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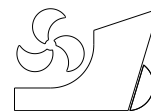
ANOTHER BLOW ON THE TORN DOWN WALL-THE INCLINING EXPERIMENT

Summary

The classical method (namely inclining experiment) has been used to estimate the vertical center of gravity (VCG or more often KG) of ships for many years. This method is based on the assumption that the metacenter is unchanged in the calculation of KG when the vessel is heeled. However, ships built today have knuckles, chines, dead-rise which may give rise to excessive change in the water-plane. The location of the metacenter is changed on these vessels when heeled. Therefore, determination of the vertical center of gravity may be somewhat erroneous. In this study, three different methods based on the assumption of unchanged metacenter have been examined. Employing the Graphical method, Polar method and Generalized method, the KG values of all vessels can be calculated without any dependence on the metacenter. The three methods mentioned in this study were studied and applied on ten different ships. Furthermore, the results from the classical method compared with those obtained from the recently developed methods. Based on this comparison, it is observed that the new methods developed have similar results to the classical method. Therefore, these methods may be a good alternative to the inclining experiment in the future. Moreover, uncertainty analyses have been performed for the results obtained from the classical method just to realize if there are any critical GM values within the margin of error.

Keywords: *Vertical center of gravity; Metacenter; Graphical method; Generalized method; Polar Method; Uncertainty analysis*

1. Introduction

For the time being, the inclining experiment is the principal method for determining center of gravity of a ship. After a vessel is launched, an inclining experiment is performed to calculate especially lightship characteristics of the vessel, KG in particular. Likewise, this test should be repeated for existing ships following any major over-haul that will alter ship's weight considerably.

The basis of the inclining experiment calculation, which is also known as the classical method, dates back to the end of 17th century. Firstly, Hoste, a professor of mathematics, suggested the concept of inclining experiment in 1697 [1]. Furthermore, a practical method for the implementation of inclining experiment was defined by Bouguer in 1746 [2]. Using this classical method, GM_0 of ships are to be obtained directly.

The classical method contains some assumptions intrinsically. Therefore, the validity of the classical method has been under discussion for many years. For small angles of heel, the immersed and emerged volumes are equal to each other when heeled. It is assumed that the metacenter is unchanged. The position of the metacenter depends on the unchanged water-plane area, equation (1).

$$KM = KB + BM = KB + \frac{I_{WP}}{\nabla} \tag{1}$$

The classical method is limited to small angles of heel. Especially, ships built in recent years have chines, knuckles and deadrise that can cause certain deviations in the metacenter. Alternative methods in calculating KG that has a great importance on ship stability have been emerged and improved in recent years.

One of the alternative methods of calculating KG of ships is the Graphical method proposed by O.O. Kanifolskyi and M.M. Konotopets in 2016 [3]. The Generalized method was proposed by R.J. Dunworth in 2013 and improved by R.J. Dunworth and A.C. Smith [5-8]. Another alternative method, the Polar method was proposed quite recently and refined by K. B. Karolius and D. Vassalos [4, 11, 12]. These alternative methods do not rely on the metacenter.

Since KG of ships is one of the most important governing parameter in stability and in turn safety, determination of it accurately is of great importance. Criticism on the inclining test has been around for so many years. Thus, pursuit for a more reliable method has gained momentum in recent years. In this study, KG of 10 vessels of various types are calculated by using three above-mentioned alternative methods and the results are compared with those from the classical method in order to lay out the differences if any. Additionally, uncertainty analyses have been employed on all 10 vessels to pinpoint the possible sources of error result from the classical method.

2. The Inclining Experiment (Classical Method)

The inclining experiment is carried out to find the vertical center of gravity of ships. The test is based on the measurement of angle of inclinations caused by moving the weights transversely placed on the deck. In some cases, ballast water may be used instead of solid weights as an alternative. In this case, selected tanks should be symmetrical (PS/SB) and trim of the ship should not change.

The inclining experiment is mandatory by the IMO's 2008 IS Code for every commercial ship above 24 m in length and all passenger ships.

2.1 Application

During the test, the vessel should float freely in water and mooring lines should be slack. The ship is then heeled by movements of test weights. These deflections are recorded by various means such as U-tubes, inclinometers or pendulums. If the inclining experiment is carried out using solid test weights, 8 shifts should be conducted as a standard required by 2008 IS Code. A sample sequence of the shifts are shown in Table 1.

Table 1 Weight shift sequence

Shift	Shift of weights	
	PS	SB
0	w2,w4,w6	w1,w3,w5
1	w4,w6	w1,w2,w3,w5
2		w1,w2,w3,w4,w5,w6
3	w6	w1,w2,w3,w4,w5
4	w2,w4,w6	w1,w3,w5
5	w1,w2,w3,w4,w6	w5
6	w1,w2,w3,w4,w5,w6	
7	w1,w2,w4,w6	w3,w5
8	w2,w4,w6	w1,w3,w5

2.2 Determination of KG

Since GM of a ship is obtained by the classical method directly, KG is calculated using equation (2).

$$KG = KM - GM \tag{2}$$

According to equation (1), the metacenter remains constant. KM in equation (2) is acquired from the hydrostatics at the test loading condition.

Ideally, prior to the inclining experiment, the ship should be in equilibrium with the weights placed symmetrically on deck without any heel and trim. When a weight is moved to a distance d, the center of gravity shifts from G to G_1, Fig. 1. Then $\tan\varphi$ is given by:

$$\tan\varphi = \frac{GG_1}{GM} \tag{3}$$

Re-arranging this equation, GM is obtained.
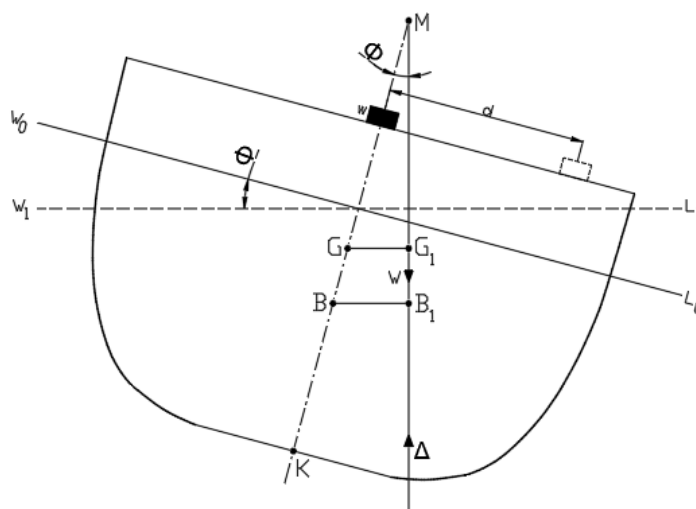
$$GM = \frac{GG_1}{\tan(\varphi)} \tag{4}$$

Fig. 1 Shift of center of gravity

When a weight is moved, the center of gravity is shifted accordingly. In this case, GG_1 is expressed by equation (5).

$$GG_1 = \frac{w.d}{\Delta} \tag{5}$$

Re-arranging equation (4), the final equation is obtained for GM.

$$GM = \frac{w.d}{\Delta.\tan(\varphi)} \tag{6}$$

In this expression, tanφ is calculated according to equation (7) for each pendulum and tanφ values calculated for each movement are averaged to minimize the error.

$$\tan(\varphi) = \frac{pen.deflection}{pen.length} \tag{7}$$

tanφ and moment values for all weight shift cases are calculated by the above equations. Then, w.d/Δ is plotted against the corresponding tanφ for each weight shift and a linear line passing through these points is fitted according to the least squares method. The slope of the linear line gives GM. Alternatively, w.d is plotted against tanφ for each weight shift, as shown in Fig.2, and GM is calculated according to equation (8).

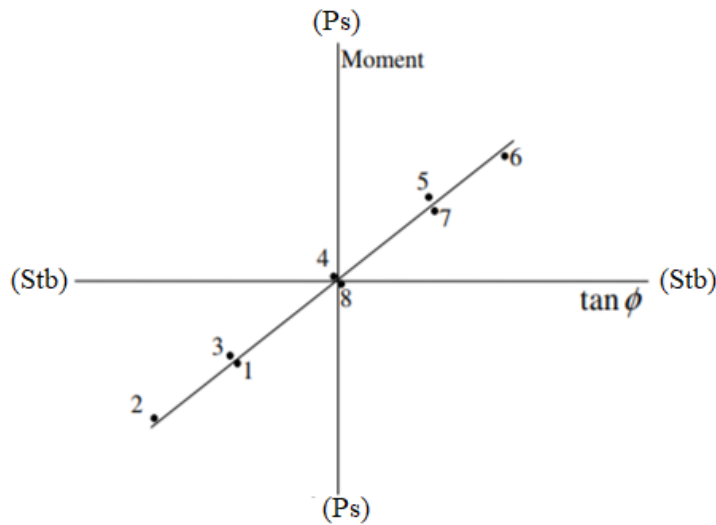$$GM = \frac{1}{\Delta.R_{slope}} \tag{8}$$

Fig. 2 Tanφ- moment linear line

In case of slack liquid in tanks, it is necessary to introduce a free surface correction. Free surface moment (FSM) is calculated as in equation (9).

$$FSM = \frac{l.b^3.\rho_t}{12.\Delta} \tag{9}$$

Finally, KG is obtained as below.

$$KG = KM - GM - FSM \tag{10}$$

In addition, LCG and TCG are calculated as below.

$$LCG = LCB - \frac{Trim.MCT.100}{\Delta} \tag{11}$$

$$TCG = TCB_{\varphi=0} + \tan(\varphi).GM \tag{12}$$

3. New Alternative Methods

3.1 Generalized method (Dunworth method)

Generalized method was first proposed by R.J. Dunworth and A.C. Smith [3]. After each weight shift, the vessel reaches an equilibrium. Therefore, the vessel's righting and heeling moments must be equal.

$$H_{moment} = R_{moment} \tag{13}$$

$$\Delta.HZ = B_M.GZ \tag{14}$$

Since displacement and buoyancy forces are equal, equation (14) reduces to equation (15).

$$HZ = GZ \tag{15}$$

Heeling arm (HZ) is obtained as below.

$$HZ = \frac{w.d.\cos(\varphi)}{\Delta} \tag{16}$$

The trigonometric relationships shown in Fig. 3 are used to obtain the righting arm of the ship.
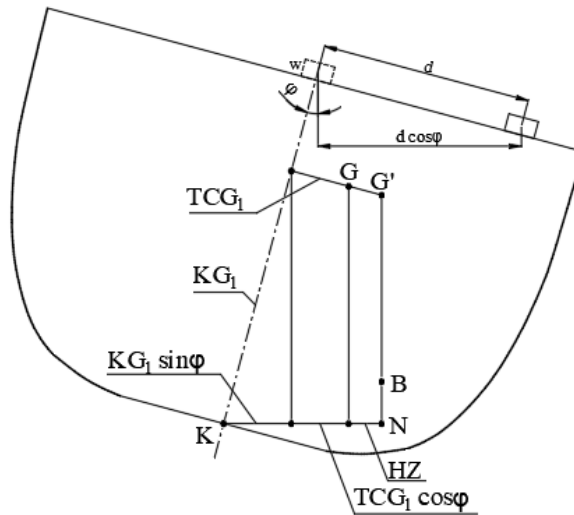

Fig. 3 Main parameters during heel [7]

$$GZ = KN - KG_1.\sin(\varphi) - TCG.\cos(\varphi) \tag{17}$$

When this equation is re-arranged using equation (15) and equation (16), KG is calculated as follows.

$$KG_1.\sin(\varphi) = KN - HZ - TCG_1.\cos(\varphi) \tag{18}$$

$$KG_1 . \sin(\varphi) = KN - \frac{w.d.\cos(\varphi)}{\Delta} - TCG_1 . \cos(\varphi) \qquad (19)$$

Equation (18) reduces to equation (20) yielding TCG_1 when $\varphi = 0$.

$$TCG_1 = KN_0 - HZ_0 \qquad (20)$$

KN_0 is obtained from the hydrostatics. Dunworth proposed that HZ_0 is calculated by plotting HZ against heel angle (φ) and a third-order polynomial passing through these points is fitted in accordance with the least squares method. When $\varphi = 0$, HZ_0 equals the intercept point on the y-axis of the trend line, Fig. 4 [7].
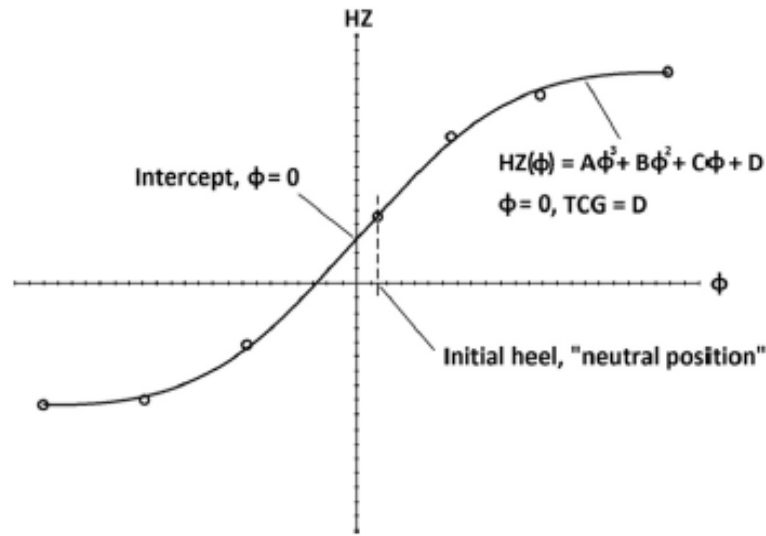

Fig. 4 Determination of HZ_0 [4]

3.2 Graphical method

Graphical method, which calculates KG directly, was proposed by Kanifolskyi and Konotopets in [3]. KN used in the calculation of KG are acquired from the hydrostatics. Kanifolskyi and Konotopets summarized the calculation of KG in 5 steps as shown in Fig. 5. The first step is to draw KN from the keel (K). The second step is to draw a perpendicular to KN. In the third step, HZ is calculated according to equation (16). In the fourth step, calculated HZ is placed in the correct position. That is, HZ arm is parallel to KN and intercepts the centerline. In the final step, KG is read off from the graph. These 5 steps must be repeated for all shifts of weights. In addition, the mean KG is calculated according to equation (21).

$$KG = \frac{\sum_{i=1}^{n} KG_i}{n} \qquad (21)$$

As can be seen in Fig. 5, trigonometric relationships are driven from the sketch. KG is calculated according to the following sequence of equations:

$$KG = \frac{KN - HZ}{\sin(\varphi)} \qquad (22)$$

$$KG.\sin(\varphi) = KN - HZ \qquad (23)$$

$$KG.\sin(\varphi) = KN - \frac{w.d.\cos(\varphi)}{\Delta} \qquad (24)$$
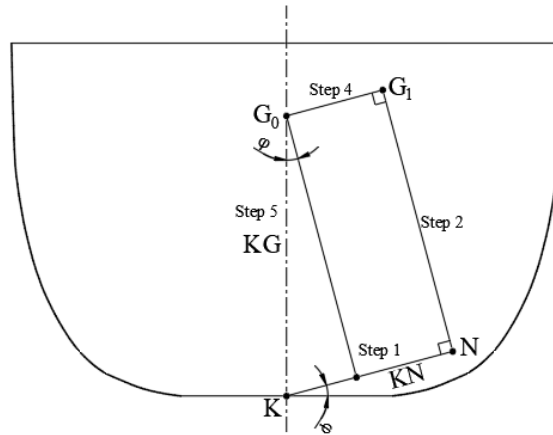
Fig. 5 Graphical method steps [4]

3.3 Polar method

Polar method is also a new method proposed by K. Karolius and D. Vassalos [4,11,12]. In this method, a line parallel to BM radius is considered. This line is shifted up to distance HZ, and represented in polar coordinates. In the initial condition, KG and TCG are located on this line. Moreover, the location of KG and TCG does not changed when each weight is shifted. Initial KG_0 and TCG_0 are fixed on this line. However, TCG is shifted up a distance G_0G_i when each weight is shifted, Fig. 6.

Fig. 6 The parameters of Polar method [4]

The equation of the straight line is given below.

$$z = x.\cos(\varphi) + y.\sin(\varphi) \tag{25}$$

Equation (25) is rearranged to get equation (26). When equation (26) and equation (27) are combined, equation (28) is attained.

$$z = TCG.\cos(\varphi) + KG.\sin(\varphi) \tag{26}$$

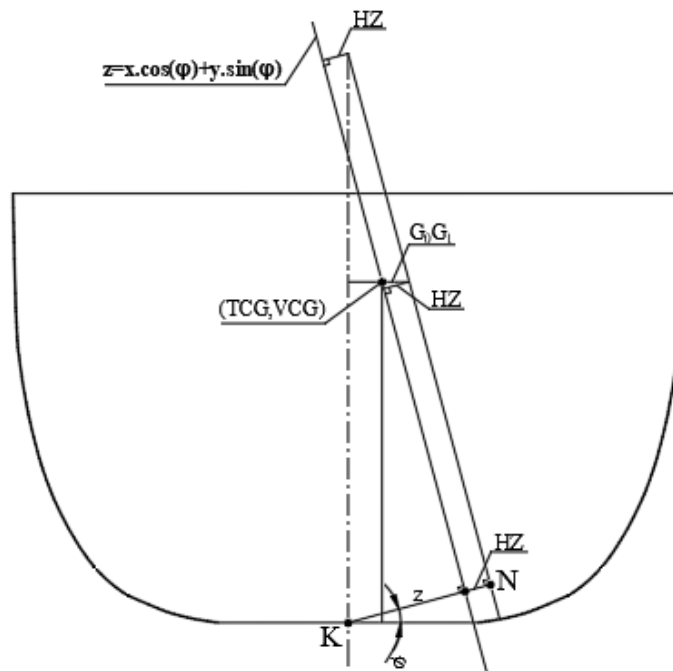$$z = KN - HZ \tag{27}$$

$$KN - HZ = TCG.\cos(\varphi) + KG.\sin(\varphi) \tag{28}$$

As mentioned above, KG_i and TCG_i for each shift i must be equal to TCG_0 and KG_0 respectively.

$$TCG_0 = TCG_i \tag{29}$$

$$KG_0 = KG_i \tag{30}$$

When equation (28) is reorganized using equations (29) and (30), KG and TCG are found as follows:

$$KG = \frac{(KN_i - HZ_i).\cos(\varphi_0) - (KN_0 - HZ_0).\cos(\varphi_i)}{\cos(\varphi_0).\sin(\varphi_i) - \sin(\varphi_0).\cos(\varphi_i)} \tag{31}$$

$$TCG = \frac{(KN_i - HZ_i).\sin(\varphi_0) - (KN_0 - HZ_0).\sin(\varphi_i)}{\cos(\varphi_i).\sin(\varphi_0) - \sin(\varphi_i).\cos(\varphi_0)} \tag{32}$$

Equation (29) and (30) are general forms which may be simplified according to state of the vessel during the inclining experiment. For example, if heeling arm in the initial position is zero, HZ_0 vanishes. Furthermore, if the initial heel angle (φ_0) is zero and ship is completely symmetrical, KN_0 is also equal zero. Therefore, equations (29) and (30) are simplified as below.

$$KG = \frac{(KN_i - HZ_i)}{\sin(\varphi_i)} \tag{33}$$

$$TCG = 0 \tag{34}$$

The above equations are the same equations seen in Graphical method. Therefore, the Graphical method is valid only for completely symmetrical vessels which have a limited heel angle in the initial condition. The KN used in the Polar method are to be obtained from a 3D stability software model of the ship. But they would still introduce additional errors. On the other hand, HZ_0 is calculated as described in Section 3.1.

4. Sample Ships

In this study, 10 different ships having different sizes, hull forms and types with approved inclining experiment results have been used in order to observe the effect of various parameters on the results by these different methods.

This set of sample ships is comprised of two chemical tankers, two asphalt tankers, a service boat, a tug, a Ro-Ro, a container ship, a research vessel and a fast boat. The axis convention applied throughout this study is given in Table 2. The general characteristics of these vessels are supplied in Table 3.

Table 2 Axis convention

Positive x-direction	Aft→ Forward
Positive y-direction	Port→Starboard

Table 3 Sample vessel characteristics

	L_{OA}	L_{BP}	B	T	D	Δ	C_B	A_{WP}
	m.	m	m.	m.	m.	tons		m^2
Chemical tanker-1	105	102.18	18	6.8	9.2	10021	0.774	1458
Chemical tanker-2	131.85	123.99	18.9	7.98	10.2	14834	0.770	1709.7
Asphalt tanker-1	119.9	116.8	19.4	7.35	10.8	14241	0.740	1649.8
Asphalt tanker-2	156.5	147.9	25	9.25	14.6	27960	0.800	2915.7
Service boat	14.9	14.225	4.8	1.275	2.31	51.09	0.517	58.375
Tug	17.55	17.55	8.4	3.1	3.25	235.95	0.520	88.45
Ro-Ro		120	19.4	4.88				1902.2
Research vessel		56.45	8.8	3.95				
Fast boat		13.06	4.04	0.765		17.355	0.6620	
Container		134.47	22.6	3.576		17887	0.620	

5. Adoption of New Methods

The calculation of KG is performed according to the methods described in Section 3 on the sample vessels. The details of the calculations are given for research vessel only as an example in the following subsections respectively. The same procedures have been applied to the other ships for all methods but only the final results were revealed for space limitation.

5.1 The graphical method

The graphical method KG calculations are shown below.

Table 4 Research vessel-Graphical method procedure

Shift	tanφ	φ	∑φ	φ	HZ	KN	Sinφ	KG.Sinφ
	mean	degree	degree	radian	m	m		
	0	0	-1.0000	-0.0175	0	-0.0754	-0.0175	-0.0754
1	0.0164	0.9423	-0.0577	-0.0010	0.0068	-0.0043	-0.0010	-0.0112
2	0.0304	1.7428	0.7428	0.0130	0.0137	0.0560	0.0130	0.0423
3	-0.0164	-0.9423	-1.9423	-0.0339	-0.0069	-0.1465	-0.0339	-0.1396
4	-0.0006	-0.0335	-1.0335	-0.0180	0.0000	-0.0779	-0.0180	-0.0779
5	-0.0149	-0.8553	-1.8553	-0.0324	-0.0068	-0.1399	-0.0324	-0.1331
6	-0.0291	-1.6663	-2.6663	-0.0465	-0.0137	-0.2011	-0.0465	-0.1874
7	0.0153	0.8753	-0.1247	-0.0022	0.0069	-0.0094	-0.0022	-0.0163
8	0.0000	0.0000	-1.0000	-0.0175	0.0000	-0.0754	-0.0175	-0.0754

tanφ values given in Table 4 were found from the ratio of pendulum deviations to the pendulum length from the inclining experiment report. Since three pendulums were used in the inclining experiment, readings from these three pendulums were averaged. In the fourth column, heel angle of the third column and initial heel angle are collected. In sixth column, HZ

is calculated according to equation (16). KN in Table 4 is obtained from ship's stability booklet. Finally, KG.sinφ is calculated according to equation (23).

Results that are obtained from Table 4 is plotted in Fig. 8. Slope of the linear line provides KG. Free surface correction, weights to be added and removed for calculations of lightship KG are shown in Table 5.
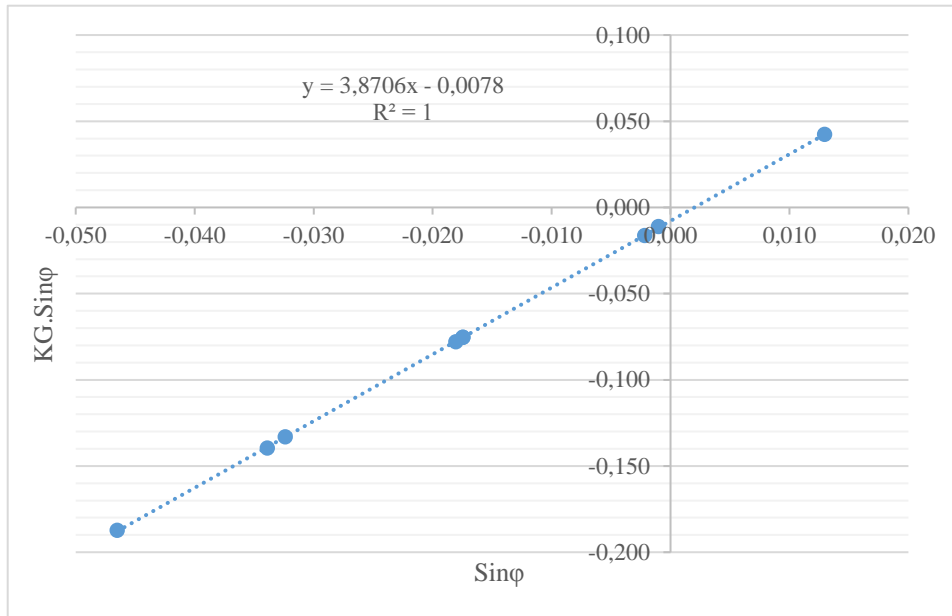

Fig. 7 Research vessel Graphical method, KG.sinφ vs sinφ

Table 5 Research vessel-Graphical method KG value

	Weight (t)	KG (m)	FSM (ton.m)
Ship as inclined	1167.602	3.8706	104.025
FS corrections		-0.089	
Fluid KG	1167.602	3.782	
Total items to remove	-295.85	2.051	
Lightship	871.75	4.369	

5.2 The Generalized method

Table 6 Research vessel-Generalized method procedure

Shift	tanφ	φ	∑φ	φ	HZ	KN	Sinφ	KG.Sinφ
	mean	degree	degree	radian	m	m		
	0.0000	0.0000	-1.0000	-0.0175	0.0000	-0.0754	-0.0175	-0.0678
1	0.0164	0.9423	-0.0577	-0.0010	0.0068	-0.0043	-0.0010	-0.0036
2	0.0304	1.7428	0.7428	0.0130	0.0137	0.0560	0.0130	0.0499
3	-0.0164	-0.9423	-1.9423	-0.0339	-0.0069	-0.1465	-0.0339	-0.1320
4	-0.0006	-0.0335	-1.0335	-0.0180	0.0000	-0.0779	-0.0180	-0.0703
5	-0.0149	-0.8553	-1.8553	-0.0324	-0.0068	-0.1399	-0.0324	-0.1255
6	-0.0291	-1.6663	-2.6663	-0.0465	-0.0137	-0.2011	-0.0465	-0.1798
7	0.0153	0.8753	-0.1247	-0.0022	0.0069	-0.0094	-0.0022	-0.0087
8	0.0000	0.0000	-1.0000	-0.0175	0.0000	-0.0754	-0.0175	-0.0678

The generalized method utilizes the spreadsheet shown in Table 6. The steps in the table are the same as the graphical method except the last column. In the last column, KG.sinφ is calculated according to equation (18). Fig. 8 is used to obtain HZ in this equation. Using a third order polynomial approximation, HZ_0 is found to be -0.0023 m.

Graphical representation of the results in Table 6 is depicted in Fig. 9. The slope of the linear line gives KG. Free surface correction, weights to be added and removed for the lightship KG are shown in Table 7.
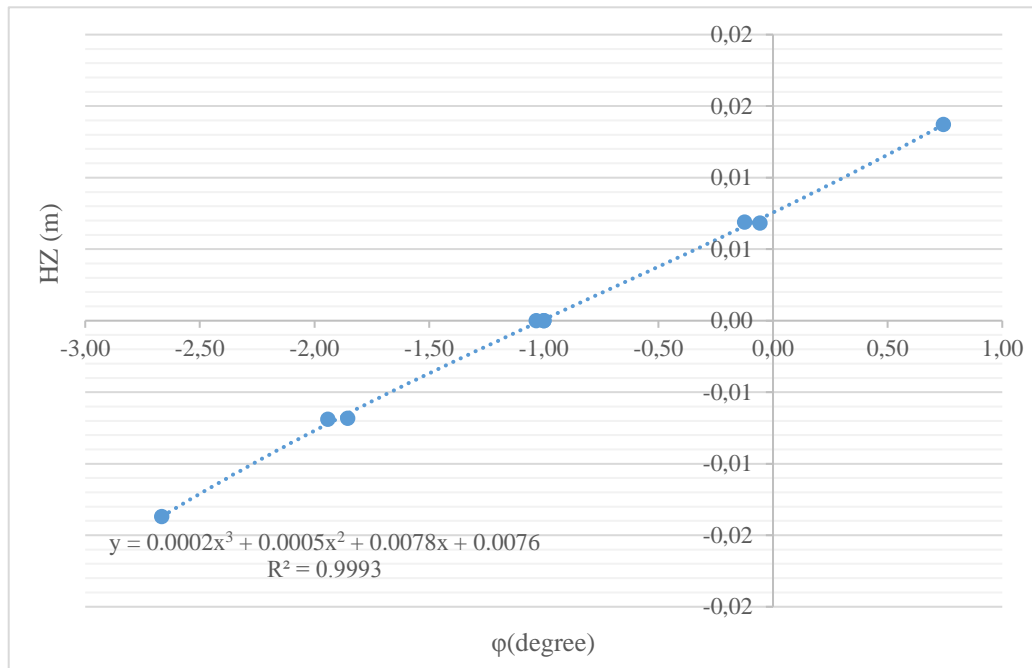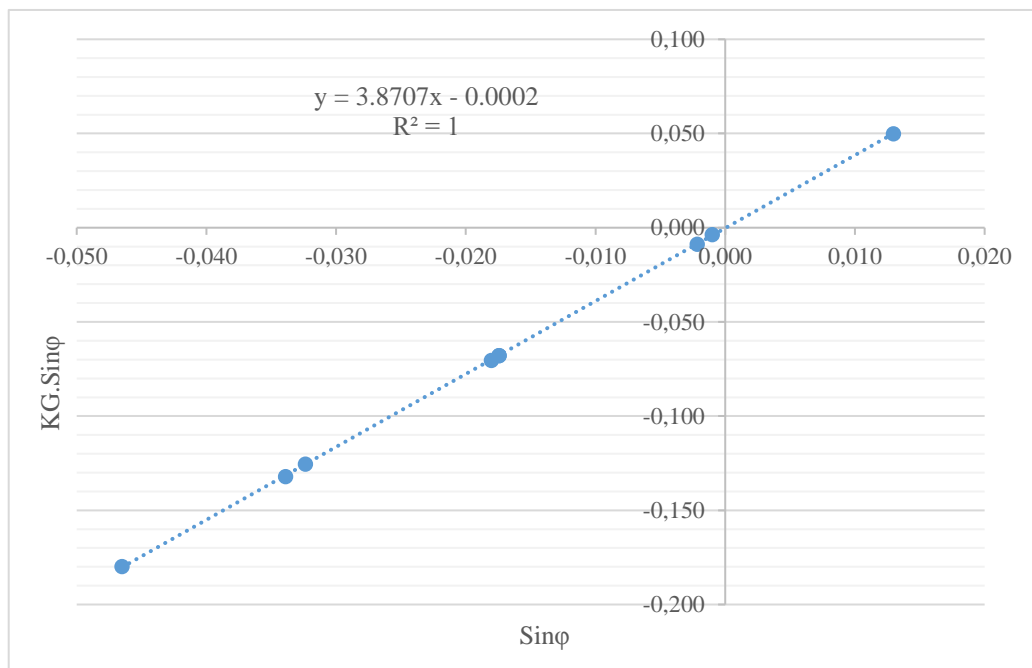

Fig. 8 Research vessel-Generalized method, HZ vs φ

Fig. 9 Research vessel-Generalized method, KG.sinφ vs sinφ

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Table 7 Research vessel-Generalized method, KG value

	Weight (t)	KG (m)	FSM (ton.m)
Ship as inclined	1167.602	3.8707	104.025
FS corrections		-0.089	
Fluid KG	1167.602	3.782	
Total items to remove	-295.85	2.051	
Lightship	871.75	4.369	

5.3 Polar method

The procedure to calculate KG in Polar method is shown in Table 8. The steps up to last two columns are similar to the graphical and generalized methods. Unlike other two methods, $\sin(\varphi_i - \varphi_0)$ is used. $KG.\sin(\varphi_i - \varphi_0)$ in the last column is calculated according to equation (31). HZ_0 in equation (31) is the same as the Generalized method and equal to -0.023m.

Table 8 Research vessel-Polar method procedure

Shift	tanφ	φ	∑φ	φ	HZ	KN	Sinφ	sin(φ_i- φ_0)	KG.sin(φ_i- φ_0)
	mean	degree	degree	radian	m	m			
0	0.0000	0.0000	-1.0000	-0.0175	0.0000	-0.0754	-0.0175	0.0000	-0.0678
1	0.0164	0.9423	-0.0577	-0.0010	0.0068	-0.0043	-0.0010	0.0164	-0.0036
2	0.0304	1.7428	0.7428	0.0130	0.0137	0.0560	0.0130	0.0304	0.0499
3	-0.0164	-0.9423	-1.9423	-0.0339	-0.0069	-0.1465	-0.0339	-0.0164	-0.1320
4	-0.0006	-0.0335	-1.0335	-0.0180	0.0000	-0.0779	-0.0180	-0.0006	-0.0703
5	-0.0149	-0.8553	-1.8553	-0.0324	-0.0068	-0.1399	-0.0324	-0.0149	-0.1255
6	-0.0291	-1.6663	-2.6663	-0.0465	-0.0137	-0.2011	-0.0465	-0.0291	-0.1797
7	0.0153	0.8753	-0.1247	-0.0022	0.0069	-0.0094	-0.0022	0.0153	-0.0087
8	0.0000	0.0000	-1.0000	-0.0175	0.0000	-0.0754	-0.0175	0.0000	-0.0678

After $KG.\sin(\varphi_i - \varphi_0)$ is calculated, results are plotted in Fig.10. Slope of linear line gives KG. Finally, free surface correction, weights to be added and removed for the lightship KG are shown in Table 9.

Table 9 Research vessel- Polar method KG value

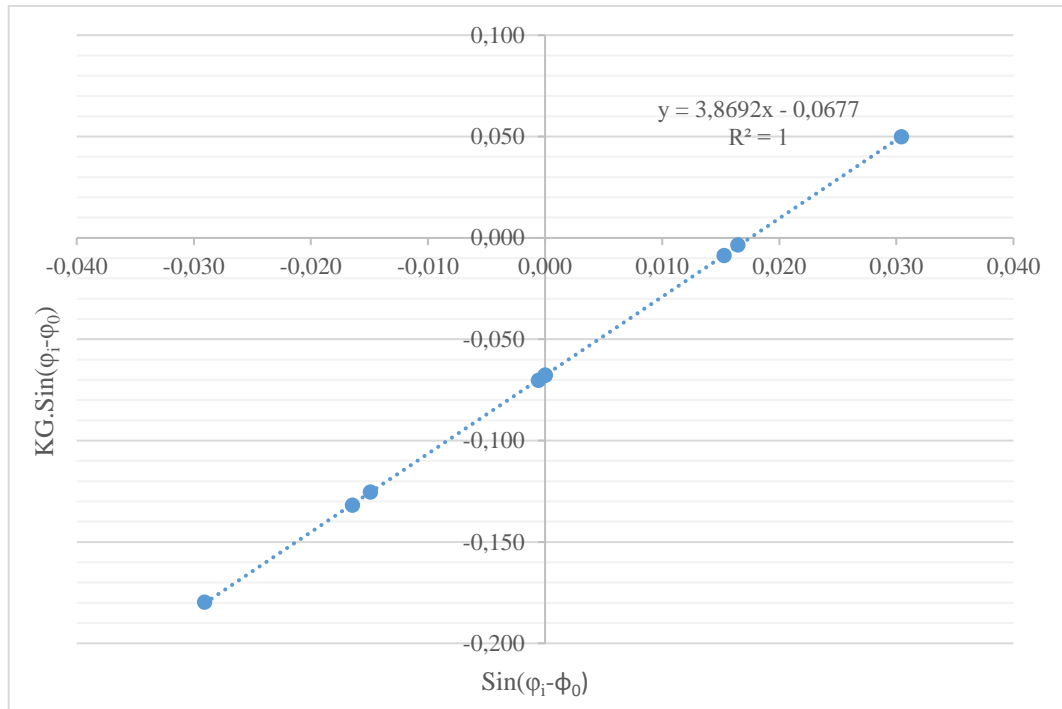|	Weight (t)	KG (m)	FSM (ton.m)
Ship as inclined	1167.602	3.8692	104.025
FS corrections		-0.089	
Fluid KG	1167.602	3.780	
Total items to remove	-295.85	2.051	
Lightship	871.75	4.367	

Fig. 10 Research vessel, Polar method KG. $\sin(\varphi_i - \varphi_0)$ vs $\sin(\varphi_i - \varphi_0)$

## 6.   Comparison and Analysis of the Results

In this section, KG of 10 different vessels obtained from four different methods are presented in Table 10. KG obtained according to the classical method is taken as a reference. The items shown as difference are the difference between the KG values obtained from the alternative methods and by the classical method.

When differences are examined, the highest difference was 8.4% between Graphical and Generalized methods in research vessel. In polar method, this difference is 8.3%. In chemical tanker (1), the difference in three methods is obtained as -6.4%. For tugboat, similar to chemical tanker (1) in the Graphical method and Generalized methods, however, a positive 6.4% difference was calculated. This difference is 6.3% in polar method. For asphalt tanker (1), in the polar and graphical method, the difference is in the order of -5.1% and in the Generalized method in the order of -5.0%. In the case of fast boat, the difference is calculated as -5.1% in the polar method and -5.0% in the graphical and Generalized methods. The difference for the container ship is 4.5% in all methods. Asphalt tanker (2) shows a difference of -3.8% in all methods. In Ro-Ro vessel and service boat, the difference is -1.8% in all methods evaluated except for polar method. This difference is -1.9% in polar method. Finally, the closest result is calculated with 0.2% difference in all methods for chemical tanker (2).

Small differences between 3 methods indicate that these methods can be applied to different ships having different hull geometries, sizes and characteristics with a good level of confidence. Graphical representation of comparison of all three methods is shown in Fig. 11.

Table 10 Comparison of results by methods

Sample Ships	Classical	Graphical			Polar			Generalized		
	KG	KG	Diff..	Diff..	KG	Dif.	Dif.	KG	Diff.	Diff.
	(m)	(m)	(mm)	(%)	(m)	(mm)	(%)	(m)	(mm)	(%)
Chemical Tanker (1)	7.449	6.969	-480	-6.4	6.969	-480	-6.4	6.969	-480	-6.4
Asphalt Tanker (1)	8.466	8.034	-431	-5.1	8.038	-428	-5.1	8.038	-427	-5.0
Service Boat	1.663	1.632	-31	-1.8	1.632	-31	-1.9	1.633	-30	-1.8
Asphalt Tanker (2)	8.501	8.177	-324	-3.8	8.176	-325	-3.8	8.177	-323	-3.8
Chemical Tanker (2)	8.153	8.170	17	0.2	8.169	16	0.2	8.170	17	0.2
Tug	3.425	3.643	218	6.4	3.641	216	6.3	3.643	218	6.4
Ro-Ro	9.413	9.241	-172	-1.8	9.240	-173	-1.9	9.241	-172	-1.8
Research	4.031	4.369	338	8.4	4.367	336	8.3	4.369	338	8.4
Fast boat	1.294	1.228	-66	-5.1	1.228	-66	-5.0	1.228	-66	-5.1
Container	9.926	10.375	449	4.5	10.375	449	4.5	10.375	449	4.5
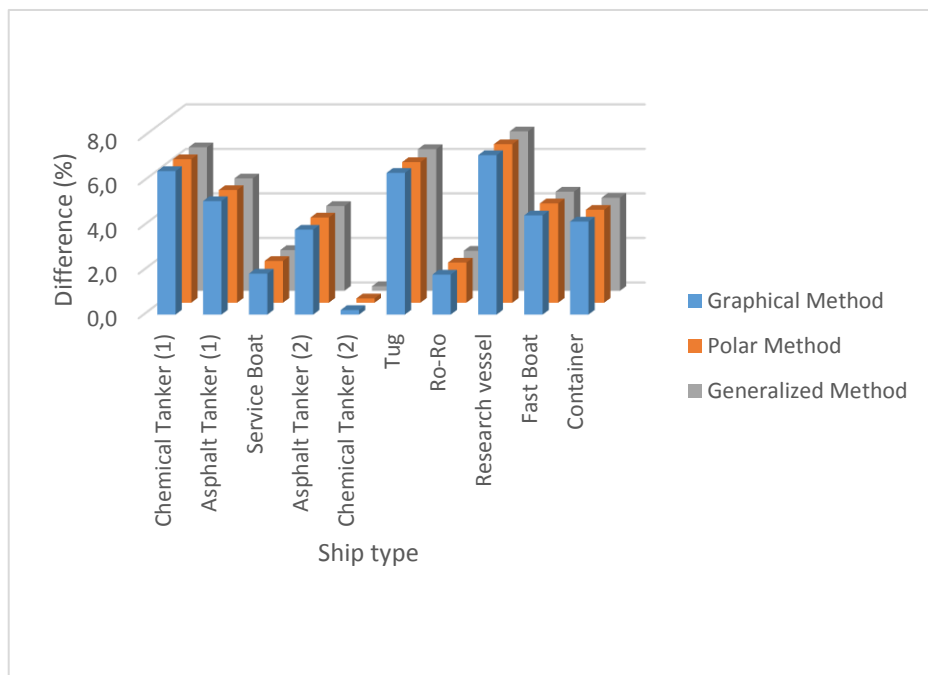
Fig. 11 Comparison of three methods

7. Uncertainty Analysis

There are a number of uncertainties in the inclining experiment conducted for the determination of the vertical center of gravity (KG) of ships. Since it involves certain measurements and human interference, this experiment more likely contains some errors. These uncertainties and errors start with draft readings before and after the test and continue with the

assumptions made during the calculations. The following may be shown as sources of error; wind, current, wave, human error, errors of measurement devices etc. All these errors may accumulate from the beginning to the end of the inclining experiment and may result in significant nonconformities in KG and GM.

It is assumed that metacenter is fixed when the vessel is heeled. However, most of the ships that are built today having knuckles, chines and dead-rise which result in deviations in the water-plane area. This in turn causes errors in the lightship KG and GM.

On the contrary, three new methods discussed in this study are not based on the assumption that the metacenter is fixed. Therefore, the uncertainties caused by the change of metacenter are eliminated.

7.1 Uncertainty analysis procedure and results

In this section, a range of source of uncertainties and errors such as draft reading, displacement, heel angle, wind, wave etc. are examined. The effect of these uncertainties has been applied to all ships used in the study [10]. Uncertainties in inclining experiment have been investigated by many other researchers such as Wilczynski et al [13] and Woodward et al [14]. They examined the uncertainty in GM which results from the bias and accuracy errors pertinent to each measured and calculated variable.

As a procedure, the uncertainty in the slope of the best fit linear line is determined first. Then, the uncertainty in displacement is calculated according to equation (8). Uncertainties of vessel draft, calculated molded volume, density, molded vs. as-built volume are reflected to the uncertainty of the displacement. The uncertainty in GM (U_{GM}) is found after the uncertainty of displacement (U_Δ) and slope of line (U_{Slope}) are determined.

$$\left(\frac{U_{GM}}{GM}\right) = \sqrt{\left(\frac{U_{slope}}{slope}\right)^2 + \left(\frac{U_\Delta}{\Delta}\right)^2} \tag{35}$$

Or, in shorthand;

$$U_{GM} = \sqrt{U_{slope}^2 + U_\Delta^2} \tag{36}$$

Since KG is composed of various hydrostatic parameters, the uncertainty in KG (as inclined) is given as;

$$U_{KG} = \sqrt{U_{KB}^2 + U_{BM}^2 + U_{GM}^2} \tag{37}$$

BM on the other hand is the ratio of moment of inertia and volume having the uncertainty of;

$$U_{BM} = \sqrt{U_I^2 + U_\nabla^2} \tag{38}$$

Similarly, uncertainty of each parameter can be determined using its constituents in the same manner as explained above.

Next, errors from KB, BM and experimental weights are calculated. Finally, Uncertainty of lightship KG is determined. The details of uncertainty analysis applied to chemical tanker (1) are given in Table 11 below.

Table 11 Result of the uncertainty analysis for research vessel

Parameter	Measured/ calculated value	Uncertainty	Units
Slope (tangent/moment)	265.550	0.316	1/t.m
Molded vs. as-built volume (v)	1143.587	0.043	m^3
Vessel drafts	3.948	0.019	m
Calculated molded volume (m)	1143.587	22.872	m^3
Displacement volume (V)	1143.587	23.612	m^3
Density	1.021	0.002	t/m^3
Displacement (Δ)	1167.602	24.171	t
As- inclined GM	0.445	0.009	m
As- inclined KG	3.529	0.076	m
Lightship KG	4.031	0.183	m

Note: All uncertainties are at 95% confidence level.

GM= 0.445±0.009 m

KG= 4.031±0.183 m

The same procedure has been applied to all sample vessels and tabulated results are supplied in Table 12 below. Uncertainties in GM and KG are also depicted graphically in Fig.12 and Fig.13 respectively.

Table 12 Comparison of uncertainty analyses for sample ships

Sample Ships	GM (m)	error (± m)	KG (m)	error (± m)
Chemical tanker-1	4.206	0.09	7.449	0.47
Chemical tanker-2	2.971	0.06	8.153	0.34
Asphalt tanker-1	2.30	0.05	8.466	0.31
Asphalt tanker-2	7.441	0.16	8.501	0.31
Service boat	1.266	0.04	1.663	0.09
Tug	1.998	0.04	3.425	0.11
Ro-Ro	1.256	0.03	9.413	0.29
Research vessel	0.445	0.01	4.031	0.18
Fast boat	0.828	0.04	1.294	0.14
Container	2.44	0.05	9.926	0.35

Uncertainty analysis provides a possible margin of error which is inherent to inclining experiment. Table 12 depicts the errors in GM and KG for the sample vessels. This may be important especially for ships whose margin of stability is critical.

Wilczynski [13] also provided a sample calculation on the uncertainty of GM of an OSV having length 44.5 m. and GM = 6.593 m. from the test. He obtained ±0.122 m. uncertainty in GM which is quite comparable with the uncertainties in chemical tanker (1) and asphalt tanker (1). Woodward et al [14] compared the uncertainties of four ships (with a fixed reference GM=0.15 m.): buoy tender 0.15±0.15 m., superyacht 0.15±0.033m., supply ship 0.15±0.047m.,

containership 0.15±0.029m. and ropax 0.15±0.077m. These results are also in line with the results obtained for the sample vessels in Table 12.
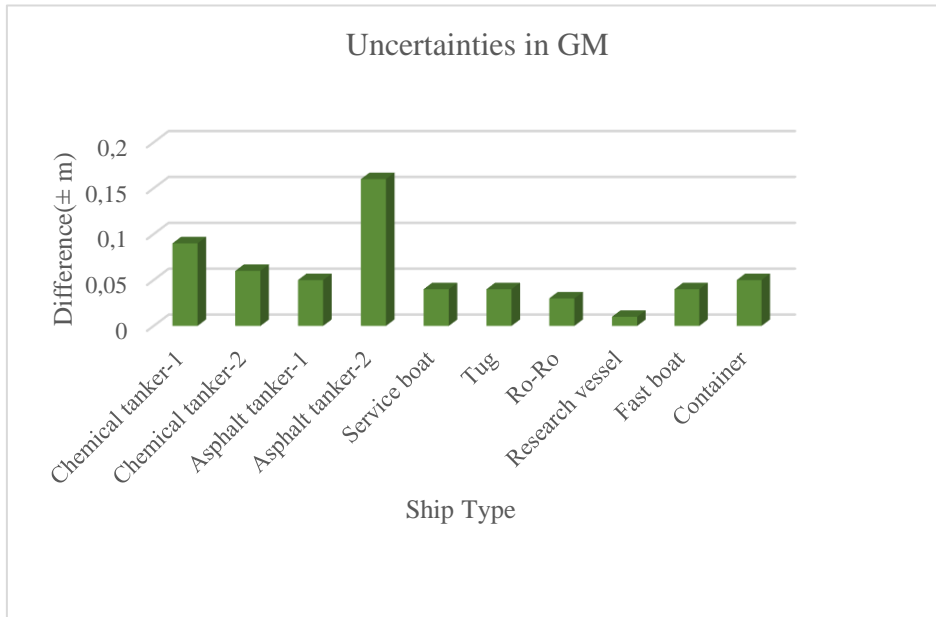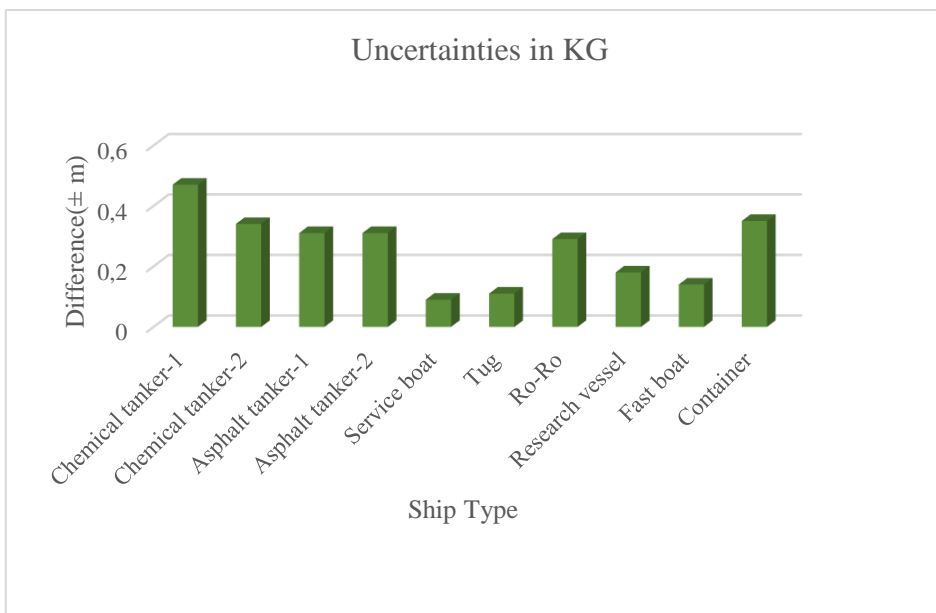

Fig. 12 Uncertainty in GM

Fig. 13 Uncertainty in KG

8.   Conclusions

In this study, Graphical method, Generalized method and Polar method developed in recent years have been applied to ten different sample ships to determine the lightship KG values and the pertinent calculations are presented in details.

When the results of the methods are evaluated, the three anew proposed methods give very similar results for KG. Having compared the classical method results with the newly developed three methods, the biggest difference is seen as 8.4% in the research vessel. This

discrepancy may be attributed to the fact that the research vessel has such a hull form which plays an important role on the water-plane area. The closest results are attained within the range of 0.2% for chemical tanker (2) having a high C_B.

The generalized method reveals slightly different results since it does not take into account the initial heel angle (φ_0). Among the three methods discussed, the most diverse results are obtained from the graphical method. This may be because the graphical method does not take into account the transverse center of gravity (TCG) and the initial heel angle (φ_0).

The outcome of the analysis on the vessels such as asphalt tankers, chemical tanker (2), Ro-Ro and container ship reveals that there is a difference of less than 5% in KG. However, there is a difference in the order of 6-8% in KG calculated with the newly developed methods. An uncertainty analysis has been carried out on the results from the inclining experiment in order to observe whether they fall within the above-mentioned margins. This fact is really important for the vessels having small GM values barely complying with the regulations.

It is the authors' belief that the difference between the results of the inclining experiment and the results of the other methods taken into consideration emanating from the assumptions made during the conventional inclining experiment in addition to the inherent errors from various sources such as draft, pendulum readings etc.

As a conclusion, certain amount of error is inevitable especially in determining KG from the inclining experiment. Therefore, in order to minimize the error, these proposed methods may be considered as an alternative since they do not depend on the uncertainties confronted in the test. Furthermore, these methods are much easier to apply to any ship regardless of hull form, size or type. However, it is not easy to draw definitive generalized conclusions from this comparative analysis.

Nevertheless, the readers should be bear in mind that the comparative analysis carried out in this study relies on the fact that the results from the inclining experiment are taken as a reference since the absolute error free GM is not known. Obviously, the main objective of this paper is not to determine the exact value of GM but to compare results of various methods using different test vessels. Although, it is mandatory to perform inclining experiment for ships today, it seems feasible that one of these methods may be considered by IMO in the future to supersede the current inclining experiment after further research and benchmark studies.

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