The Influence of Receiver Selection Strategy on Packet Success Probability in Ad Hoc Network

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SUMMARY

Considering the importance of the receiver (RX) selection strategy for the packet success probability (PSP) in ad hoc network, this paper probes into the PSPs with nearest RX selection strategy and farthest RX selection strategy and determines the number of hops with the two strategies. Next, the performance of the successful transmission probability (STP) and PSP were discussed through numerical simulation with the above mentioned two strategies. The simulation results show that the PSP is affected by the terminal density, the RX selection strategy, the packet length and the STP; the number of hops mainly depends on the terminal density, the RX selection strategy, the length between the source TX and the destination RX. Furthermore, the nearest RX selection strategy and the farthest RX selection strategy differ insignificantly in the packet transmission duration between source TX to destination RX at a small terminal density.

KEY WORDS: Successful transmission probability (STP); packet success probability (PSP); number of hops; ad hoc network.

1. INTRODUCTION

In ad hoc network, the source packet has to take several hops to reach the destination. The success of each hop relies on the successful transmission probability (STP) [1] and the easy-to-measure packet length [2]. Many scholars have explored the STP [3-5]. For instance, Webber [6] built an ad hoc network model based on stochastic geometry, and studied the STP with no control over power and channel inversion power. Zhang [7] derived the expression of the lower bound on the capacity of wireless ad hoc network. Qin [8] analysed the transport capacity of full duplex ad hoc network. Bojd [9] designed a new routing algorithm to improve the delay and capacity in mobile ad hoc network. Yin [10] defined the overlaid ad hoc network model for the research into the STP of primary and secondary networks. Chen [11] developed an efficient and closed-form approximation of the throughput capacity in ad hoc network. To finalize the expression of the STP, all the above researches adopt the random receiver (RX) selection strategy [12], which determines the signal-to-interference ratio (SIR) at the RX.
Hence, it is very essential to study how different RX selection strategies affect the packet success probability (PSP).

In light of the above, this paper probes into the PSPs with nearest RX selection strategy and farthest RX selection strategy. The rest of this paper is organized as follows: Section 2 defines the network model; Section 3 investigates the STP; Section 4 verifies the performance of the PSP through the numerical simulation; Section 5 wraps up this paper with some useful conclusions.

2. NETWORK MODEL

For a wireless ad hoc network on a 2D plane, the terminal location can be viewed as a homogeneous Poisson point process (PPP) $\lambda$ with spatial density $\lambda$ [13]. Let $p$ be the probability for a terminal to send a packet. Then, the transmitters (TXs) could form a PPP $\Pi$ with spatial density $p\lambda$, and the other terminals could be taken as the RX which form a PPP $\Pi$ with spatial density $(1-p)\lambda$. For the Poisson network of intensity $\lambda$, the number of nodes in the area $D$, denoted as $N_D$, is Poisson with mean $\lambda D$:

$$P(N_D=k) = e^{-\lambda D} \frac{(\lambda D)^k}{k!}$$

To measure the network performance, a reference RX was placed at the origin, and the communication pair of the reference RX and its associated TX was denoted as communication pair $0$. Actually, each TX in the network corresponds to at least one RX. TX $i$ and its associated RX $i$ constitute a communication pair $i$. Considering the stationarity of the Poisson process, RX$0$ must have the same statistics as any other RX. For the signals received at RX $0$, all those transmitted from the TX other than TX $0$ are regarded as interferences. Compared with these interferences, the ambient noise is so small that it is negligible.

During propagation, the signals in the wireless ad-hoc network are subjected to path loss attenuation $d^{-\alpha}$ for the distance $d$ with exponent $\alpha > 2$. Let $P_i$ be the power of the signals transmitted from TX $i$; $H_{ij}$ the fading power factor between TX $i$ and RX $j$; $X_{ij}$ the distance between TX $i$ and RX $j$; $R_i$ the transmission distance between TX $i$ and RX $i$, and $I_{\Pi_i}$ the cumulative interference at RX $0$. Then, the cumulative interference at RX $0$, denoted as $I_{\Pi_i}$, can be expressed as:

$$I_{\Pi_i} = \sum_{i\in\Pi_i} P_i H_{ij} X_{ij}^{-\alpha}$$

According to the channel inversion power control strategy, each TX will adjust its transmission power $P_i$ to ensure the constancy of the power $P_r$ of the signals received at its associated RX. The value of $P_i$ can be calculated as:

$$P_i = P_r H_{ii}^{-1} R_i^\alpha$$

Let $\beta$ be the SIR threshold, that is, the signals from TX $i$ can not be received successfully at RX $i$ if the SIR at RX $i$ is below $\beta$. Since the signal power from TX $0$ is a constant $P$, the SIR at RX $0$ can be obtained from Eqs. (1) and (2):

$$SIR = \frac{1}{\sum_{i\in\Pi_0} R_i^\alpha H_{i0}^{-1} H_{i0} X_{i0}^{-\alpha}}$$
The STP can be expressed as:

$$S(\lambda) = \Pr(SIR \geq \beta) = \Pr\left( \sum_{i \in \Pi_i} R_i^a H_i^{-1} H_{i0} X_{i0}^{-\alpha} \leq \beta^{-1} \right)$$  \hspace{1cm} (4)$$

This paper considers a typical multi-hop transmission scenario. The destination RX is assumed to be sufficiently far from the source TX that the distance between them is infinite. The selected RX for each hop must be closer to the destination than the source. Therefore, each TX is allowed to select a terminal from a region of candidate RXs that does not send packet in the same slot as its associated RX. The RX selection region is defined as a semicircle with radius $R_m$, which equals the maximum transmission distance in that hop. As shown in Figure 1, the line $AB$ crosses the centre of the semicircle, and points towards the direction of the destination RX.

![Fig.1 RX selection region](image)

The nearest RX selection strategy and the farthest RX selection strategy were studied in detail. The former selects the nearest RX within the RX selection region as the relay, while the latter selects the farthest RX within the RX selection region as the relay.

To calculate the PSP, it is assumed that the level of cumulative interference is constant during the transmission of a packet. The symbol errors are independent, and the packet length is denoted as $N$. Thus, the PSP can be expressed as:

$$P_s(\lambda) = [S(\lambda)]^N$$  \hspace{1cm} (5)$$

Furthermore, the network is assumed to have Rayleigh fading, whose fading power factor $H_{ij}$ has an exponential distribution with the parameter $\tau$.

### 3. ANALYSIS OF AD HOC NETWORKS WITH DIFFERENT RX SELECTION STRATEGIES

#### 3.1 STP

Eq. (4) can be rewritten as:

$$S(\lambda) = \Pr(I \leq \beta^{-1}) = F_I(\beta^{-1})$$  \hspace{1cm} (6)$$

where:

$$I = \sum_{i \in \Pi_i} R_i^a H_i^{-1} H_{i0} X_{i0}^{-\alpha}$$  \hspace{1cm} (7)$$

and $F_I(*)$ is the cumulative distribution function (CDF) of $I$. The first step to obtain the value of $F_I(*)$ is to determine the characteristic function of $I$. 

...
Inspired by [14], the characteristic function of $I$ can be established as:

$$
\Phi(w) = e^{-\lambda \pi E(R_m^2)E(H_{i0}^\gamma)E(H_{ii}^\gamma)\Gamma(1-\gamma)\left|w\right|^\gamma e^{-\frac{1}{2}\alpha\left|\sin(\gamma\pi)\right|}}
$$

(8)

where $\gamma=2/\alpha$; $\Gamma(*)$ is Gamma function. Since $H_{i0}$ and $H_{ii}$ have exponential distributions with parameter $\tau$, we have:

$$
E(H_{i0}^\gamma) = \int_0^\infty h^\gamma \tau \exp(-\tau h) dh = \Gamma(1+\gamma)
$$

(9)

and

$$
E(H_{ii}^\gamma) = \int_0^\infty h^\gamma \tau \exp(-\tau h) dh = \Gamma(1-\gamma)
$$

(10)

According Eqs. (9) and (10), we have:

$$
E(H_{i0}^\gamma)E(H_{ii}^\gamma) = \gamma \pi \csc\csc(\gamma\pi)
$$

(11)

Substituting (11) into (8), we have:

$$
\Phi(w) = \exp\left(-\frac{\lambda \pi E(R_m^2)\Gamma(1-\gamma)E(R_i^2)}{\sin(\gamma\pi)}e^{-\frac{1}{2}\alpha\left|\sin(\gamma\pi)\right|}\left|w\right|^\gamma\right)
$$

(12)

Substituting $\gamma=0.5 (\alpha=4)$ into (12), probability of density function (PDF) of $I$ can be obtained through inverse transformation:

$$
f_i(x) = \frac{\lambda \pi^2 E(R_i^2)}{4} x^{-1.5} e^{-\frac{x^2}{16}}
$$

(13)

Integrating the PDF (13), the CDF can be obtained from:

$$
F_I(x) = \text{Erfc}\left(\frac{1}{4\lambda E(R_i^2)\pi^{2.5} x^{-0.5}}\right)
$$

(14)

Substituting (14) into (6), we have:

$$
S(\lambda) = \text{Erfc}\left(\frac{1}{4\lambda E(R_i^2)\pi^{2.5} \beta^{-0.5}}\right)
$$

(15)

### 3.2 NETWORK PERFORMANCE WITH NEAREST RX SELECTION STRATEGY

Let $c(x,y)$ be the circle whose inside radius is $x$ and outside radius is $y$. Then the RX selection region can be expressed as $c(0, R_m)$. If $\Pi_r$ is the set of all RX s, then $\Pi_r \cap c(x,y) \neq \emptyset$ means there is at least one RX in $c(x,y)$ and $\Pi_r \cap c(x,y) = \emptyset$ means there is no RX in $c(x,y)$. With the nearest RX selection strategy, the CDF of $R_i$ can be expressed as:

$$
F_{i0}(r) = 1 - \text{Pr}(\Pi_r \cap b(0,r) = \emptyset | \Pi_r \cap b(0,R_m) \neq \emptyset) = \frac{1 - \exp[-(1-p)\lambda \pi r^2]}{1 - \exp[-(1-p)\lambda \pi R_m^2]}
$$

(16)
Then, $E(R_i^2)$, the expectation of $R_i^2$, can be expressed as:

$$E(R_i^2) = \int_0^{R_m} r^2 dF_r(r)$$  \hspace{1cm} (17)

Let $K_n$ be $E(R_i^2)$. Substituting (16) into (17), we have:

$$K_n = \frac{1}{(1-p) \lambda \pi} \frac{R_m^2 \exp[-(1-p) \lambda \pi R_m^2]}{1 - \exp[-(1-p) \lambda \pi R_m^2]}$$  \hspace{1cm} (18)

Substituting (18) in (6), we have:

$$P_n^f(\lambda) = \left[ \text{Erfc} \left( \frac{1}{4} \lambda K_n \pi^{2.5} \beta^{0.5} \right) \right]^N$$  \hspace{1cm} (19)

Then, the expected value of $R_i$ can be expressed as:

$$E(R_i) = \int_0^{R_m} r dF_r(r)$$  \hspace{1cm} (20)

Let $D_n$ be $E(R_i)$. Substituting (16) into (20), we have:

$$D_n = \frac{\text{Erf} \left( \sqrt{(1-p) \lambda \pi R_m} - 2 \sqrt{(1-p) \lambda \pi R_m e^{-(1-p) \lambda \pi R_m^2}} \right)}{2 \sqrt{(1-p) \lambda} \left[ 1 - e^{-(1-p) \lambda \pi R_m^2} \right]}$$  \hspace{1cm} (21)

Let $L$ be the length between the source TX and the destination RX. Then, the total number of hops with the nearest RX selection strategy can be obtained from:

$$N_n = \text{Ceiling} \left( \frac{L(1 - e^{-(1-p) \lambda \pi R_m^2})}{\text{Erf} \left( \sqrt{(1-p) \lambda \pi R_m} \right) - R_m e^{-(1-p) \lambda \pi R_m^2}} \right)$$  \hspace{1cm} (22)

where Ceiling$(x)$ is the smallest integer greater than or equal to $x$.

### 3.3 THE PERFORMANCE OF AD HOC NETWORKS WITH FARthest RX SELECTION STRATEGY

With the farthest RX selection strategy, the CDF of $R_i$ can be expressed as:

$$F_n^f(r) = \Pr(\Pi_r \cap b(r, R_m) = \emptyset | \Pi_r \cap b(0, R_m) \neq \emptyset) = \frac{e^{\lambda(1-p) \pi r^2} - 1}{e^{\lambda(1-p) \pi R_m^2} - 1}$$  \hspace{1cm} (23)

Let $K_f$ be $E(R_i^2)$. Substituting (23) into (17), we have:

$$K_f = \frac{1 + \left[ (1-p) \lambda \pi R_m^2 - 1 \right] e^{(1-p) \lambda \pi R_m^2}}{(1-p) \lambda \pi (e^{(1-p) \lambda \pi R_m^2} - 1)}$$  \hspace{1cm} (24)
Substituting (24) into (6), we have:

\[ P_s^f(\lambda) = \left[ \text{Erfc}\left( \frac{1}{4} \lambda K \pi^2 0.5^2 \rho \right) \right]^N \]  

Let \( D_N \) be \( E(R_j) \). Substituting (16) into (20), we have:

\[ D_f = \frac{2\sqrt{(1-p)\lambda R_m e^{(1-p)\lambda R_m^2}} - \text{Erfi}\left( \sqrt{(1-p)\lambda R_m} \right)}{2\sqrt{(1-p)\lambda e^{(1-p)\lambda R_m^2}} - \text{Erfi}\left( \sqrt{(1-p)\lambda R_m} \right)} \]  

where \( \text{Erfi}(\cdot) \) is the imaginary error function. Then, the total number of hops with the farthest RX selection strategy can be obtained from:

\[ N_f = \text{Ceiling} \left( \frac{L\left( e^{(1-p)\lambda R_m^2} - 1 \right)}{R_m e^{(1-p)\lambda R_m^2} - \text{Erfi}\left( \sqrt{(1-p)\lambda R_m} \right)} \right) \]  

4. NUMERICAL SIMULATION

This section evaluates the performance of the STP and PSP. Unless specified otherwise, the network parameters are set as follows: \( p = 0.1, R_m = 20 \text{ m}, \beta = 3, N = 10 \) and \( \alpha = 4, L = 2000 \text{ m} \).

![Figure 2](image_url)

**Fig. 2** Relationship between STP and terminal density \( \lambda \) at \( p=0.1 \) and \( p=0.5 \)

Figure 2 shows how STP varies with terminal densities \( \lambda \) at \( p=0.1 \) and \( p=0.5 \). It can be seen that the STP decreases with the terminal density \( \lambda \), and that the nearest RX selection strategy boasts a higher STP than the farthest RX selection strategy. The growth in terminal density \( \lambda \) means the interference power in the ad hoc network is on the rise. As a result, the SIR of the constant power of the received signals decreases, leading to a reduction of the STP.
When the terminal density $\lambda$ remains constant, there is no change to the interference power. In this case, the nearest RX selection strategy produces greater SIR than the farthest RX selection strategy, because of its relatively small transmission distance.

With the nearest RX selection strategy, the transmission distance is smaller at $p=0.5$ than at $p=0.1$; with the farthest RX selection strategy, the transmission distance is greater at $p=0.5$ than at $p=0.1$. The STP of random RX selection strategy must fall between the results of these two strategies.

Figure 3 shows how PTP varies with terminal densities $\lambda$ when the packet length is $N=10$ or $N=5$. It can be seen that the PTP decreases with terminal density $\lambda$. When the terminal density remains constant, the PSP is greater at $N=5$ than at $N=10$, and it is greater with the nearest RX selection strategy than with the farthest RX selection strategy. The reason for these results is as follows: The PSP is mainly dependent on the STP. A greater packet length means more signals received successfully in the same slot. The PTP of random RX selection strategy must fall between the results of these two strategies.

![Fig. 3 Relationship between PTP and terminal density $\lambda$ at N=10 and N=5](image1)

![Fig. 4 Relationship between number of hops and terminal density $\lambda$](image2)
Figure 4 shows the variation in the number of hops with terminal densities $\lambda$. It can be seen that the number of hops increases with the terminal density $\lambda$ with the nearest RX selection strategy but decreases with the terminal density $\lambda$ with the farthest RX selection strategy. When the terminal density $\lambda$ remains constant, the number of hops is greater with the nearest RX selection strategy than with the farthest RX selection strategy. This is because the growth in terminal density $\lambda$ means more TXs in the same slot, and the transmission distance per hop is reduced with the nearest RX selection strategy (and increased with the other strategy).

After a packet is transmitted from a TX, it will not be resent to the next slot if it is not received successfully at the associated RX. Let $T$ be the duration of one slot. Then, the time to transmit a packet from the source TX to the destination RX can be expressed as:

$$T_n = \frac{N_n}{P_s(\lambda)}$$ (28)

Figure 5 compares the $T_f - \lambda$ curve and $T_n - \lambda$ curve. As shown in the figure, $T_f$ is slightly below $T_n$ at a small terminal density $\lambda$, and much higher at a high terminal density. These trends can be attributed to the following facts: since the interference power is close to zero at a small terminal density, the STP is almost the same with whichever RX selection strategy, and the number of hops is greater with the farthest RX selection strategy than the nearest one; the interference power cannot be neglected at a high terminal density and the packet transmission duration from source TX to destination RX is mainly determined by the STP.

5. CONCLUSION

Through the above analysis, a trade-off was found between transmission duration and the PSP. The nearest RX selection strategy should be chosen to yield a high PSP, while the farthest RX selection strategy should be applied to shorten the transmission duration. As for the random RX selection strategy, its PSP and transmission duration must fall between those of the nearest RX selection strategy and the farthest RX selection strategy.
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7. REFERENCES


