

# ESTIMATION OF PROCESS CAPABILITY BASED ON CONTINUOUS AND ATTRIBUTE DATA

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**Abstract:** In the paper, a process capability and process performance analyses for continuous and attribute data are conducted by using an industry example. The inner diameter of the bearing rings is analysed by applying the normal capability analysis for the continuous data and by applying the binomial capability analysis for the attribute data. In order to quantify the process performance and process capability for continuous and attribute data, the sigma level or the process sigma is calculated. This represents an alternative method to calculating process capability indices and process performance indices.

**Keywords:** attribute data; continuous data; process capability; process performance; sigma level

## 1 INTRODUCTION

Process capability can be defined as the ability of the process to produce products/services which satisfy the requirements and needs determined by the customers themselves. Process capability studies can be conducted by using either the continuous or attribute data [1]. Continuous data is quantitative data and can contain an infinite number of values. Attribute data is data that has the quality or attribute characteristic that can be counted for recording and for the analysis.

Before process capability could be assessed, the process needed to be brought under control [2-4]. In the paper, the  $p$  control chart and  $\bar{X}$  -  $R$  control chart were used to bring and keep the process under statistical control. The capability of attribute data was estimated by using the binomial distribution while the capability of continuous data was estimated by using the normal distribution. In order to compare a variable process to an attribute process, a sigma level ( $Z$ -score and  $Z$ .Bench) was used [5-7].

## 2 PROCES CAPABILITY ANALYSIS

The paper tracks the daily production of rings ( $N = 100$  pcs) for a period of 25 days. The control of the production process was carried out through the application of the "100% control" process on the inner diameter of the bearing rings ( $LSL = 62.943 \text{ mm} \leq \text{inner diameter} \leq USL = 63.050 \text{ mm}$ ). The diameter of the ring was controlled through the use of the "go – no go" method and the number of non-conforming units was ascertained (Tab. 1). The results of the non-conforming units under the lower specification limits (LSL) and above the upper specification limits (USL) are presented in Tab. 1.

Table 1 Number of non-conforming units

Number of non-conforming units											
LSL	2	0	0	0	0	0	0	0	0	1	0
USL	0	1	2	0	2	0	1	2	1	0	1
Number of non-conforming units											
LSL	0	0	0	0	0	1	0	0	0	1	0
USL	1	1	0	0	0	1	2	0	1	0	1

Considering the attributive nature of the data, the  $p$  control chart was used to ascertain whether the process was

Table 2 Inner diameter in mm

Sample No.				
1.	2.	3.	4.	5.
62.986	62.997	62.980	62.967	63.017
62.963	63.024	63.012	62.989	62.992
62.980	63.013	62.971	63.017	62.994
62.996	63.010	62.974	62.989	63.010
62.973	63.014	62.996	62.968	62.995
Sample No.				
6.	7.	8.	9.	10.
62.980	63.017	62.997	62.997	62.999
63.018	62.975	62.981	62.985	62.983
63.017	62.981	63.043	63.020	62.994
62.989	62.998	63.008	63.013	63.022
62.967	62.990	63.017	62.964	62.995
Sample No.				
11.	12.	13.	14.	15.
63.026	62.969	63.043	62.984	63.006
63.026	63.013	63.046	62.958	62.993
62.977	62.993	62.987	63.010	63.002
62.984	63.010	63.027	63.028	62.958
62.999	62.998	62.986	62.961	63.014
Sample No.				
16.	17.	18.	19.	20.
62.990	62.986	62.993	63.044	62.985
62.987	62.993	63.002	62.926	62.989
62.988	62.994	62.989	63.033	63.021
63.009	63.010	63.031	63.003	63.008
63.003	63.001	62.979	63.009	63.014
Sample No.				
21.	22.	23.	24.	25.
63.013	63.009	62.973	62.973	63.044
62.978	63.028	62.987	62.987	63.008
63.018	62.998	62.939	62.939	62.992
63.017	63.001	63.003	63.003	62.999
62.974	62.980	62.998	62.998	62.989

stable and in control. Parallel to these actions, five rings were randomly selected from the process each day and their inner diameter was measured. Results are presented in Tab. 2.

Taking into account the fact that this analysis deals with continuous data, the  $\bar{X}$  -  $R$  control chart was used to ascertain the stability of the process. The capability of attribute data was calculated by using the binomial

distribution, while the capability of continuous data was calculated by using the normal distribution via the Minitab trial version. The Binomial Process Capability Report is presented in Fig. 1.

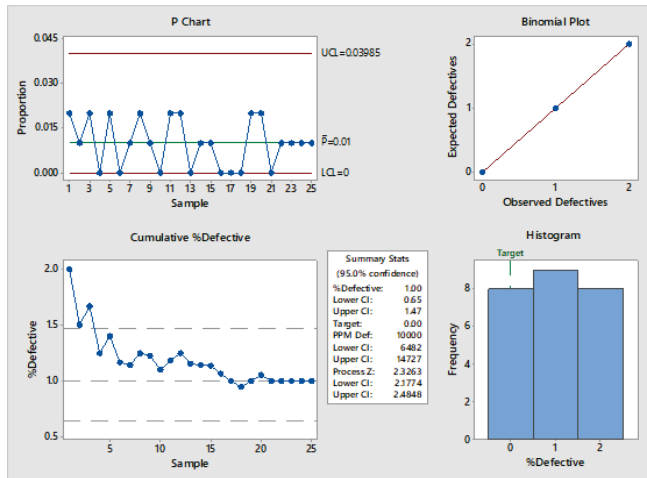


Figure 1 Binomial Process Capability Report

The *p* chart indicates that there are no data points exceeding the control limits. The chart of the cumulative estimate of the mean percentage of defective items (% defect) demonstrates that the estimate of the overall defective rate appears to be stabilizing around 1%, but additional data may need to be collected in order to verify this. The process *Z* equals 2.33. The process *Z* (the sigma capability of a process) is the capability index. The larger the process *Z* is, the better the process is performing. Process *Z* is computed from the average probability *P* by determining the value from a standard normal (0, 1) distribution so that the area to the right of the value is the average *P* [8-9].

The results of the process control using the *X* bar – *R* control chart are shown in Figs. 2a) and 2b), while the process capability report (normal) is shown in Fig. 3.

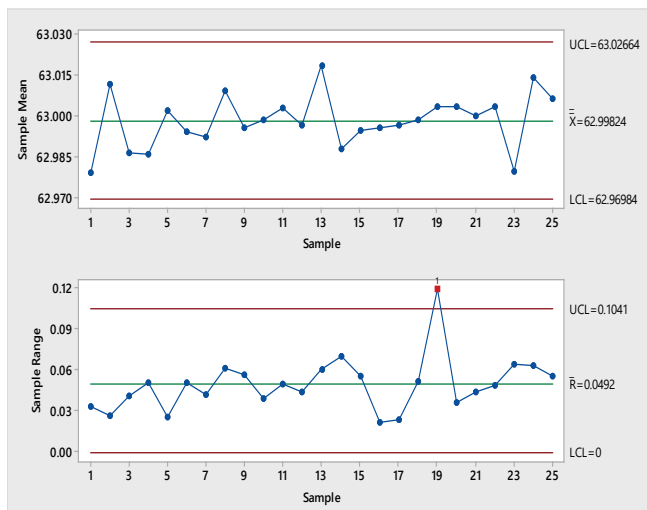


Figure 2 a) X bar – R control charts - Process not under control

In Fig. 2a), the displayed process is not stable, hence it makes no sense to calculate its capability. In such a case, the

cause of process instability needs to be investigated and removed. Fig. 2b), on the other hand, displays a process which is stable and under control.

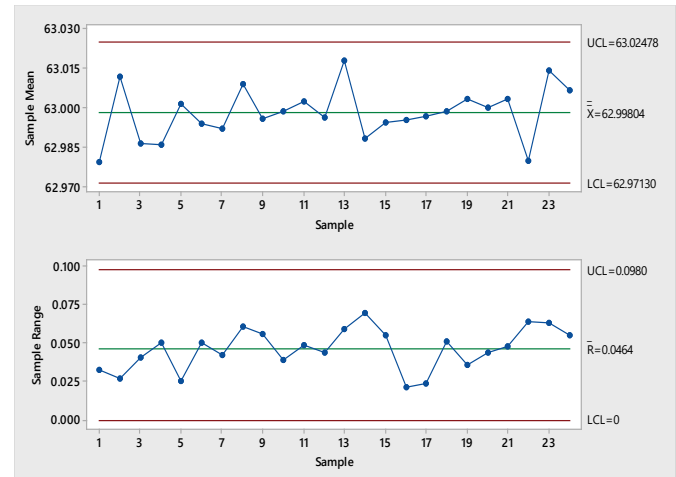


Figure 2 b) X bar – R control charts - Process under control

In accordance with the process capability report (Fig. 3), capability assessments were conducted by calculating the potential capability (process capability) and overall capability (process performance). As shown in Fig. 3, the data follow a normal distribution.

Instead of potential capability indices *C<sub>p</sub>* and process performance indices *P<sub>p</sub>*, the sigma level *Z*.Bench (potential) and sigma level *Z*.Bench (overall) were calculated.

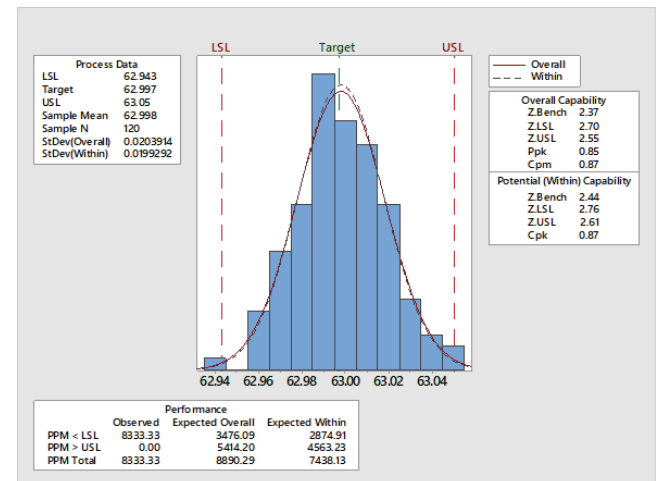


Figure 3 Process capability report (normal)

*Z*. Bench is defined by the Eq. (1).

$$Z. Bench = \phi^{-1}(1 - P_1 - P_2) \tag{1}$$

Where:

$$P_1 = \text{Prob}(X < LSL) = 1 - \phi(Z. LSL) \tag{2}$$

$$P_2 = \text{Prob}(X > USL) = 1 - \phi(Z. USL) \tag{3}$$

$\phi$  - cumulative distribution function of a standard normal distribution

$\phi^{-1}$  - inverse of a cumulative distribution function of a standard normal distribution

$Z.LSL$  - measure of sigma capability in relation to the upper specification limit

$Z.USL$  - measure of sigma capability in relation to the lower specification limit

Sigma capability in relation to the lower specification limit  $Z.LSL$  and sigma capability in relation to the upper specification limit  $Z.USL$  are defined by Eqs. (4) and (5).

$$Z.LSL = \frac{\bar{x} - LSL}{\sigma}, \quad (4)$$

$$Z.USL = \frac{USL - \bar{x}}{\sigma}. \quad (5)$$

Where:  $\bar{x}$  – arithmetic mean,  $\sigma$  – standard deviation.

The Process capability report (Fig. 3) shows that the process mean (62.998 mm) is slightly larger than the target (62.996 mm) and both tails of the distribution fall outside the specification limits. This means that there are rings whose diameter is below the lower specification of 63.943 mm or greater than the upper specification of 63.050 mm. The  $Ppk$  index equals 0.85, indicating that the process has to be improved by reducing variability and by centering the process on the target. In the case of this production process, its large variation represents the main challenge in assuring process quality, while the process being slightly off target is a secondary issue [10-11]. The benchmark  $Z$  statistics is computed by determining the  $Z$  value using the standard normal (0.1) distribution [8-9]. The  $Z$  benchmark describes the sigma capability of a process.  $Z.LSL$  and  $Z.USL$  measure how the process is performing against the specification limits [8-9].  $Z.Bench$  equals 2.37.

The  $Z$  score resulting from the analysis of the attribute data amounts to 2.33 and it is very close to the value of the  $Z.Bench$  obtained from the analysis of the continuous data which is equal to 2.37. It can therefore be concluded that both analyses reached similar conclusions in regard to the sigma level of the process.

### 3 CONCLUSION

The procedure for estimating process capability depends on the data types used. In the first step, it is necessary to check whether the data type is continuous or attributive. In the second step, stability has to be checked by using control charts for continuous or attribute data. Depending on the type of data, the normal capability or binomial capability procedure should be applied. The process capability and process performance can be conducted via the calculation of process capability indices, as well as process performance indices and the sigma level. The sigma level or process sigma represents a good measuring tool that can be used to quantify the ability of process performance of the continuous or attribute data. The process sigma or sigma level is a universal benchmark that can be used to measure the quality of different processes.

### 4 REFERENCES

- [1] <https://support.minitab.com/en-us/minitab/18/help-and-how-to/quality-and-process-improvement/capability-analysis/supporting-topics/data-and-data-assumptions/data-types-for-capability-analysis/>
- [2] Deming, W. E. (1982). *Quality, Productivity and Competitive Position*, MIT Press, Cambridge.
- [3] Quentin, B. (2010). *Lean Six Sigma & Minitab: The Complete Toolbox Guide for All Lean Six Sigma Practitioners*, OPEX Resources, Winchester.
- [4] Antony, J., Antony, F. J., Kumar, M., & Cho, B. R. (2007). Six Sigma in Service Organizations: Benefits, Challenges and Difficulties, Common Myths, Empirical Observation and Success Factors. *International Journal of Quality & Reliability Management*, 24(3), 294-311. <https://doi.org/10.1108/02656710710730889>.
- [5] Kotz, S. & Johnson, N. L. (1993). *Process Capability Indices*, Chapman & Hall, London. <https://doi.org/10.1007/978-1-4899-4465-8>
- [6] Harris, T. J. & Ross, W. H. (1991). Statistical Process Control Procedures for Correlated Observations, *Canadian Journal of Chemical Engineering*, 69(1), 48-57. <https://doi.org/10.1002/cjce.5450690106>
- [7] Runje, B., Medić, S., Kondić, Ž., Alar, Ž., & Tunjić, Đ. (2012). Statistical process control of fire-resistant coating production based on thickness measurements. *Tehnicki Vjesnik-Technical Gazette*. 19(3), 589-593.
- [8] <https://analyse-it.com/docs/user-guide/process-capability/z-benchmark>
- [9] <https://support.minitab.com/en-us/minitab/18/help-and-how-to/quality-and-process-improvement/capability-analysis/supporting-topics/capability-metrics/process-z-values-for-attribute-capability/>
- [10] Sommerville, S. & Montgomery, D. C. (1996). Process Capability Indices and Non-Normal Distribution. *Quality Engineering*, 9(2), 305-316. <https://doi.org/10.1080/08982119608919047>
- [11] Relyea, D. B. (2011). *The Practical Application of the Process Capability Study: Evolving from Product Control to Process Control*, CRC Press, Boca Raton.

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