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Factor substitution in rice production function: the case of Vietnam

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ABSTRACT

Vietnamese rice production has achieved remarkable success over the last couple of decades. This is due to land and market reforms, known as 'Doi Moi'. There were noticeable changes in policies, such as land and production systems, which were transformed from a collective to an individual contract system in the 1980s. Vietnam made progress in rice production through the legalisation of the privatisation of farm properties and a huge investment in irrigation systems. The country not only ensured its domestic demand, but also started exporting rice and gradually became the second largest exporter in the world. An estimate of the Constant Elasticity of Substitution function (CES) for Vietnam's rice production is essential for the government to design effective policy on agricultural production. This study makes the first attempt to estimate the nested CES model for Vietnamese rice production in 2012. The paper finds that the elasticity of substitution of Vietnam's nested CES model lies between 0.44 and 0.46. The results indicate the weak substitutability between land and the capital-labour composite in the nested CES model. This also suggests that it is impossible to take labour as the substitutable factor for land and capital.

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1. Introduction

In the Vietnamese agricultural sector, rice production is considered the most important industry since it plays a vital role in agricultural and rural economic development. Rice is an absolutely indispensable source of nutrition to millions of Vietnamese people and rice production contributes noticeably to ensuring national food security. Vietnamese rice production has achieved remarkable success over the last decade. Before 1980, rice production could not meet the consumption demands of the country; therefore, the country had to import rice exceeding 1.5 million tons during the 1970s (GSO, 1995). However, through extensive land and market reforms, known as

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'Doi Moi', there were noticeable changes through policies such as land and production systems. The changes included transformation from a collective to an individual contract system in the 1980s, the process of the legal privatisation of farm properties and a huge investment in irrigation systems by which Vietnam made progress in rice production (Kompas, Che, Nguyen, & Nguyen, 2012; McCaig & Pavcnik, 2013). The country not only ensured its domestic demand, but also started exporting rice, leading it to become one of the largest rice exporters in the world.

Despite these outstanding achievements, rice production still reveals many limitations and shortcomings. The growth in rice production over the past couple of decades is mainly due to the development in width, without attaching special importance to depth; thus, rice planting uses huge resources with less economic efficiency and has a negative impact on the environment (ADB, 2012). Since rice production had still been heavily based on traditional technological methods, the result was the low productivity of the production process. Therefore, the income of the rice grower was not remarkably improved, one of the main factors leading to the movement of labour from the agricultural sector to other industries. Moreover, poor land management as well as progress in industrialisation and urbanisation brought the challenges of reducing the rice-planted area in the following years (Kompas et al., 2012).

To handle these limitations and shortcomings, the need for maximising productivity and utilising capacity in the demand for inputs in rice production is considered one of the primary issues for the development of Vietnamese agricultural policy. The main concerns are: (i) what are the appropriate forms of CES function for rice production in Vietnam?; (ii) what is the value of the elasticity of substitution of these CES models?; (iii) which policy implications could be deduced from this in regards to Vietnam? and (iv) are there any further issues during the estimation of the CES function of rice production in Vietnam?

The aim of this study is to address these questions by first identifying which CES models are appropriate for Vietnam's rice production. Then, the study estimates the elasticity of substitution of these CES models. In order to identify the CES models, it is necessary to test possible nested structures of three inputs: capital; labour; and land. The method adopted for this study is the nonlinear least square approach through the Levenberg-Marquardt method.

The remainder of the paper is structured as follows. Section 2 reviews the literature. Section 3 describes the data collection and outlines the variable construction process as well as the model specification to estimate the elasticity of substitution of rice production in Vietnam in 2012. Section 4 discusses the results and concludes the study.

2. Literature review

CES production function was first estimated by Bodkin and Klein (1967). The authors employed a nonlinear maximum likelihood method and incorporated the correlation of the proportion of two marginal productivities. The results significantly indicated the increasing returns to scale and proved that the estimated value of the elasticity of substitution lies between 0 and 1. However, Bodkin and Klein (1967) admitted that the limitation of their research is that the results were very influenced by serial correlation in the residuals and based on the assumption that both outputs and inputs were exogenous variables. Based on a different approach, Kmenta (1967) estimated the CES production function by using the linear approximation of CES production function through the Taylor's series approximation. His idea was that since CES production function is the generalisation of the Cobb-Douglas function which allows any positive elasticity of substitution, it could be estimated through a single equation after taking ordinary least squares. Zarembka (1970) applied the Kmenta method to examine the case of US manufacturing. He found that in this case the Cobb-Douglas is more empirically appropriate than the CES production function.

However, this basic CES production function has the limitation of defaulting equal substitution elasticity between input factors. Thus, Sato (1967) introduced the nested CES functions to remove this limitation. The general idea of the nested models is to group the inputs with the same substitution and combine them with other input groups. In other words, nested models aim to create more levels of CES which are divided into upper level and lower level. The inputs in the upper-level group in CES function could be replaced with other inputs in the lower-level group. The nested CES model is now becoming popular when we try to examine the input factors that might have further differentiation (Sato, 1967). Recent studies investigated nested CES production function at either an aggregate level or industry level through different estimation approaches. Most of them examined the structure of CES production function with three inputs: capital; labour; and energy. Dissou, Karnizova, and Sun (2015) employed the cost minimization method to estimate the nested CES production function with capital, labour and energy as inputs for 10 manufacturing industries in Canada in the period 1962–1997. The study showed that the structure which fits the data best is the nested structure K-(LE). This structure is set following an order: labour (L) and energy (E) are firstly combined to form a composite and then it is put together with capital (K) in the upper CES function. The authors also found evidence of the Cobb-Douglas technology for four manufacturing industries. Okagawa and Ban (2008) estimated the elasticity of substitution for nested CES production function using a cost minimization method using three inputs (capital, labour and energy) for 19 industries from 14 countries in the period 1995 to 2004, in which the elasticity of substitution for agricultural industries was estimated at 0.39. Kemfert (1998) assessed the nested CES function with three different structures of capital, labour and energy for both aggregate and seven industries in Germany. He proved that for the aggregate production function the nested structure (KE)-L in which capital and energy are composite is suitable and fits the data best. For the case of each industry, the structure form of (KL)-E is more appropriate.

Among the methods for the estimation of Constant Elasticity of Substitution, the maximum likelihood estimation, linear Taylor-series approximation and nonlinear least squares are the most popular. Compared to two other methods, the nonlinear least square approach is considered more effective, especially for the case of having more than two independent variables in CES (Hoff, 2004). In linear Taylor-series approximation, the Kmenta approximation is the best representative (Henningsen & Henningsen, 2012). Kmenta (1967) derived the linear approximation of two-input

CES production function by logarithmising the CES function and then using the second-order Taylor series expansion. The advantage of the Kmenta approximation is that from the linear approximation we could estimate the CES production function by ordinary least squares techniques. This is also efficient to test whether the CES production function is under Cobb-Douglas form or not. However, applying the approximation methods to linearise the CES function exposes many problems. Kmenta (1967) himself confirms that if the elasticity of substitution is at the extreme (very low or very high), then his estimation might not produce reliable results. Others also prove this approximation is unsuitable when examining that the CES production is not under the form of the Cobb-Douglas function (Maddala and Kadane, 1967; Thursby and Lovell, 1978; Henningsen and Henningsen, 2012). To overcome these problems of Kmenta's approximation approach, many researchers have tried to estimate the linear system of equations which derived from a cost minimization approach. However, this estimation often needs comprehensive price data which is, in most cases, usually difficult to get and might create additional measurement errors. The nonlinear least square approach exposes numerous advantages when addressing the estimation of CES production function with n-inputs (Henningsen and Henningsen, 2012). Also, in the nonlinear least square approach, the Levenberg-Marquardt method is considered the most effective method for estimating the CES function (Hoff, 2004).

Estimating the CES production function plays an important role in assessing the structure of production function in many industries. In the case of Vietnam's rice production, this paper makes the first attempt to estimate its CES production function with the objective of finding out the elasticity of substitution for it. The nonlinear least square method with the Levenberg-Marquardt approach was applied to estimate the CES function. The rice production output is used as dependent variable and the variables – capital, labour and land – are used as explanatory variables. This study adds to the literature by a construction of a Labour Index for Vietnamese rice production function. Furthermore, it examines the nested CES model to assess which nested structures are appropriate for Vietnam's rice production. In addition, it performs the grid search method to re-assess the results of CES estimation and also alleviates some limitations of the non-linear least square method.

3. Method

3.1. Model specification

3.1.1. Nested CES models

The Cobb-Douglas function is the best-known functional form of production function in economics by Cobb and Douglas (1928) that is used to express the technological correlation between inputs and outputs. The inputs here are often capital and labour. However, this form is employed under the strong presumption of an elasticity of substitution which must be an equal one. In other words, this production function has constant returns to scale (Cobb & Douglas, 1928). This means that when we double the consumption of capital and labour, output is also doubled. Therefore, Arrow, Chenery, Minhas, and Solow (1961) point out that the Cobb-Douglas function with this strong presumption of a unity elasticity of substitution was unacceptable in the empirical studies.

The CES function generalises the Cobb-Douglas function and accepts any positive elasticity of substitution while still remains an accurate economic interpretation of the parameters. This is the main advantage of the CES function compared to the Cobb-Douglas function. It was first introduced by Solow (1956) and then developed by Arrow et al. (1961). One of the noteworthy limitations of CES function is that the more complicated CES function often exposes much more problematic estimations since the estimation process of CES needs the simultaneous estimation of a non-linear system (Henningsen & Henningsen, 2011). Moreover, the inputs in CES production are likely to be correlated, creating the phenomenon of multicollinearity (Nerlove, 1967). Another problem of the CES function is that in many cases, the endogenous independent variables could result in the problem of simultaneity bias (Miller, 2008). This bias becomes more problematic especially when factor-biased technical change was added (de La Grandville, 1997).

The CES function with two-input formula is specified as:

$$Q = \gamma\left[\delta {x_1}^{-\rho} + (1-\delta] {x_2}^{-\rho}\right]^{-\frac{\upsilon}{\rho}} \tag{1}$$

Where Q is the output quantity; $x_1,\ x_2$: input quantities; $\gamma, \delta,\ \rho$: are parameters; γ indicates the productivity ($\gamma\ \epsilon\ [0,\infty)$); $\delta\epsilon\ [0,\infty)$ indicates the input optimal distribution; ρ ranges between $[-1,0)$ U $(0,\infty)$and ρ will define the elasticity of substitution $\sigma = \frac{1}{1+\rho}$; parameter $\upsilon\ \epsilon[0,\infty)$ is the elasticity of scale. One point that should be noted here is that the CES production function introduced by Arrow et al. (1961) only considers the constant return to scale. Therefore, to allow for an increasing or decreasing return to scale Kmenta (1967) put the parameter υ. The CES function will have increasing returns to scale if $\upsilon > 1$ and decreasing returns to scale if $\upsilon < 1$.

For multiple inputs, the general formula is identified as:

$$y = \gamma\left(\sum_{i=1}^{n} \delta_i x_i\right)^{-\frac{\vartheta}{\rho}} \tag{2}$$

Where n is number of input factors, x_i is the quantity of input i.

The three-input nested CES function could be specified as:

$$y = \gamma\left[\delta(\delta_1 {x_1}^{-\rho_1} + (1-\delta_1){x_2}^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta){x_3}^{-\rho}\right]^{-\upsilon/\rho} \tag{3}$$

Where y is output; $x_1,\ x_2,\ x_3$ are inputs; $\gamma, \delta, \delta_1, \rho, \rho_1$ are parameters; γ indicates the productivity ($\gamma\ \epsilon\ [0,\infty)$); $\delta, \delta_1\epsilon\ [0,1]$ indicate the inputs' optimal distribution; ρ, ρ_1 range between $[-1,0)$ U $(0,\infty)$ and ρ will define the elasticity of substitution $= \frac{1}{1+\rho}$; υ indicates the elasticity of scale.

3.1.2. Nested CES models of Vietnam's rice production

In this section, we examine a two-level production function with three inputs: capital (KA); labour (LB); and land (LA). The first-level function of nested CES function with input KA and LB is specified as:

$$\mathrm{M} = \left(\delta_1 KA^{-\rho_1} + (1-\delta_1)LB^{-\rho_1}\right)^{-\frac{1}{\rho_1}} \tag{4}$$

This first-level function will be nested with input variable LA to form the second level of CES function:

$$QX = \ \gamma[\delta M^{-\rho} + (1-\delta)LA^{-\rho}]^{-\upsilon/\rho} \tag{5}$$

Putting Equation (4) into (5) yields the two-level nested CES function with three inputs: capital; labour; and land. We denote this structure as (KA,LB)LA:

$$QX = \gamma\left[\delta(\delta_1 KA^{-\rho_1} + (1-\delta_1)LB^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)LA^{-\rho}\right]^{-\upsilon/\rho} \tag{6}$$

Where, QX is the output quantity; KA, LB, LA are input quantities of capital, labour and land respectively; $\gamma, \delta, \delta_1, \rho, \rho_1$:are parameters; γ indicates the productivity, or an index of technological efficiency ($\gamma \in [0,\infty)$); $\delta, \delta_1 \in [0,1]$ indicates the inputs' optimal distribution; ρ, ρ_1 ranges between $[-1,0)$ U $(0,\infty)$ and ρ will define the elasticity of substitution $\sigma = \frac{1}{1+\rho}$

Similarly, the other two nested structures (LA,LB)KA and (KA,LA)LB are respectively identified as:

$$QX = \gamma\left[\delta(\delta_1 LA^{-\rho_1} + (1-\delta_1)LB^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)KA^{-\rho}\right]^{-\upsilon/\rho} \tag{7}$$

$$QX = \gamma\left[\delta(\delta_1 KA^{-\rho_1} + (1-\delta_1)LA^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)LB^{-\rho}\right]^{-\upsilon/\rho} \tag{8}$$

The study examines all three nested CES functions for Vietnam's rice production to estimate the elasticity of substitution of nested CES function $\sigma = \frac{1}{1+\rho}$. Specifically, it estimates two kinds of elasticity of substitution: the Hicks-McFadden elasticity of substitution and the Allen-Uzawa elasticity of substitution. According to McFadden (1963), the Hicks-McFadden elasticity of substitution is the direct elasticity of substitution which measures the elasticity of substitution only between the nested inputs. For example, in the nested model as (KA,LB)LA in Equation (6), it measures the elasticity of substitution between capital and labour and it is specified as:

$$\sigma_{KA,LB} = \frac{1}{1+\rho_1} \tag{9}$$

Allen-Uzawa's elasticity of substitution, on the other hand, evaluates the partial elasticity of substitution which measures the elasticity of substitution between the nested inputs and the third input (Uzawa, 1962). For the nested model (KA,LB)LA, it measures the elasticity of substitution between the nest (capital, labour) and land. Allen-Uzawa's elasticity of substitution is defined as:

$$\sigma_{KALB,\ LA} = \frac{1}{1+\rho} \tag{10}$$

One thing that should be noted here is that the parameter γ implies the technological change, which is assumed to be Hicks neutral. Hicks (1932) is the first person to introduce the concept of Hicks neutrality. He claimed that a change is regarded as Hicks neutral if the change does not have influence on the marginal rate of substitution (MRS) between any inputs in the production function (Hicks, 1932). Therefore, it also means that the Hicks-neutral technological change has no impact on the proportion of marginal products for a given capital–labour ratio (Antras, 2004).

3.2. Data

This study uses the latest full data set from the Vietnam Household Living Standard Survey (VHLSS) of 2012. This survey was conducted by the Vietnam General Statistics Office (GSO) to collect information on the living standards of many households over the provinces and cities of the country based on detailed questionnaires. The original data set included more than 27,000 surveyed households. By filtering data, a number of households with unavailable data were eliminated, thus reducing the size of data to be used in this study, which is now from more than 14,000 households. The available data for constructing the capital variable is taken from each household's cost of rice production. Therefore, the capital variable is the sum of following costs: seedlings; saplings; small equipment; chemical fertilizer; insecticide; herbicide chemicals; electricity; gasoline; repair; maintenance; and other costs. The available data for land includes land used for ordinary rice and glutinous rice production. The available data for output variable is comprised of outputs of ordinary rice and glutinous rice production. The available data for labour variables includes the amount of money that each household used for hiring additional labour and the number of working days in rice production for each member of the household. They are used in constructing the Labour Index which is described in detail in section 3.3.

3.3. Construction of the Labour Index for the Vietnamese rice production function

The VHLSS of 2012 provides the information on labour involved in rice production which includes: (1) the amount of money that one household used for hiring additional labour and; (2) the number of working days in rice production for each member in one household (GSO, 2012a). Based on this information, the study constructs a Labour Index for Vietnamese rice production function in the year 2012 which is as follows:

Step 1: Summing up the working days all people from each household spent on rice production, we obtained WORKDAY_i where i denotes each household.

Step 2: Summing up all working days of all households in rice production, we obtained the total time consumption on rice production for all household members: $\sum_i^N \text{WORKDAY}_i$ where i denotes each household and N denotes total number of households.

Step 3: Dividing the total working days of all households by the number of households, we obtained the average time spent on rice production. We set this as a working hour's standard for a standard labour or one unit of labour index. In this paper, a standard labour in rice production is assumed as the person who spends 173.67 days per year on rice production.

$$\overline{WORKDAY} = \frac{\sum_i^n \text{WORKDAY}_i}{N} = 173.67 \text{ days}$$

Step 4: Dividing the total working days of each household spent on rice production by the average time consumed by rice production to have the *Labourindex*1

$$Labourindex1_{\boldsymbol{i}} = \frac{\text{WORKDAY}_i}{\overline{WORKDAY}}$$

Step 5: Besides each household member participating in rice production, many households hired additional outside labour to supplement their production. *Labourindex*2 calculates the hired outside labour for the rice production of each household. The available data in VHLSS files contains the annual rent for the outside labour of each household: $RENTLABOUR_i$

First, we convert the cost of outside labour into feasible working days until outside labour supplements each household's rice production. To do this, we must calculate the daily average income that the normal farmers could have. According to GSO (2012b), the average monthly earnings of a worker in the agricultural sector in 2012 was 2,543,000 VND. Thus, the standard daily income that the farmers in rice production have should equal $(2,543,000 \text{ VND x12months})/365\text{days} = 83,605.48 \text{ VND}$

Then, dividing the annual cost of outside labour of each household by $83,605.48$ VND, we obtain the Labourindex2 or the working days that the additional labour added to the households' rice production.

Step 6: Finally, the *Labourindex* of household i is calculated as the sum of the *Labourindex*1 and *Labourindex*2 of this household.

$$Labourindex_i = Labourindex1_{\boldsymbol{i}} + Labourindex2_i$$

3.4. The Levenberg-Marquardt method

The paper will employ the Levenberg-Marquardt curve-fitting method as the main approach to estimate the CES parameters. The Levenberg-Marquardt curve-fitting method is fundamentally the connection of two methods' approaches: the Gradient Descent method and the Gauss-Newton method (Marquardt, 1963). This method procedure is described as follows:

Assume a non-linear function (CES function) to be estimated has the form: $\hat{y}(x_i; b)$ for the function $y(x_i)$ wherex_i are independent variables and b is the parameters vector. The target is that we should minimize the sum of weighted residuals between the data points and curve-fit functions, or, in other words, to perform a least squares estimation.

Consider the chi-squared error criterion:

$$\chi^2(b) = \sum_{i=1}^{m} \left[\frac{y(x_i) - \hat{y}(x_i; b)}{w_i}\right]^2 = (y - \hat{y}(b))^T W (y - \hat{y}(b)) = y^T W y - 2y^T W \hat{y} + \hat{y}^T W \hat{y} \tag{11}$$

Where w_i is an error measurement of $y(x_i\ )$, W is the weighted matrix in which W_{ii} equals $\frac{1}{w_i^2}$. This criterion aims to seek the perturbation $\boldsymbol{\theta}$ for the parameters b which will minimize $\chi^2(b)$.

The Gradient Descent Approach: The purpose of this method is to force the updating parameters in the opposite sign of the gradient from the chi-square objective function.

$$\frac{\partial}{\partial b}\chi^2 = (y - \hat{y}(b))^T W \frac{\partial}{\partial b}(y - \hat{y}(b)) = -(y - \hat{y}(b))^T W \left(\frac{\partial \hat{y}(b)}{\partial b}\right) = -(y - \hat{y}(b))^T W \boldsymbol{J} \tag{12}$$

In which $\boldsymbol{J} = \frac{\partial \hat{y}(b)}{\partial b}$ is the Jacobian matrix that reflects the marginal changes in $\hat{y}(b)$ to the variation of parameters b. From this, the perturbation $\boldsymbol{\theta} = \alpha \boldsymbol{J}^{\boldsymbol{T}} W(\boldsymbol{y} - \hat{y})$ will update the parameter b toward the steepest decent. A scalar α here is the determination of steps in the steepest-descent control.

The Gauss-Newton Approach: This method aims at minimizing the sum-of-squares of a function which is assumed to be nearly quadratic with parameters at the points near the optimal bound. We have the estimated function in which the parameters are perturbed as $\hat{y}(b + \theta)$. From the first-order Taylor approximation series expansion, it is specified as:

$$\hat{y}(b + \theta) \approx \hat{y}(b) + \left[\frac{\partial \hat{y}}{\partial b}\right]\theta = \hat{y} + J\theta \tag{13}$$

Replace $\hat{y}(b)$ with $\hat{y}(b + \theta)$ in the Equation (11):

$$\chi^2(b + \theta) \approx y^T W y + \hat{y}^T W \hat{y} - 2y^T W \hat{y} - 2(y - \hat{y})^T W J \theta + \theta^T J^T W J \theta \tag{14}$$

This equation indicates χ^2 is nearly quadratic in the θ and the Hessian in this case is approximate $J^T W J$. Then the estimated θ which minimize $\chi^2(p + \theta)$ is identified as:

$$\frac{\partial}{\partial b}\chi^2(b+\theta) \approx -2(y-\hat{y})^T WJ + 2\theta^T J^T WJ = 0 \tag{15}$$

Or

$$\left[J^T WJ\right]\theta = J^T W(y-\hat{y})^T \tag{16}$$

Equation (16) will give us the estimated Gauss-Newton perturbation.

The Levenberg-Marquardt Method will combine both the Gradient Descent approach and the Gauss-Newton approach; therefore, it lets the parameters move in the range that updated from both the Gradient Descent method and the Gauss-Newton method. Specifically, we have:

$$\left[J^T WJ + \lambda I\right]\theta_{LM} = J^T W(y-\hat{y}) \tag{17}$$

Where, λ is the algorithmic parameter. From this, we have the Levenberg-Marquardt solution for non-linear least squares by the relationship:

$$\left[J^T WJ + \lambda diag\left(J^T WJ\right)\right]\theta_{LM} = J^T W(y-\hat{y}) \tag{18}$$

3.5. Grid search

Estimating CES function through the non-linear least square method could lead to the problem of flattening the surface of the area around the minimal value of residuals when using the wide range of substitution parameters ρ, ρ_0, ρ_1. Grid search is the advanced and efficient method to alleviate this problem. The process of grid search is the following. First, the grid of values for ρ, ρ_0, ρ_1 is pre-selected which, as expected, results in the smallest sum of the square of residuals. Then, holding those pre-selected values of substitution parameters fixed, we estimate the remaining parameters by using the non-least square (Henningsen & Henningsen, 2011). The purpose of the grid search is to try to find the best value of the remaining parameters which result in the smallest sum of the square of residuals. The idea of this method is that the pre-selected grid values of substitution parameters will actively limit the area around the minimum; through this, when the non-linear least square operates again, the remaining parameters will consequently be produced on the ground of this limited surface around minimum.

4. Results

4.1. Descriptive statistics

The lists of variables used in the model and the summary statistics for Vietnam's rice production quantity (kilograms), weighted index for labour and the area of land for rice production (metres) in 2012 can be seen in Table 1 and Table 2.

Table 1. Lists of variables used in the model.

Variable	Short	Definition	Source	Unit
Output	QX	Gross quantities of rice production	VHLSS files	Kilograms
Capital	KA	Total fixed capital stock	VHLSS files	Thousands VND
Labour	LB	Labour Index	VHLSS files	Unit of labour
Land	LA	Total lands used	VHLSS files	Square metres

Source: Authors.

Table 2. Summary statistics for Vietnam's rice production, 2012.

Variable	Mean	Standard deviation	Minimum	Maximum
Output quantity (QX), kilograms	4729.88	9943.24	65	333450
Labour (LB), Labour Index	1.28	1.38	0.00172	14.79
Capital (KA), '000 VND	13491.59	29678.9	109	924098
Land (LA), square metres	8731.93	15938.28	120	507000

Source: Authors' calculation.

In 2012, on average, one Vietnamese household produced around 4729 kilograms of rice per annum; the highest output quantity is 333,450 kilograms and the lowest only 65 kilograms. The Labour Index has the highest value at 14.79 and the lowest value at 0.00172. The average Labour Index is 1.28, which means that on average each household has over one standard labour working in rice production. A standard labour or one unit of Labour Index in rice production is assumed in our model as the person spending on average 173.67 days per annum on rice production as explained in section 2.3. Annually, on average, each household used 8731 square metres of land per annum for rice production and spent 13.491 million VND on rice production. To estimate Vietnam's nested CES production function, the rice production output is used as a dependent variable and the variables – capital, labour and land – are used as explanatory variables.

4.2. Estimating Vietnam's nested CES rice production function

In this study, the statistical software R project (version 3.2.2) was used to perform the CES estimation and grid search process. Specifically, in the R project, we use the R-package micEconCES created by Henningsen and Henningsen (2011). The R-package micEconCES is mainly designed to estimate the CES production function and currently this is one of the most up-to-date and efficient programs for estimating the CES (Shen & Whalley, 2013). The estimated results are represented as follows. First, the general results of estimating CES production function with a different nested structure will be checked to find out the suitable nested structures. Then, the detailed results for each suitable nested structure will be examined. Second, the grid search is performed to provide more accurate results for CES estimation. One point should be noted: in all estimation processes, the grid search is performed by the Levenberg-Marquardt method programmed under R-package micEconCES.

4.2.1. Choosing a suitable nested CES structure for Vietnam's rice production function

The task of this part is to find out which nested CES structures are suitable for Vietnam's rice production. Table 3 compares the estimated results of CES function

Table 3. Estimating Vietnam's CES rice production function with different nested structures.

	Constant return to scale (with $\upsilon = 1$)				Variable return to scale (without $\upsilon = 1$)			
Nest Structure	Estimate of $\sigma_{12,3}$	Standard error of $\sigma_{12,3}$	R^2	Residual standard error	Estimate of $\sigma_{12,3}$	Standard error of $\sigma_{12,3}$	R^2	Residual standard error
(Ka,Lb)La	0.443	0.0154	0.975	1554.22	0.445	0.0151	0.977	1508.80
(La,Lb)Ka	0.4401	0.0132	0.975	1554.73	0.445	0.0125	0.977	1508.368
(Ka,La)Lb	2.829	84.245	0.975	1554.658	0.880	0.0280	0.369	7899.023

Note: Only $\sigma_{12,3}$ Allen-Uzawa (partial) elasticity of substitution is checked.
$\sigma_{12,3}$ is the elasticity of substitution in which input 1 and input 2 are nested but input 3 is not.
Source: Authors' calculation.

Table 4. Estimated nested CES function with constant return to scale.

	Estimates		Standard error		T value		Pr (>t)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.377	0.453	1.361	16.221	0.277	0.028	0.782	0.978
delta_1(δ_1)	−0.281	−0.145	4.444	64.886	−0.063	−0.002	0.950	0.998
Delta (δ)	0.613	0.597	1.748	18.351	0.351	0.033	0.726	0.974
rho_1(ρ_1)	−0.468	−0.824	1.377	44.77	−0.340	−0.018	0.734	0.985
Rho (ρ)	1.255	1.272	0.0782	0.0679	16.050	18.726	<2e-16***	<2e-16***
$\sigma_{1,2}$	1.879	5.683						
$\sigma_{12,3}$	0.443	0.44						

	Nested CES structures	
	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1554.22	1554.73
R^2	0.975	0.975

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution.
$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution.
***indicates significance at a 1% level.
Source: Authors' calculation.

Table 5. Estimated nested CES rice production function with variable return to scale.

	Estimates		Standard error		T value		Pr (>t)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.5018	0.618	0.30	0.258	1.671	2.393	0.0948	0.0167*
delta_1(δ_1)	0.91	0.627	2.224	0.812	0.407	0.772	0.684	0.439
Delta (δ)	0.04	0.149	0.685	0.431	0.05	0.347	0.953	0.728
rho_1(ρ_1)	−1.055	−0.575	3.856	0.366	−0.274	−1.572	0.784	0.116
Rho (ρ)	1.25	1.248	0.0762	0.0634	16.340	19.677	<2e-16***	<2e-16***
Nu (ν)	1.036	1.0376	0.00126	0.00136	821.497	763.918	<2e-16***	<2e-16***
$\sigma_{1,2}$	NA	NA						
$\sigma_{12,3}$	0.445	0.445						

	Nested CES structures	
	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1508.80	1508.368
R^2	0.977	0.977

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution.
$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution.
*** and * indicates significance at a 1% level and a 5% level respectively.
Source: Authors' calculation.

with different nested structures in both cases of constant return to scale and variable return to scale. The CES function with constant return to scale has the assumption that the parameter υ is unity whereas the CES function with constant return to scale has no condition on υ. The results show that in both cases of the constant return to scale and the variable return to scale, the nested structure (Ka,Lb)La and (La,Lb)Ka are suitable, while the structure (Ka,La)Lb is not reliable and reflects less economic meaning for Vietnam's CES rice production function.

In the case of constant return to scale, the results from Table 4 prove that the nested structure (Ka,La)Lb does not suit the case of nested CES function while the nested structures (Ka,Lb)La and (La,Lb)Ka are the suitable models. All three nest structures have high value of R^2 which is rounded to 0.975. The R^2 has the value of 0.975, which means that approximately 97.5% of variations in the Vietnam's rice production output could be explained by the variables capital, labour and land. Thus, the higher the value of R^2, the better the models fit the data (Heij, Boer, Franses, Kloek, & Dijk, 2004). However, in the nest structure (Ka,La)Lb the estimate of the Allen-Uzawa (partial) elasticity of substitution $\sigma_{12,3}$ is considerably high at around 2.829. Normally, to have economic meaning, $\sigma_{12,3}$ should lie in the range [0,1] (Henningsen & Henningsen, 2011). Such a high value of $\sigma_{12,3}$ has no application to reality. Moreover, standard error of $\sigma_{12,3}$ in the nest (Ka,La)Lb is estimated at approximately 84.245 which is much higher than the ones in the other two nested structures. Theoretically, high standard errors usually imply that the estimates might have high variances or might be a result of the small sample size (Heij et al., 2004). In this study, the data is quite large and sufficient enough; thus, the high variances of estimates would be the main problem. Therefore, this evidence indicates that the nested structure (Ka,La)Lb does not suit the case of the nested CES function with constant return to scale. Also, this suggests the CES production function with the nested structures (Ka,Lb)La and (La,Lb)Ka are the suitable models for the case of Vietnam's rice production function in 2012.

Table 5 indicates that the nested CES production function with variable return to scale has the same conclusion as the case of constant return to scale. The nested structure (Ka,La)Lb is inappropriate for Vietnam's nested CES rice production function while the nested structures (Ka,Lb)La and (La,Lb)Ka are proved to be the suitable ones. From Table 2, both the nested structures (Ka,Lb)La and (La,Lb)Ka have the same goodness of fit (R^2) which is at a high value of 0.977. However, the nested structure (Ka,La)Lb has low R^2 which is approximately 0.369. Such a low value of R^2 means only 36.9% of variations in Vietnam's rice production output could be explained by the variables capital, labour and land. These poor results indicate that the estimation of the nest structure (Ka,La)Lb is unreliable and this structure will not be chosen for estimating nested CES function.

From the results stated above, the study chooses the nested structures (Ka,Lb)La and (La,Lb)Ka for further assessment to estimate Vietnam's nested CES production function. The nested structure (Ka,La)Lb is not suitable due to its unreliable and meaningless results. The next section will focus on examining in detail the estimates of two nested structures (Ka,Lb)La and (La,Lb)Ka for both cases of constant return to scale and variable return to scale.

4.2.2 Estimating Vietnam's nested CES rice production function with constant return to scale

After choosing the suitable nested structures (Ka,Lb)La and (La,Lb)Ka, the next step is to examine their estimation results in detail.

There is not much difference in estimates of CES production function with the nested structure (Ka,Lb)La and (La,Lb)Ka. They both have the Allen-Uzawa (partial) elasticity of substitution around 0.44. Two nested structures have the same goodness of fit (R^2) and residual standard errors. The R^2 value of 0.975 means that approximately 97.5% of variations in Vietnam's rice production output could be explained by the variables capital, labour and land. This proves that the estimates are relatively reliable and the models fit well with the data. The Allen-Uzawa (partial) elasticity of substitution is smaller than one which implies the weak substitutability between land and the nest (capital and labour).

4.2.3. Estimating Vietnam's nested CES rice production function with variable return to scale

The Allen-Uzawa (partial) elasticity of substitution is estimated at around 0.445 for both nested structures (Ka,Lb)La and (La,Lb)Ka. Again, it indicates that land and the nest (capital and labour) seems to be difficult to substitute with each other. The estimates are relatively reliable with a high value of R^2. Land, labour and capital together contribute around 97.7% of the variation in rice output quantities.

Moreover, in nested CES rice production (La,Lb)Ka, the Hick-neutral technological change γ has an estimate of coefficient of 0.618 which is significant at the 5% level. Since γ varies between 0 and 1, this high value reflects the considerably positive impact of technological change γ on outputs. This means the increase in technical change will significantly lead to the increase in outputs. However, as noted in section 3.1.2, this Hicks-neutral technical change does not affect the ratio of marginal products for a given capital-labour ratio. Therefore, the change in parameter γ solely influences the outputs of production function.

4.3. Grid search for Vietnam's CES rice production function

This section examines the nested CES production function again with the attempt to see how estimated results change when we perform the grid search method. Usually, one problem that estimating CES production often has is that if we allow substitution parameters ρ, ρ_0, ρ_1 to run in the wide ranges of number, this consequently tends to flatten the optimal area of residuals. Thus, this might distort the estimates. The grid search method alleviates this problem through narrowing down the range of substitution parameters and using this range to re-estimate CES function.

4.3.1. Choosing a pre-selected grid of values for substitution parameters

There is no software or program designed to find out automatically the pre-selected values for substitution which will be used as inputs for the grid search method. The proper pre-selected values for substitution are usually chosen based on graphs of grid search and comparison of the results (Henningsen & Henningsen, 2011).

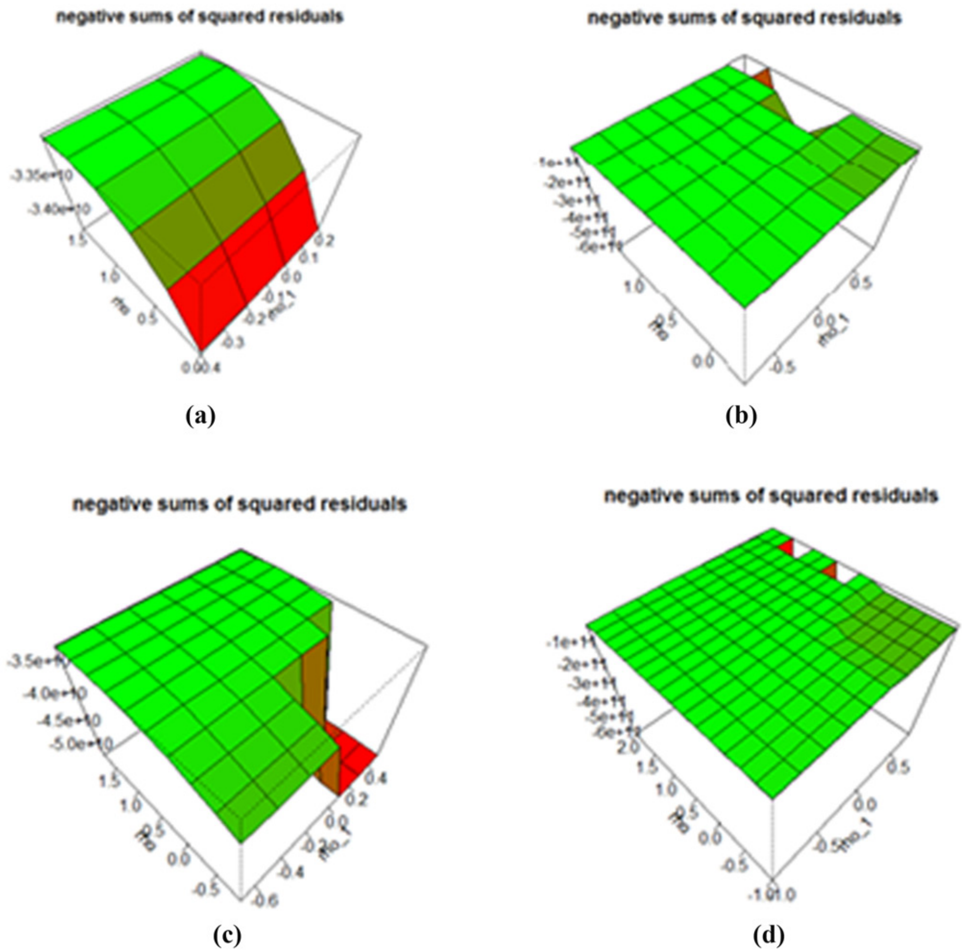

Figure 1. Grid search for values of substitution parameters.
Note:
(a): rho1(ρ_1) (from −0.4 to 0.3, with an increment 0.2); rho (ρ) (from 0 to 1.5, with an increment 0.3).
(b): rho1(ρ_1) (from −0.8 to 0.9, with an increment 0.2); rho (ρ) (from −0.4 to 1.5, with an increment 0.3).
(c): rho1(ρ_1) (from −0.7 to 0.6, with an increment 0.2); rho (ρ) (from −0.9 to 2, with an increment 0.3).
(d): rho1(ρ_1) (from −1 to 0.8, with an increment 0.2); rho (ρ) (from −1 to 2, with an increment 0.2).
Source: Authors.

Figure 1 describes the different combinations of negative sum of squares and arbitrary grid values of substitution parameters (ρ, ρ_1). Each graph is constructed of three axes. The vertical axis measures the negative sum of square residuals and the other two measure the value of ρ_1 and ρ .The ranges of grid values of substitution parameters is selected randomly and is also guessed based on the normal CES estimates. While graphs (b), (c), (d) reflects the flatter surface around the maximum of negative sum of square, the graph (a) expresses the optimal area of the negative sum of square. The larger negative sum of square means the smaller absolute value of the sum of square residuals. Thus, graph (a) with a concave shape shows that the sum of square is smallest and achieves optimal value when the value of ρ_1 ranges from −0.4 to 0.3

with an increment of 0.2 and the value ρ from 0 to 1.5 with an increment of 0.3. Hence, the range values of (ρ, ρ_1) from graph (a) is the most proper pre-selected values for grid search to find out the best value for the elasticity of substitution.

4.3.2. Re-estimating nested CES function with grid search method

After selecting the pre-selected grid values of substitution parameters, the next step is to re-estimate CES function using these pre-selected values to produce a more accurate value of σ. The pre-selected range of values that the value of ρ_1 ranges from -0.4 to 0.3 with an increment of 0.2 and the value of ρ_0 from 0 to 1.5 with an increment 0.3 will be chosen in this section. The re-estimated results from Table 6 and Table 7

Table 6. Re-estimated nested CES function with constant return to scale.

	Estimates		Standard error		T value		Pr (>t)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.158	1.02e-05	0.164	2.747e-03	0.965	0.004	0.335	0.99704
delta_1(δ_1)	−0.527	−9.19e + 01	0.632	1.014e + 04	−0.835	−0.009	0.404	0.99277
Delta (δ)	0.789	1.000e + 00	0.188	3.403e-04	4.188	2938.858	2.82e-05***	<2e-16***
rho_1(ρ_1)	−0.20	−4.000e-01	0.0776	1.225e-1	−2.575	−3.265	0.010*	0.00109**
Rho (ρ)	0.90	1.200e + 00	0.0642	6.687e-02	14.029	17.945	< 2e-16***	<2e-16***
$\sigma_{1,2}$	1.25	1.667	0.121	0.34	10.30	4.898	<2e-16***	9.68e-07***
$\sigma_{12,3}$	0.526	0.454	0.0178	0.0138	29.62	32.9	<2e-16***	<2e-16***

Nested CES models	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1549.768	1549.689
R^2	0.976	0.976

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution.
$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution.
*** and * indicates significance at a 1% level and a 5% level respectively.
Source: Authors' calculation.

Table 7. Re-estimated nested CES function with variable return to scale.

	Estimates		Standard error		T value		Pr (>t)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.334	0.455	0.0113	0.355	29.485	1.284	<2e-16***	0.199
delta_1(δ_1)	−0.00125	0.788	0.0065	0.771	−0.192	1.021	0.848	0.307
Delta (δ)	0.412	0.427	0.0225	0.520	18.269	0.821	<2e-16***	0.412
rho_1(ρ_1)	0.20	−0.40	0.461	0.467	0.434	−0.857	0.664	0.391
Rho (ρ)	1.20	1.20	0.0698	0.0621	17.189	19.318	<2e-16***	<2e-16***
Nu (ν)	1.035	1.037	0.00133	0.00138	779.928	753.156	<2e-16***	<2e-16***
$\sigma_{1,2}$	0.833		0.320		2.603		0.00923	
$\sigma_{12,3}$	0.454	0.454	0.0144		31.514		< 2e-16***	

Nested CES models	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1508.50	1508.55
R^2	0.977	0.977

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution.
$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution.
*** and * indicates significance at a 1% level and a 5% level respectively.
Source: Authors' calculation.

show that the value of the Allen-Uzawa (partial) elasticity of substitution lies between 0.45 to 0.53. This again confirmed the weak substitution of land and the nest (labour, capital). The high value of R^2, which is around 0.98, proves the best fit for estimated models. These estimates re-affirm the accuracy of previous results before using grid search. Therefore, it helps to see that in this case, the problem of flattening the optimal area of residuals is that the CES estimation might not affect the estimates.

5. Conclusion

The study results show that the estimated value of the elasticity of substitution for Vietnamese rice production lies between 0.44 and 0.46, which indicates the weak substitutability between land and the nest (labour, capital). This result suggests that the chance to use additional land as the substitutable factor for the shortage of labour and capital is very low. In other words, it is difficult to compensate the shortage of labour and capital only by expanding the area of land. Moreover, compared with other industries such as the textile and real-estate sectors, the elasticity of substitution for rice production is lower. This implies that in rice production, it is relatively difficult to have substitutability between its input factors compared to other sectors. The study also finds that the nested structure in which capital with land are combined composite while labour plays a role as the third input is rejected. This finding suggests it is impossible to take labour as the substitutable factor for land and capital. Moreover, in the estimates of nested CES rice production (La,Lb)Ka with variable return to scale, the Hick-neutral technological change γ has the estimates of coefficient as 0.618, which is significant at the 5% level. Since γ varies between 0 and 1, this high value reflects the considerably positive impact of technological change γ on outputs. This means the increase in technical change will significantly lead to the increase in outputs.

These findings have useful policy implications. An estimate of constant elasticity of substitution for Vietnam's rice production is necessary to address the existing weakness of rice production in Vietnam and partly contributes to providing the useful empirical evidence for designing appropriate policies for rice production. First, the weak elasticity of substitution suggests that with Vietnamese rice production, is not easy to have substitutability between its inputs. Given that rice production is managed by the Vietnamese government, these findings will partly help policy-makers design the appropriate policies on rice production with the efficient proportion of input factors in order to get the optimal output. Secondly, the conclusion on rejected structures suggests that it is impossible to compensate the shortage in capital and land by only expanding the amount of labour. A feasible policy that might be considered is to increase productivity through enhancing the application of technology. Thirdly, the weaker substitution of inputs in rice production compared with other industries will partly help the Vietnamese government plan how to allocate inputs and resources between rice production and other industries. Fourthly, the positive impact of technical change on outputs has important considerations for agricultural policy. Given the ratio of marginal products of inputs, to increase the output's productivity it is necessary to increase technical change. The technical change could come from two main factors: (1) improvement in the quality of the inputs (capital, labour, management and so on); and (2) investment in research and development, as a means of stimulating the creation of new technologies. The quality of physical capital could be enhanced through investments in infrastructure (communication, roads and so on). A better infrastructure system consequently has positive effects on the effectiveness of physical capital in rice production. The quality of labour could be improved through education, experience and training. These improvements will lead to an increase in technical change, which in turn will stimulate productivity growth. Finally, the estimates for Vietnam's CES production function could be taken as references in future research on the topic of Computable General Equilibrium (CGE) modelling.

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