ISSN 1330-3651 (Print), ISSN 1848-6339 (Online)
https://doi.org/10.17559/TV-20171121122241
Original scientific paper

Analysis and Computation Method of Torsional Compliance of Worm Gear

Slavko PAVLENKO, Mária KAČALOVÁ, Róbert FAČKOVEC, Tomáš CORANIČ

Abstract: This paper deals with the issue of the torsional compliance of the worm gear. Addressing the issue is one of the most important parts of research and design in manufacturing technology, because demands on machine parameters are increasing and with them the dynamics of processes that are taking place in them. The main objective was to point out the numerical calculation of the compliance of the worm gear. The purpose of this research is to evaluate the compliance of a warm support, eventually of a worm wheel supports, which has a dominant influence on the total torsional compliance of the gearing.

Keywords: equivalent torsional compliance; linear compliance; worm gearing

1 INTRODUCTION

Modern trends in the development of structural education aim at systematizing the process, taking into account all the factors influencing the complex construction process. The growing economic pressures a back increased demand on the optimality of the proposed solutions. Increasingly higher demands on machine parameters are reflected in the growth of dynamism loads which are caused in particular by increasing the absolute speed of individual parts in knots kinematic chains. It is necessary to consider the action of machines with internal and external forces, as well as the dynamics of the processes that take place in them [1-3].

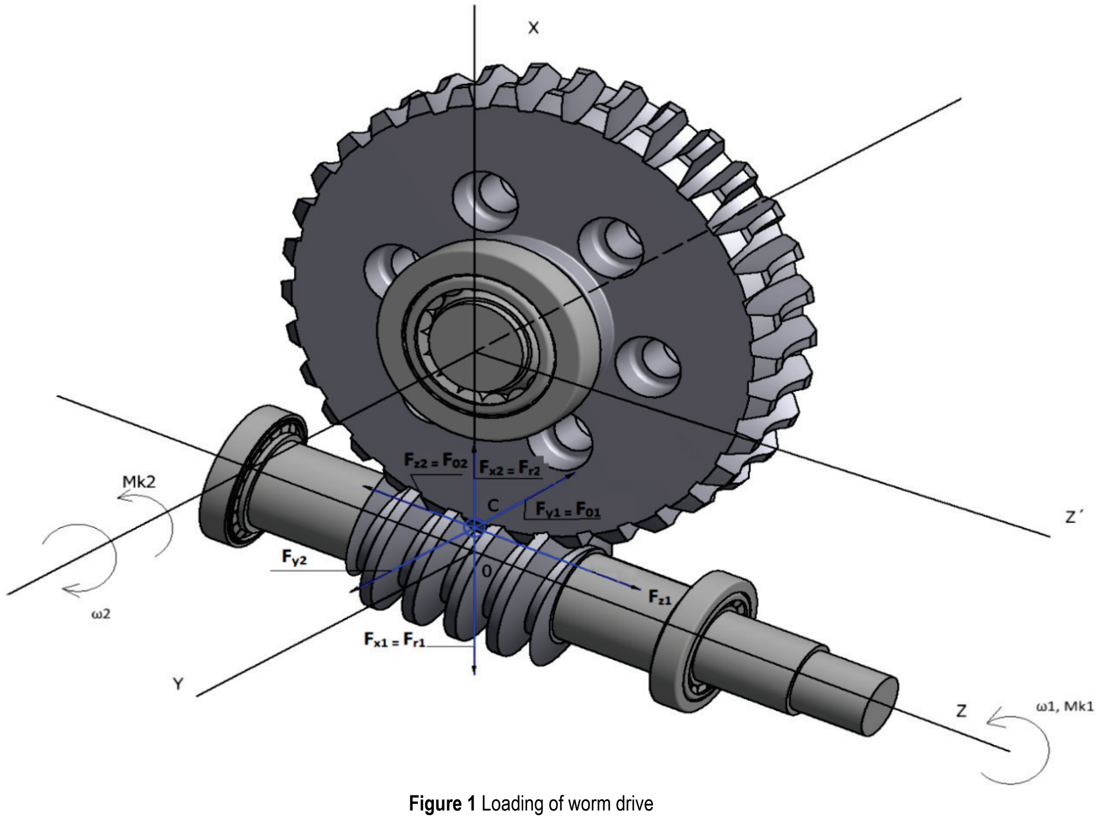

Figure 1 Loading of worm drive

Modern scientific research in the field of design and construction machine parts is moving in the same direction in the development of methods for the analysis of dynamic phenomena occurring in machines and mechanisms [4-8]. Worm gears are one of the main knots for drive machines and are widely used in various fields of industrial practice. Increasing their operability is paid much attention constantly. Most often, this problem is solved by traditional methods and by improving the quality and heat treatment of materials for worm and worm wheel using the present lubricant compositions [9, 10]. For heavier modes associated with high start, stop and reverse frequencies, effective material results do not reach the material properties of worm gear elements [12, 13]. For this reason, research was aimed at analysing the stiff parameters of the worm gear. Due to the load, individual worm gears deform and occupy a new position [14, 15]. Since the teeth contact of the cylindrical worm gears runs along a spatial curve (touch line), it is not possible to predict a uniform load distribution along these lines. Therefore, the simplification which has been used so far, and the load of the worm gears was concentrated at the rolling point C shown in Fig. 1. By introducing the coordinate system 0 (x, y, z), the force sizes defined in the direction of the individual co-ordinate axes are loaded with the worm gear. As shown in Fig. 1, the warm shaft and the warm wheel are loaded with the three forces F_{x1}, F_{y1}, F_{z1} (F_{x2}, F_{y2}, F_{z2}), due to the stress concentrating in three mutually perpendicular forces, it is not important in this case to examine the tooth loads in the normal plane research focused on the force acting in a direction tangential to the pitch circle radius F_{02} ($F_{z2} = F_{z1}$). We assume that the shaft loaded by the forces F_{x1}, F_{y1}, F_{z1} is deformed and this deformation is manifested in terms of the torsional compliance of the transmission, so it is necessary to perform the analysis in individual planes.

2 COMPLIANCE OF SUPPORTS

Fig. 1 shows the loading of worm gearing components which is at certain simplified assumption concentrated in the central point C. Influenced by this loading a displacement of the gearing components occurs which is proportional to the compliance of their supports [1, 23, 24, 25].

Compliance of worm supports can be expressed as:

$$\delta_1 = \delta_{p1} = \delta_{01} \tag{1}$$

where: δ_{p1} – compliance of bearing (m/N); δ_{01} – bending compliance of a worm shaft (m/N).

By means of the expression (1) it is possible to describe the compliance of worm supports, or eventually of worm wheel supports (index 2) in the planes x-z, y-z, or y'-z' which are perpendicular to each other.

3 BENDING COMPLIANCE OF SHAFTS

A shaft of a worm and of a worm wheel is loaded by three forces F_{x1}, F_{y1}, F_{z1} (F_{x2}, F_{y2}, F_{z2}). Loading F_{x1} acting in the plane x-z cause the deflection Y_{xz1} (Fig. 2).

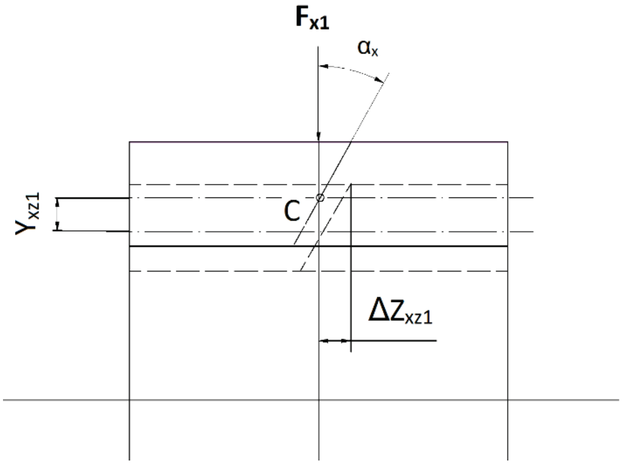

Figure 2 The deflection Y_{xz1} and "clearance" ΔZ_{xz1}

The deflection may be expressed as the following:

$$Y_{xz1} = \delta_{01} F_{x1} \tag{2}$$

Similarly, this way we can define the deflection of a worm shaft in the plane y-z, or eventually of a worm-gear shaft in the planes x-y, x'-z'.

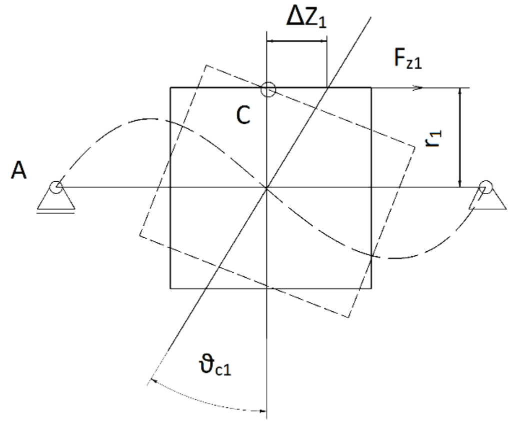

Figure 3 The displacement ΔZ_1

Axial loads F_{z1}, F_{z2} do not cause deflections in the point C but they cause rotation of a shaft cross section (Fig. 3) by the angle ϑ_1 and the displacement corresponding to this rotation, which we write as:

$$\Delta Z_1 = \vartheta_1 r_1 \tag{3}$$

where: r_1 – radius of a pitch cylinder of a worm (m); ϑ_1 – rotation of a shaft cross section (rad), and for small angles of ration we assume $\sin \vartheta_1 = \vartheta_1$.

4 EQUIVALENT TORSIONAL COMPLIANCE

Equivalent torsional compliance is considered the compliance causing the rotation of the gearing components equal to that deformation of shafts and bearings. In defining such a compliance, it is necessary to take into account kinematic link among elements of a worm gearing.

The deflection Y_{xz1} shown in Fig. 2 causes a "clearance" ΔZ_{xz1} (comparing with the position of a worm in an unloaded state), for which it applies that:

$$\Delta Z_{xz1} = Y_{xz1} \tan \alpha_x \tag{4}$$

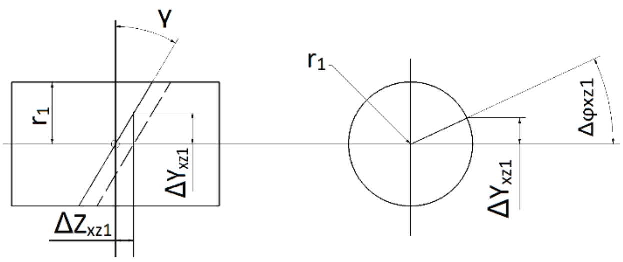

Figure 4a **Figure 4b**

To this value in the plane y-z (Fig. 4a) in the direction of axis y corresponds the value ΔY_{xz1}. We can express it as:

$$\Delta Y_{xz1} = \frac{\Delta Z_{xz1}}{\tan \gamma} \tag{5}$$

and for small angles we can write (Fig. 4b):

$$\tan \varphi_{xz1} = \Delta\varphi_{xz1} = \frac{Y_{xz1}}{r_1} \tag{6}$$

where: $\Delta\varphi_{xz1}$ – additional rotation of a worm compensating "the clearance" caused by bending strain Y_{xz1} (rad).

Then Y_{xz1} is expressed depending upon Mk_1 in the form:

$$Y_{xz1} = \delta_1 Mk_1 = \frac{\tan \alpha_n \cos \rho}{r_1 \sin (\gamma + \rho)} \tag{7}$$

where: α_n – pressure angle in the normal plane (deg); γ – lead angle of a helix on a pitch cylinder (deg); ρ – angle of friction (deg).

Using relation (6) and (7), we get:

$$\Delta\varphi_{xz1} = \frac{\delta_1 Mk_1}{r_1 \tan \gamma} \tag{8}$$

Let is denote

$$\delta_{xz1} = \frac{\delta_{xz1}}{r_1 \tan \gamma} A_{xz1} \tag{9}$$

where: δ_{xz1} – equivalent torsional compliance corresponding to the compliance of worm supports in the plane *x-z* (rad/Nm).

$$A_{xz1} = \frac{\tan^2 \alpha_n \cos \rho}{r_1 \sin (\gamma + \rho)}, \text{m}^{-1} \tag{10}$$

Similarly, the equivalent torsional compliance caused by loading deformation F_{y1} in the plane *y-z* will be:

$$\delta_{yz1} = \frac{\delta_1}{r_1} A_{yz1} \text{ where } A_{yz1} = \frac{1}{r_1}, \text{m}^{-1} \tag{11}$$

Event, the equivalent torsional compliance due loading deformation F_{z1} (Fig. 3) is:

$$\delta_{Fz1} = \frac{\delta_{01}}{\tan \gamma} A_{Fz1} \text{ where } A_{Fz1} = \frac{4}{l_1^2 \tan (\gamma + \rho)}, \text{m}^{-2} \tag{12}$$

where: l_1 – bearing distance (m); δ_{01} – bending compliance of a worm shaft (in calculation δ_{01} we can take into consideration the influence of the kind of a bearing and the way the shaft placement) (m/N).

Resultant equivalent torsional compliance corresponding to the compliance of worm supports is:

$$\delta_{0h1} = \delta_{xz1} + \delta_{yz1} + \delta_{Fz1} \tag{13}$$

Similarly, for worm wheel supports we write:

$$\delta_{0h2} = \delta_{xz2} + \delta_{y'z'2} + \delta_{Fz2} \tag{14}$$

where:

$$\delta_{xz2} = \frac{\delta_2}{r_2} A_{xy2}, A_{xy2} = \frac{\tan^2 \alpha_n \cos \rho}{r_2 \cos (\gamma + \rho) \cos \rho} \tag{15}$$

where: r_2 – radius of a pitch cylinder of a worm wheel (m); δ_2 – compliance of worm wheel supports in the terms of the relation (1) (m/N),

In the plane *y′-z′* (Fig. 1) from the loading F_{z2}

$$\delta_{y'z'2} = \frac{\delta_2}{r_2} A_{y'z'2} \text{ where } A_{y'z'2} = \frac{1}{r_2}, \text{m}^{-1} \tag{16}$$

and from the rotation of a worm wheel due to loading F_{y2}:

$$\delta_{Fy2} = \delta_{02} \tan \gamma A_{Fy2},$$
$$\text{where } A_{Fy2} = \frac{4}{l_2^2 \tan (\gamma + \rho)}, \text{m}^{-2} \tag{17}$$

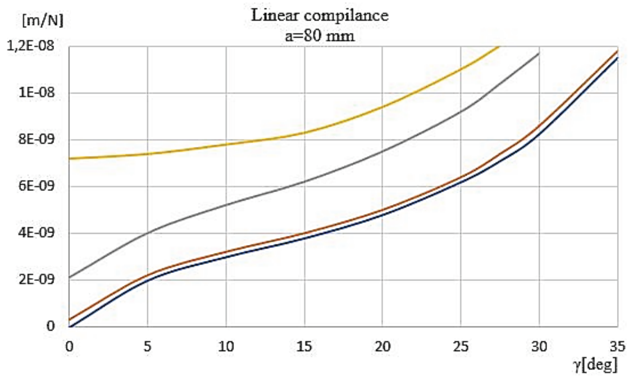

Figure 5 Linear compliance when a = 80 mm

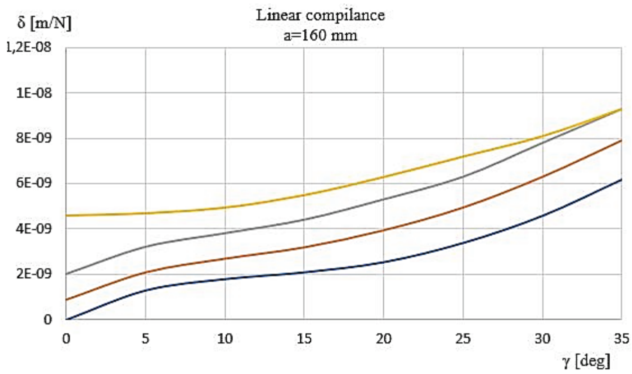

Figure 6 Linear compliance when a = 160 mm

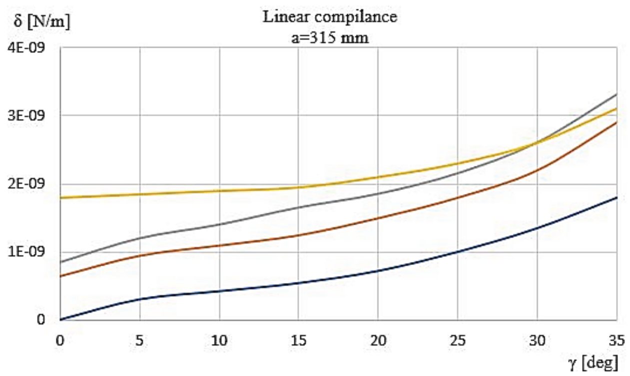

Figure 7 Linear compliance when a = 315 mm

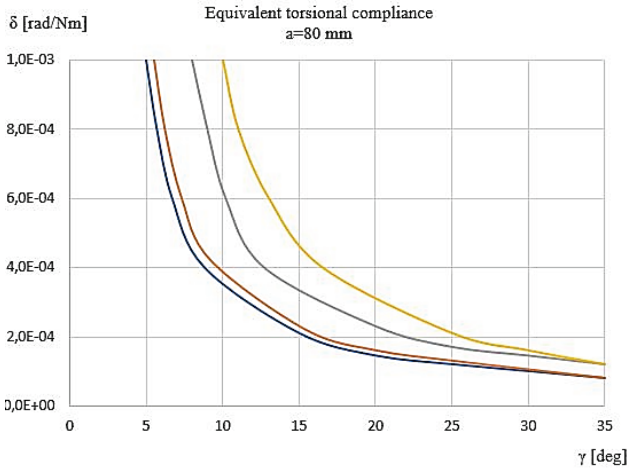

Figure 8 Equivalent torsional compliance when a = 80 mm

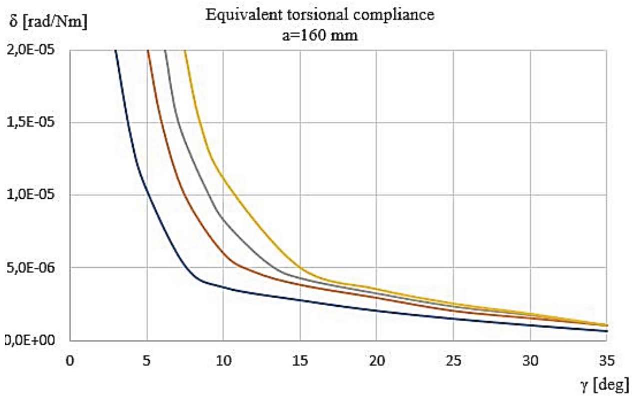

Figure 9 Equivalent torsional compliance when a = 160 mm

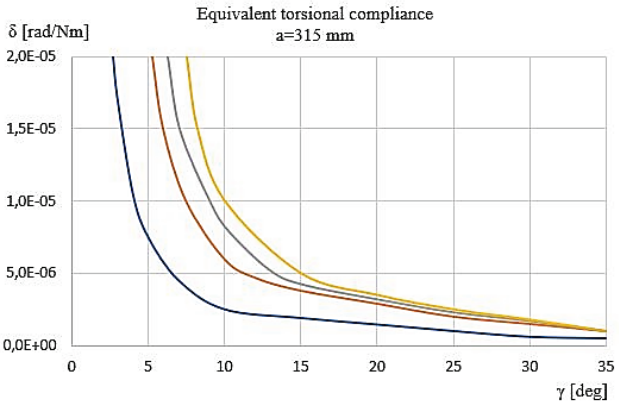

Figure 10 Equivalent torsional compliance when a = 315 mm

5 CONCLUSION

The main task of the article was to develop a method of calculating the torsional compliance of a worm gear and to clarify the effect of the worm shaft size on the linear compliance and the equivalent torsional compliance. The proposed method is versatile and can also be used for other types of worm gear geometry. The results can be used both for dynamic calculation, determination of the mean stiffness of teeth, for determining the course of contact stresses along the touch lines as well as for determining the optimum length of the worm shaft, where it is possible to save machine time in production. It is well known that worm gears are used in various areas of industrial practice, so they are constantly paying attention to increasing their serviceability. For heavier modes associated with high start, stop and reversing frequencies, we can eliminate these adverse effects.

Description of the method of torsional compliance determination of a cylindrical worm gearing caused by the compliance of supports is compared on the plot in Fig. 5 – 10. The courses are for three sizes of cylindrical worm gearings (axial distance a = 80 mm – Fig. 5, 8; a = 160 mm – Fig. 6, 9; a = 315 mm – Fig. 7, 10). The curve 4 is drawn according to the formula given in (1):

$$\delta_{H5} = \frac{l_1 b_2 \left[8 d_1^2 + 1,33 l_1 \left(\tan^2\gamma + \tan^2 \frac{\alpha_n}{\cos^2\gamma} \right) \right]}{\pi E_1 d_{\mathrm{f1}}^4} \qquad (18)$$

where: $d_1 = 2r_1$, d_{f1} – diameter of the root cylinder of a worm (m); E_1 – modulus of elasticity of the worm shaft material (N/m^2), b_1 – tooth width of a worm wheel (m).

The compliance according to formula (18) is a linear compliance due to bending strain of a worm shaft in the tangential direction towards a pitch cylinder of a worm wheel and in the parallel direction with the axis of a worm for a unit of a wheel width.

In Fig. 5, 6, 7 the courses of linear compliance due to the supports compliance in the above-mentioned direction are shown. In Fig 8, 9, 10 the course of equivalent torsional compliance based on a high – speed shaft is shown.

The curve 1 depicts δ_{0h1} according to the relation (13). The curve 2 is the course of the sum $\delta_{0h1} + \delta_{0h2}$. When calculating it we consider the compliance of the bearing $\delta_{p1} = \delta_{p2} = 0$. The curve 3 is calculated similarly as the curve 2 but $\delta_{p1} = \delta_{p2} = 0,1\delta_{01}$. The compliances are calculated for concrete values of worms commonly manufactured (gear ratio u = 10, teeth number $z_1 = 4$, $z_2 = 41$, with axial moduli m_{x80} = 3,15 mm, m_{x160} = 6,3 mm, m_{x315} = 12,5 mm, $b_{2(80)}$ = 22 mm, $b_{2(160)}$ = 45 mm, $b_{2(315)}$ = 90 mm), while lead angle of a helix for all sizes is γ = 20,8°.

As can be seen from the linear compliance shown in Fig. 5, 6, 7, it is obvious that linear compliance does not change significantly with the change of parameter a, which is the axial distance. The opposite effect occurs in torsional equivalent compliance as shown in Fig. 8, 9, 10, that is, when the parameter a (axial distance) changes, the value of the torsional equivalent compliance is changing radically, which only applies to a certain diameter, where the changes are only minimal when it is exceeded.

In calculating torsional compliance of a worm gearing the described method enables taking into consideration the influence of individual parameters more precisely than the formula (18).

6 REFERENCES

[1] Pavlenko, S. (2006). *Dynamické zaťaženie závitovkových prevodov*. TUKE Fakulta výrobných technológií v Prešove, DATAPRESS s.r.o. Prešov.

[2] Bodzas, S. & Dudas, I. (2010). Designing of smoother hob. *Hungarian Journal of Industrial Chemistry, Veszprem, 38*(2), 89-94.

[3] Trubachev, E., Savelveva, T., & Pushkareva, T. (2018). Practice of design and production of worm gears with localized contact. *Mechanism and Machine Science*, 51, 327-343. https://doi.org/10.1007/978-3-319-60399-5_16

[4] Lagutin, S. (2016). Analogues of axes of meshing in general type worm gearing. *Mechanism and Machine Science*, 34, 145-158. https://doi.org/10.1007/978-3-319-19740-1_8
[5] Sohn, J. & Park, N. (2017). Study on the influence of gear hobbing and shaft misalignments on the geometric interfence of cylindrical worm gear set. *Proceedings of the Institution of Mechanical Engineers*, Vol. 231, 4646-4654.
[6] Nieszporek, T. & Gołębski, R. (2017). Analysis of the wormwheel toothing accuracy. *Tehnicki vjesnik, 24*(4), 993-1000.
[7] Ramadani, R., Belsak, A., Kegl, M., Predan, J., & Pehan, S. (2018). Topology Optimization Based Design of Lightweight and Low Vibration Gear Bodies. International *Journal of Simulation Modelling, 17*(1), 92-104. https://doi.org/10.2507/IJSIMM17(1)419
[8] Zeng, Q. L., Wang, K., Wan, L. R., & Zhang, X. (2017). Accurate Modelling and Transient Meshing Analysis of Involute Spur Gear Based on thePrinciple of Gear Shaping. *International Journal of Simulation Modelling, 16*(2), 322-333. https://doi.org/10.2507/IJSIMM16(2)CO7
[9] Miltenovic, A., Banicm M., & Miltenovic, D. (2017). Load capacity of worm gear transmission from aspect of maximal use of available resources. *MATEC Web Conferences*, Vol. 121. https://doi.org/10.1051/matecconf/201712101009
[10] Sobek, M., Baier, A., & Grabowski, L. (2017). The impact of various distance between axes of worm gear on torque value. *IOP Conference Series: Materials Science and Engineering*, Vol. 227. https://doi.or/10.1088/1757-899X/227/1/012118
[11] Sabinik. H. G. (2017). Testing worm gears with cooperating elements made of different materials. *Proceedings of the Institution of Mechanical Engineers*, Vol. 23, 341-346. https://doi.org/10.1177/1350650116656982
[12] Roth, P., Sigmund, W., Born, S. , Kadach, D., & Stahl, K. (2017). A numerical approach to the calculation of the surface temperature distribution of worm gears. *ASME 2017 International Design Engineering Technical Conference and Computers and Information in Engineering Conference*, Vol. 10. https://doi.org/10.1115/DETC2017-67049
[13] Guingand, M., Jbily. D., & De Vaujany, J. P. (2016). A wear model for worm gear. *Proceedings of the Institution of Mechanical Engineering*, Vol. 230, 1290-1302.
[14] Bruzzese, C. & Santini. E. (2016). Study of cage torsional resonance failures in inverter-fed fabricated-cage induction motors used in traction drives. *Proceedings – 2016 IEEE International Power and Electronics and Motion Control Conference*. https://doi.org/10.1109/EPEPEMC.2016.7752071
[15] Li, X. Y., Yang, M. S., Zhou, X. L., & Guo, J. (2017). Torsion fatigue characteristics and crack propagation behaviour of 15Cr14Co12Mo5Ni2 gear steel. *Chinese Society for Metals*, Vol. 52, 84-91.
[16] Zhang, W. & Ding, Q. (2014). Torsion vibration and parametric instability analysis of a spur gear system with time-varying and square nonlinearities. *International Journal of Applied Mechanics*, Vol. 6. https://doi.org/10.1142/S1758825114500070
[17] Balambica, V. et al. (2014). Finite Element Application of Gear Tooth Analysis. *Advanced Materials Research*, Vols. 889-890, 527-531. https://doi.org/10.4028/www.scientific.net/AMR.889-890.527
[18] Valenčík, Š., Stejskal, T., Kmec, J., Bičejová, Ľ., & Gombár, M. (2016). Manufacturing systems building and developing. *Key Engineering Materials*, 669, 514-522. https://doi.org/10.4028/www.scientific.net/KEM.669.514
[19] Vojtko, I., Kočiško, M., Šmeringaiová, A., & Adamčík, P. (2013). Vibration of Worm Gear Boxes. *Applied Mechanics and Materials*, 308, 45-49. https://doi.org/10.4028/www.scientific.net/AMM.308.45
[20] Panić, N., Katana, B., & Mrčela, Z. I. (2012). Determining an Allowable Wear of Worm Wheels. *Tehnicki vjesnik, 41*(2), 55-60.
[21] Bičejová, Ľ. (2016). Influence of gas pressure change on character of cutting surface in material laser cutting. *MM Science Journal*, 669, 969-972. https://doi.org/10.17973/MMSJ.2016_09_201644
[22] Brůžek, B. & Leidlich, E. (2003). Numerical Simulation of Stresses in Thin-rimmed Spur Gears with Keyway. *Acta polytechnica, Journal of Advanced Engineering, 43*(5), 47-53.
[23] Tuplin, A. W. (1964). *Namáhaní ozubených kol*, Státní nakladatelství technickej literatúry. Praha.
[24] Klimenko, A. A., Romanenko. P. A. (1974). *Izgibnaja zestkosť zubjev cilindriceskoj cervjacnoj speredaci*. Sb. Detalimašin, Vyp. 20. Kiev, Technika.
[25] Rivin, J. I. (1966). *Dinamika privoda stankov*, Mašinostrojenije, Moskava.

Contact information:

Slavko PAVLENKO, Prof. Ing. CSc.
Technical University of Košice,
Faculty of Manufacturing Technologies,
Department of Technological Systems Design,
Bayerova 1, 080 01 Prešov, Slovak Republic
slavko.pavlenko@tuke.sk

Mária KAČALOVÁ, Ing.
Technical University of Košice,
Faculty of Manufacturing Technologies,
Department of Technological Systems Design,
Bayerova 1, 080 01 Prešov, Slovak Republic
maria.kacalova@tuke.sk

Róbert FAČKOVEC, Dipl. Ing.
Technical University of Košice,
Faculty of Manufacturing Technologies,
Department of Technological Systems Design,
Bayerova 1, 080 01 Prešov, Slovak Republic

Tomáš CORANIČ, Ing.
Technical University of Košice,
Faculty of Manufacturing Technologies,
Department of Technological Systems Design,
Bayerova 1, 080 01 Prešov, Slovak Republic
tomas.coranic@tuke.sk