

**Hrvoje
Volarević****KREIRANJE OPTIMALNE
IZVEDBE INVESTICIJSKOG
PROJEKTA****CREATION OF OPTIMAL
PERFORMANCE OF THE
INVESTMENT PROJECT**

SAŽETAK: Izbor investicijskog projekta promatra se kao problem višekriterijskog odlučivanja. U ovom radu donositelj odluka koristi se sa šest atributa odnosno kriterija koja najviše koriste međunarodna poduzeća u praksi (neto sadašnja vrijednost, interna stopa rentabilnosti, vrijeme povrata, računovodstvena stopa povrata, operativna profitna marža i povrat na vlasnički kapital).

Konstruiraju se individualne funkcije korisnosti za svaki atribut zasebno te globalna funkcija korisnosti kao ponderirani zbroj pojedinačnih funkcija korisnosti. Za svaki kriterij određuje se konačan skup uređenih parova odnosno točaka korisnosti na temelju procjena donositelja odluke. Potom se dobivene točke aproksimiraju funkcijom korisnosti.

Konačno, rješava se problem optimizacije u svrhu dobivanja optimalne izvedbe selektiranog projekta po mišljenju donositelje odluke. Postupkom pregovaranja ponuđene izvedbe približavaju se optimalnoj izvedbi odabranog projekta u svrhu dogovora donositelja odluke i investitora.

KLJUČNE RIJEČI: investicijski projekt, višekriterijsko odlučivanje, funkcija korisnosti, pregovaranje

ABSTRACT: The selection of an investment project is seen as a problem of multi-criteria decision-making. In this paper, a decision-maker uses six attributes i.e. criteria most used by the international companies in practice (net present value, internal rate of return, payback period, accounting rate of return, operating profit margin and return on equity).

Individual utility functions are made for each attribute separately and the global utility function as a weighted sum of individual utility functions. For each criterion a final set of arranged pairs i.e. points of utility is determined based on the decision-maker's assessments. Then, the points obtained are approximated by the utility function.

Finally, the optimization issue solved in order to obtain the optimal performance of the selected project according to decision-maker's opinion. The negotiation procedure enables the offered performances to approach optimal performance of the selected project aimed at decision-maker and investor reaching an agreement.

KEY WORDS: investment project, multi-criteria decision-making, utility function, negotiations

UVOD

Izbor investicijskog projekta u okviru istog poduzeća je problem u kojem su za donesenu odluku zainteresirane dvije strane, donositelj odluke (glavni menadžment poduzeća) i investitor (menadžment u okviru investicijskog centra odgovornosti u poduzeću). U situaciji ograničenog kapitalnog budžeta u poduzeću (gdje u pravilu postoji mogućnost za pokretanje samo jedne investicije u dužem vremenskom razdoblju), donositelj odluke će biti spreman investirati vlastita novčana sredstva u selektirani investicijski projekt odnosno financirati ga zajmom banke (Alkaraan, Northcott, 2006). Zbog toga oba participanta imaju identični cilj – prihvaćanje tog investicijskog projekta.

Njihova razmišljanja razlikuju se u izvedbama pokazatelja efikasnosti odabranog projekta, odnosno na izvedbu projekta donositelj odluke postavlja zahtjeve za koje očekuje da će ih investitor prihvatiti. Time problem prelazi u problem pregovaranja dva participanta. Odabrani investicijski projekt može imati više različitih izvedbi. U ovom radu investitor predlaže izvedbe pokazatelja efikasnosti tako da ni jedna od izvedbi ne dominira nad preostalim izvedbama. To znači da se vrijednost jednog pokazatelja ne može poboljšati bez da se vrijednost drugog pokazatelja pogorša, kao što to pretpostavlja koncept Pareto efikasnosti (Heiskanen, 2001). Za sve predložene izvedbe odabranog projekta konstruira se funkcija korisnosti za svaki atribut zasebno. Problem optimizacije je maksimizacija skora, globalne funkcije korisnosti, za odabrani projekt odnosno alternativu. U pregovore donositelj odluke ulazi s optimalnom izvedbom projekta (Li, Tesauro, 2003).

Ovaj rad prezentira integrirani konceptualni model u čije oblikovanje su uključeni višekriterijska teorija korisnosti (eng. *multi-attribute utility theory* – MAUT) i postupak pregovaranja. MAUT model omogućuje razmatranje faktora koji se mjere u različitim veličinama i različitoj relativnoj važnosti za donošenje odluka (Min, 1994).

INTRODUCTION

The selection of an investment project within the same company is a problem where two parties, the decision-maker (company principal management) and the investor (management within the company's investment responsibility center) are interested in the made decision. In the situation of a limited capital budget in a company (where, as a rule, the launching of only one investment over a longer period of time is possible), the decision-maker will be willing to invest his or her own funds into a selected investment project i.e. finance it with a bank loan (Alkaraan, Northcott, 2006). For this reason, both participants have the identical goal – accepting this investment project.

Their considerations differ in performances of the selected project's efficiency indicator i.e. the decision-maker sets requirements related to the project performance and expect the investor to accept them. Following the said, the problem turns into a problem of negotiating between the two participants. The selected investment project may have several different performances. In this paper, the investor proposes such efficiency indicator performances where no performance dominates the others. This means that the value of an indicator cannot be improved without having the value of another indicator worsened as implied by the Pareto efficiency concept (Heiskanen, 2001). For each proposed performance of the selected project, the utility function is made separately for each attribute. The optimization problem is maximization of score, the global utility function, for the selected project or alternative. Decision-maker enters the negotiations with the optimal performance of the project (Li, Tesauro, 2003).

This paper presents an integrated conceptual model whose shaping includes multi-attribute utility theory (MAUT) and negotiation procedure. The MAUT model allows for consideration of factors measured in different sizes and different relative importance for decision-making (Min, 1994).

Ostatak rada prezentiran je na sljedeći način. U drugom poglavlju predočeno je svih šest kriterija, zatim formulacija višekriterijskog modela optimizacije te cilj rada i radna hipoteza. U trećem poglavlju prikazana je konstrukcija specifičnih funkcija korisnosti od strane donositelja odluka. Pristup u rješavanju ovakvog modela ilustriran je u četvrtom poglavlju na temelju primjera koji uključuje odabrani projekt i šest atributa odnosno kriterija. Na kraju četvrtog poglavlja pojašnjeno je ostvarenje osnovnog cilja rada kao i potvrda radne hipoteze. U zaključnom poglavlju predočene su prednosti i nedostaci korištenja ovakvog modela u praktičnoj upotrebi.

IZBOR INVESTICIJSKOG PROJEKTA

Izbor investicijskog projekta je klasični problem višekriterijskog odlučivanja. Ulaganje u investicijske projekte predstavlja proces identificiranja i selektiranja investicija u dugotrajnu imovinu ili u imovinu od koje se očekuje stvaranje ekonomskih koristi u razdoblju dužem od godine dana (Van Horne, Wachowich, 2002). U ovom radu analizira se selektirani investicijski projekt unutar jednog poduzeća čiji je definiran vijek trajanja 10 godina, što znači da takvi projekti pripadaju procesu dugoročnog poslovnog planiranja.

Osnovni cilj ovog rada je pokazati mogućnost praktične upotrebe višekriterijskog matematičkog modela prilikom rješavanja različitih problema iz područja ekonomije, u ovom slučaju odabranog investicijskog projekta čija konačna izvedba se optimizira te time predstavlja podlogu za daljnji postupak pregovaranja u poduzeću. U biti, cilj ovog rada mogao bi se povezati i s mogućnošću znanstvenog odnosno stručnog doprinosa u okviru interdisciplinarnog područja koje obuhvaća poslovne financije, upravljačko računovodstvo i operacijska istraživanja. Sukladno navedenom, u ovom radu može se postaviti sljedeća radna hipoteza:

H1: *Kreiranje optimalne izvedbe investicijskog projekta može se formulirati kao matematički model*

The rest of the paper is presented in the following way. In the second chapter all six criteria are presented, followed by the formulation of a multi-criteria optimization model, working goal and working hypothesis. In the third chapter, the structure of specific utility functions is presented by the decision-maker. The approach to solving such a model is illustrated in the fourth chapter based on the example involving the selected project and six attributes i.e. criteria. At the end of the fourth chapter, the achievement of basic working goal as well as the confirmation of working hypothesis was clarified. The final chapter presents the advantages and disadvantages of using such a model in practice.

SELECTION OF INVESTMENT PROJECT

The selection of investment project is a classic issue of multi-criteria decision-making. Investment in investment projects represents the process of identifying and selecting investments into fixed assets or assets that are expected to generate economic benefits over a period of more than one year (Van Horne, Wachowich, 2002). An investment project selected within a company for the defined life span of 10 years, meaning that such projects belong to the process of long-term business planning is analyzed herein.

The main purpose of this paper is to show the possibility of practical use of a multi-criteria mathematical model in solving various problems in the field of economy, in this case the selected investment project whose final performance is optimized and thus represents the basis for further negotiation within the company. Actually, the aim of this paper could also be connected with the possibility of scientific or expert contribution within an interdisciplinary area that includes corporate finance, managerial accounting and operational research. In accordance with the above, the following working hypothesis can be set out in this paper:

H1: *Creating the investment project optimal performance can be formulated as a mathematical*

višekriterijskog odlučivanja. Rješenje matematičkog modela pridonosi kvalitetnijem postupku pregovaranja između donositelja odluke i investitora.

Kao prvi korak u definiranju matematičkog modela višekriterijskog odlučivanja, potrebno je izabrati kriterije na temelju kojih će se vršiti ocjena odabranog investicijskog projekta. Pretpostavimo da imamo jedan odabrani projekt (alternativu) i m mogućih izvedbi tog projekta te varijable odluke koje označavamo sa $x_i, i=1, \dots, m$. Te varijable su binarne sa značenjem da ako je $x_i=1$, izvedba projekta i je prihvaćena, te ako je $x_i=0$, izvedba projekta i nije prihvaćena. Kako je odabrana samo jedna izvedba, imamo ograničenje

$$\sum_{i=1}^m x_i = 1.$$

Izbor kriterija ili atributa vrši se na temelju konzultacija s donositeljem odluke. U ovom radu, nakon obavljene detaljne analize o praktičnoj upotrebi kriterija za procjenu efikasnosti investicijskih projekata u okviru međunarodnih odnosno američkih kompanija, u užu izbor su ušla četiri kriterija (neto sadašnja vrijednost, interna stopa rentabilnosti, vrijeme povrata i računovodstvena stopa povrata). Na temelju dostupnih podataka iz provedenog istraživanja (Graham, Campbell, 2002), dobiven je grafički prikaz na slici broj 1, kojim se ilustrira intenzitet praktične upotrebe četiri najvažnija kriterija kod odabranih američkih kompanija (s obzirom na općepoznatu primjenu ovih kriterija u svjetskoj praksi, ovako dobiveni rezultati mogu se očekivati i kod nekih drugih međunarodnih kompanija koje nisu predmet ove analize).

Preostala dva pokazatelja koja se koriste u radu također su odabrana na temelju njihove praktične upotrebe, a odnose se na pojedinačne pokazatelje profitabilnosti – operativnu profitnu maržu i povrat na vlasnički kapital. Radi se o pokazateljima koji se i danas intenzivno koriste za mjerenje rezultata poslovanja u sklopu investicijskih centara odgovornosti u poduzeću (Berrios, 2013). Kako smo u ovom radu investicijski centar

model of multi-criteria decision-making. The solution to the mathematical model contributes to a more qualitative negotiation process between the decision-maker and the investor.

As the first step in defining the mathematical model of multi-criteria decision-making, it is necessary to choose the criteria based on which the assessment of the selected investment project will be made. Suppose we have one selected project (alternative) and m of possible performances of that project and the decision variables we denote with $x_i, i=1, \dots, m$. These variables are binary with the meaning that if $x_i=1$, the performance of the project i is accepted, and if $x_i=0$, the execution of the project i is not accepted. As only one performance is selected, we have a restriction

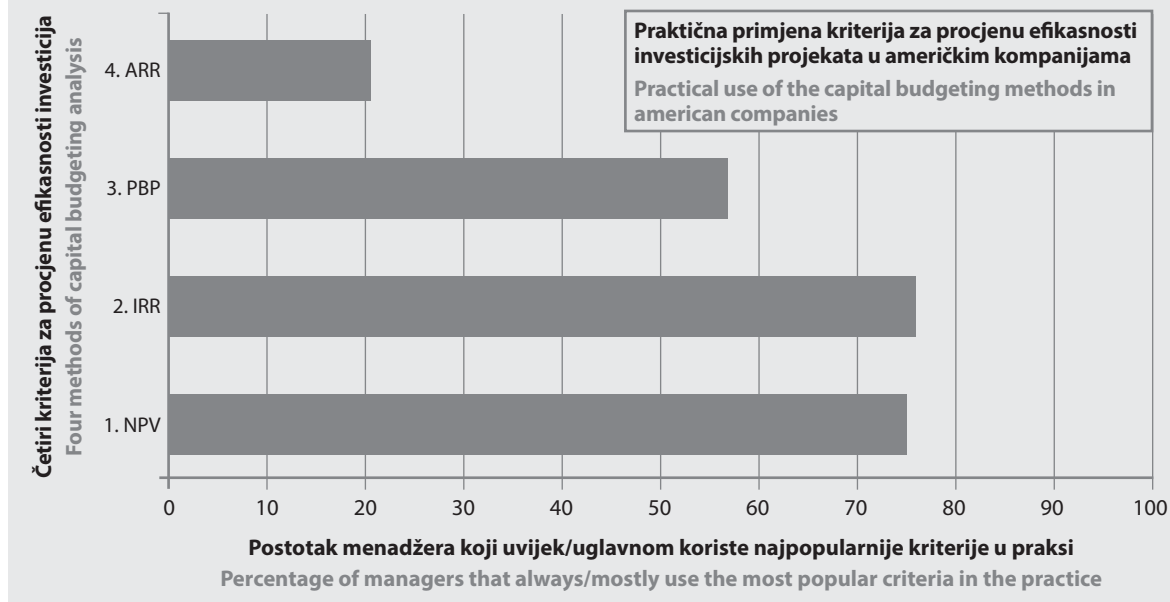
$$\sum_{i=1}^m x_i = 1.$$

The choice of criteria or attributes is made based on the consultation with the decision-maker. In this paper, after a detailed analysis of practical use of criteria for assessing the efficiency of investment projects within international i.e. U.S. companies, four criteria have been introduced in the shortlist (net present value, internal rate of return, payback period and accounting rate of return). Based on available data from the survey (Graham, Campbell, 2002), a graphical representation shown in Figure 1 that illustrates the intensity of practical use of the four most important criteria for the selected U.S. companies has been obtained (given the generally known application of these criteria in world practice, the results obtained can be expected in some other international companies that are not subject to this analysis).

The remaining two indicators used in the paper are also selected based on their practical use, referring to individual profitability indicators – operating profit margin and return on equity. These are indicators that are still intensively used for measuring the results of business operations within the investment responsibility centers of the company (Berrios, 2013). As we have defined in this paper

SLIKA 1. POSTOTAK PRAKTIČNE PRIMJENE KRITERIJA U AMERIČKIM TVRTKAMA

FIGURE 1. PERCENTAGE OF PRACTICAL APPLICATION OF CRITERIA IN THE U.S. COMPANIES



Izvor/Source: John Graham, Campbell Harvey: "How Do CFOs Make Capital Budgeting and Capital Structure Decisions", *Journal of Applied Corporate Finance*, Volume 15, Number 1, page 11, 2002.

odgovornosti definirali kao investitora (jednu od dvije zainteresirane strane u problemu), sukladno tome poželjno je u daljnji izračun uključiti i ova dva pokazatelja. Na taj način ćemo u izračunu imati uključene i historijske podatke o dosadašnjoj profitabilnosti poslovanja investitora, što u određenoj mjeri može biti indikativno i za njegovo buduće poslovanje sa stanovišta pokretanja investicija (Adler, 2000). Oni ipak neće imati značajnu težinu u daljnjem izračunu kao preostala četiri pokazatelja jer se ipak ne radi o kriterijima za procjenu efikasnosti investicijskih projekata, ali će svakako biti dio budućih kalkulacija (Burns, Walker, 2009).

Sukladno navedenom, sada možemo pristupiti pojašnjenju svih šest odabranih pokazatelja (Brealey, Myers, Marcus, 2007). Neka je t životni vijek

the investment responsibility center as an investor (one of the two interested parties in the problem), it is desirable, therefore, to include these two indicators in further calculation as well. This way, we will include in the calculation historical data on the previous profitability of the investor's business, which may, to a certain extent, also be indicative of its future business from the standpoint of investment launch (Adler, 2000). However, they will not have a significant weight in further calculation like the remaining four indicators, as it is not the criterion for assessing-appraising the efficiency of investment projects but it will certainly be a part of future calculations (Burns, Walker, 2009).

Accordingly, we can now approach the clarification of all six selected indicators (Brealey, Myers, Marcus, 2007). Let t be the life span of investment project,

investicijskog projekta, NNT_n neto novčani tijek investicijskog projekta u n -toj godini i r diskontna stopa. U tom slučaju:

1. Neto sadašnja vrijednost (eng. *net present value* – NPV) predstavlja zbroj svih neto novčanih tijekova investicijskog projekta svedenih na sadašnju vrijednost postupkom diskontiranja. Ako je I_0 početno ulaganje u investiciju, tada je NPV neto sadašnja vrijednost investicijskog projekta jednaka:

$$NPV = -I_0 + \sum_{n=1}^t \left(\frac{NNT_n}{(1+r)^n} \right) \quad (1)$$

Ako sa NPV_i označimo neto sadašnju vrijednost izvedbe i odabranog projekta, dobivamo sljedeću funkciju cilja:

$$f_1(x) = \sum_{i=1}^m NPV_i x_i \quad (2)$$

2. Interna stopa rentabilnosti (eng. *internal rate of return* – IRR) je diskontna stopa koja neto sadašnju vrijednost investicijskog projekta svodi na nulu (radi se o maksimalno prihvatljivoj stopi profitabilnosti, najvećoj što je investicijski projekt može prihvatiti). Računamo je na sljedeći način:

$$NPV = -I_0 + \sum_{n=1}^t \left(\frac{NNT_n}{(1+IRR)^n} \right) = 0 \quad (3)$$

Ako sa IRR_i označimo internu stopu rentabilnosti izvedbe i odabranog projekta, dobivamo sljedeću funkciju cilja:

$$f_2(x) = \sum_{i=1}^m IRR_i x_i \quad (4)$$

3. Vrijeme povrata (eng. *payback period* – PBP) predstavlja broj razdoblja (godina) tijekom kojih je potrebno ostvariti takav neto novčani tijek investicijskog projekta kojim bi se unutar njegovog životnog vijeka povratila ukupna vrijednost

NNT_n the net cash flow of investment project in the year n and r the discount rate. In that case:

1. Net Present Value (NPV) represents the sum of all net cash flows of an investment project reduced to the present value by the discounting method. If I_0 is the initial investing in the investment, then NPV net present value of the investment project is equal to:

$$NPV = -I_0 + \sum_{n=1}^t \left(\frac{NNT_n}{(1+r)^n} \right) \quad (1)$$

If we mark with NPV_i the net present value of performance i of the selected project, we get the following objective function:

$$f_1(x) = \sum_{i=1}^m NPV_i x_i \quad (2)$$

2. Internal rate of return (IRR) is the discount rate that reduces the net present value of investment project to zero (it is the maximum acceptable rate of profitability, the maximum that an investment project can accept). We calculate it as follows:

$$NPV = -I_0 + \sum_{n=1}^t \left(\frac{NNT_n}{(1+IRR)^n} \right) = 0 \quad (3)$$

If we mark the internal rate of return of the performance i of the selected project with IRR_i , we get the following objective function:

$$f_2(x) = \sum_{i=1}^m IRR_i x_i \quad (4)$$

3. Payback period (PBP) is the number of periods (years) during which such an investment project net cash flow must be achieved that will restore the total value of realized investment within its life span. If the value of the investment in the year n of investment project life span is I_n , then investment project payback period t_p (PBP) is calculated from:

realizirane investicije. Ako je I_n vrijednost investicije u n -toj godini životnog vijeka investicijskog projekta, tada se t_p (PBP) vrijeme povrata investicijskog projekta računa iz:

$$\sum_{n=0}^t I_n = \sum_{n=0}^{t_p} NNT_n \quad (5)$$

Ako sa PBP_i označimo vrijeme povrata izvedbe i odabranog projekta, dobivamo sljedeću funkciju cilja:

$$f_3(x) = \sum_{i=1}^m PBP_i x_i \quad (6)$$

4. Računovodstvena stopa povrata (eng. *accounting rate of return* – ARR) predstavlja omjer prosječne vrijednosti svih budućih računovodstvenih neto dobitaka/gubitaka poduzeća za vrijeme životnog vijeka investicijskog projekta i ukupne vrijednosti investicije realizirane u tom istom vremenskom razdoblju. Prosječna vrijednost svih budućih računovodstvenih neto dobitaka/gubitaka poduzeća dobiva se na način da se zbroj budućih računovodstvenih neto dobitaka/gubitaka poduzeća dijeli s ukupnim brojem godina u okviru kojih su oni i ostvareni (tj. s brojem godina životnog vijeka investicijskog projekta). Ako je $\pm ND_n$ računovodstveni neto dobitak/gubitak (+/-) poduzeća u n -toj godini, tada je ARR računovodstvena stopa povrata jednaka:

$$ARR = \frac{\sum_{n=1}^t ND_n}{\sum_{n=0}^{t_i} I_n} \quad (7)$$

Ako sa ARR_i označimo računovodstvenu stopu povrata izvedbe i odabranog projekta, dobivamo sljedeću funkciju cilja:

$$f_4(x) = \sum_{i=1}^m ARR_i x_i \quad (8)$$

$$\sum_{n=0}^t I_n = \sum_{n=0}^{t_p} NNT_n \quad (5)$$

If we mark with PBP_i the payback period of performance i of the selected project, we get the following objective function:

$$f_3(x) = \sum_{i=1}^m PBP_i x_i \quad (6)$$

4. Accounting rate of return (ARR) is the ratio of average value of all future net accounting gains/losses of a company during the investment project life span and total value of the investment realized in that same time period. The average value of all future net accounting gains/losses of a company is obtained by dividing the sum of all future accounting net gains/losses of a company with a total number of years within which they are realized (i.e. with the number of years of the investment project life span). If $\pm ND_n$ is the net accounting gain/loss (+/-) of the company in the year n , then the ARR accounting rate of return equals to:

$$ARR = \frac{\sum_{n=1}^t ND_n}{\sum_{n=0}^{t_i} I_n} \quad (7)$$

If we mark with ARR_i the accounting rate of return of performance i of the selected project, we obtain the following objective function:

$$f_4(x) = \sum_{i=1}^m ARR_i x_i \quad (8)$$

5. Operating profit margin (OPM) represents the ratio between the realized operating profit $EBIT_t$ (earnings before interest and taxes) in the reporting period of the investor t and total revenue of the

5. Operativna profitna marža (eng. *operating profit margin* – OPM) predstavlja omjer između ostvarene operativne dobiti $EBIT_t$ (eng. *earnings before interest and taxes*) u izvještajnom razdoblju investitora t , i ukupnih prihoda poduzeća TR_t u izvještajnom razdoblju investitora t , što se računa:

$$OPM = \frac{EBIT_t}{TR_t} \quad (9)$$

Ako sa OPM_i označimo povrat na investiciju izvedbe i odabranog projekta, dobivamo sljedeću funkciju cilja:

$$f_5(x) = \sum_{i=1}^m OPM_i x_i \quad (10)$$

6. Povrat na vlasnički kapital (eng. *return on equity* – ROE) predstavlja omjer između ostvarene neto dobiti NP_t u izvještajnom razdoblju investitora t , i ukupne vrijednosti vlasničkog kapitala E_t u izvještajnom razdoblju investitora t , što se računa:

$$ROE = \frac{NP_t}{E_t} \quad (11)$$

Ako sa ROE_i označimo povrat na investiciju izvedbe i odabranog projekta, dobivamo sljedeću funkciju cilja:

$$f_6(x) = \sum_{i=1}^m ROE_i x_i \quad (12)$$

FUNKCIJA KORISNOSTI

U drugom poglavlju je uvedeno šest atributa pomoću kojih mjerimo izvedbu ili vrijednosti alternativa. Kako imamo multidimenzionalan problem višekriterijskog odlučivanja, potrebno ga je svesti na jednodimenzionalan. Općenito, funkcije korisnosti za različite kriterije u odlučivanju nisu eksplicitno poznate (Sycara, 1988). Uvodimo

company TR_t in the reporting period of the investor t , which is calculated as follows:

$$OPM = \frac{EBIT_t}{TR_t} \quad (9)$$

If we mark return on investment of performance i of selected project with OPM_i , we get the following objective function:

$$f_5(x) = \sum_{i=1}^m OPM_i x_i \quad (10)$$

6. Return on equity (ROE) is the ratio between the realized net profit NP_t in the reporting period of the investor t and total value of equity E_t of investor t in the reporting period, which is calculated as follows:

$$ROE = \frac{NP_t}{E_t} \quad (11)$$

If we mark with ROE_i the return on performance i of the selected project, we get the following objective function:

$$f_6(x) = \sum_{i=1}^m ROE_i x_i \quad (12)$$

UTILITY FUNCTION

In the second chapter six attributes are introduced for measuring alternatives - performance or values. Since we have a multidimensional problem of multi-criteria decision/making, it is necessary to reduce it to one-dimensional. In general, utility functions for different decision-making criteria are not explicitly known (Sycara, 1988). We introduce a specific decision maker's utility function for each individual objective function i.e. attribute. The structure is made in such a way that gives a set of pairs of different points that we call the

specifičnu funkciju korisnosti donositelja odluke za svaku pojedinu funkciju cilja odnosno atribut. Konstrukcija se vrši na način da je dan skup parova različitih točaka koje zovemo točke korisnosti (y_i, u_i) , $i=1, \dots, n$ za svaku funkciju cilja, gdje je y_i vrijednost funkcije cilja (atributa, pokazatelja) i u_i je njoj pridružena korist. Korist u_i od izvedbe y_i proizvoljno određuje donositelj odluke. Pomoću točaka korisnosti konstruiraju se pojedine funkcije korisnosti. Različite tehnike koriste se za tu konstrukciju. Pojedini autori predlažu interpolaciju linearnom, kvadratnom ili kubnom funkcijom te tehnike izgladivanja oko odabranih točaka korisnosti (Ehrgott, Klamroth, Schwehm, 2004).

Investitor predlaže više različitih izvedbi odabranog projekta. Donositelj odluke izabrat će optimalnu izvedbu odabranog projekta. Sve izvedbe generirane su slučajnim odabirom oko točke koja predstavlja početnu izvedbu odabrane alternative. Na temelju različitih izvedbi odabranog investicijskog projekta konstruiraju se lokalne funkcije korisnosti vrijednosti svakog atributa. Općenito generirane izvedbe odabrane alternative nisu dominirane nekom izvedbom. Dobivene točke korisnosti aproksimiramo funkcijom pomoću metode najmanjih kvadrata.

Za globalnu funkciju korisnosti (eng. *utility* – U) uzimamo skor S koji odgovara:

$$S = \sum_{j=1}^6 w_j u_j (f_j(x)) \quad (13)$$

gdje je S skor izvedbe odabrane alternative. Varijabla $x_i=1$ ako je izabrana izvedba i , $x_i=0$ ako izvedba i nije izabrana. Ako je $0 < x_i < 1$ za neko i , konstruirana je nova izvedba odabrane alternative (konveksna kombinacija više različitih izvedbi) te se investitoru predlaže ispitivanje je li ta izvedba odabrane alternative moguća. Parametar w_j predstavlja težinu koja je pridružena atributu tj. kriteriju j . Suma težina pridružena svim kriterijima jednaka je 1. Pretpostavimo da imamo m izvedbi odabrane alternative odnosno projekta. Rješavamo problem optimizacije (R):

points of utility (y_i, u_i) , $i=1, \dots, n$ for each objective function, where y_i is the value of the function of target (attribute, indicator) point and u_i represents its associated utility. Utility u_i of the performance y_i is determined arbitrarily by the decision-maker. Utility points are used for creating certain utility functions. Various techniques are used for this structure. Some authors suggest interpolation by linear, squared or cubic function and techniques of smoothing around selected utility points (Ehrgott, Klamroth, Schwehm, 2004).

The investor proposes several different performances of the selected project. The decision-maker will choose the optimal performance of the selected project. All performances are generated by random selection around the point that represents the initial performance of the selected alternative. Based on different performances of the selected investment project, local utility functions of each attribute's value are created. Generally generated performances of the selected alternative are not dominated by any performance. Then we approximate the acquired utility points with function by using the least squares method.

For the global utility function (U) we take the score S that corresponds:

$$S = \sum_{j=1}^6 w_j u_j (f_j(x)) \quad (13)$$

where S represents the score of selected alternative performance. Variable $x_i=1$ if selected performance is i and $x_i=0$ if the selected performance is not i . If $0 < x_i < 1$ for a certain i , new performance of the selected alternative (a convex combination of several different performances) has been made and the investor is suggested to check if that performance of the selected alternative is possible. The parameter w_j represents the weight associated with the attribute i.e. criterion j . The sum of weights associated with all criteria is equal to 1. Assume that we have m performances of the selected alternative or project. We solve the optimization problem (R):

$$\begin{aligned} \max \sum_{j=1}^6 w_j u_j(f_j(x)) \\ \sum_{i=1}^m x_i = 1 \\ x_i \geq 0, i = 1, \dots, m \end{aligned} \quad (14)$$

Primijetimo da smo uvjet binarnosti varijabli zamijenili uvjetom nenegativnosti. Time dozvoljavamo da osim kreiranih ponuda izvedbe odabranog projekta optimalna izvedba projekta može biti naknadno kreirana u dogovoru s investitorom. Optimalno rješenje daje optimalnu izvedbu projekta. S ovom izvedbom ulazi se u pregovore s investitorom projekta.

IMPLEMENTACIJA

Prije nego se krene na samu implementaciju problema optimizacije (14), potrebno je definirati težine w_j koje se pridružuju svakom atributu tj. odabranom kriteriju. U Tablici 1 su predočene težine koje će se koristiti u ovom matematičkom modelu za svih šest kriterija (NPV, IRR, PBP, ARR, OPM i ROE).

Valja napomenuti da je kod određivanja težina odnosno pondera za prva četiri kriterija (NPV, IRR, PBP i ARR) presudnu ulogu imala učestalost njihove primjene u praksi kod odabranih kompanija (vidi Sliku 1). Iz Tablice 1 vidljivo je kako NPV i IRR kriterij imaju najveću težinu, što je i logično s obzirom na to da se daleko najviše koriste u praksi i da su u pitanju dva superiorna i u pravilu

$$\begin{aligned} \max \sum_{j=1}^6 w_j u_j(f_j(x)) \\ \sum_{i=1}^m x_i = 1 \\ x_i \geq 0, i = 1, \dots, m \end{aligned} \quad (14)$$

Note that we have replaced the binary variables condition with the non-negativity condition. With this we allow, in addition to the created offers of the selected project performances, the subsequent creation of project optimal performance in agreement with the investor. The optimal solution provides the optimal performance of the project. With this performance, negotiations are entered with the investor of the project.

IMPLEMENTATION

Prior to the problem optimization implementation (14), it is necessary to define the weights w_j assigned to each attribute i.e. the selected criterion. Table 1 shows the weights to be used in this mathematical model for all six criteria (NPV, IRR, PBP, ARR, OPM and ROE).

It should be noted that in determining the weights i.e. weights for the first four criteria (NPV, IRR, PBP and ARR), the frequency of their application in practice at selected companies was crucial (see figure 1). Table 1 shows that the NPV and IRR criteria have the highest weight, which is logical since they are far most widely used in practice and that they are two superior and, in principle, evenly

TABLICA 1. PROCJENA TEŽINA ZA SVIH ŠEST KRITERIJA U MODELU
TABLE 1. ASSESSMENT OF WEIGHTS FOR ALL SIX CRITERIA IN THE MODEL

KRITERIJ / CRITERION	1. NPV	2. IRR	3. PBP	4. ARR	5. OPM	6. ROE
PONDER (w) / WEIGHT	0,30	0,30	0,20	0,10	0,05	0,05

Izvor: Izradio autor. / Source: Prepared by the author.

ravnomjerno zastupljena kriterija s obzirom na karakteristike koje iskazuju (njihovim korištenjem postiže se maksimizacija bogatstva dioničara). Treće mjesto po učestalosti korištenja drži PBP kriterij (prvenstveno zbog svoje razumljivosti i jednostavnosti u izračunu), bez obzira na primjetne nedostatke (npr. ne koristi diskontirane novčane tijekove u kalkulaciji). Posljednje mjesto rezervirano je za ARR kriterij, koji ima drugačiji koncept izračuna u odnosu na preostala tri kriterija (koristi računovodstvenu dobit umjesto neto novčanih tijekova), zbog čega ga i izbjegava većina donositelja odluka u odabranim kompanijama. Za kraj, ostavljena su posljednja dva kriterija (OPM i ROE), kojima je dodijeljena najmanja vrijednost težina (5%) s obzirom na simboličnu ulogu koju imaju u ovom izračunu, što je ranije u tekstu detaljno pojašnjeno.

Donositelj odluka je u predloženi problem optimizacije uključio svih šest odabranih kriterija – neto sadašnju vrijednost (NPV), internu stopu rentabilnosti (IRR), vrijeme povrata (PBP), računovodstvenu stopu povrata (ARR), operativnu profitnu maržu (OPM) i povrat na vlasnički kapital (ROE) – na temelju njihove praktične upotrebe u odabranim kompanijama. Glavni menadžment poduzeća odobrio je ograničeni kapitalni budžet za investiranje (3.200.000,00 USD) i na temelju toga selektirao je samo jedan dugoročni investicijski projekt koji ima projekciju budućih neto novčanih tijekova u USD, kao što je to prikazano u Tablici 2 (početna tj. nulta godina projekta je 2019. godina, a životni vijek projekta u kojem se analizira njegova efikasnost iznosi 10 godina – od 2020. do 2029. godine). Treba napomenuti kako je kod ovog investicijskog projekta vrijednost investicijskog novčanog tijeka (eng. *investment cash flows* – ICF) u početnoj tj. nultoj godini negativna jer se radi o novčanim ulaganjima u investiciju, dok je u posljednjoj godini pozitivna jer predstavlja prodajnu odnosno tržišnu vrijednost investicije nakon deset godina životnog vijeka. S druge strane, vrijednost operativnog novčanog tijeka (eng. *operating cash flows* – OCF) je svih deset godina pozitivna i u konstantnom je porastu.

represented criteria based on the characteristics they express (by using them, the maximization of shareholders' wealth is achieved). The PBP criterion holds the third place by frequency of use (primarily because of its comprehensibility and simplicity in the calculation) regardless of noticeable deficiencies (e.g. does not use discounted cash flows in calculation). The last place is reserved for the ARR criterion that has a different concept of calculation in relation to the remaining three criteria (it uses the accounting profit instead of net cash flow), which is why most decision-makers in the selected companies are avoiding it. In the end, the last two criteria (OPM and ROE) have been assigned the lowest weight value (5%) given the symbolic role they play in this calculation, as previously explained in detail.

The decision-maker has included in the proposed optimization problem all six selected criteria – net present value (NPV), internal rate of return (IRR), payback time (PBP), accounting rate of return (ARR), operating profit margin (OPM) and return on equity (ROE) – based on their practical use in the selected companies. The company principal management approved a limited capital budget (USD 3,200,000.00) for investment and accordingly selected only one long-term investment project that has a projection of future net cash flows in USD as shown in Table 2 (the initial i.e. zero year of the project is 2019 and the project life span during which its efficacy is being analyzed is 10 years – from 2020 to 2029). It should be noted that in this investment project the value of the *investment cash flows* (ICF) is negative in the initial i.e. zero year, given the fact that these are cash investments into investment, while in the last year it is positive because it represents the sales i.e. market value of the investment after ten years of its life span. On the other hand, the value of *operating cash flows* (OCF) is positive during the entire ten years and is steadily increasing.

Table 3 shows the values of all six attributes for the selected alternative (investment project A) for its initial performance, which were obtained based

TABLICA 2. PROJEKCIJA BUDUĆIH NOVČANIH TOKOVA ZA ODABRANU ALTERNATIVU (PROJEKT)
TABLE 2. PROJECTION OF FUTURE CASH FLOWS FOR THE SELECTED ALTERNATIVE (PROJECT)

GODINA / YEAR	2019.	2020.	2021.	2022.	2023.	2024.	2025.	2026.	2027.	2028.	2029.
Razdoblje / Period	0	1	2	3	4	5	6	7	8	9	10
ICF	-3.200.000,00	-	-	-	-	-	-	-	-	-	5.763.864,00
OCF	-	150.655,00	256.958,00	282.875,00	355.478,00	414.835,00	480.210,00	521.849,00	518.162,00	574.777,00	576.386,00
Neto novčani tijekovi Net cash flows	-3.200.000,00	150.655,00	256.958,00	282.875,00	355.478,00	414.835,00	480.210,00	521.849,00	518.162,00	574.777,00	6.340.250,00

Izvor: Izradio autor. / Source: Prepared by the author.

TABLICA 3. VRIJEDNOSTI SVIH ŠEST ATRIBUTA (KRITERIJA) ZA ODABRANU ALTERNATIVU (PROJEKT)
TABLE 3. VALUES OF ALL SIX ATTRIBUTES (CRITERIA) FOR THE SELECTED ALTERNATIVE (PROJECT)

KRITERIJ x_j / CRITERION x_j	x_1	x_2	x_3	x_4	x_5	x_6
Vrijednost za investiciju A Value for the investment A	2,45	15,12%	8,38	9,04%	12,71%	5,18%
Kriterij (atribut) / Criterion (attribute)	1. NPV	2. IRR	3. PBP	4. ARR	5. OPM	6. ROE
Funkcija cilja / Objective function	max	max	max	max	max	max

Izvor: Izradio autor. / Source: Prepared by the author.

U tablici broj 3 navedene su vrijednosti svih šest atributa za odabranu alternativu (investicijski projekt A) za njenu početnu izvedbu koji su dobiveni na temelju ukupne vrijednosti investicije i budućih novčanih tokova u razdoblju od 10 godina. Također, definirana je i funkcija cilja za svaki atribut (kojom se maksimizira odnosno minimizira vrijednost svakog atributa).

Donositelj odluke je proizvoljno odredio točke korisnosti za navedene podatke (u_j). U Tablici 4 u stupcu U_1 navedena je korist od odgovarajuće neto sadašnje vrijednosti u iznosu od 8,00. U stupcu U_2 navedena je korist od vrijednosti interne stope povrata u iznosu od 7,00. U stupcu U_3 navedena je korist od vrijednosti vremena povrata u iznosu od 4,50. U stupcu U_4 navedena je korist od vrijednosti računovodstvene stope povrata u iznosu od 3,00. U stupcu U_5 navedena je korist od

total investment value and future cash flows over a period of 10 years. Also, objective function for each attribute (which maximizes or minimizes the value of each attribute) is defined.

The decision maker has arbitrarily determined the utility points for the given data (u_j). Table 4 column U_1 indicates the benefit of the corresponding net present value of 8.00. Column U_2 indicates a benefit of the internal rate of return of 7.00. Column U_3 indicates a benefit of payback time value of 4.50. Column U_4 indicates a benefit of the accounting rate of return value of 3.00. Column U_5 indicates a benefit of operating profit margin value of 5.50. Column U_6 indicates a benefit of return on equity value of 3.50. Finally, the score S for the investment project A is calculated with defined weights for all six criteria (w_j) as follows:

vrijednosti operativne profitne marže u iznosu od 5,50. U stupcu U_6 navedena je korist od vrijednosti povrata na vlasnički kapital u iznosu od 3,50. Na kraju je izračunat skor S za investicijski projekt A uz definirane težine za svih šest kriterija (w_j) kako slijedi:

$$S = w_1 \times u_1 + w_2 \times u_2 + w_3 \times u_3 + w_4 \times u_4 + w_5 \times u_5 + w_6 \times u_6$$

$$S = 0,30 \times 8,00 + 0,30 \times 7,00 + 0,20 \times 4,50 + 0,10 \times 3,00 + 0,05 \times 5,50 + 0,05 \times 3,50$$

$$S = 2,40 + 2,10 + 0,90 + 0,30 + 0,275 + 0,175 = 6,15$$

Nadalje, za odabranu alternativu investitor konstruira njemu prihvatljive izvedbe svakog atributa (kriterija) te time dobiva niz izvedbi odabrane alternative. Za odabranu alternativu (projekt) investitor je predložio deset izvedbi atributa u okolini početne izvedbe. Donositelj odluke je svakoj izvedbi svih korištenih atributa (*NPV*, *IRR*, *PBP*, *ARR*, *OPM* i *ROE*) pridružio točke korisnosti te procjenjuje da je u tom slučaju najbolja aproksimacija funkcije korisnosti opadajuća odnosno rastuća kvadratna funkcija (što primarno ovisi o funkciji cilja kojom se minimizira odnosno maksimizira vrijednost svakog atributa). Iz dobivenih točaka metodom najmanjih kvadrata (radi se o kvadratnoj zavisnosti koja se grafički prikazuje parabolom) dobivena je aproksimacija funkcije korisnosti ($U_1 - U_6$), kao što je to i prikazano na slikama broj 2, 3, 4, 5, 6 i 7.

Iz šest grafičkih prikaza odnosno aproksimacija funkcija korisnosti vidljivo je kako su sve monotone tj. konkavne te da imamo pet rastućih (*NPV*, *IRR*, *ARR*, *OPM* i *ROE*) odnosno samo jednu

$$S = w_1 \times u_1 + w_2 \times u_2 + w_3 \times u_3 + w_4 \times u_4 + w_5 \times u_5 + w_6 \times u_6$$

$$S = 0,30 \times 8,00 + 0,30 \times 7,00 + 0,20 \times 4,50 + 0,10 \times 3,00 + 0,05 \times 5,50 + 0,05 \times 3,50$$

$$S = 2,40 + 2,10 + 0,90 + 0,30 + 0,275 + 0,175 = 6,15$$

Furthermore, for the selected alternative, the investor creates for him acceptable performance of each attribute (criteria) and thus obtains a variety of alternatives. For the selected alternative (project), the investor has proposed ten performance attributes around the initial performance. The decision-maker assigned each performance of all the attributes used (*NPV*, *IRR*, *PBP*, *ARR*, *OPM*, and *ROE*) the utility points and estimates that in that case the best approximation of the utility function is decreasing or increasing quadratic function (which primarily depends on the objective function used for minimizing i.e. maximizing the value of each attribute). From the obtained points by the least squares method (quadratic dependence shown graphically with a parabola), the approximation of utility function ($U_1 - U_6$) is obtained as shown in Figures 2, 3, 4, 5, 6 and 7.

From six graphic representations, that is, approximations of utility functions, it is clear that they are all monotone i.e. concave and that we have five increasing (*NPV*, *IRR*, *ARR*, *OPM* and *ROE*) and only one decreasing utility function (*PBP*). The decision-maker assesses-appraises based on the results obtained for all ten attribute

TABLICA 4. SKOR FUNKCIJA KORISNOSTI ZA ODABRANU IZVEDBU

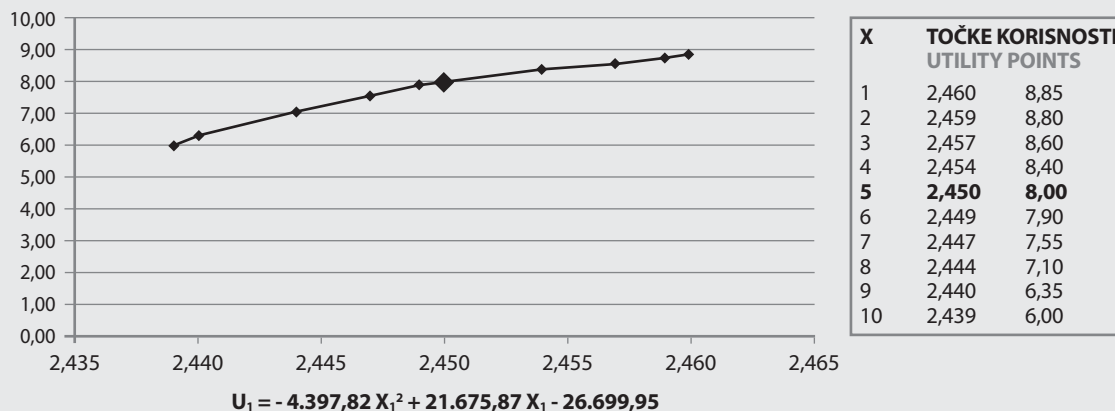
TABLE 4. SCORE OF UTILITY FUNCTION FOR SELECTED PERFORMANCE

INVESTICIJSKI PROJEKT INVESTMENT PROJECT	NPV	U_1	IRR	U_2	PBP	U_3	ARR	U_4	OPM	U_5	ROE	U_6	SKOR (S)
Investicija A Investment A	2,45	8,00	15,12%	7,00	8,38	4,50	9,04	3,00	12,71%	5,50	5,18%	3,50	6,15
Ponder (w) / Weight		0,30		0,30		0,20		0,10		0,05		0,05	1,0

Izvor: Izradio autor. / Source: Prepared by the author.

SLIKA 2. APROKSIMACIJA FUNKCIJE KORISNOSTI (U_1) ZA PRVI ATRIBUT (NPV)

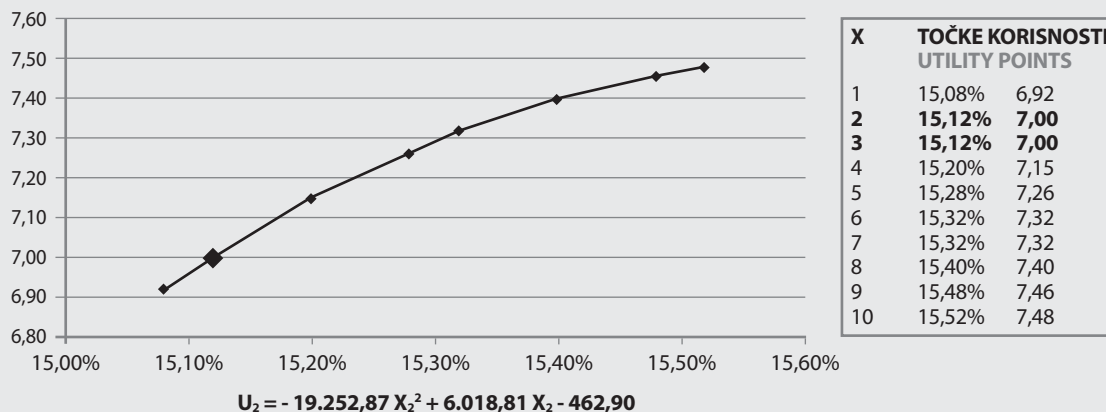
FIGURE 2. APPROXIMATION OF UTILITY FUNCTION (U_1) FOR THE FIRST ATTRIBUTE (NPV)



Izvor: Izradio autor. / Source: Prepared by the author.

SLIKA 3. APROKSIMACIJA FUNKCIJE KORISNOSTI (U_2) ZA DRUGI ATRIBUT (IRR)

FIGURE 3. APPROXIMATION OF UTILITY FUNCTION (U_2) FOR THE SECOND ATTRIBUTE (IRR)



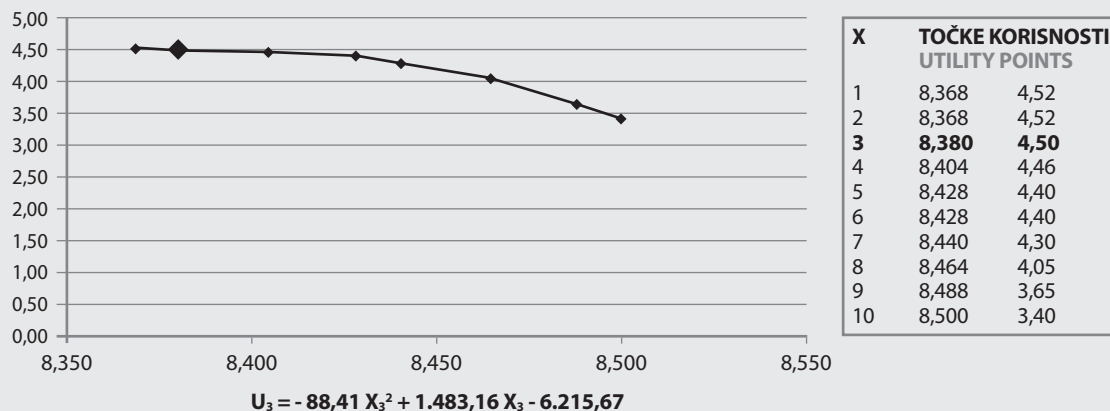
Izvor: Izradio autor. / Source: Prepared by the author.

opadajuću funkciju korisnosti (PBP). Donositelj odluke procjenjuje na temelju dobivenih rezultata za svih deset izvedbi atributa (kriterija) kako

performances (criteria) how in increasing approximations the function is rising ever slower or dropping ever faster compared to the initial

SLIKA 4. APROKSIMACIJA FUNKCIJE KORISNOSTI (U_3) ZA TREĆI ATRIBUT (PBP)

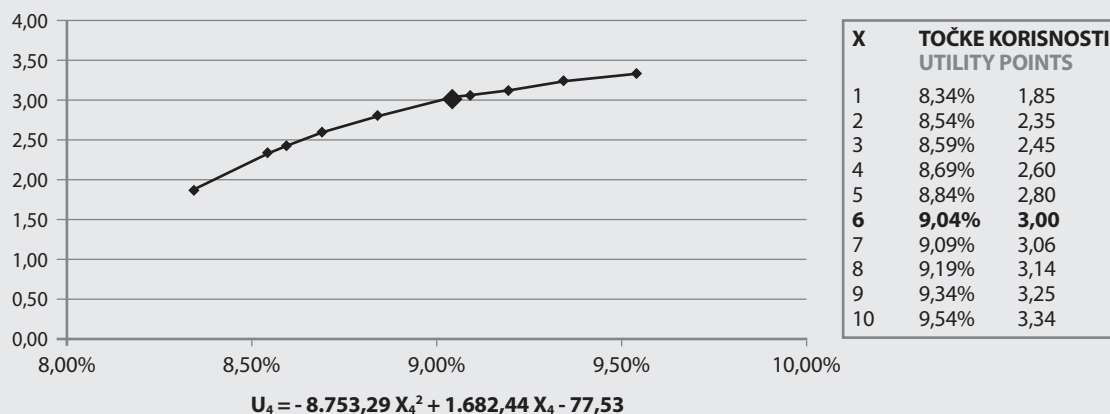
FIGURE 4. APPROXIMATION OF UTILITY FUNCTION (U_3) FOR THE THIRD ATTRIBUTE (PBP)



Izvor: Izradio autor. / Source: Prepared by the author.

SLIKA 5. APROKSIMACIJA FUNKCIJE KORISNOSTI (U_4) ZA ČETVRTI ATRIBUT (ARR)

FIGURE 5. APPROXIMATION OF UTILITY FUNCTION (U_4) FOR THE FORTH ATTRIBUTE (ARR)



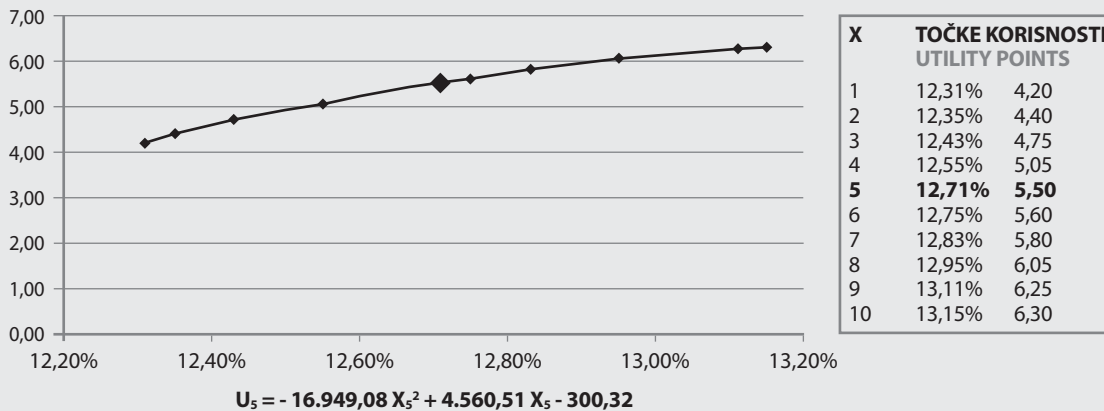
Izvor: Izradio autor. / Source: Prepared by the author.

korisnost kod rastućih aproksimacija funkcija raste sve sporije odnosno pada sve brže u odnosu na početnu izvedbu (isto se može zaključiti i za

performance (the same can be concluded for the only decreasing approximation of the function). In such a way obtained utility function results in

SLIKA 6. APROKSIMACIJA FUNKCIJE KORISNOSTI (U_5) ZA PETI ATRIBUT (OPM)

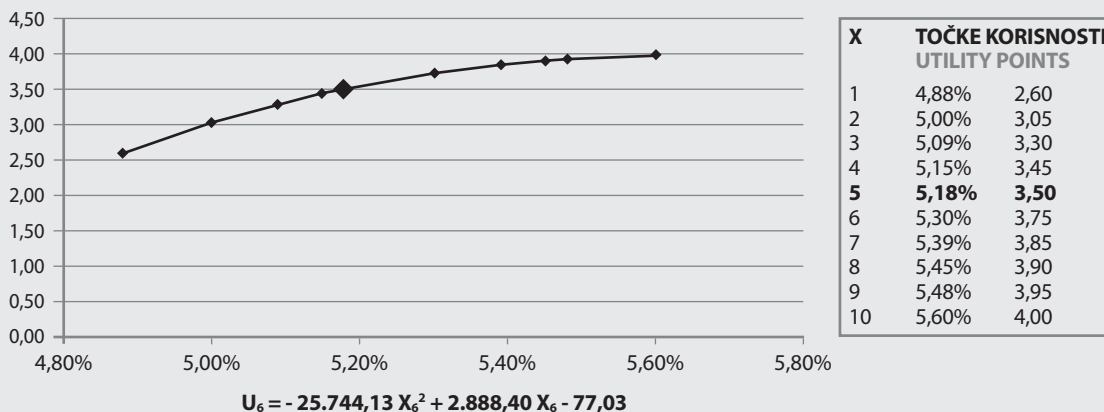
FIGURE 6. APPROXIMATION OF UTILITY FUNCTION (U_5) FOR THE FIFTH ATTRIBUTE (OPM)



Izvor: Izradio autor. / Source: Prepared by the author.

SLIKA 7. APROKSIMACIJA FUNKCIJE KORISNOSTI (U_6) ZA ŠESTI ATRIBUT (ROE)

FIGURE 7. APPROXIMATION OF UTILITY FUNCTION (U_6) FOR THE SIXTH ATTRIBUTE (ROE)



Izvor: Izradio autor. / Source: Prepared by the author.

jedinu opadajuću aproksimaciju funkcije). Ovako dobiveni rezultati funkcija korisnosti u odnosu na deset definiranih izvedbi atributa predstavljaju

relation to ten defined attribute performances represents a quantitative basis for negotiations between the decision-maker and the investor.

TABLICA 5. VRIJEDNOSTI SVIH DESET IZVEDBI ATRIBUTA (KRITERIJA) I REZULTATI OSTVARENIH KORISNOSTI ZA SVAKU APROKSIMACIJU FUNKCIJE KORISNOSTI
TABLE 5. VALUES OF ALL TEN ATTRIBUTE PERFORMANCES (CRITERIA) AND RESULTS OF REALIZED UTILITIES FOR EACH APPROXIMATION OF THE UTILITY FUNCTION

TOČKA IZVEDBE ATRIBUTA POINT OF ATTRIBUTE PERFORMANCE	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	X ₁₀
1. FUNKCIJA KORISNOSTI U₁ (NPV) / 1. UTILITY FUNCTION U₁ (NPV)										
• vrijednost kriterija (atributa) • value of criterion (attribute)	2,4600	2,4590	2,4570	2,4540	2,4500	2,4490	2,4470	2,4440	2,4400	2,4390
• ostvarena korisnost • realised utility	8,8186	8,7756	8,6633	8,4288	7,9931	7,8621	7,5739	7,0755	6,2879	6,0690
2. FUNKCIJA KORISNOSTI U₂ (IRR) / 2. UTILITY FUNCTION U₂ (IRR)										
• vrijednost kriterija (atributa) • value of criterion (attribute)	0,1512	0,1508	0,1512	0,1520	0,1532	0,1528	0,1532	0,1540	0,1552	0,1548
• ostvarena korisnost • realised utility	7,0010	6,9192	7,0010	7,1461	7,3175	7,2665	7,3175	7,4010	7,4800	7,4598
3. FUNKCIJA KORISNOSTI U₃ (PBP) / 3. UTILITY FUNCTION U₃ (PBP)										
• vrijednost kriterija (atributa) • value of criterion (attribute)	8,5000	8,4880	8,4640	8,4280	8,4400	8,4280	8,4040	8,3680	8,3800	8,3680
• ostvarena korisnost • realised utility	3,4229	3,6482	4,0226	4,3931	4,2951	4,3931	4,5128	4,5014	4,5307	4,5014
4. FUNKCIJA KORISNOSTI U₄ (ARR) / 4. UTILITY FUNCTION U₄ (ARR)										
• vrijednost kriterija (atributa) • value of criterion (attribute)	0,0954	0,0934	0,0919	0,0909	0,0904	0,0884	0,0869	0,0859	0,0854	0,0834
• ostvarena korisnost • realised utility	3,3129	3,2532	3,1626	3,0802	3,0325	2,7978	2,5758	2,4060	2,3145	1,9047
5. FUNKCIJA KORISNOSTI U₅ (OPM) / 5. UTILITY FUNCTION U₅ (OPM)										
• vrijednost kriterija (atributa) • value of criterion (attribute)	0,1231	0,1235	0,1243	0,1255	0,1271	0,1275	0,1283	0,1295	0,1311	0,1315
• ostvarena korisnost • realised utility	4,2396	4,3920	4,6804	5,0724	5,5190	5,6171	5,7971	6,0263	6,2561	6,2999
6. FUNKCIJA KORISNOSTI U₆ (ROE) / 6. UTILITY FUNCTION U₆ (ROE)										
• vrijednost kriterija (atributa) • value of criterion (attribute)	0,0488	0,0500	0,0509	0,0515	0,0518	0,0530	0,0539	0,0545	0,0548	0,0560
• ostvarena korisnost • realised utility	2,6163	3,0302	3,2919	3,4432	3,5119	3,7404	3,8631	3,9218	3,9442	3,9873

Izvor: Izradio autor. / Source: Prepared by the author.

kvantitativnu podlogu za pregovore između donositelja odluke i investitora.

U Tablici 5 prikazane su kreirane vrijednosti svih deset izvedbi atributa (x_i) prema redoslijedu pojavljivanja (od 1. do 10. točke redom) i pripadajući rezultati korisnosti (U_j) za svaku aproksimaciju funkcije korisnosti odabranih atributa (NPV, IRR, PBP, ARR, OPM i ROE).

U sljedećem koraku definirat će se globalna konkavna funkcija korisnosti koja će predstavljati ponderirani zbroj šest pojedinačnih tj. lokalnih konkavnih funkcija iz prethodnih grafičkih aproksimacija funkcija korisnosti (za svaki definirani atribut na temelju njegovih deset izvedbi).

Za dodijeljene težine (w), formuliramo sljedeći problem optimizacije (R_1):

$$\begin{aligned} \max \sum_{j=1}^6 w_j u_j(f_j(x)) \\ \sum_{i=1}^{10} x_i = 1 \\ x_i \geq 0, i = 1, \dots, 10 \end{aligned} \quad (15)$$

Kad se uvrste empirijske točke korisnosti u model (15) dobiva se sljedeći matematički model:

$$\begin{aligned} S = \max [0,3 * (-4397,82y_1^2 + 21675,87y_1 - 26699,95) \\ + 0,3 * (-19252,87y_2^2 + 6018,81y_2 - 462,9) \\ + 0,2 * (-88,41y_3^2 + 1483,16y_3 - 6215,67) \\ + 0,1 * (-8753,29y_4^2 + 1682,44y_4 - 77,53) \\ + 0,05 * (-16949,08y_5^2 + 4560,51y_5 - 300,32) \\ + 0,05 * (-25744,13y_6^2 + 2888,40y_6 - 77,03)] \end{aligned}$$

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1 \\ 2,4600x_1 + 2,4590x_2 + 2,4570x_3 + 2,4540x_4 \\ + 2,4500x_5 + 2,4490x_6 + 2,4470x_7 \\ + 2,4440x_8 + 2,4400x_9 + 2,4390x_{10} = y_1 \\ 0,1512x_1 + 0,1508x_2 + 0,1512x_3 + 0,1520x_4 \\ + 0,1532x_5 + 0,1528x_6 + 0,1532x_7 \\ + 0,1540x_8 + 0,1552x_9 + 0,1548x_{10} = y_2 \\ 8,5000x_1 + 8,4880x_2 + 8,4640x_3 + 8,4280x_4 \\ + 8,4400x_5 + 8,4280x_6 + 8,4040x_7 \\ + 8,3680x_8 + 8,3800x_9 + 8,3680x_{10} = y_3 \end{aligned}$$

Table 5 shows the created values of all ten attribute performances (x_i) according to the order of occurrence (from points 1 to 10 in sequence) and the corresponding utility results (U_j) for each approximation of the utility function of selected attributes (NPV, IRR, PBP, ARR OPM and ROE).

The next step will define a global concave utility function that will represent the weighted sum of six individual, that is, local concave functions from previous graphical approximations of utility functions (for each defined attribute based on its ten performances).

For the given weights (w), we formulate the following optimization problem (R_1):

$$\begin{aligned} \max \sum_{j=1}^6 w_j u_j(f_j(x)) \\ \sum_{i=1}^{10} x_i = 1 \\ x_i \geq 0, i = 1, \dots, 10 \end{aligned} \quad (15)$$

When empirical utility points are included in the model (15), the following mathematical model is obtained:

$$\begin{aligned} S = \max [0,3 * (-4397,82y_1^2 + 21675,87y_1 - 26699,95) \\ + 0,3 * (-19252,87y_2^2 + 6018,81y_2 - 462,9) \\ + 0,2 * (-88,41y_3^2 + 1483,16y_3 - 6215,67) \\ + 0,1 * (-8753,29y_4^2 + 1682,44y_4 - 77,53) \\ + 0,05 * (-16949,08y_5^2 + 4560,51y_5 - 300,32) \\ + 0,05 * (-25744,13y_6^2 + 2888,40y_6 - 77,03)] \end{aligned}$$

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1 \\ 2,4600x_1 + 2,4590x_2 + 2,4570x_3 + 2,4540x_4 \\ + 2,4500x_5 + 2,4490x_6 + 2,4470x_7 \\ + 2,4440x_8 + 2,4400x_9 + 2,4390x_{10} = y_1 \\ 0,1512x_1 + 0,1508x_2 + 0,1512x_3 + 0,1520x_4 \\ + 0,1532x_5 + 0,1528x_6 + 0,1532x_7 \\ + 0,1540x_8 + 0,1552x_9 + 0,1548x_{10} = y_2 \\ 8,5000x_1 + 8,4880x_2 + 8,4640x_3 + 8,4280x_4 \\ + 8,4400x_5 + 8,4280x_6 + 8,4040x_7 \\ + 8,3680x_8 + 8,3800x_9 + 8,3680x_{10} = y_3 \end{aligned}$$

$$\begin{aligned}
 &0,0954x_1 + 0,0934x_2 + 0,0919x_3 + 0,0909x_4 \\
 &\quad + 0,0904x_5 + 0,0884x_6 + 0,0869x_7 \\
 &\quad + 0,0859x_8 + 0,0854x_9 + 0,0834x_{10} = y_4 \\
 &0,1231x_1 + 0,1235x_2 + 0,1243x_3 + 0,1255x_4 \\
 &\quad + 0,1271x_5 + 0,1275x_6 + 0,1283x_7 \\
 &\quad + 0,1295x_8 + 0,1311x_9 + 0,1315x_{10} = y_5 \\
 &0,0488x_1 + 0,0500x_2 + 0,0509x_3 + 0,0515x_4 \\
 &\quad + 0,0518x_5 + 0,0530x_6 + 0,0539x_7 \\
 &\quad + 0,0545x_8 + 0,0548x_9 + 0,0560x_{10} = y_6 \\
 &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0
 \end{aligned} \tag{16}$$

Nakon sređivanja funkcije cilja u modelu (16), matematički model glasi:

$$\begin{aligned}
 S = \max & \left[-1319,35y_1^2 + 6502,76y_1 - 5775,86y_2^2 \right. \\
 & + 1805,64y_2 - 17,68y_3^2 + 296,63y_3 - 875,33y_4^2 \\
 & + 168,24y_4 - 847,45y_5^2 + 228,03y_5 - 1287,21y_6^2 \\
 & \left. + 144,42y_6 - 9418,61 \right] \\
 &x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1 \\
 &2,4600x_1 + 2,4590x_2 + 2,4570x_3 + 2,4540x_4 \\
 &\quad + 2,4500x_5 + 2,4490x_6 + 2,4470x_7 \\
 &\quad + 2,4440x_8 + 2,4400x_9 + 2,4390x_{10} = y_1 \\
 &0,1512x_1 + 0,1508x_2 + 0,1512x_3 + 0,1520x_4 \\
 &\quad + 0,1532x_5 + 0,1528x_6 + 0,1532x_7 \\
 &\quad + 0,1540x_8 + 0,1552x_9 + 0,1548x_{10} = y_2 \\
 &8,5000x_1 + 8,4880x_2 + 8,4640x_3 + 8,4280x_4 \\
 &\quad + 8,4400x_5 + 8,4280x_6 + 8,4040x_7 \\
 &\quad + 8,3680x_8 + 8,3800x_9 + 8,3680x_{10} = y_3 \\
 &0,0954x_1 + 0,0934x_2 + 0,0919x_3 + 0,0909x_4 \\
 &\quad + 0,0904x_5 + 0,0884x_6 + 0,0869x_7 \\
 &\quad + 0,0859x_8 + 0,0854x_9 + 0,0834x_{10} = y_4 \\
 &0,1231x_1 + 0,1235x_2 + 0,1243x_3 + 0,1255x_4 \\
 &\quad + 0,1271x_5 + 0,1275x_6 + 0,1283x_7 \\
 &\quad + 0,1295x_8 + 0,1311x_9 + 0,1315x_{10} = y_5 \\
 &0,0488x_1 + 0,0500x_2 + 0,0509x_3 + 0,0515x_4 \\
 &\quad + 0,0518x_5 + 0,0530x_6 + 0,0539x_7 \\
 &\quad + 0,0545x_8 + 0,0548x_9 + 0,0560x_{10} = y_6 \\
 &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0
 \end{aligned} \tag{17}$$

Konačno, dobiveni skor S za optimalnu izvedbu investicijskog projekta uz definirane težine za svih šest kriterija (w_i) jednak je:

$$\begin{aligned}
 &0,0954x_1 + 0,0934x_2 + 0,0919x_3 + 0,0909x_4 \\
 &\quad + 0,0904x_5 + 0,0884x_6 + 0,0869x_7 \\
 &\quad + 0,0859x_8 + 0,0854x_9 + 0,0834x_{10} = y_4 \\
 &0,1231x_1 + 0,1235x_2 + 0,1243x_3 + 0,1255x_4 \\
 &\quad + 0,1271x_5 + 0,1275x_6 + 0,1283x_7 \\
 &\quad + 0,1295x_8 + 0,1311x_9 + 0,1315x_{10} = y_5 \\
 &0,0488x_1 + 0,0500x_2 + 0,0509x_3 + 0,0515x_4 \\
 &\quad + 0,0518x_5 + 0,0530x_6 + 0,0539x_7 \\
 &\quad + 0,0545x_8 + 0,0548x_9 + 0,0560x_{10} = y_6 \\
 &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0
 \end{aligned} \tag{16}$$

After determining the objective function in the model (16), the mathematical model reads:

$$\begin{aligned}
 S = \max & \left[-1319,35y_1^2 + 6502,76y_1 - 5775,86y_2^2 \right. \\
 & + 1805,64y_2 - 17,68y_3^2 + 296,63y_3 - 875,33y_4^2 \\
 & + 168,24y_4 - 847,45y_5^2 + 228,03y_5 - 1287,21y_6^2 \\
 & \left. + 144,42y_6 - 9418,61 \right] \\
 &x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 1 \\
 &2,4600x_1 + 2,4590x_2 + 2,4570x_3 + 2,4540x_4 \\
 &\quad + 2,4500x_5 + 2,4490x_6 + 2,4470x_7 \\
 &\quad + 2,4440x_8 + 2,4400x_9 + 2,4390x_{10} = y_1 \\
 &0,1512x_1 + 0,1508x_2 + 0,1512x_3 + 0,1520x_4 \\
 &\quad + 0,1532x_5 + 0,1528x_6 + 0,1532x_7 \\
 &\quad + 0,1540x_8 + 0,1552x_9 + 0,1548x_{10} = y_2 \\
 &8,5000x_1 + 8,4880x_2 + 8,4640x_3 + 8,4280x_4 \\
 &\quad + 8,4400x_5 + 8,4280x_6 + 8,4040x_7 \\
 &\quad + 8,3680x_8 + 8,3800x_9 + 8,3680x_{10} = y_3 \\
 &0,0954x_1 + 0,0934x_2 + 0,0919x_3 + 0,0909x_4 \\
 &\quad + 0,0904x_5 + 0,0884x_6 + 0,0869x_7 \\
 &\quad + 0,0859x_8 + 0,0854x_9 + 0,0834x_{10} = y_4 \\
 &0,1231x_1 + 0,1235x_2 + 0,1243x_3 + 0,1255x_4 \\
 &\quad + 0,1271x_5 + 0,1275x_6 + 0,1283x_7 \\
 &\quad + 0,1295x_8 + 0,1311x_9 + 0,1315x_{10} = y_5 \\
 &0,0488x_1 + 0,0500x_2 + 0,0509x_3 + 0,0515x_4 \\
 &\quad + 0,0518x_5 + 0,0530x_6 + 0,0539x_7 \\
 &\quad + 0,0545x_8 + 0,0548x_9 + 0,0560x_{10} = y_6 \\
 &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \geq 0
 \end{aligned} \tag{17}$$

Finally, the obtained score S for the investment project optimal performance with defined weights for all the six criteria (x_i) equals:

$$S = w_1 \times u_1 + w_2 \times u_2 + w_3 \times u_3 + w_4 \times u_4 + w_5 \times u_5 + w_6 \times u_6$$

$$S = 0,30 \times 8,4288 + 0,30 \times 7,1461 + 0,20 \times 4,3931 + 0,10 \times 3,0802 + 0,05 \times 5,0724 + 0,05 \times 3,4432$$

$$S = 2,5287 + 2,1438 + 0,8786 + 0,3080 + 0,2536 + 0,1722 = 6,2849$$

Za navedeni problem optimizacije (R_1) dobili smo da je optimalni skor $S = 6,2849$ te možemo zaključiti da je najbolja njegova četvrta izvedba po redu, kao što je to vidljivo iz Tablice 6, što znači da je varijabla $x_4=1$.

Na temelju dobivenih rezultata donositelj odluke ima pravo na preferencijalnu odluku kojom utječe na izvedbu projekta. Tablica 6 je podloga za pregovaranje između donositelja odluke i investitora u koje donositelj odluke ulazi s optimalnom izvedbom. Osim navedenih izvedbi odabranog projekta, može se ići u daljnje pregovaranje i nove procjene. Kompromis između donositelja odluke i investitora postiže se s obzirom na ustupke koje su pregovarači voljni napraviti. Iz Tablice 6 vidi se kako donositelj odluke za odabranu alternativu poboljšava vrijednosti tri atributa (NPV, IRR i ARR), dok istovremeno za preostala tri manje značajna atributa (PBP, OPM i ROE) dozvoljava podbačaj, čime se potvrđuje ostvarena Pareto efikasnost modela.

Na temelju ostvarenog optimalnog skora funkcije korisnosti iz Tablice 6 koji je dobiven primjenom odgovarajućeg matematičkog modela (17), može

$$S = w_1 \times u_1 + w_2 \times u_2 + w_3 \times u_3 + w_4 \times u_4 + w_5 \times u_5 + w_6 \times u_6$$

$$S = 0,30 \times 8,4288 + 0,30 \times 7,1461 + 0,20 \times 4,3931 + 0,10 \times 3,0802 + 0,05 \times 5,0724 + 0,05 \times 3,4432$$

$$S = 2,5287 + 2,1438 + 0,8786 + 0,3080 + 0,2536 + 0,1722 = 6,2849$$

For the mentioned optimization problem (R_t) we have obtained the optimal score $S = 6,2849$ and we can conclude that the best performance is the fourth one as shown in Table 6, which means that the variable is $x_4=1$.

Based on the results obtained, the decision-maker is entitled to a preferential decision affecting the performance of the project. Table 6 serves as basis for negotiations between the decision-maker and the investor in which the decision-maker enters with the optimal performance. In addition to the abovementioned performance of the selected project, further negotiation and new estimates can be made. A compromise between decision-maker and investor is achieved concerning the concessions the negotiators are willing to make. Table 6 shows that for the selected alternative the decision-maker improves the value of three attributes (NPV, IRR and ARR), while at the same time for the remaining three less significant attributes (PBP, OPM and ROE) a subtraction is allowed, confirming the realized Pareto efficiency of the model.

Based on the realized optimal score of the utility function from the Table 6, which was obtained by

TABLICA 6. OPTIMALNI SKOR FUNKCIJE KORISNOSTI ZA ODABRANU INVESTICIJU
TABLE 6. OPTIMAL SCORE OF UTILITY FUNCTION FOR SELECTED INVESTMENT

INVESTICIJSKI PROJEKT INVESTMENT PROJECT	NPV	U ₁	IRR	U ₂	PBP	U ₃	ARR	U ₄	OPM	U ₅	ROE	U ₆	SKOR (S)
Investicija A Investment A	2,45	8,00	15,12%	7,00	8,38	4,50	9,04%	3,00	12,71%	5,50	5,18%	3,50	6,1500
Optimalna izvedba Optimal performance	2,4540	8,4288	15,20%	7,1461	8,4280	4,3931	9,09%	3,0802	12,55%	5,0724	5,15%	3,4432	6,2849
Ponder (w) / Weight		0,30		0,30		0,20		0,10		0,05		0,05	1,0

Izvor: Izradio autor. / Source: Prepared by the author.

se zaključiti kako je potvrđena osnovna hipoteza ovog rada. Zaključak je kako je moguće formirati matematički model višekriterijskog odlučivanja koji je primjenjiv u praksi. Također, dobivena optimalna izvedba investicijskog projekta predstavlja polaznu osnovu za daljnji postupak pregovaranja između donositelja odluke i investitora. U konačnici, ostvaren je i osnovni cilj ovog rada, kojim se potvrđuje kako je rješavanje različitih problema iz područja ekonomije moguće kroz adekvatnu primjenu matematičkog modela višekriterijskog odlučivanja, čime se potiče interdisciplinarni pristup u procesu donošenja odluka u poduzećima.

ZAKLJUČAK

Definirajući problem izbora izvedbe investicijskog projekta kao problema višekriterijskog programiranja određena je optimalna izvedba odabranog investicijskog projekta. Vrijednost globalne funkcije korisnosti (skor) poboljšana je sa 6,15 za proizvoljno odabranu izvedbu na 6,2849 za optimalnu izvedbu investicijskog projekta.

Poboljšanje konačnog rezultata je posljedica povećanja ili smanjenja vrijednosti svih atributa uz pretpostavku zadržavanja Pareto efikasnosti samog modela, što u ovom slučaju znači isti ograničeni kapitalni budžet za investiranje, identično trajanje životnog vijeka investicijskog projekta te nepromijenjenost njegovih budućih novčanih tijekova. Može se dakle zaključiti kako je ovakav matematički model postigao svoju svrhu jer je na temelju istih ograničenih resursa investitora donositelju odluke omogućio kreiranje optimalnog investicijskog projekta, što predstavlja startnu poziciju za njihove buduće poslovne aktivnosti.

Predloženi pristup optimalne izvedbe investicijskog projekta uključuje angažman donositelja odluke u svim fazama ovakvog postupka. Na taj način njegove preferencije postaju osnova za pregovore uz ustupke koje je pri tome spreman napraviti. Ovakva vrsta pristupa primjenjiva je na probleme gdje je

using the appropriate mathematical model (17), it can be concluded that the basic hypothesis of this paper is confirmed. The conclusion is that it is possible to form a mathematical model of multi-criteria decision-making, applicable in practice. Also, the obtained optimized performance of investment project represents a starting point for further negotiation between the decision-maker and the investor. Ultimately, the main purpose of this paper was to demonstrate that solving various problems in the field of economy is possible through proper application of mathematical model of multi-criteria decision-making, which promotes an interdisciplinary approach to the decision-making process in the companies.

CONCLUSION

Defining the problem of selection of an investment project performance as a multi-criteria programming problem, the optimal performance of the selected investment project has been determined. The value of the global utility function (score) was improved from 6.15 for the arbitrarily selected performance at 6.2849 for optimal performance of the investment project.

Improvement of final result is a consequence of increase or decrease of values of all attributes, assuming maintaining the Pareto efficiency of the model itself, which in this case means the same limited capital budget for investing, the same life span of an investment project and having its future cash flows unchanged. It can therefore be concluded that such a mathematical model achieved its purpose because it has enabled the decision-maker to create an optimal investment project based on the same limited investor's resources that represents a starting point for their future business activities.

The proposed approach to optimizing the performance of an investment project involves the engagement of decision-makers at all stages of such a process. This way, his preferences become the basis for negotiations with the concessions he is

postupak pregovaranja ključan za donošenje svih vrsta poslovnih odluka.

Prednosti korištenja ovakvog matematičkog koncepta u procesu donošenja odluka sagledavaju se kroz daljnje poticanje razvoja znanstvene misli o potrebi povezivanja interdisciplinarnog područja koje obuhvaća poslovne financije, upravljačko računovodstvo i operacijska istraživanja te kroz razvijanje znanstvene misli o potrebi donošenja poslovnih odluka vezanih za investicijske projekte na temelju formuliranog matematičkog modela višekriterijskog odlučivanja. Nedostaci koji se pri tome mogu pojaviti vezani su uz problematiku kvalitetnog interpretiranja dobivenih numeričkih rezultata, što može biti posljedica neadekvatnog znanja donositelja odluka odnosno nedovoljne kvantitativne podrške u procesu odlučivanja. Sukladno tome, poželjno je za neka nova istraživanja u okviru ovog interdisciplinarnog područja uključiti sve relevantne poznavatelje ove materije, a sve u cilju snažnijeg povezivanja znanosti i gospodarstva.

willing to make. This kind of approach is applicable to the problems where negotiating procedure is the key to making all kinds of business decisions.

The advantages of using such a mathematical concept in the decision-making process are seen through further encouraging of development of scientific thought about the need of linking an interdisciplinary area that encompasses corporate finance, managerial accounting and operational research and about the need of making business decisions related to investment projects based on formulated mathematical multi-criteria decision model through scientific thinking development. The disadvantages that may arise are related to the issue of quality interpretation of the obtained numerical results that may be due to decision-makers' inadequate knowledge or insufficient quantitative support in the decision-making process. Accordingly, it is desirable for some new researches within this interdisciplinary area to include all the relevant experts in this subject matter, all with the goal of strengthening the link between science and economy.

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