

Evaluation of Eigenvalue and Block Diagonalization Beamforming Precoding Performance for 5G Technology over Rician Channel

Cebraıl ÇİFLİKLI, Musaab AL-OBAIDI

Abstract: In traditional wireless cellular, at the same cell, users can cause co-channel interference (CCI) between each other; CCI can deteriorate the channel's capacity. A multiple-input multiple-output (MIMO) system with beamforming technology solves this CCI problem. Exploiting the channel state information (CSI) in a multi-user MIMO (MU-MIMO) system can improve the performance of the channel link by designing the precoding vectors for every user. A linear precoder has multiple methods, like Block diagonalization precoding (BDP) and Eigenvalue precoding (EP) that facilitate its use. This paper evaluates the symbol-detection performance for BDP and EP in MU-MIMO beamforming over a Rayleigh fading channel. Then, the channel matrix replaces the typical channel assumption with its correlated realistic Rician fading channel. Simulation results show that the Rician fading channel has performance improvement until with low Rician factor value, compared to a conventional channel. The high value of the Rician factor can reduce the error rate.

Keywords: beamforming; block diagonalization; CSI; eigenvalue; fading; MU-MIMO

1 INTRODUCTION

At the moment, it is not yet clear which technologies will do the most for 5G in the long run, but a few early favourites have emerged. The front-runners include beamforming, millimeter waves, small cells, massive MIMO and full duplex [1] [2]. Therefore, we started evaluating the beamforming technology as one of 5G technologies. In wireless mobile networks, transmitting data through a wireless channel may lead to the possibility of changes to the data, causing errors. The error probability for a fading channel is inversely proportional to the signal-to-noise ratio (SNR). Each wireless channel has an individual and independent fading channel that is different from the other channels [3]. Therefore, information that is transmitted over a wireless channel has different distributions, depending on the broadcast environment. When a system transmits information in a line-of-sight (LOS) environment, the wireless channel has a Rician distribution as its probability density function (PDF). On the other hand, a wireless channel has a Rayleigh distribution as its PDF in a non-line-of-sight (NLOS) environment [4]. Based on space-time theory, the performance of any wireless system is improved through a powerful tool generated by using suitable techniques (at the receiver node) and signalling (at the transmitter node), together with a number of antennas at both sides of the wireless communication network nodes [5]. Increasing numbers of wireless communication customers lead to reuse frequency for the efficient use of the available spectrum. Unfortunately, the reason for decreasing system capacity and service quality may be because of interference that comes from increased frequency reuse. Therefore, the MIMO system has captured significant attention, considering its possible ability to improve the capacity of wireless channel networks. For this reason, MIMO systems have been extensively studied due to their features. Nevertheless, it has depended on the unreality models of channels such as Rayleigh and the Rician fading channel, which is presumption and theoretical channel to evaluate the performance of MIMO systems [6]. In multi-user MIMO (MU-MIMO) scenarios, numerous mobile stations in the same cells in the same time or frequency slots share

the same base station and try to contact it. In this situation, many co-channels occur. According to the participation fact of reuse the sources in MU-MIMO systems, the system suffers from multi-user interference. The main challenge in MU-MIMO communications is that the transmitter has the ability to coordinate transmissions from all of its antenna elements, while the users are typically unable to coordinate with each other [7]. Therefore, the operation of multi-user interference cancellation is often done on the BS side. Linear precoding at the wireless communications station, which is installed at a fixed location, and decoding at the user approaches are necessary to solve this problem [8]. In order to mitigate the co-channel interference that occurs at mobile stations, the designer must create a transmitter that can send information to several mobile stations via beamforming vectors. Particularly, there has been significant attention given to MU-MIMO systems as a future technology for LTE Advanced, which promises to provide optimal execution to wireless communication systems [9]. It should be mentioned that at any time, a sender of a MIMO system has no information about the CSI, neither multi-user diversity nor spatial multiplexing can be achieved [10]. On the other hand, if the CSI of all mobile stations is available at the transmitter, then the precoder is capable of fully removing CCI. By removing CCI, each user can communicate with the transmitter over an interference-free, single-user channel [11].

Therefore, through an imperfect feedback channel, reconnaissance of limited CSI and employment of CSI are critical points for a MIMO system [10]. In cases when the channel state information at the transmitter (CSIT) is on hand and using a MIMO system with a linear precoding technique, it can accrue additional profits [13]. CSIT is very important, because when it is fully available at the base station, the MIMO system performs best in numerous ways via using the precoding method. For example, to mitigate symbol interference, precoding can be used with spatial diversity and spatial multiplexing provided by the MIMO system. Besides high gain coding, if space-time codes can be combined with precoding, maximum gain diversity is available [14]. The BD algorithm that supports multiple-stream transmissions for MU-MIMO systems in which every user has a number of antennas trying to

connect with the base station can eliminate the CCI completely [15]. A BDP scheme can be used for designing transmit beamforming vectors in an uncomplicated way. This technique has been proposed to obtain a precoding matrix for each mobile station. This matrix is forced to lie in the null space of other mobile stations' channel matrices. This means that the beamforming technique depends on the spatial information of each mobile station. Unfortunately, the desired power of the signal reception will be reduced [16]. On the other hand, the EP algorithm can get the beamforming technique. This algorithm can design a beamforming precoding matrix based on maximizing the signal-to-leakage ratio (SLR). This means that it maximizes the useful power of received signals at the mobile station, while minimizing the overall interference power caused by the mobile station at all other co-channel mobile stations [17]. Compared to the BDP scheme, the EP algorithm maximizes the desired signal power for each mobile station. Unfortunately, it also takes no action to lower CCI, making it an interference-limited system with high SNRs [18, 16]. In this paper, we analyse and present an evaluation of both the BDP and EP at the transmitter for the wireless fading channel as transmission media to the signal and ZF detection at the receiver when CSI exists. We also discuss our simulation of the performance of an MU-MIMO beamforming system that employs BD with ZF and EP with a maximum-likelihood detection scheme and transmits the signal over a Rayleigh fading channel (in the NLOS environment) and then over a correlated realistic Rician fading channel (in the LOS environment).

2 MU-MIMO BEAMFORMING SYSTEM MODEL

In MU-MIMO system, we have considered an environment of U geographically sparse mobile stations as multi-user (MU) communicates with the MIMO base station (BS) which has M antennas. In such an environment system, each mobile station is independent and employs N_i antennas of user i . These users will receive their own signal, as shown in Fig. 1. The total users' antennas number is defined as;

$$N_T = \sum_{u=1}^U N_u \quad (1)$$

In addition, this system has an operation condition, which is $N_T \leq M$ with the independent channels of flat fading. The meant message signal for the i^{th} user is the scalar s_i . Thereby, the transmitted symbol vector to U users is:

$$\mathbf{S} = [s_1, s_2, \dots, s_U]^T \quad (2)$$

In the second step, we denote to precoding matrix step as:

$$\mathbf{W} = [w_1, w_2, \dots, w_U] \quad (3)$$

where $w_i \in C^{N_i \times M}$ is the joint beamforming coefficients for i^{th} user.

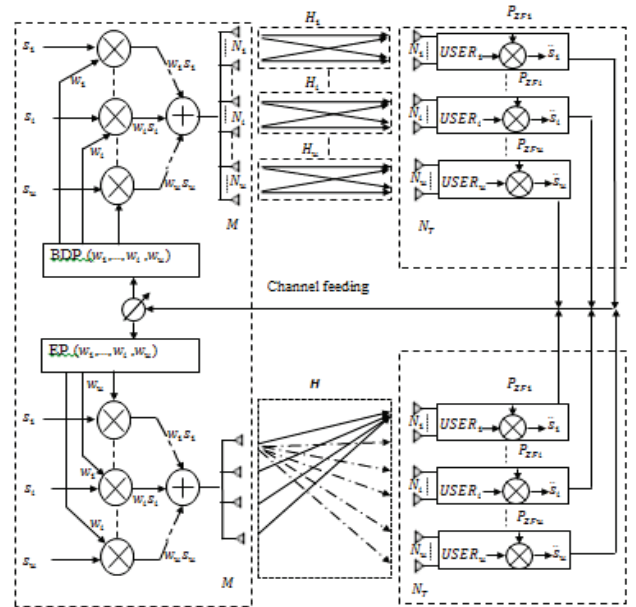


Figure 1 Block diagram of the MU-MIMO beamforming system

Then the transmitted symbol vector is multiplied by the precoding matrix as the third step to produce precoding data as:

$$\mathbf{X} = \sum_{u=1}^U w_u s_u = \mathbf{W}\mathbf{S} \quad (4)$$

The symbol s_i and the coefficients of beamforming precoding w_i will be normalized as follows:

$$E|s_u|^2 = 1, \|w_u\|^2 = 1 \text{ for } u = \{1, \dots, U\}.$$

In the broadcast step, we assumed that signals $\mathbf{W}\mathbf{S} \in C^{N_i \times M}$ are broadcast over the channels denoted as:

$$\mathbf{H} = [H_1^T, H_2^T, \dots, H_U^T]^T \quad (5)$$

where $H_i \in C^{N_i \times M}$ describes the channel coefficients between N_i receiver antenna at i^{th} user and BS antennas as:

$$H_i = \begin{bmatrix} h_i^{(1,1)} & \dots & h_i^{(1,M)} \\ \vdots & \ddots & \vdots \\ h_i^{(N_u,1)} & \dots & h_i^{(N_u,M)} \end{bmatrix} \quad (6)$$

where $h_i^{(n,m)}$ denotes the channel matrix ingredient which is located between the m^{th} transmitter array antenna of base station and the n^{th} receiver array antenna of i^{th} user.

Thus, at users' antennas the received signals as:

$$\mathbf{y} = [y_1^T, y_2^T, \dots, y_U^T]^T = \mathbf{H}\mathbf{W}\mathbf{S} + \mathbf{n} \quad (7)$$

where $y_i \in C^{N_i \times M}$ represents the signal which is received at i^{th} recipient, whilst for the additive noise is denoted by \mathbf{n} . When we have given careful consideration to each user

separately, we will find the received signal at an i^{th} recipient as:

$$\begin{aligned}
 y_i &= H_i \sum_{u=1}^U w_u s_u + n_i \\
 y_i &= H_i w_i s_i + H_i \sum_{u=1, u \neq i}^U w_u s_u + n_i \\
 y_i &= H_i x_i + H_i \sum_{u=1, u \neq i}^U w_u + n_i
 \end{aligned} \quad (8)$$

where x_i is precoding data of i^{th} user and w_u is precoding data of other users as interference for the i^{th} user. The H_i vector has complex Gaussian variables components with unit-variance and zero-mean. Moreover, the components of the additive noise n_i have distribution as $N(0, \sigma_i^2)$ and are temporarily white and spatial.

3 DOWNLINK CHANNEL MODEL

Due to LOS propagation the strongest propagation component of MIMO channel corresponds to deterministic component (also referred to as specular components). On the other hand, all the other components are random components (due to NLOS also referred to as scattering components) [3]. The broadcast channel distribution has been following the Rayleigh channel distribution which is Gaussian distribution with a variance of σ^2 and zero mean. That means there is no component of LOS ($K = 0$):

$$\sigma = \sqrt{\frac{1}{K+1}}.$$

On the other hand, when there is any

component of LOS (for $K > 0$) the broadcast channel distribution has been following the Gaussian distribution with a variance of σ^2 and mean of q or Rician distribution when K increases as:

$$q = \sqrt{\frac{K}{K+1}}, \quad \sigma = \sqrt{\frac{1}{K+1}}.$$

Therefore, in this work, channel matrix of MIMO system tends to be described as [19]:

$$\mathbf{H} = \sqrt{\frac{K}{K+1}} H_d + \sqrt{\frac{1}{K+1}} H_r. \quad (9)$$

where H_d represents the component of the normalized deterministic channel matrix, while H_r represents the component of random channel matrix, with $\|H_d\|^2 = N_T M$, $E\{|[H_r]_{i,j}|^2\} = 1$, $i = 1:N_T, j = 1:ME$ [19]. While K is known as factor of the Rician channel which is the relation between the component of the specular power c^2 and the component of scattering power $2\sigma^2$, displayed as [3]:

$$K = \frac{\|H_d\|^2}{E\{|[H_r]_{i,j}|^2\}} = \frac{c^2}{2\sigma^2}. \quad (10)$$

4 PRECODING DESIGN OF DOWNLINK CHANNEL

Precoding can be used in MIMO to mitigate or cancel the user interference. For lower complexity, we concentrate on linear precoding methods in this work. BDP and EP are linear precoding methods. To design BDP and EP it needs to CSI. Therefore, we suppose that the CSI of the wireless communication network in the cell is recognized to BS in Frequency-division duplexing (FDD) or Time-division duplexing (TDD) systems. In FDD systems, BS can get the downlink CSI by feedback of the users at the same cell. While in TDD systems, according to channel reciprocity, BS can correctly guess the downlink CSI due to the uplink CSI.

4.1 Block Diagonalization Precoding (BDP)

BDP method is compatible with the multiple users, every user has multiple antennas. By the precoding process of this method, the interference signal which is coming from other user signals will be canceled. Therefore, MU-MIMO channel model will be converted into multiple independent single user MIMO channels model by BD method [3].

Briefly, definition of BDP matrix starts from the channel of all users except the i^{th} user as:

$$\tilde{H}_i = [H_1, \dots, H_{i-1}, H_{i+1}, \dots, H_U]^T \quad (11)$$

where \tilde{H}_i is H without H_i , then we should compute the null space \tilde{V}_i^{ns} of all users except the i^{th} user by *SVD* to \tilde{H}_i :

$$\text{SVD of } \tilde{H}_i = \tilde{U}_i \tilde{\Lambda}_i [\tilde{V}_i^b \tilde{V}_i^{ns}]^H \quad (12)$$

where $(.)^H$ denotes Hermitian transposition. To prevent other users interference multiplies H_i by \tilde{V}_i^{ns} , and then uses *SVD* again:

$$\text{SVD of } H_i \tilde{V}_i^{ns} = U_i A [V_i^b V_i^{ns}]^H \quad (13)$$

where V_i^{ns} is the null space of the i^{th} user, while V_i^b is the beam of the i^{th} user. Therefore, we can get the precoding matrix w_i for the i^{th} user from \tilde{V}_i^{ns} and V_i^b as:

$$w_i = [V_i^b \tilde{V}_i^{ns}] \quad (14)$$

However, it imposes a strong condition on the system configuration in terms of the number of antennas [3]:

$$N_T \geq M \quad (15)$$

Now under the condition $\tilde{H}_i \tilde{V}_i^{ns} = 0$, we substitute (14) into (8), we can obtain:

$$y_i = H_i x_i + 0 + n_i \quad (16)$$

where y_i represents the received signal which consists of the required signal of the i^{th} user without multiuser interference. At user i , ZF approach will be used as detection method to estimate x_i from the received signal by the following weight matrix [3]:

$$P_{ZF} = (H^H H)^{-1} H^H \quad (17)$$

The received signal y_i is multiplied by P_{ZF} from the left as space filter output:

$$\begin{aligned} P_{ZF} y_i &= P_{ZF} (H_i x_i + n_i) = \\ &= x_i + (H^H H)^{-1} H^H n_i = x_i + n_{ZF} \end{aligned} \quad (18)$$

4.2 Eigenvalue Precoding (EP)

This scheme computes the maximum beamforming precoding (w_i^o) of each user from the maximum signal-to-leakage ratio (SLR) of these users [17] as follow:

$$SLR = \frac{\|H_i w_i\|^2}{\sum_{u=1, u \neq i}^U \|H_u w_i\|^2} \quad (19)$$

Then

$$w_i^o = \arg \max \frac{\|H_i w_i\|^2}{\sum_{u=1, u \neq i}^U \|H_u w_i\|^2} \quad (20)$$

where $\|H_i w_i\|^2$ represents the required signal power of user i , while $\sum_{u=1, u \neq i}^U \|H_u w_i\|^2$ represents the total leakage power from the total power of user i as an interference on the other users under the condition [17]:

$$N_T \leq M \quad (21)$$

By substituting (11) into (19), we can obtain

$$SLR = \frac{\|H_i w_i\|^2}{\| \tilde{H}_i w_i \|^2} = \frac{w_i^* H_i^* H_i w_i}{w_i^* \tilde{H}_i^* \tilde{H}_i w_i} \quad (22)$$

Depending on [15, 18] we can solve (20) as:

$$\frac{w_i^* H_i^* H_i w_i}{w_i^* \tilde{H}_i^* \tilde{H}_i w_i} \leq \lambda_{\max}(H_i^* H_i, \tilde{H}_i^* \tilde{H}_i) \quad (23)$$

where λ_{\max} is the largest generalized eigenvalue. Equality occurs if w_i is proportional to a generalized eigenvector that corresponds to the largest generalized eigenvalue.

Then

$$w_i^o \propto \max \text{ gen. eigenvector}(H_i^* H_i, \tilde{H}_i^* \tilde{H}_i) \quad (24)$$

The proportionality constant is chosen to normalize the norm of w_i^o to unity. At user i , maximum-likelihood detection scheme will be used to estimate s_i from the received signal as the following [17]:

$$\tilde{y}_i = \frac{w_i^* H_i^*}{\|H_i w_i\|^2} y_i$$

Then

$$\tilde{y}_i = s_i + \frac{w_i^* H_i^* \sum_{u=1, u \neq i}^U H_i w_u s_u}{\|H_i w_i\|^2} + \frac{w_i^* H_i^*}{\|H_i w_i\|^2} n_i \quad (25)$$

5 SIMULATION RESULTS AND EVALUATION

In this section, we present our evaluation of the signal-to-noise ratio (SNR) against the bit error rate (BER) as a scale of precoding efficiency. A typical MU-MIMO scheme was imitated to estimate the performance of the suggested MU-MIMO beamforming-precoding scheme over a Rician fading channel in comparison to the same scheme over a Rayleigh fading channel.

The samples of these parameters are set up to 10000 with elements generated as zero-mean for Rayleigh fading channel while m-mean for Rician fading channel and unit-variance independent and identically distributed (i.i.d) complex Gaussian random variables. For EP we simulate $\{2, 2, 2\} \times 4$ and $\{2, 2, 2, 2\} \times 6$ under the condition (21), while for BDP we simulate $\{2, 2, 2\} \times 6$ and $\{2, 2, 2, 2\} \times 8$ MU-MIMO system under the condition (15), because in BDP imposes a strong condition on the system configuration in terms of the number of antennas (BS antenna $\geq \Sigma$ user antenna). For EP, in order for this problem to be well posed, one needs to require: BS antenna $\leq \Sigma$ user antenna. Interestingly, it is found through simulations that in BDP:BS antenna = Σ user antenna is able to (at least nearly) meet the best result. In EP:BS antenna $\leq (\Sigma$ user antenna $- 2)$.

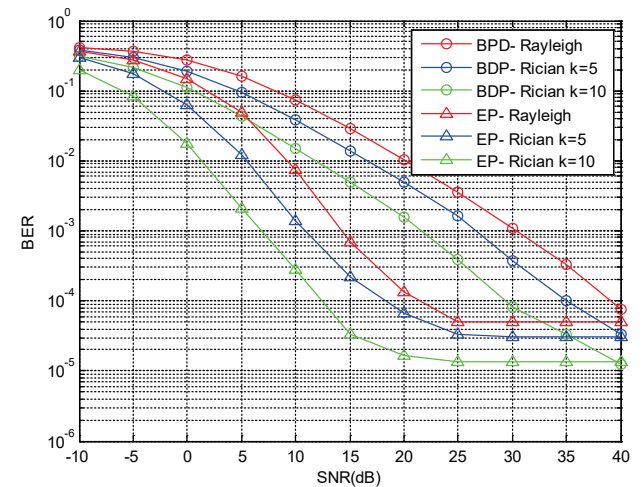


Figure 2 A MU-MIMO system with $M = 6$ for BDP and $M = 4$ for EP and $U = 3$, each equipped with $N_r = 2$ receive antennas

We compared the BD method of [3, 16] which used zero forcing (ZF) algorithms, with EP method of [17],

which used classical maximum-likelihood detection scheme algorithm.

The average BER is taken of BDP approach at BS while the receiver was using a ZF approach. Then, the average BER is taken of EP approach at BS while the receiver was using a maximum-likelihood approach. The BS transmitted by M antennas to each user over the noise and flat fading channel, while each user employed N_u antennas to receive the signal.

A QPSK signal constellation was used as a broadcast modulation in all simulations, and the results are averaged through several channel investigations.

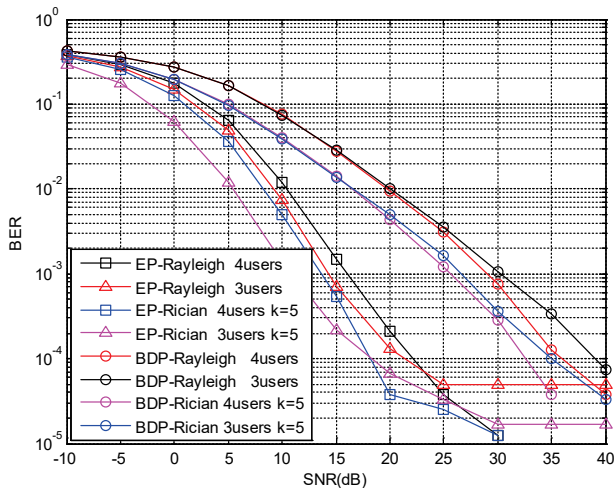


Figure 3 A MU-MIMO system with $M = U \cdot N_t$ for BDP and $M = (U \cdot N_t - 2)$ for EP, each user equipped with $N_r = 2$ receive antennas

For all receivers, the noise variance per receiver antenna should equal $\sigma_1^2 = \dots = \sigma_K^2 = \sigma^2$. As seen in Fig. 2, in the case of BDP, the performance of the system gradually and continuously improved with increases in the values of SNRs, although in the case of EP, system performance was dramatically improved at significantly lower values of SNRs. Unfortunately, when the values of SNRs increased, performance improvement slowly stabilized even at high values of SNRs. That is because of noise and inter-user interference factors. For the EP scheme, which ignores inter-user interference and maximizes useful power, the system performed better than the BDP scheme at low values of SNRs because the effect of noise was bigger than the effect of inter-user interference. Therefore, at low values of SNRs, the major factor limiting the performance of the system was the noise. On the other hand, the BDP scheme, which cancelled multi-user interference, outperformed the EP scheme at high values of SNRs because the effect of inter-user interference was greater than the effect of noise on the system. Therefore, at high values of SNRs, the major factor limiting the performance of the system was inter-user interference. In addition, it can be seen that the MU-MIMO beamforming system (in the case of both the BDP and EP schemes) over conventional channels produced adverse results with regard to BER Fig. 3; the realistic channel gives a suitable BER in the existence of three dynamic users. To best comprehend the behaviour of the suggested schemes, the BER was plotted for different values of K ($K > 0$), compared to the Rayleigh fading channel. Fig. 3

illustrates that to achieve better performance with different numbers of users, it needs high numbers of users for the BDP scheme at all values of SNRs, while low numbers of users for the EP scheme at low values of SNRs and high numbers of users at high values of SNRs. This means that depending on the number of users, the BDP scheme is more stable than the EP scheme even when the system broadcasts over a realistic channel.

6 CONCLUSION

The significant topic of various MIMO downlink fading channel models using pragmatic beamforming by BDP and EP is addressed here. This paper presents an evaluation and analysis of the MU-MIMO downlink channel system's precoding performance over the Rayleigh and Rician fading channels and its parameters, given estimated CSI. The simulation results show that the BER performance of EP schemes with maximum-likelihood detection clearly outpaces BDP schemes with ZF detection, with low values of SNRs. The results also show how MU-MIMO system precoding could achieve significant performance improvements over realistic channels for different Rician factors ($K > 0$) compared to conventional channels, for example, the Rayleigh fading channel. At high values of SNRs, EP scheme performance improvement is limited, whereas for BDP, performance improvement persists.

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Contact information:

Cebrail ÇİFLİKLİ, Prof. Dr.
 Electronic Technology Program/Electronics and Automation,
 Erciyes University,
 38039, Melikgazi/Kayseri, Turkey
 E-mail: cebrailc@erciyes.edu.tr

Musaab AL-OBAIDI, PhD student
 Electrical and Electronic Engineering,
 Erciyes University,
 38039, Melikgazi/Kayseri, Turkey
 E-mail: musaab_sami2000@yahoo.com