystok in the residential category, the ranking calculation is as follows:

$$\begin{aligned} social\ ranking\ for\ Bialystok: &\ 0.0039 + 0.0002 + 0.0003 + 0.0261 + 0.0038 + 0.0002 \\ &+ 0.0063 + 0.0298 + 0.0079 + 0.0102 + 0.0038 \\ &+ 0.0153 + 0.0071 + 0.0091 + 0.0041 + 0.0524 \\ &+ 0.0064 + 0.0072 + 0.0000 + 0.0000 + 0.0000 = 0.1943 \end{aligned}$$

The result can be read in the first row of Table 3.

**Table 3.** Ranking and rating for category of subsets for Polish markets.

| City/markets | Economic and Politic Ranking scale | Social Ranking scale | Spatial and Location Ranking scale | Residential Ranking scale | Residential Rating scale |
|---|---|---|---|---|---|
| Bialystok | 0.3634 | 0.1919 | 0.394 | 0.1943 | C+ |
| Bydgoszcz | 0.2455 | 0.3968 | 0.3091 | 0.3241 | CCC– |
| Gdansk | 0.5168 | 0.6432 | 0.2247 | 0.4899 | B+ |
| Katowice | 0.446 | 0.5393 | 0.4143 | 0.4784 | B+ |
| Kielce | 0.3368 | 0.3342 | 0.2551 | 0.2637 | CC |
| Cracow | 0.4264 | 0.7236 | 0.2773 | 0.4692 | B |
| Lodz | 0.3471 | 0.1684 | 0.2415 | 0.199 | C+ |
| Lublin | 0.3506 | 0.4159 | 0.2907 | 0.2757 | CC+ |
| Olsztyn | 0.3428 | 0.5702 | 0.3200 | 0.3742 | CCC+ |
| Opole | 0.3896 | 0.4008 | 0.3676 | 0.3091 | CCC– |
| Poznan | 0.4649 | 0.6788 | 0.2991 | 0.5405 | BB– |
| Rzeszow | 0.4611 | 0.2198 | 0.2467 | 0.2646 | CC |
| Szczecin | 0.3275 | 0.3678 | 0.2695 | 0.3246 | CCC– |
| Warsaw | 0.5965 | 0.7979 | 0.6261 | 0.7292 | A– |
| Wroclaw | 0.4414 | 0.7010 | 0.3833 | 0.5361 | BB– |
| Zielona Gora | 0.4897 | 0.5935 | 0.3704 | 0.3786 | CCC+ |
| *k – threshold* (unbiased standard deviation $\hat{\sigma}$ for c4 = 0.9835) | 0.0888 | 0.2031 | 0.1002 | | |

*Source:* Own calculation.

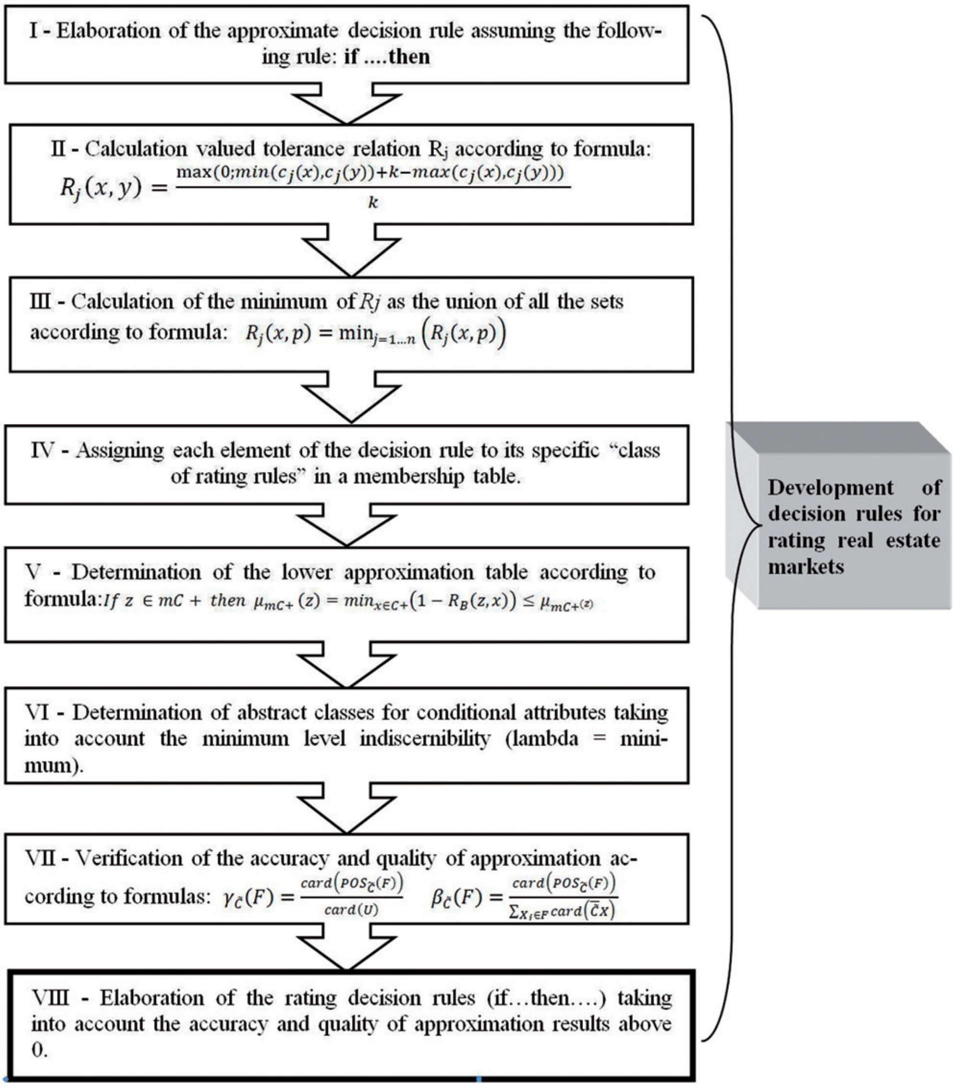


**Figure 3.** Analytical procedure of elaborating decision rules for real estate market ratings. Source: own elaboration.

Therefore, the rating scale in Table 2 was finally applied to the ranking scale obtained for each city. For example, for Bialystok in the first row of Table 3, the rating was indicated as follows: ranking scale = 0.1943; therefore, according to Table 2 the assigned rating will be C+.

### *3.4. Development of decision rules for rating real estate markets using Rough Set Theory*

In this part of the analysis, the decision rules (see Figure 2) were developed using Rough Set Theory. The authors proposed an analytical procedure based on RST which consisted of several stages (Figure 3).

The first stage of the analysis is concerned with the relationship between objects and their features in the form of an 'informative system', whose rows represent 'universe units' or objects. The informative system $S$ is expressed in formal terms, as in the following equation:

$$S = \left\langle U, Q, V_q, f \right\rangle \qquad [7]$$

where $U$ = universe or finite element set (all residential real estate markets); $Q$ = finite set of features (main categories of data that classify the conditions of real estate markets); $V_q$ = feature with a $q$ domain; $f$ = information function, that describes the relationship between object and features belong to the $Q$ set.

In this theory it is very important to highlight the assumptions whether two objects are considered as indiscernible (similar) in the concept of granular information. When $U$ is the universe, $X$ is a universe object set (real estate markets with known rating classification), $Q$ is the features (ranking categories of data) that classify the conditions of real estate markets (that belongs to $U$ universe), and $C$ is a features subset. Assuming that $S_c = \left\langle U, \widetilde{C} \right\rangle$ is the *approximation* area, and any set $C \subseteq U$ then the lower approximation and at the same time positive area of the set $X$ $w$ $S_c$ is the set:

$$\underline{\widetilde{C}}X = \left\{ x \in U : [x]_{\widetilde{C}} \subseteq X \right\} = POS_C(X) \qquad (8)$$

the upper approximation $X$ $w$ $S_c$ is the set:

$$\overline{\widetilde{C}}X = \left\{ x \in U : [x]_{\widetilde{C}} \cap X \right\} \neq 0 \qquad (9)$$

whereas the boundary region is expressed as:

$$BN_C(X) = \overline{\widetilde{C}}X - \underline{\widetilde{C}}X \qquad (10).$$

Due to this fact, for any $X \subseteq U$, $X$ is $\widetilde{C}$-accurate if and only if $\underline{\widetilde{C}}X = \overline{\widetilde{C}}X$, whereas $X$ is approximated when $\underline{\widetilde{C}}X = \overline{\widetilde{C}}X$. The approximation of families of sets, as well as single sets, can be characterised with the use of the following measures:

- quality of approximation of family $F$ in the space of approximation $S$ relative to a set of attributes $C$:

$$\gamma_{\widetilde{C}}(F) = \frac{card\left(POS_{\widetilde{C}}(F)\right)}{card(U)} \qquad (11)$$

- accuracy of approximation of family $F$ in the space of approximation $S$ relative to a set of attributes $C$:

$$\beta_{\widetilde{C}}(F) = \frac{card\left(POS_{\widetilde{C}}(F)\right)}{\sum_{X_i \in F} card\left(\overline{\widetilde{C}}X_i\right)} \qquad (12)$$

The term card ($POS_C(F)$) is cardinality (card) and is a measure of the number of the elements in the set $POS_C(F)$. For these factors, the following regularities occur: $0 \leq \beta_{\tilde{C}}(F) \leq \gamma_{\tilde{C}}(F) \leq 1$ and if all the elements of the family $F$ are $C$-accurate sets, then: $\beta_{\tilde{C}}(F) = \gamma_{\tilde{C}}(F) = 1$.

Usually the information systems are presented within tables of information, by dividing the features in conditional ($C$) and decisional ($D$) set. Each object $u \in U$ in the table of decisions $TD = (U,C,D,V,f)$ can be presented in the form of a conditional sentence (If … Then …) and regarded as a decision rule. There are two general types of decisional rules. One of them is 'exact decisional or deterministic rule', where the decisional set contains the conditional attributes with quality and accuracy of approximation equal to 1 concerning crisp indiscernibility relation. The second is 'approximate decisional rule', in which the decisional set contains the conditional attributes with quality and accuracy of approximation lower than 1 but higher than 0 concerning the more vague, fuzzy relation. In this case, casual relationship between features expressed by the social, economic and special condition of the area and residential market are definitely more imprecise, vague and fuzzy. Due to the application value tolerance relation (Stefanowski & Tsoukias, 2000) to the conventional RST based on a crisp indiscernibility relation, the more flexible way to deal with the indiscernibility relation was obtained and better matched the real estate market analysis.

The value tolerance relation is expressed in the formula:

$$R_j(x,y) = \frac{\max\left(0; min\left(c_j(x), c_j(y)\right) + k - max\left(c_j(x), c_j(y)\right)\right)}{k} \tag{13}$$

where $R_j(x,y)$ = relation between sets with membership function [0,1]; $c_j(x)$, $c_j(y)$ = feature of the analysed real estate market; $k$ = threshold for the feature set of a given real estate market, considering objects as indiscernible without having identical values. The formula is used to compare two data sets and obtained result within the 0–1 range marks the level of the indiscernibility relation. In the formula output, there is a level of indiscernibility relation between the object and the rule, assuming a $k$ level of threshold for the measure of the attribute. In this research, the authors proposed the $k$-threshold as the unbiased standard deviation (proposed by d'Amato, 2007). The formula, developed and discussed by Stefanowski and Tsoukias (2000), has been applied in real estate market analyses by d'Amato (2002, 2007, 2015) and Renigier-Biłozor Wiśniewski, Biłozor, & Kaklauskas (2014).

According to RST, the first step of the procedure for developing decision rules for rating real estate markets (see Figure 3) is elaboration of the approximate decision rule (Figure 3, step I). The approximate decision rule assumes the division of the economic, social and special features of the markets as a conditional part and residential features as a decision part in the following rule:

$$\textbf{if}(\text{ECONOMIC and POLITIC} = \text{C}_1) \text{ and } (\text{SOCIAL} = \text{C}_2) \textbf{ and } (\text{SPATIAL and LOCATION} = \text{C}_3) \textbf{ then } (\text{RESIDENTIAL} = \text{D})$$

**Table 4.** Ranking and rating for category of subsets for Italian markets.

| City/markets | Economic and Politic Ranking scale | Social Ranking scale | Spatial and Location Ranking scale | Residential Ranking scale | Residential Rating scale |
|---|---|---|---|---|---|
| Ancona | 0.4098 | 0.5428 | 0.1432 | 0.3513 | CCC |
| Aosta | 0.4205 | 0.5084 | 0.4376 | 0.4111 | B– |
| Bari | 0.4503 | 0.4153 | 0.1703 | 0.2978 | CC+ |
| Bologna | 0.4744 | 0.6763 | 0.3108 | 0.4567 | B |
| Cagliari | 0.5240 | 0.4894 | 0.2395 | 0.3821 | CCC+ |
| Campobasso | 0.3720 | 0.3752 | 0.2328 | 0.3566 | CCC |
| Catanzaro | 0.5157 | 0.3904 | 0.2154 | 0.3546 | CCC |
| Firenze | 0.4832 | 0.7217 | 0.3125 | 0.4418 | B |
| Genova | 0.4187 | 0.4918 | 0.3156 | 0.3642 | CCC |
| l'Aquila | 0.3917 | 0.4252 | 0.2003 | 0.2511 | CC |
| Milano | 0.8081 | 0.6139 | 0.4885 | 0.5641 | BB |
| Napoli | 0.3735 | 0.2383 | 0.3143 | 0.1826 | C+ |
| Palermo | 0.3776 | 0.2809 | 0.3047 | 0.3676 | CCC |
| Perugia | 0.3472 | 0.6324 | 0.1659 | 0.3529 | CCC |
| Potenza | 0.4426 | 0.3238 | 0.2577 | 0.2385 | CC |
| Roma | 0.4976 | 0.5789 | 0.3292 | 0.4116 | B– |
| Torino | 0.3595 | 0.4740 | 0.4273 | 0.3849 | CCC+ |
| Trento | 0.4363 | 0.7160 | 0.2657 | 0.4223 | B– |
| Trieste | 0.2866 | 0.6947 | 0.3847 | 0.4125 | B– |
| Venezia | 0.5005 | 0.3759 | 0.0617 | 0.1819 | C+ |
| *k – threshold* (unbiased standard deviation $\hat{\sigma}$ for $c4 = 0.9869$) | 0.1075 | 0.1484 | 0.1078 | | |

*Source:* Own calculation.

The next part of the procedure involves calculation of the tolerance relation $R_j$ according to formula [13] (Figure 3, step II). The calculation of the object comparison through the tolerance relation values for Polish and Italian markets is presented in Appendix B and C (Tables 2.1–2.3 and Tables 3.1–3.3). Due to the small number of observations, the standard deviation was calculated using unbiased estimation, according to the Cochran theorem that the C4 factor for 16 observations (Polish markets) is 0.98348 and for 20 (Italian markets) it is 0.98693 (see Tables 3 and 4). For example, for the set spatial and location (Polish markets, Table 2.3), the comparison between object 1 and object 2 has the value 0.1968. This value is obtained as follows:

$$R_j(x_1, y_2) = \frac{\max(0; 0.3091 + 0.1002 - 0.3940)}{0.1002} = 0.1528$$

A further step assumes the calculation of the minimum of $R_j$ (Figure 3, step III) as the union of all the sets and will give a final result of the comparison (Appendix B and C, Tables 2.4 and 3.4). For example, the minimum of the comparison between object 1 and object 2 (Polish markets) was calculated as follows:

$$R_j(x, p) = \min_{j=1\ldots n} \left(R_j(x, p)\right) = \min(0; 0; 0.1528) = 0 \tag{14}$$

In the following step, each element of the decision rule is assigned to its specific class in the membership table presented in Tables 5 and 6. In this step, each element of the rule sample is assigned to its specific decision attribute class (Figure 3, step IV) on the basis of Tables 3 and 4. For example, the first decision attribute class for

**Table 5.** Membership table of attribute decisions for Polish markets.

| Objects/markets | mC+ | mCC | mCC+ | mCCC− | mCCC+ | mB | mB+ | mBB− | mA− |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 7 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 12 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 16 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

*Source:* Own elaboration.

**Table 6.** Membership table of attribute decisions for Italian markets.

| Objects/markets | mC+ | mCC | mCC+ | mCCC | mCCC+ | mB– | mB | mBB |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 9 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 12 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 15 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 20 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Source:* Own elaboration.

Polish markets is $mC+$. Table 3 indicates that Bialystok and Lodz belong to this class, thus they are objects 1 and 7 (see Table 5).

This table highlights the membership of each observation (market) to a specific and clear real estate market rating class (attribute decision).

The next step of the procedure involves the determination of the lower approximation table and, according to d'Amato (2008), calculating the minimum of the complement of $R_j$ derived from a comparison between the object and the elements that do not belong to the decision class taken into account (Figure 3, step V). For example, the lower approximation of the decision class $mC+$ for Polish markets was calculated as follows:

$$If\ \ z\ \ \in mC+\ \ then\ \ \mu_{mC+}(z) = min_{x \in C+}\left(1 - R_B(z,\ \ x)\right) \leq \mu_{mC+^{(z)}}\ \ = 0.8162 \qquad (15)$$

**Table 7.** Lower approximation table for Polish markets.

| Objects/markets | mC+ | mCC | mCC+ | mCCC– | mCCC+ | mB | mB+ | mBB– | mA– |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | – | – | – | – | – | – | – | – |
| 2 | – | – | – | 0.9230 | – | – | – | – | – |
| 3 | – | – | – | – | – | – | 0.7424 | – | – |
| 4 | – | – | – | – | – | – | 0.4919 | – | – |
| 5 | – | 0.1654 | – | – | – | – | – | – | – |
| 6 | – | – | – | – | – | 0.4333 | – | – | – |
| 7 | 0.8162 | – | – | – | – | – | – | – | – |
| 8 | – | – | 0.2600 | – | – | – | – | – | – |
| 9 | – | – | – | – | 0.7596 | – | – | – | – |
| 10 | – | – | – | 0,6818 | – | – | – | – | – |
| 11 | – | – | – | – | – | – | – | 0.4333 | – |
| 12 | – | 1 | – | – | – | – | – | – | – |
| 13 | – | – | – | 0.1654 | – | – | – | – | – |
| 14 | – | – | – | – | – | – | – | – | 1 |
| 15 | – | – | – | – | – | – | – | 0.5437 | – |
| 16 | – | – | – | – | 0.4919 | – | – | – | – |
| LAMBDA = minimum | 0.8162 | 0.1654 | 0.2600 | 0.1654 | 0.4919 | 0.4333 | 0.4919 | 0.4333 | 1 |

*Source:* Own elaboration.

**Table 8.** Lower approximation table for Italian markets.

| Objects/markets | mC+ | mCC | mCC+ | mCCC | mCCC+ | mB– | mB | mBB |
|---|---|---|---|---|---|---|---|---|
| 1 | – | – | – | 0.6033 | – | – | – | – |
| 2 | – | – | – | – | – | 0.5682 | – | – |
| 3 | – | – | 0.5452 | – | – | – | – | – |
| 4 | – | – | – | – | – | – | 0.3054 | – |
| 5 | – | – | – | – | 0.6673 | – | – | – |
| 6 | – | – | – | 0.3366 | – | – | – | – |
| 7 | – | – | – | 0.6081 | – | – | – | – |
| 8 | – | – | – | – | – | – | 0.3054 | – |
| 9 | – | – | – | 0.7339 | – | – | – | – |
| 10 | – | 0.3366 | – | – | – | – | – | – |
| 11 | – | – | – | – | – | – | – | 1 |
| 12 | 0.2871 | – | – | – | – | – | – | – |
| 13 | – | – | – | 0.2871 | – | – | – | – |
| 14 | – | – | – | 0.6033 | – | – | – | – |
| 15 | – | 0.6054 | – | – | – | – | – | – |
| 16 | – | – | – | – | – | 0.6564 | – | – |
| 17 | – | – | – | – | 0.5682 | – | – | – |
| 18 | – | – | – | – | – | 0.4184 | – | – |
| 19 | – | – | – | – | – | 1 | – | – |
| 20 | 1 | – | – | – | – | – | – | – |
| LAMBDA = minimum | 0.2871 | 0.3366 | 0.5452 | 0.2871 | 0.5682 | 0.4184 | 0.3054 | 1 |

*Source:* Own elaboration.

In Tables 7 and 8, the lower approximations have been calculated for each decision class.

The next stage involves determining abstract classes for conditional attributes in each decision class, taking into account the minimum level indiscernibility (Figure 3, step VI). It was assumed as the level of the minimum lambda (Tables 7 and 8) for each class decision separately, based on which the reduction of objects in abstraction classes was conducted. Objects that are above the minimum level (lambda minimum) are considered indiscernible and enter the individual class of abstraction.

For example, decision class *mC+* (Polish markets) belongs to two objects, 1 and 7, so the class abstraction determined on the basis of Tables 2.4 and 3.4 (Appendix B

**Table 9.** Abstract class for all decision classes for Polish markets.

| Decision class | Objects in particular abstract class | Abstract class | | |
|---|---|---|---|---|
| mC+ | U / $IND_{SI}$ (C) = {$C_1$ i $C_7$} | $C_1$ = {1} | $C_7$ = {7} | — |
| mCC | U / $IND_{SI}$ (C) = {$C_5$ i $C_{12}$} | $C_5$ = {5,7,8,13} | $C_{12}$ = {12} | — |
| mCC+ | U / $IND_{SI}$ (C) = {$C_8$} | $C_8$ = {8,5,13} | — | — |
| mCCC– | U / $IND_{SI}$ (C) = {$C_2$ i $C_{10}$ i $C_{13}$ } | $C_2$ = {2} | $C_{10}$ = {4,8,10} | $C_{13}$ = {5,8,13} |
| mCCC+ | U / $IND_{SI}$ (C) = {$C_9$ i $C_{16}$} | $C_9$ = {9} | $C_{16}$ = {4,16} | — |
| mB | U / $IND_{SI}$ (C) = {$C_6$ } | $C_6$ = {6,11} | — | — |
| mB+ | U / $IND_{SI}$ (C) = {$C_3$ i $C_4$} | $C_3$ = {3,4} | $C_4$ = {3,4} | — |
| mBB– | U / $IND_{SI}$ (C) = {$C_{11}$ i $C_{15}$ } | $C_{11}$ = {6,11} | $C_{15}$ = {15} | — |
| mA– | U / $IND_{SI}$ (C) = {$C_{14}$ } | $C_{14}$ = {14} | — | — |

*Source:* Own elaboration.

**Table 10.** Abstract class for all decision classes for Italian markets.

| Decision class | Objects in particular abstract class | Abstract class | | |
|---|---|---|---|---|
| mC+ | U / $IND_{SI}$ (C) = {$C_{12}$ i $C_{20}$} | $C_{12}$ = {12,13,15} | $C_{20}$ = {20} | – |
| mCC | U / $IND_{SI}$ (C) = {$C_{10}$ i $C_{15}$} | $C_{10}$ = {3,6,10} | $C_{15}$ = {6,12,13,15} | – |
| mCC+ | U / $IND_{SI}$ (C) = {$C_3$} | $C_3$ = {3} | – | – |
| mCCC | U / $IND_{SI}$ (C) = {$C_1$ i $C_6$ i $C_7$ i $C_9$ i $C_{13}$ i $C_{14}$ } | $C_1$ = {1,14} | $C_6$ = {6,10,13,15} | $C_7$ = {3,5,7,15} |
| | | $C_9$ = {9} | $C_{13}$ = {6,12,13,15} | $C_{14}$ = {1,14} |
| mCCC+ | U / $IND_{SI}$ (C) = {$C_5$ i $C_{17}$} | $C_5$ = {5} | $C_{17}$ = {17} | – |
| mB– | U / $IND_{SI}$ (C) = {$C_2$ i $C_{16}$ i $C_{18}$ i $C_{19}$} | $C_2$ = {2,17} | $C_{16}$ = {16} | $C_{18}$= {4,8,18} |
| | | $C_{19}$ = {19} | – | – |
| mB | U / $IND_{SI}$ (C) = {$C_4$ i $C_8$} | $C_4$ = {4,8,16,18} | $C_8$ = {4,8} | – |
| mBB | U / $IND_{SI}$ (C) = {$C_{11}$ } | $C_{11}$ = {11} | – | – |

*Source:* Own elaboration.

**Table 11.** The decision attribute approximation of classification sets for Polish markets.

| Decision classes | Number of objects in indiscernibility class of decision attributes | Number of objects in lower approximation | Number of objects in upper approximation | Accuracy of approximation | Quality of approximation |
|---|---|---|---|---|---|
| mC+ | 2 | 2 | 2 | 1 | 1 |
| mCC | 2 | 1 | 5 | (1/5) 0.20 | (1/2) 0.50 |
| mCC+ | 1 | 0 | 3 | 0 | 0 |
| mCCC– | 2 | 1 | 7 | 0.14 | 0.50 |
| mCCC+ | 2 | 1 | 3 | 0.33 | 0.50 |
| mB | 1 | 0 | 2 | 0 | 0 |
| mB+ | 2 | 2 | 2 | 1 | 1 |
| mBB– | 2 | 1 | 3 | 0.33 | 0.50 |
| mA– | 1 | 1 | 1 | 1 | 1 |

*Source:* Own elaboration

and C) is defined as in Table 7: C5 = 1 = {rule 1 (1); and C7 = {7 (1)}. The analysis therefore leads to the following classes of abstraction for each decision rule (see Tables 9 and 10).

In the next stage, the accuracy and approximation quality for individual decision rules were verified (Figure 3, step VII). The calculations are in Tables 11 and 12.

Taking into account the assumptions of the 'approximate decision rule' for fuzzy relations, the decision rules were chosen with accuracy and approximation qualities above 0. According to Tables 13 and 14, the representation of the indiscernibility class of decision attributes constitutes the decision rules (Figure 3, step VIII).

The results indicated the relations between a particular social, economic, political, spatial or location rating and a residential rating in established decision rules.

**Table 12.** The decision attributes approximation of sets classification for Italian markets.

| Decision classes | Number of objects in indiscernibility class of decision attributes | Number of objects in lower approximation | Number of objects in upper approximation | Accuracy of approximation | Quality of approximation |
|---|---|---|---|---|---|
| mC+ | 2 | 1 | 4 | 0.25 | 0.50 |
| mCC | 2 | 0 | 7 | 0 | 0 |
| mCC+ | 1 | 1 | 1 | 1 | 1 |
| mCCC | 6 | 3 | 11 | 0.27 | 0.50 |
| mCCC+ | 2 | 2 | 2 | 1 | 1 |
| B– | 4 | 2 | 7 | 0.29 | 0.50 |
| B | 2 | 2 | 4 | 0.50 | 1 |
| BB | 1 | 1 | 1/1 | 1 | 1 |

*Source:* Own elaboration.

**Table 13.** Established decision rules for Polish markets.

| Rating decision rules for Polish markets | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Economic and Politic | | Social | | Spatial and Location | | | Residential | |
| No. decision rules | | 1 | 2 | 3 | 4 | 5 | 6 | | 7 | 8 |
| 1 | If | 0.3634 | CCC | 0.1919 | C+ | 0.3940 | CCC+ | than | 0.1943 | mC+ |
| 7 | If | 0.3471 | CCC | 0.1684 | C | 0.2415 | CC | than | 0.1990 | |
| 12 | If | 0.4611 | B | 0.2198 | CC– | 0.2467 | CC | than | 0.2646 | mCC+ |
| 2 | If | 0.2455 | CC | 0.3968 | CC+ | 0.3091 | CCC– | than | 0.3241 | mCCC– |
| 9 | If | 0.3428 | CCC | 0.5702 | BB | 0.3200 | CCC– | than | 0.3742 | mCCC+ |
| 3 | If | 0.5168 | BB– | 0.6432 | BBB– | 0.2247 | CC– | than | 0.4899 | mB+ |
| 4 | If | 0.4460 | B | 0.5393 | BB– | 0.4143 | B– | than | 0.4784 | mB+ |
| 15 | If | 0.4414 | B | 0.7010 | BBB+ | 0.3833 | CCC+ | than | 0.5361 | mBB– |
| 14 | If | 0.5965 | BB+ | 0.7979 | A+ | 0.6261 | BBB– | than | 0.7292 | mA– |

*Source:* Own elaboration.

**Table 14.** Established decision rules for Italian markets.

| Rating decision rules for Italian markets | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Economic and politic | | Social | | Spatial and location | | | Residential | |
| No. decision rules | | 1 | 2 | 3 | 4 | 5 | 6 | | 7 | 8 |
| 20 | If | 0.5005 | B+ | 0.3759 | CCC+ | 0.0617 | D | than | 0.1819 | mC+ |
| 3 | If | 0.4503 | B | 0.4153 | B– | 0.1703 | C+ | than | 0.2978 | mCC+ |
| 1 | If | 0.4098 | B– | 0.5428 | BB– | 0.1432 | C | than | 0.3513 | mCCC |
| 14 | If | 0.3472 | CCC | 0.6324 | BBB– | 0.1659 | C | than | 0.3529 | |
| 9 | If | 0.4187 | B– | 0.4918 | B+ | 0.3156 | CCC– | than | 0.3642 | |
| 5 | If | 0.5240 | BB– | 0.4894 | B+ | 0.2395 | CC | than | 0.3821 | mCCC+ |
| 17 | If | 0.3595 | CCC | 0.4740 | B | 0.4273 | B– | than | 0.3849 | |
| 16 | If | 0.4976 | B+ | 0.5789 | BB+ | 0.3292 | CCC– | than | 0.4116 | mB– |
| 19 | If | 0.2866 | CC+ | 0.6947 | BBB+ | 0.3847 | CCC+ | than | 0.4125 | |
| 8 | If | 0.4832 | B+ | 0.7217 | A– | 0.3125 | CCC– | than | 0.4418 | mB |
| 11 | If | 0.8081 | A+ | 0.6139 | BBB– | 0.4885 | B+ | than | 0.5641 | mBB |

*Source:* Own elaboration.

In analysing the individual results, we can see that in Poland and Italy are there no markets that deserve a crisis level rating (*D*). This probably indicates that they were the capital cities of the regions. At the same time, Polish markets are at an investment level (*A*), due to the fact that Poland is a very rapidly developing country with positive perspectives. Simultaneously, both markets are at a stagnant level (*C*), probably due to the very unstable global economic and social situation. Additionally, Italy is experiencing a significant crisis, with a reduced number of transactions that have only recently begun increasing.

## 4. Conclusions and future directions of research

The last global financial crisis (2007–2008), primarily initiated by the insolvency of mortgage borrowers, underlined the importance of monitoring the real estate market as an absolute requirement to maintain balance, increase security and minimise the risk of crisis in urban spaces. One of the methods to monitor the real estate markets is to analyse and monitor the market continuously using comprehensive classification. For this reason, the authors proposed a rating market classification that provides quick, objective, reliable and updated information. The elaborated procedure can be implemented in every domain, especially with objects/attributes that are uncertain, imperfect or fuzzy.

The analysis found that Poland is generally in quite good condition and that Polish markets are not divided into particular regions. Warsaw almost always obtained the highest score, denoting its capital role, unlike Bialystok, Lodz and Kielce, which had the lowest scores. The situation is slightly different in Italy, where Rome does not denote its capital role and differences between the northern part (more developed) and the southern part of the country (less developed) were noted. Trento obtained the highest score in residential market segment, probably due to the link between the residential market and quality of life in urban spaces. Trento was classified as the best in 2014 for the quality of life.

The analysis allows establishing residential market decision rules which reflect the social, economic and spatial conditions of an urban area which represent quality-of-life features. A correlation analysis (Table 15) indicated that there is a statistically significant relationship between the analysed sphere of urban areas and residential market decision rules. It also confirmed that the selection of these two market areas for a comparison was appropriate.

The results also confirmed the significant link between social conditions in the residential markets in Poland, whereas in Italy the spatial and location sphere had the biggest impact on the residential market. This is due to the fact that residential housing is an important aspect of the quality of life in any community and people have many different needs primarily related to the aspects of shelter and, on the other hand, many varied needs which must be fulfilled by the real estate remaining in an inseparable relation to the surrounding space and its condition. For the rating analysis, because the decision rules associated with it rely on specific factors, which are

**Table 15.** Correlation results for Polish and Italian markets.

| | Polish markets | | | Italian markets | | |
|---|---|---|---|---|---|---|
| Pearson correlation results | Economic and politic | Social | Spatial and location | Economic and politic | Social | Spatial and location |
| Residential | 0.6853 | 0.9283 | 0.7337 | 0.4602 | 0.6918 | 0.8481 |

*Source*: Own calculation.

rather constant over a longer period, one could expect a forecasting function for the developed rules.

Classification of real estate market potential based on the conditions and specific character of the analysed urban space allows for its evaluation as well as for inspiring its development and adjustment to current and future needs. The proposed decision-making system procedure is versatile and can be implemented in any domain, especially with imprecise or vague data analysis. The presented procedure is dynamic and not static, as the set of indicators is in constant change. In many countries, the availability of information is increasing to improve transparency. More indicators are now available than in the past for real estate market analysis. The methods used and the databases are always changing. Even the concepts and the methods of classification can change over the time. Future directions of research may include spatial measures that may enrich the analysis. Integrating spatial analysis with market ratings at different levels may provide richer information integrating visual and discrete data.

It is well known that there are no ideal analytical methods without some limits. This method also has possible limits, including an increase in the analysis complexity when considering a large number of objects, as well as the possible occurrence of instability. Due to this fact, the development of automated procedures would be helpful, which will be studied in the next step in this research.

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## Appendix A

### Social Set

| No. ind. | Social type of indicator | Determinant or destimulant |
|---|---|---|
| 1 | Quality of life | Determinant |
| 2 | Number of deaths older than 50, per 1000 residents | Determinant |
| 3 | Forecasting of population number for 2020 (percentage in comparison with 2014) 2014 = 100% | Determinant |
| 4 | Forecasting of population number for 2035 (percentage in comparison with 2014) 2013 = 100% | Determinant |
| 5 | Number of private cars (per 10 residents) | Determinant |
| 6 | Unemployment rate | Destimulant |
| 7 | Unemployment rate (average from 5 last years, 2010–2014) | Destimulant |
| 8 | Difference between regional and local unemployment rate | Determinant |
| 9 | Population growth (per 1000 residents) | Determinant |
| 10 | Net migration rate (per 1000 residents) | Determinant |
| 11 | Number of marriages (per 1000 residents) | Determinant |
| 12 | Number of students (per 1000 residents) | Determinant |
| 13 | Contribution of people in the productive age (%) | Determinant |
| 14 | Contribution of people in the pre-productive age group (%) | Determinant |
| 15 | Contribution of people in the post-productive age group (%) | Destimulant |
| 16 | Number of sports clubs (per 10,000 residents) | Determinant |
| 17 | Number of cultural centres (per 100,000 residents) | Determinant |
| 18 | Number of cinemas (per 100,000 residents) | Determinant |
| 19 | Number of hypermarkets (per 100,000 residents) | Determinant |

### Economic and Political Set

| No. ind. | Economic and political type of indicator | Determinant or destimulant |
|---|---|---|
| 20 | Fuel prices per litre (€/litre) | Destimulant |
| 21 | Number of science and technology parks determinants | Determinant |
| 22 | Number of suspended business activities (per 1000 residents) | Destimulant |
| 23 | Number of new registered businesses (per 1000 residents) | Determinant |
| 24 | Number of businesses employing workers (per 10,000 residents) | Determinant |
| 25 | Local government income (€ per resident) | Determinant |
| 26 | Local government's spending (€ per resident) | Determinant |
| 27 | Difference between the national average salary and the average salary on the local market | Determinant |

### Spatial and Location Set

| No. ind. | Spatial and location type of indicator | Determinant or destimulant |
|---|---|---|
| 28 | Level of retail area ($m^2$/1000 residents) | Determinant |
| 29 | Supply of office area ($m^2$/1000 residents) | Determinant |
| 30 | Supply of warehouse area ($m^2$/1000 residents) | Determinant |
| 31 | Percent of green areas (%) | Determinant |
| 32 | Cycle path (per 10,000 residents) | Determinant |
| 33 | Roads with hard surface (km per 10,000 residents) | Determinant |
| 34 | Roads with hard surface (km per $km^2$ of city) | Determinant |
| 35 | Number of green parks in the region | Determinant |
| 36 | Population density (per $km^2$) | Determinant |
| 37 | Number of buses (per 1000 residents) | Determinant |
| 38 | Number of high schools (per 100,000 residents) | Determinant |

## Residential Set

| No. ind. | Residential type of indicator | Determinant or destimulant |
|---|---|---|
| 39 | Number of apartments (per 1000 residents) | Determinant |
| 40 | Usable space in dwelling (per resident) | Determinant |
| 41 | Average number of rooms in a dwelling | Determinant |
| 42 | Value of new mortgage agreement (€ per resident) | Determinant |
| 43 | Total number of issued construction permits (per 10,000 residents) | Determinant |
| 44 | The average area of room (per $m^2$) | Determinant |
| 45 | Number of property transactions (per 10,000 residents) | Determinant |
| 46 | Value of property transactions (€ per 1000 residents) | Determinant |
| 47 | Number of property offers – average from the most popular websites (per 1000 residents) | Determinant |
| 48 | The average number of persons in apartment | Determinant |
| 49 | Availability of apartments on primary market in terms of average salary ($m^2$) | Determinant |
| 50 | Availability of apartments on secondary market in terms of average salary ($m^2$) | Determinant |
| 51 | Offered purchasing power on the local housing market (average salary on the local market / average price per 1 $m^2$ of property on the local market) | Determinant |
| 52 | Transaction purchasing power on the local housing market (average salary on the local market / average price per 1 $m^2$ of property on the local market) | Determinant |
| 53 | Number of real estate agents on the local market (per 10,000 residents) | Determinant |
| 54 | Number of real estate appraisers on the local market (per 10,000 residents) | Determinant |
| 55 | Average time on the secondary market (in days) | Destimulant |
| 56 | Average difference between the average offered and transaction price of $m^2$ real estate on the primary and secondary market (%) | Destimulant |
| 57 | Changes in local property offered prices (percentage) | Determinant |
| 58 | Changes in local property transaction prices (%) | Determinant |
| 59 | Average difference between changes in offered and transaction prices on the secondary and primary market (percentage) | Destimulant |
| 60 | Difference between low and high standard for offered prices (€/$m^2$) | Determinant |

*Sources of information:* Eurostat-Local Data Bank, NBP reports, PAIiIZ (Polish Information and Foreign Investment Agency), Otodom.pl, gratka.pl, Polityka (quality of life ranking), E-petrol.pl, geoportal, MSiPM (municipal spatial information system), the study of conditions and directions of spatial management; Apsti.it (Associazione Parchi Scientifici Tecnologici Italiani), Camera di Commercio d'Italia; Eurostat; FIAIP-Federazione Italiana Agenti Immobiliari Professionali (online database of real estate agents); Il sole 24 ore-CASA 24 plus (website processing real estate data); Immobiliare.it (website of real estate brokerage); ISTAT (The National Institute of Statistics, Italy); Italiaoggi.it (economic and political online newspaper), Local government rankings; Minister delle infrastrutture e dei trasporti (The Ministry for Transport and Infrastructure); Ministero dello sviluppo economico (The Ministry of Economic Development); Miur-Ministero dell'istruzione della università e della ricerca (The Ministry of Education, Universities and Research); OMI-Osservatorio del mercato Immobiliare-Agenzia delle entrate (database of real estate prices); Paginegialle.it (online database of different business); Pisteciclabili.com (online database of Italian cycle path); Soldipubblici.gov.it (online database of local government spending); Stimatrixcity.it (online database of appraisers); Tecnocasa.it (websites of real estate brokerage).

## Appendix B

### Table 2.1.

$R_j$ for ECONOMIC and POLITICAL feature – Polish market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_j$ for ECONOMIC and POLITICAL feature – Polish market. | | | | | | | | | | | | | | | | |
| 1 | 1 | 0.0000 | 0.0000 | 0.0703 | 0.7006 | 0.2909 | 0.8165 | 0.8559 | 0.7681 | 0.7051 | 0.0000 | 0.0000 | 0.5959 | 0.0000 | 0.1221 | 0.0000 |
| 2 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0770 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 0.0000 | 1 | 0.2031 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4158 | 0.3731 | 0.0000 | 0.1029 | 0.1513 | 0.6950 |
| 4 | 0.0703 | 0.0000 | 0.2031 | 1 | 0.0000 | 0.7794 | 0.0000 | 0.0000 | 0.0000 | 0.3652 | 0.7873 | 0.8300 | 0.0000 | 0.0000 | 0.9482 | 0.5081 |
| 5 | 0.7006 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.8841 | 0.8447 | 0.9325 | 0.4057 | 0.0000 | 0.0000 | 0.8953 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.2909 | 0.0000 | 0.0000 | 0.7794 | 0.0000 | 1 | 0.1074 | 0.1468 | 0.0590 | 0.5858 | 0.5667 | 0.6094 | 0.0000 | 0.0000 | 0.8312 | 0.2875 |
| 7 | 0.8165 | 0.0000 | 0.0000 | 0.0000 | 0.8841 | 0.1074 | 1 | 0.9606 | 0.9516 | 0.5216 | 0.0000 | 0.0000 | 0.7794 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.8559 | 0.0000 | 0.0000 | 0.0000 | 0.8447 | 0.1468 | 0.9606 | 1 | 0.9122 | 0.5610 | 0.0000 | 0.0000 | 0.7400 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 0.7681 | 0.0000 | 0.0000 | 0.0000 | 0.9325 | 0.0590 | 0.9516 | 0.9122 | 1 | 0.4732 | 0.0000 | 0.0000 | 0.8278 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 0.7051 | 0.0000 | 0.0000 | 0.3652 | 0.4057 | 0.5858 | 0.5216 | 0.5610 | 0.4732 | 1 | 0.1524 | 0.1952 | 0.3010 | 0.0000 | 0.4170 | 0.0000 |
| 11 | 0.0000 | 0.0000 | 0.4158 | 0.7873 | 0.0000 | 0.5667 | 0.0000 | 0.0000 | 0.0000 | 0.1524 | 1 | 0.9572 | 0.0000 | 0.0000 | 0.7355 | 0.7209 |
| 12 | 0.0000 | 0.0000 | 0.3731 | 0.8300 | 0.0000 | 0.6094 | 0.0000 | 0.0000 | 0.0000 | 0.1952 | 0.9572 | 1 | 0.0000 | 0.0000 | 0.7783 | 0.6781 |
| 13 | 0.5959 | 0.0770 | 0.0000 | 0.0000 | 0.8953 | 0.0000 | 0.7794 | 0.7400 | 0.8278 | 0.3010 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 0.0000 | 0.0000 | 0.1029 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 |
| 15 | 0.1221 | 0.0000 | 0.1513 | 0.9482 | 0.0000 | 0.8312 | 0.0000 | 0.0000 | 0.0000 | 0.4170 | 0.7355 | 0.7783 | 0.0000 | 0.0000 | 1 | 0.4563 |
| 16 | 0.0000 | 0.0000 | 0.6950 | 0.5081 | 0.0000 | 0.2875 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.7209 | 0.6781 | 0.0000 | 0.0000 | 0.4563 | 1 |

### Table 2.2.

$R_j$ for SOCIAL feature – Polish market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.2995 | 0.0000 | 0.8843 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.8626 | 0.1340 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 1 | 0.0000 | 0.2985 | 0.6918 | 0.0000 | 0.0000 | 0.9060 | 0.1464 | 0.9803 | 0.0000 | 0.1286 | 0.8572 | 0.0000 | 0.0000 | 0.0316 |
| 3 | 0.0000 | 0.0000 | 1 | 0.4885 | 0.0000 | 0.6042 | 0.0000 | 0.0000 | 0.6406 | 0.0000 | 0.8247 | 0.0000 | 0.0000 | 0.2384 | 0.7155 | 0.7553 |
| 4 | 0.0000 | 0.2985 | 0.4885 | 1 | 0.0000 | 0.0927 | 0.0000 | 0.3925 | 0.8479 | 0.3182 | 0.3132 | 0.0000 | 0.1557 | 0.0000 | 0.2039 | 0.7332 |
| 5 | 0.2995 | 0.6918 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.1838 | 0.5978 | 0.0000 | 0.6721 | 0.0000 | 0.4368 | 0.8346 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0000 | 0.0000 | 0.6042 | 0.0927 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.2448 | 0.0000 | 0.7794 | 0.0000 | 0.0000 | 0.6342 | 0.8887 | 0.3595 |
| 7 | 0.8843 | 0.0000 | 0.0000 | 0.0000 | 0.1838 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.7470 | 0.0184 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0000 | 0.9060 | 0.0000 | 0.3925 | 0.5978 | 0.0000 | 0.0000 | 1 | 0.2404 | 0.9257 | 0.0000 | 0.0346 | 0.7632 | 0.0000 | 0.0000 | 0.1257 |
| 9 | 0.0000 | 0.1464 | 0.6406 | 0.8479 | 0.0000 | 0.2448 | 0.0000 | 0.2404 | 1 | 0.1660 | 0.4654 | 0.0000 | 0.0036 | 0.0000 | 0.3561 | 0.8853 |
| 10 | 0.0000 | 0.9803 | 0.0000 | 0.3182 | 0.6721 | 0.0000 | 0.0000 | 0.9257 | 0.1660 | 1 | 0.0000 | 0.1089 | 0.8375 | 0.0000 | 0.0000 | 0.0513 |
| 11 | 0.0000 | 0.0000 | 0.8247 | 0.3132 | 0.0000 | 0.7794 | 0.0000 | 0.0000 | 0.4654 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.4137 | 0.8907 | 0.5801 |
| 12 | 0.8626 | 0.1286 | 0.0000 | 0.0000 | 0.4368 | 0.0000 | 0.7470 | 0.0346 | 0.0000 | 0.1089 | 0.0000 | 1 | 0.2714 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 0.1340 | 0.8572 | 0.0000 | 0.1557 | 0.8346 | 0.0000 | 0.0184 | 0.7632 | 0.0036 | 0.8375 | 0.0000 | 0.2714 | 1 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 0.0000 | 0.0000 | 0.2384 | 0.0000 | 0.0000 | 0.6342 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4137 | 0.0000 | 0.0000 | 1 | 0.5230 | 0.0000 |
| 15 | 0.0000 | 0.0000 | 0.7155 | 0.2039 | 0.0000 | 0.8887 | 0.0000 | 0.0000 | 0.3561 | 0.0000 | 0.8907 | 0.0000 | 0.0000 | 0.5230 | 1 | 0.4708 |
| 16 | 0.0000 | 0.0316 | 0.7553 | 0.7332 | 0.0000 | 0.3595 | 0.0000 | 0.1257 | 0.8853 | 0.0513 | 0.5801 | 0.0000 | 0.0000 | 0.0000 | 0.4708 | 1 |

**Table 2.3.**

$R_j$ for Spatial and location feature – Polish market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.1528 | 0.0000 | 0.7974 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2616 | 0.7366 | 0.0530 | 0.0000 | 0.0000 | 0.0000 | 0.8932 | 0.7645 |
| 2 | 0.1528 | 1 | 0.1578 | 0.0000 | 0.4611 | 0.6827 | 0.3254 | 0.8164 | 0.8912 | 0.4162 | 0.9002 | 0.3773 | 0.6048 | 0.0000 | 0.2596 | 0.3883 |
| 3 | 0.0000 | 0.1578 | 1 | 0.0000 | 0.6966 | 0.4751 | 0.8324 | 0.3414 | 0.0490 | 0.0000 | 0.2576 | 0.7805 | 0.5529 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.7974 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0590 | 0.5340 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6907 | 0.5619 |
| 5 | 0.0000 | 0.4611 | 0.6966 | 0.0000 | 1 | 0.7785 | 0.8643 | 0.6448 | 0.3524 | 0.0000 | 0.5609 | 0.9162 | 0.8563 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0000 | 0.6827 | 0.4751 | 0.0000 | 0.7785 | 1 | 0.6428 | 0.8663 | 0.5739 | 0.0989 | 0.7825 | 0.6946 | 0.9222 | 0.0000 | 0.0000 | 0.0710 |
| 7 | 0.0000 | 0.3254 | 0.8324 | 0.0000 | 0.8643 | 0.6428 | 1 | 0.5090 | 0.2167 | 0.0000 | 0.4252 | 0.9481 | 0.7206 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0000 | 0.8164 | 0.3414 | 0.0000 | 0.6448 | 0.8663 | 0.5090 | 1 | 0.7076 | 0.2326 | 0.9162 | 0.5609 | 0.7884 | 0.0000 | 0.0760 | 0.2047 |
| 9 | 0.2616 | 0.8912 | 0.0490 | 0.0590 | 0.3524 | 0.5739 | 0.2167 | 0.7076 | 1 | 0.5250 | 0.7914 | 0.2686 | 0.4961 | 0.0000 | 0.3683 | 0.4971 |
| 10 | 0.7366 | 0.4162 | 0.0000 | 0.5340 | 0.0000 | 0.0989 | 0.0000 | 0.2326 | 0.5250 | 1 | 0.3165 | 0.0000 | 0.0211 | 0.0000 | 0.8433 | 0.9721 |
| 11 | 0.0530 | 0.9002 | 0.2576 | 0.0000 | 0.5609 | 0.7825 | 0.4252 | 0.9162 | 0.7914 | 0.3165 | 1 | 0.4771 | 0.7046 | 0.0000 | 0.1598 | 0.2885 |
| 12 | 0.0000 | 0.3773 | 0.7805 | 0.0000 | 0.9162 | 0.6946 | 0.9481 | 0.5609 | 0.2686 | 0.0000 | 0.4771 | 1 | 0.7725 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 0.0000 | 0.6048 | 0.5529 | 0.0000 | 0.8563 | 0.9222 | 0.7206 | 0.7884 | 0.4961 | 0.0211 | 0.7046 | 0.7725 | 1 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 |
| 15 | 0.8932 | 0.2596 | 0.0000 | 0.6907 | 0.0000 | 0.0000 | 0.0000 | 0.0760 | 0.3683 | 0.8433 | 0.1598 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.8713 |
| 16 | 0.7645 | 0.3883 | 0.0000 | 0.5619 | 0.0000 | 0.0710 | 0.0000 | 0.2047 | 0.4971 | 0.9721 | 0.2885 | 0.0000 | 0.0000 | 0.0000 | 0.8713 | 1 |

**Table 2.4.**

Min $R_j$ – Polish market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0770 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2576 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3182 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2039 | 0.5081 |
| 5 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.1838 | 0.5978 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.8346 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0590 | 0.0000 | 0.5667 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0710 |
| 7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1838 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0184 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5978 | 0.0000 | 0.0000 | 1.0000 | 0.2404 | 0.2326 | 0.0000 | 0.0000 | 0.7400 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0590 | 0.0000 | 0.2404 | 1.0000 | 0.1660 | 0.0000 | 0.0000 | 0.0036 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 0.0000 | 0.0000 | 0.0000 | 0.3182 | 0.0000 | 0.0000 | 0.0000 | 0.2326 | 0.1660 | 1.0000 | 0.0000 | 0.0000 | 0.0211 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 0.0000 | 0.0000 | 0.2576 | 0.0000 | 0.0000 | 0.5667 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1598 | 0.2885 |
| 12 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 0.0000 | 0.0770 | 0.0000 | 0.0000 | 0.8346 | 0.0000 | 0.0184 | 0.7400 | 0.0036 | 0.0211 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| 15 | 0.0000 | 0.0000 | 0.0000 | 0.2039 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1598 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.4563 |
| 16 | 0.0000 | 0.0000 | 0.0000 | 0.5081 | 0.0000 | 0.0710 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2885 | 0.0000 | 0.0000 | 0.0000 | 0.4563 | 1.0000 |

## Appendix C

$R_j$ for ECONOMIC and POLITICAL feature – Italian market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.9004 | 0.6232 | 0.3990 | 0.0000 | 0.6476 | 0.0150 | 0.3175 | 0.9170 | 0.8316 | 0.0000 | 0.6617 | 0.7000 | 0.4168 | 0.6947 | 0.1831 | 0.5314 | 0.7534 | 0.0000 | 0.1559 |
| 2 | 0.9004 | 1 | 0.7227 | 0.4986 | 0.0374 | 0.5480 | 0.1146 | 0.4171 | 0.9834 | 0.7321 | 0.0000 | 0.5621 | 0.6004 | 0.3172 | 0.7942 | 0.2826 | 0.4318 | 0.8530 | 0.0000 | 0.2554 |
| 3 | 0.6232 | 0.7227 | 1 | 0.7759 | 0.3147 | 0.2708 | 0.3919 | 0.6944 | 0.7061 | 0.4548 | 0.0000 | 0.2848 | 0.3231 | 0.0400 | 0.9285 | 0.5599 | 0.1545 | 0.8698 | 0.0000 | 0.5327 |
| 4 | 0.3990 | 0.4986 | 0.7759 | 1 | 0.5388 | 0.0466 | 0.616) | 0.9185 | 0.4820 | 0.2306 | 0.0000 | 0.0607 | 0.0990 | 0.0000 | 0.7043 | 0.7841 | 0.0000 | 0.6456 | 0.0000 | 0.7569 |
| 5 | 0.0000 | 0.0374 | 0.3147 | 0.5388 | 1 | 0.0000 | 0.9228 | 0.6203 | 0.0208 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2432 | 0.7548 | 0.0000 | 0.1844 | 0.0000 | 0.7820 |
| 6 | 0.6476 | 0.5480 | 0.2708 | 0.0466 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.5646 | 0.8160 | 0.0000 | 0.9859 | 0.9476 | 0.7692 | 0.3423 | 0.0000 | 0.8838 | 0.4010 | 0.2058 | 0.0000 |
| 7 | 0.0150 | 0.1146 | 0.3919 | 0.6160 | 0.9228 | 0.0000 | 1 | 0.6975 | 0.0980 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3203 | 0.8319 | 0.0000 | 0.2616 | 0.0000 | 0.8591 |
| 8 | 0.3175 | 0.4171 | 0.6944 | 0.9185 | 0.6203 | 0.0000 | 0.6975 | 1 | 0.4005 | 0.1492 | 0.0000 | 0.0000 | 0.0175 | 0.0000 | 0.6229 | 0.8655 | 0.0000 | 0.5641 | 0.0000 | 0.8383 |
| 9 | 0.9170 | 0.9834 | 0.7061 | 0.4820 | 0.0208 | 0.5646 | 0.0980 | 0.4005 | 1 | 0.7486 | 0.0000 | 0.5787 | 0.6170 | 0.3338 | 0.7776 | 0.2661 | 0.4484 | 0.8364 | 0.0000 | 0.2389 |
| 10 | 0.8316 | 0.7321 | 0.4548 | 0.2306 | 0.0000 | 0.8160 | 0.0000 | 0.1492 | 0.7486 | 1 | 0.0000 | 0.8301 | 0.8684 | 0.5852 | 0.5263 | 0.0147 | 0.6998 | 0.5850 | 0.0218 | 0.0000 |
| 11 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 0.6617 | 0.5621 | 0.2848 | 0.0607 | 0.0000 | 0.9859 | 0.0000 | 0.0000 | 0.5787 | 0.8301 | 0.0000 | 1 | 0.9617 | 0.7551 | 0.3563 | 0.0000 | 0.8697 | 0.4151 | **0.1917** | 0.0000 |
| 13 | 0.7000 | 0.6004 | 0.3231 | 0.0990 | 0.0000 | 0.9476 | 0.0000 | 0.0175 | 0.6170 | 0.8684 | 0.0000 | 0.9617 | 1 | 0.7168 | 0.3946 | 0.0000 | 0.8314 | 0.4534 | 0.1534 | 0.0000 |
| 14 | 0.4168 | 0.3172 | 0.0400 | 0.0000 | 0.0000 | 0.7692 | 0.0000 | 0.0000 | 0.3338 | 0.5852 | 0.0000 | 0.7551 | 0.7168 | 1 | 0.1115 | 0.0000 | 0.8854 | 0.1702 | 0.4366 | 0.0000 |
| 15 | 0.6947 | 0.7942 | 0.9285 | 0.7043 | 0.2432 | 0.3423 | 0.3203 | 0.6229 | 0.7776 | 0.5263 | 0.0000 | 0.3563 | 0.3946 | 0.1115 | 1 | 0.4884 | 0.2260 | 0.9413 | 0.0000 | 0.4612 |
| 16 | 0.1831 | 0.2826 | 0.5599 | 0.7841 | 0.7548 | 0.0000 | 0.8319 | 0.8655 | 0.2661 | 0.0147 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4884 | 1 | 0.0000 | 0.4297 | 0.0000 | 0.9728 |
| 17 | 0.5314 | 0.4318 | 0.1545 | 0.0000 | 0.0000 | 0.8838 | 0.0000 | 0.0000 | 0.4484 | 0.6998 | 0.0000 | 0.8697 | 0.8314 | 0.8854 | 0.2260 | 0.0000 | 1 | 0.2848 | 0.3220 | 0.0000 |
| 18 | 0.7534 | 0.8530 | 0.8698 | 0.6456 | 0.1844 | 0.4010 | 0.2616 | 0.5641 | 0.8364 | 0.5850 | 0.0000 | 0.4151 | 0.4534 | 0.1702 | 0.9413 | 0.4297 | 0.2848 | 1 | 0.0000 | 0.4025 |
| 19 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2058 | 0.0000 | 0.0000 | 0.0000 | 0.0218 | 0.0000 | 0.1917 | 0.1534 | 0.4366 | 0.0000 | 0.0000 | 0.3220 | 0.0000 | 1 | 0.0000 |
| 20 | 0.1559 | 0.2554 | 0.5327 | 0.7569 | 0.7820 | 0.0000 | 0.8591 | 0.8383 | 0.2389 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4612 | 0.9728 | 0.0000 | 0.4025 | 0.0000 | 1 |

**Table 3.2.**

$R_j$ for SOCIAL feature – Italian market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.7682 | 0.1409 | 0.1003 | 0.6399 | 0.0000 | 0.0000 | 0.0000 | 0.6564 | 0.2070 | 0.5213 | 0.0000 | 0.0000 | 0.3967 | 0.0000 | 0.7567 | 0.5362 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.7682 | 1 | 0.3727 | 0.0000 | 0.8718 | 0.1023 | 0.2045 | 0.0000 | 0.8882 | 0.4388 | 0.2895 | 0.0000 | 0.0000 | 0.1649 | 0.0000 | 0.5249 | 0.7681 | 0.0000 | 0.0000 | 0.1067 |
| 3 | 0.1409 | 0.3727 | 1 | 0.0000 | 0.5009 | 0.7296 | 0.8318 | 0.0000 | 0.4845 | 0.9338 | 0.0000 | 0.0000 | 0.0939 | 0.0000 | 0.3830 | 0.0000 | 0.6046 | 0.0000 | 0.0000 | 0.7340 |
| 4 | 0.1003 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.6946 | 0.0000 | 0.0000 | 0.5790 | 0.0000 | 0.0000 | 0.7036 | 0.0000 | 0.3436 | 0.0000 | 0.7326 | 0.8766 | 0.0000 |
| 5 | 0.6399 | 0.8718 | 0.5009 | 0.0000 | 1 | 0.2305 | 0.3327 | 0.0000 | 0.9836 | 0.5671 | 0.1613 | 0.0000 | 0.0000 | 0.0366 | 0.0000 | 0.3966 | 0.8963 | 0.0000 | 0.0000 | 0.2350 |
| 6 | 0.0000 | 0.1023 | 0.7296 | 0.0000 | 0.2305 | 1 | 0.8978 | 0.0000 | 0.2141 | 0.6634 | 0.0000 | 0.0772 | 0.3643 | 0.0000 | 0.6534 | 0.0000 | 0.3342 | 0.0000 | 0.0000 | 0.9956 |
| 7 | 0.0000 | 0.2045 | 0.8318 | 0.0000 | 0.3327 | 0.8978 | 1 | 0.0000 | 0.3163 | 0.7656 | 0.0000 | 0.0000 | 0.2622 | 0.0000 | 0.5512 | 0.0000 | 0.4364 | 0.0000 | 0.0000 | 0.9022 |
| 8 | 0.0000 | 0.0000 | 0.0000 | 0.6946 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.2736 | 0.0000 | 0.0000 | 0.3982 | 0.0000 | 0.0382 | 0.0000 | 0.9620 | 0.8180 | 0.0000 |
| 9 | 0.6564 | 0.8882 | 0.4845 | 0.0000 | 0.9836 | 0.2141 | 0.3163 | 0.0000 | 1 | 0.5506 | 0.1777 | 0.0000 | 0.0000 | 0.0531 | 0.0000 | 0.4131 | 0.8798 | 0.0000 | 0.0000 | 0.2185 |
| 10 | 0.2070 | 0.4388 | 0.9338 | 0.0000 | 0.5671 | 0.6634 | 0.7656 | 0.0000 | 0.5506 | 1 | 0.0000 | 0.0000 | 0.0278 | 0.0000 | 0.3168 | 0.0000 | 0.6708 | 0.0000 | 0.0000 | 0.6679 |
| 11 | 0.5213 | 0.2895 | 0.0000 | 0.5790 | 0.1613 | 0.0000 | 0.0000 | 0.2736 | 0.1777 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.8754 | 0.0000 | 0.7646 | 0.0576 | 0.3116 | 0.4556 | 0.0000 |
| 12 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0772 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.7129 | 0.0000 | 0.4238 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0728 |
| 13 | 0.0000 | 0.0000 | 0.0939 | 0.0000 | 0.0000 | 0.3643 | 0.2622 | 0.0000 | 0.0000 | 0.0278 | 0.0000 | 0.7129 | 1 | 0.0000 | 0.7110 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3599 |
| 14 | 0.3967 | 0.1649 | 0.0000 | 0.7036 | 0.0366 | 0.0000 | 0.0000 | 0.3982 | 0.0531 | 0.0000 | 0.8754 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.6400 | 0.0000 | 0.4362 | 0.5802 | 0.0000 |
| 15 | 0.0000 | 0.0000 | 0.3830 | 0.0000 | 0.0000 | 0.6534 | 0.5512 | 0.0000 | 0.0000 | 0.3168 | 0.0000 | 0.4238 | 0.7110 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6489 |
| 16 | 0.7567 | 0.5249 | 0.0000 | 0.3436 | 0.3966 | 0.0000 | 0.0000 | 0.0382 | 0.4131 | 0.0000 | 0.7646 | 0.0000 | 0.0000 | 0.6400 | 0.0000 | 1 | 0.2929 | 0.0762 | 0.2202 | 0.0000 |
| 17 | 0.5362 | 0.7681 | 0.6046 | 0.0000 | 0.8963 | 0.3342 | 0.4364 | 0.0000 | 0.8798 | 0.6708 | 0.0576 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2929 | 1 | 0.0000 | 0.0000 | 0.3387 |
| 18 | 0.0000 | 0.0000 | 0.0000 | 0.7326 | 0.0000 | 0.0000 | 0.0000 | 0.9620 | 0.0000 | 0.0000 | 0.3116 | 0.0000 | 0.0000 | 0.4362 | 0.0000 | 0.0762 | 0.0000 | 1 | 0.8560 | 0.0000 |
| 19 | 0.0000 | 0.0000 | 0.0000 | 0.8766 | 0.0000 | 0.0000 | 0.0000 | 0.8180 | 0.0000 | 0.0000 | 0.4556 | 0.0000 | 0.0000 | 0.5802 | 0.0000 | 0.2202 | 0.0000 | 0.8560 | 1 | 0.C000 |
| 20 | 0.0000 | 0.1067 | 0.7340 | 0.0000 | 0.2350 | 0.9956 | 0.9022 | 0.0000 | 0.2185 | 0.6679 | 0.0000 | 0.0728 | 0.3599 | 0.0000 | 0.6489 | 0.0000 | 0.3387 | 0.0000 | 0.0000 | 1 |

**Table 3.3.**

$R_j$ for SPATIAL AND LOCATION feature – Italian market

| On. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.0000 | 0.7484 | 0.0000 | 0.1068 | 0.1693 | 0.3299 | 0.0000 | 0.0000 | 0.4704 | 0.0000 | 0.0000 | 0.0000 | 0.7897 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2438 |
| 2 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5273 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.9042 | 0.0000 | 0.5093 | 0.0000 |
| 3 | 0.7484 | 0.0000 | 1 | 0.0000 | 0.3584 | 0.4209 | 0.5815 | 0.0000 | 0.0000 | 0.7220 | 0.0000 | 0.0000 | 0.0000 | 0.9587 | 0.1898 | 0.0000 | 0.0000 | 0.1147 | 0.0000 | 0.0000 |
| 4 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.3379 | 0.2754 | 0.1148 | 0.9841 | 0.9556 | 0.0000 | 0.0000 | 0.9679 | 0.9432 | 0.0000 | 0.5065 | 0.8294 | 0.0000 | 0.5816 | 0.3145 | 0.0000 |
| 5 | 0.1068 | 0.0000 | 0.3584 | 0.3379 | 1 | 0.9375 | 0.7769 | 0.3219 | 0.2935 | 0.6365 | 0.0000 | 0.3057 | 0.3946 | 0.3171 | 0.8313 | 0.1673 | 0.0000 | 0.7563 | 0.0000 | 0.0000 |
| 6 | 0.1693 | 0.0000 | 0.4209 | 0.2754 | 0.9375 | 1 | 0.8393 | 0.2595 | 0.2310 | 0.6989 | 0.0000 | 0.2433 | 0.3322 | 0.3795 | 0.7689 | 0.1048 | 0.0000 | 0.6938 | 0.0000 | 0.0000 |
| 7 | 0.3299 | 0.0000 | 0.5815 | 0.1148 | 0.7769 | 0.8393 | 1 | 0.0988 | 0.0703 | 0.8596 | 0.0000 | 0.0826 | 0.1715 | 0.5402 | 0.6082 | 0.0000 | 0.0000 | 0.5332 | 0.0000 | 0.0000 |
| 8 | 0.0000 | 0.0000 | 0.0000 | 0.9841 | 0.3219 | 0.2595 | 0.0988 | 1 | 0.9715 | 0.0000 | 0.0000 | 0.9838 | 0.9273 | 0.0000 | 0.4906 | 0.8453 | 0.0000 | 0.5657 | 0.3305 | 0.0000 |
| 9 | 0.0000 | 0.0000 | 0.0000 | 0.9556 | 0.2935 | 0.2310 | 0.0703 | 0.9715 | 1 | 0.0000 | 0.0000 | 0.9877 | 0.8988 | 0.0000 | 0.4621 | 0.8738 | 0.0000 | 0.5372 | 0.3589 | 0.0000 |
| 10 | 0.4704 | 0.0000 | 0.7220 | 0.0000 | 0.6365 | 0.6989 | 0.8596 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0311 | 0.6806 | 0.4678 | 0.0000 | 0.0000 | 0.3927 | 0.0000 | 0.0000 |
| 11 | 0.0000 | 0.5273 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4314 | 0.0000 | 0.0365 | 0.0000 |
| 12 | 0.0000 | 0.0000 | 0.0000 | 0.9679 | 0.3057 | 0.2433 | 0.0826 | 0.9838 | 0.9877 | 0.0000 | 0.0000 | 1 | 0.9111 | 0.0000 | 0.4744 | 0.8615 | 0.0000 | 0.5495 | 0.3467 | 0.0000 |
| 13 | 0.0000 | 0.0000 | 0.0000 | 0.9432 | 0.3946 | 0.3322 | 0.1715 | 0.9273 | 0.8988 | 0.0311 | 0.0000 | 0.9111 | 1 | 0.0000 | 0.5633 | 0.7726 | 0.0000 | 0.6384 | 0.2578 | 0.0000 |
| 14 | 0.7897 | 0.0000 | 0.9587 | 0.0000 | 0.3171 | 0.3795 | 0.5402 | 0.0000 | 0.0000 | 0.6806 | 0.0000 | 0.0000 | 0.0000 | 1 | 0.1484 | 0.0000 | 0.0000 | 0.0733 | 0.0000 | 0.0336 |
| 15 | 0.0000 | 0.0000 | 0.1898 | 0.5065 | 0.8313 | 0.7689 | 0.6082 | 0.4906 | 0.4621 | 0.4678 | 0.0000 | 0.4744 | 0.5633 | 0.1484 | 1 | 0.3359 | 0.0000 | 0.9249 | 0.0000 | 0.0000 |
| 16 | 0.0000 | 0.0000 | 0.0000 | 0.8294 | 0.1673 | 0.1048 | 0.0000 | 0.8453 | 0.8738 | 0.0000 | 0.0000 | 0.8615 | 0.7726 | 0.0000 | 0.3359 | 1 | 0.0903 | 0.4110 | 0.4851 | 0.0000 |
| 17 | 0.0000 | 0.9042 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4314 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0903 | 1 | 0.0000 | 0.6051 | 0.0000 |
| 18 | 0.0000 | 0.0000 | 0.1147 | 0.5816 | 0.7563 | 0.6938 | 0.5332 | 0.5657 | 0.5372 | 0.3927 | 0.0000 | 0.5495 | 0.6384 | 0.0733 | 0.9249 | 0.4110 | 0.0000 | 1 | 0.0000 | 0.0000 |
| 19 | 0.0000 | 0.5093 | 0.0000 | 0.3145 | 0.0000 | 0.0000 | 0.0000 | 0.3305 | 0.3589 | 0.0000 | 0.0365 | 0.3467 | 0.2578 | 0.0000 | 0.0000 | 0.4851 | 0.6051 | 0.0000 | 1 | 0.0000 |
| 20 | 0.2438 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0336 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1 |

**Table 3.4.**

min $R_j$ – Italian market

| O.n. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0000 | 0.0000 | 0.1409 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2070 | 0.0000 | 0.0000 | 0.0000 | 0.3967 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4318 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.1409 | 0.0000 | 0.0000 | 0.0000 | 0.3147 | 0.2708 | 0.3919 | 0.0000 | 0.0000 | 0.4548 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1898 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6946 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3436 | 0.0000 | 0.5816 | 0.0000 | 0.0000 |
| 5 | 0.0000 | 0.0000 | 0.3147 | 0.0000 | 0.0000 | 0.0000 | 0.3327 | 0.0000 | 0.0208 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1673 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.0000 | 0.0000 | 0.2708 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2141 | 0.6634 | 0.0000 | 0.0772 | 0.3322 | 0.0000 | 0.3423 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.0000 | 0.0000 | 0.3919 | 0.0000 | 0.3327 | 0.0000 | 0.0000 | 0.0000 | 0.0703 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3203 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 0.0000 | 0.0000 | 0.0000 | 0.6946 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0382 | 0.0000 | 0.5641 | 0.0000 | 0.0000 |
| 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0208 | 0.2141 | 0.0703 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2661 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 0.2070 | 0.0000 | 0.4548 | 0.0000 | 0.0000 | 0.6634 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0278 | 0.0000 | 0.3168 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0772 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.7129 | 0.0000 | 0.3563 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.3322 | 0.0000 | 0.0000 | 0.0000 | 0.0278 | 0.0000 | 0.7129 | 0.0000 | 0.0000 | 0.3946 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 0.3967 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0733 | 0.0000 | 0.0000 |
| 15 | 0.0000 | 0.0000 | 0.1898 | 0.0000 | 0.0000 | 0.3423 | 0.3203 | 0.0000 | 0.0000 | 0.3168 | 0.0000 | 0.3563 | 0.3946 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 16 | 0.0000 | 0.0000 | 0.0000 | 0.3436 | 0.1673 | 0.0000 | 0.0000 | 0.0382 | 0.2661 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0762 | 0.0000 | 0.0000 |
| 17 | 0.0000 | 0.4318 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 18 | 0.0000 | 0.0000 | 0.0000 | 0.5816 | 0.0000 | 0.0000 | 0.0000 | 0.5641 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0733 | 0.0000 | 0.0762 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 19 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |