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Modeling Latent Heat Fluxes of Water in Logs during their Freezing

Modeliranje latentnih toplinskih tokova vode u trupcima tijekom zamrzavanja

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ABSTRACT • *This paper suggests a methodology for mathematical modeling and research of two interconnected problems: 2D non-stationary temperature distribution in logs subjected to freezing and change in the latent heat fluxes of the free and bound water in logs during the freezing process. For the purpose of this methodology, a 2-dimensional mathematical model has been created, solved, and verified for the transient non-linear heat conduction in logs during their freezing at convective boundary conditions. The model includes a mathematical description of the specific latent heat fluxes, $q_{LHv\text{-}fw}$ and $q_{LHv\text{-}bw}$, formed by the freezing of the free and bound water in the logs, respectively. The paper presents solutions of the model with explicit form of the finite-difference method in the calculation environment of Visual Fortran Professional and its verification in accordance with our own experimental studies. The paper presents the results of simulation analysis of 2D non-stationary temperature distribution in the longitudinal section of pine log with a diameter of 0.24 m, length of 0.48 m, and moisture content above the hygroscopic range during its 30-hour freezing in a freezer at the temperature of the processing air medium of approximately –30 °C. The change in the latent heat fluxes $q_{LHv\text{-}fw}$ and $q_{LHv\text{-}bw}$ during the log freezing is presented, visualized, and analyzed.*

Keywords: *2D mathematical model, pine logs, freezing, latent heat sources, free water, bound water*

SAŽETAK • *Predložena je metodologija za matematičko modeliranje i istraživanje dvaju međusobno povezanih problema: dvodimenzionalne nestacionarne raspodjele temperature u smrznutim trupcima i promjene u protoku latentne topline slobodne i vezane vode u trupcima tijekom zamrzavanja. Za realizaciju metodologije izrađen je i provjeren dvodimenzionalni matematički model za nelinearno provođenje topline u trupcima tijekom zamrzavanja pri konvekcijskim rubnim uvjetima. Model obuhvaća matematički opis specifičnih latentnih tokova topline $q_{LHv\text{-}fw}$ i $q_{LHv\text{-}bw}$, koji nastaju zamrzavanjem slobodne i vezane vode u trupcima. U radu su prikazana rješenja modela s eksplicitnim oblikom metode konačnih razlika u računalnom sučelju Visual Fortran Professional i njegova provjera prema vlastitim eksperimentalnim istraživanjima autora. Predstavljeni su rezultati simulacijskog istraživanja dvodimenzionalne nestacionarne raspodjele temperature u uzdužnom presjeku borovih trupaca promjera 0,24 m, dužine 0,48 m i sadržaja vode iznad higroskopskog raspona tijekom 30 sati smrzavanja u zamrzivaču, pri temperaturi zraka oko –30 °C. Također su prezentirane i analizirane promjene protoka latentne topline $q_{LHv\text{-}fw}$ i $q_{LHv\text{-}bw}$ tijekom zamrzavanja trupaca.*

Ključne riječi: *dvodimenzionalni matematički model, borovi trupci, zamrzavanje, latentni izvori topline, slobodna voda, vezana voda*

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1 INTRODUCTION
1. UVOD

It is known that the duration and energy consumption of the thermal treatment of frozen logs in the winter, aimed at their plasticizing for the production of veneer, depend on the degree of the logs' icing (Chudinov, 1966, 1968, 1984; Kollmann and Côté, 1984; Shubin, 1990; Požgaj *et al*., 1997; Trebula and Klement, 2002; Videlov, 2003; Pervan, 2009; Deliiski and Dzurenda, 2010; Deliiski, 2011, 2013b). In the available specialized literature, there are limited reports about the temperature distribution in frozen logs subjected to defrosting (Steinhagen, 1986, 1991; Steinhagen and Lee, 1988; Khattabi and Steinhagen, 1992, 1993, 1995; Deliiski, 2004, 2009, 2011; Deliiski and Dzurenda, 2010; Deliiski *et al*., 2015; Hadjiski and Deliiski, 2015, 2016) and there is very little information about research of the temperature distribution in logs during their freezing (Deliiski and Tumbarkova, 2016, 2017, 2018). That is why the modeling and the multi-parameter study of the freezing process of logs are of considerable scientific and practical interest.

For different engineering and technological calculations, it is necessary to be able to determine the non-stationary temperature field in logs depending on the temperature of the gaseous or liquid medium and on the duration of their staying in this medium. Such calculations are carried out using mathematical models, which describe adequately the complex processes of the freezing of both free and bound water in the wood. The specific latent heat fluxes are an important part of these models, and they are formed by the freezing of the free and bound water in the wood. These fluxes are opposed to the external cooling heat flux whose purpose is to freeze the logs. The joint action of the fluxes affects the duration and energy consumption of the logs' freezing process. In the available literature for hydrothermal treatment of frozen wood materials, there is no information about quantitative determination of the specific latent heat fluxes of water in logs during their freezing.

The aim of the present paper is to suggest a methodology for mathematical modeling and research of two interconnected problems: 2D non-stationary temperature distribution in logs subjected to freezing and change in the specific latent heat fluxes of the free and bound water in the logs during the freezing process.

2 MATERIALS AND METHODS
2. MATERIJALI I METODE

2.1 Mathematical model of 2D temperature distribution in logs during their freezing
2.1. Matematički model dvodimenzionalne raspodjele temperature u trupcima tijekom zamrzavanja

The mechanism of the temperature distribution in logs during their freezing can be described by the equation of heat conduction. When the length of the logs does not exceed their diameter by at least 3 ÷ 4 times, then the heat transfer through the frontal sides of the logs cannot be neglected, because it influences the change in temperature of their cross sections, which are equally distant from the frontal sides (Chudinov, 1966, 1968; Shubin, 1990; Deliiski, 2011). In such cases, for the calculation of the change in the temperature in longitudinal sections of the logs (i.e. along the coordinates *r* and *z* of these sections) during their freezing in air processing medium, the following 2D model can be used (Deliiski and Tumbarkova, 2017):

$$c_{\mathrm{we}}\rho_{\mathrm{w}}\frac{\partial T(r,z,\tau)}{\partial\tau}=\lambda_{\mathrm{wr}}\left[\frac{\partial^2 T(r,z,\tau)}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T(r,z,\tau)}{\partial r}\right]+$$
$$+\frac{\partial\lambda_{\mathrm{wr}}}{\partial T}\left[\frac{\partial T(r,z,\tau)}{\partial r}\right]^2+\lambda_{\mathrm{wp}}\frac{\partial^2 T(r,z,\tau)}{\partial z^2}+$$
$$+\frac{\partial\lambda_{\mathrm{wp}}}{\partial T}\left[\frac{\partial T(r,z,\tau)}{\partial z}\right]^2+q_{\mathrm{v}} \qquad (1)$$

with an initial condition

$$T(r,z,0)=T_{\mathrm{w0}} \qquad (2)$$

and boundary conditions for convective heat transfer:

- along the radial coordinate *r* on the logs' frontal surface during the freezing process:

$$\frac{\partial T(r,0,\tau)}{\partial r}=-\frac{\alpha_{\mathrm{wp\text{-}fr}}(r,0,\tau)}{\lambda_{\mathrm{wp}}(r,0,\tau)}\left[T(r,0,\tau)-T_{\mathrm{m\text{-}fr}}(\tau)\right], \qquad (3)$$

- along the longitudinal coordinate *z* on the logs' cylindrical surface during the freezing process:

$$\frac{\partial T(0,z,\tau)}{\partial z}=-\frac{\alpha_{\mathrm{wr\text{-}fr}}(0,z,\tau)}{\lambda_{\mathrm{wr}}(0,z,\tau)}\left[T(0,z,\tau)-T_{\mathrm{m\text{-}fr}}(\tau)\right]. \qquad (4)$$

Equations 1 to 4 represent a common form of a mathematical model of 2D temperature distribution in logs during their freezing. Mathematical descriptions of individual variables in the model needed for its solution are discussed below.

2.2 Mathematical description of the latent heat fluxes in logs during their freezing
2.2. Matematički opis toka latentne topline u trupcima tijekom zamrzavanja

In Eq. 1, the specific heat flux in the logs' volume, q_{v}, reflects the influence of the latent heat of water in 1 m^3 of wood on the freezing process of logs. As pointed out above, in the available literature for hydrothermal treatment of frozen wood materials, no information can be found on the quantitative determination of the specific heat flux q_{v}. That is why a methodology for the determination of q_{v} during the freezing of logs, which has already been applied for the determination of the specific heat flux, q_{Mv}, during the process of solidification of melted metal (Salcudean and Abdullah, 1988; Dantzig, 1989; Hu and Argyropoulos, 1996; Mihailov and Petkov, 2010), is used in (Deliiski and Tumbarkova, 2018). According to this methodology, the heat source q_{Mv} is equal to

$$q_{\mathrm{Mv}}=\rho_{\mathrm{MS}}\cdot L_{\mathrm{crM}}\frac{\partial\psi_{\mathrm{MS}}}{\partial\tau} \qquad (5)$$

In solving the models of metal solidification, the current value of metal density has been calculated according to the following equation:

$$\rho_M = \Psi_{MS} \cdot \rho_{MS} + (1 - \Psi_{MS}) \cdot \rho_{ML} \quad (6)$$

The following details of the solidification process have been reflected in Eq. 6: at the beginning of that process, the metal is in a liquid state and consequently $\Psi_{MS} = 0$ and $\rho_M = \rho_{ML}$; at the end of the process, the metal is in a solid state and then $\Psi_{MS} = 1$ and $\rho_M = \rho_{MS}$.

The specifics of the wood freezing process differ significantly from those of the metal solidification process. During wood cooling at temperatures $t > 0$ °C, both free and bound water are in a liquid state. Our experiments have proven that, in the range from 0 °C to -1 °C, the crystallization of free water occurs and the relative icing degree of the wood, which is formed from the freezing of that water, $\Psi_{ice\text{-}fw}$, changes from $\Psi_{ice\text{-}fw} = 0$ to $\Psi_{ice\text{-}fw} = 1$ (Deliiski and Tumbarkova, 2016, 2017).

When the wood temperature decreases below -1 °C, the gradual freezing of the bound water begins. In his doctoral thesis, Chudinov (1966) determined that even at an extremely low climate temperature on the earth, a certain portion (approximately 0.12 kg/kg) of the bound water in the wood remains in a liquid state. This means that the relative icing degree of the wood, which is only caused by the freezing of bound water, $\Psi_{ice\text{-}fw}$, changes from $\Psi_{ice\text{-}fw} = 0$ to $\Psi_{ice\text{-}fw} < 1$ (refer to p. 3.4 below).

It is known that the ice formed freely at atmospheric pressure has a density less than 8 % compared to that of liquid water (https://bg.wikipedia.org/wiki/%D0%9B%D0% B5%D0%B4). It is also known that the lignin sharply reduces the elasticity of the cell walls and, therefore, the conditions for ice formation by both free and bound water deteriorate. As a consequence, the ice in the wood has crystal lattice modifications, which are denser in comparison to the crystal lattice of the ice formed freely.

In the specialized literature on wood thermal treatment, it has been determined and accepted that the density of the wood that contains ice is practically equal to the density of the wood containing liquid water. This allows us to use the equality of the density ρ_{wS} and ρ_W in the mathematical description of the latent heat flux in logs subjected to freezing, q_{LHv}, i.e.

$$\rho_{wS} = \rho_W \quad (7)$$

Where ρ_{wS} is the density of the wood containing frozen water, kg/m^3, ρ_W – density of the non-frozen wood above the hygroscopic range, which can be calculated (in kg/m^3) according to the below equation, which is widely accepted in the specialized literature (Chudinov, 1968; Shubin, 1990; Požgaj *et al.*, 1997; Pervan, 2009; Deliiski *et al.*, 2015; Hrčka, 2017)

$$\rho_W = \rho_b \cdot (1+u) \quad (8)$$

ρ_b – basic density of the wood based on dry mass divided by green volume, kg/m^3;
u – wood moisture content, kg/kg.

Thus, in accordance with Eq. 5, it could be written for the latent heat flux of the water in the wood, q_v, which participates in (1), that

$$q_{\text{Lat-v}} = \rho_{wS} \cdot L_{\text{cr-ice}} \frac{\partial \Psi_{\text{ice}}}{\partial \tau} = \rho_w \cdot L_{\text{cr-ice}} \frac{\partial \psi_{\text{ice}}}{\partial \tau} \quad (9)$$

Where $L_{cr\text{-}ice}$ is the latent heat of the water, also known as the "heat of crystallization". This heat is released in the wood during the water freezing and it is equal to $L_{cr\text{-}ice} = 3.34 \cdot 10^5$ J/kg for both free and bound water (Chudinov, 1966, 1968; Efimov, 1985; Rogers and Yau, 1989; Pahi, 2010; Deliiski and Tumbakova, 2016, 2018); $\Psi_{ice\text{-}fw}$ – relative icing degree of logs subjected to freezing.

Analogously to eq. (9), during the freezing of the free water in the log, the consequently formed specific latent heat flux in its volume, $q_{LHv\text{-}fw}$, is equal to

$$q_{\text{LHv-fw}} = \rho_w \cdot L_{\text{cr-ice}} \frac{\partial \psi_{\text{ice-fw}}}{\partial \tau} \quad (10)$$

Where $\Psi_{ice\text{-}fw}$ is the relative icing degree of the log, which results from the freezing of the free water in it. An approach and an algorithm for the calculation of $\Psi_{ice\text{-}fw}$ is given in Deliiski and Tumbarkova (2017).

Using Eq. 10, it is possible to calculate the specific latent heat flux $q_{LHv\text{-}fw}$ for the frozen state of the whole amount of the free water in 1 m^3 of the log. The current value of $q_{LHv\text{-}fw}$ for each moment $n \cdot \Delta\tau$ of the freezing process of logs can be calculated according to the following equation, simultaneously with the solving of the mathematical model (1) ÷ (4):

$$q^n_{\text{LHv-fw}} = \rho_w \cdot L_{\text{cr-ice}} \frac{\partial \psi^n_{\text{ice-fw}}}{\partial \tau} \cdot \frac{N^n_{\text{ice-fw}}}{N_{\text{total}}} \quad (11)$$

Where N^n_{ice-fw} is the current number of knots of the calculation mesh for solving of the model (1) to (4), in which the temperature has already been decreased below 273.15 K (i.e. below 0 °C) and then temperature conditions for crystallization of the free water have been arisen separately for each knot; N_{total} – total number of knots of the calculation mesh; n – time level: n = 0, 1, 2; $\Delta\tau$ – interval between time levels, i.e. step along the time coordinate, used for solving the model.

In accordance with Eq. 10, during the freezing of the bound water in the log, the consequently formed specific latent heat flux in its volume, $q_{LHv\text{-}fw}$ is equal to

$$q_{\text{LHv-bw}} = \rho_w \cdot L_{\text{cr-ice}} \frac{\partial \psi_{\text{ice-bw}}}{\partial \tau} \quad (12)$$

Where $\Psi_{ice\text{-}bw}$ is the relative icing degree of the log, which results from the freezing of certain portion of the bound water in it, depending on $T < 271.15$ K. An approach and an algorithm for the calculation of $\Psi_{ice\text{-}bw}$ are given in Deliiski and Tumbarkova (2017).

With the help of Eq. 12, it is possible to calculate the heat flux, $q_{LHv\text{-}bw}$, which corresponds to the whole amount of the frozen bound water in 1 m^3, which is in a frozen state in the log at the end of its freezing. The current value of $q_{LHv\text{-}bw}$ for each moment $n \cdot \Delta\tau$ of the freezing process of logs can be calculated according to the following equation, simultaneously with solving the model (1) ÷ (4):

$$q^n_{\text{LHv-bw}} = \rho_w \cdot L_{\text{cr-ice}} \frac{\partial \psi^n_{\text{ice-bw}}}{\partial \tau} \cdot \frac{N^n_{\text{ice-bw}}}{N_{\text{total}}} \quad (13)$$

Where N^n_{ice-bw} is the current value of the knots of the calculation mesh for solving the model (1) to (4), in which the temperature has already been decreased

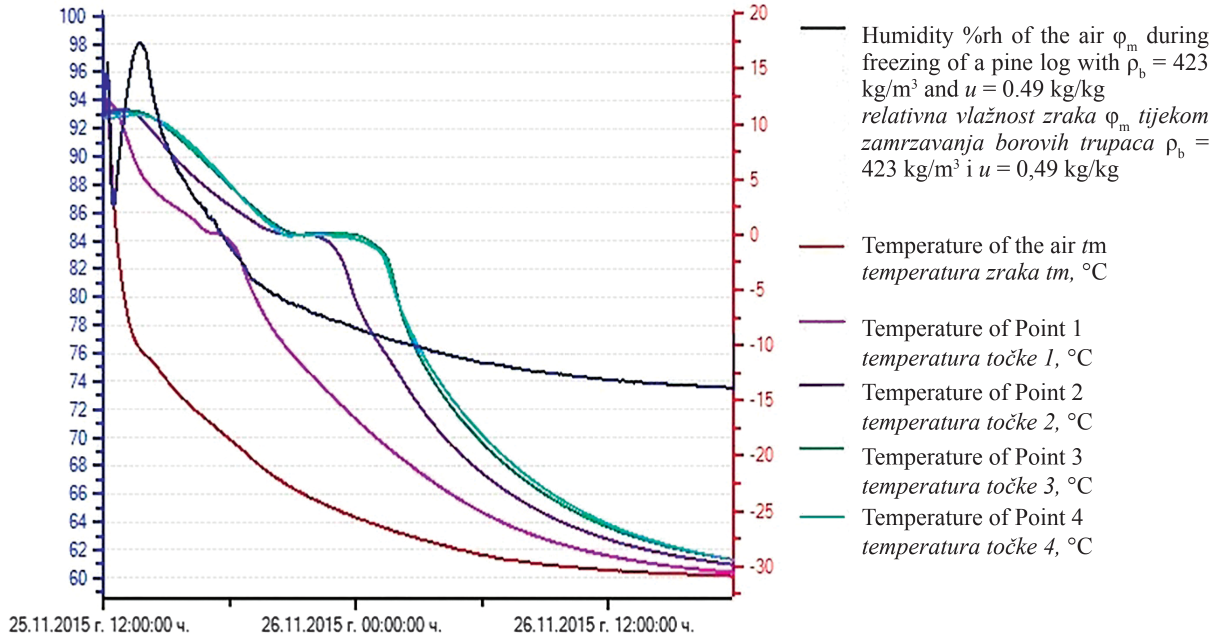

Figure 1 Experimentally determined change in t_m, φ_m, and t at four points of the pine log during its 30 h freezing in a freezer
Slika 1. Eksperimentalno utvrđene promjene parametara t_m, φ_m i t u četiri točke istraživanih borovih trupaca tijekom 30 sati zamrzavanja

below 272.15 K (i.e. below -1 ˚C) and then conditions for the crystallization of the bound water have been arisen separately for each knot.

2.3 Experimental research of the freezing process of pine logs
2.3. Istraživanje procesa zamrzavanja borovih trupaca

In order to verify the above suggested mathematical model, we needed experimentally obtained data about the change in the temperature field in logs during their freezing. That is why we carried out such experiments. The logs subjected to experimental research had the following characteristics: diameter D = 240 mm, length L = 480 mm, and moisture content above the hygroscopic range. They were produced from the sapwood of a freshly felled pine trunk (*Pinus sylvestris* L.). Before the experiments, 4 holes with diameters of 6 mm and different lengths were drilled in each log parallel to its axis until reaching the characteristic points of the log (Deliiski and Tumbarkova, 2016). Sensors with long metal casings were placed in these 4 holes for the measurement of wood temperature during the experiments.

The coordinates of the log points are as follows: Point 1: along the radius r = 30 mm and along the length z = 120 mm; Point 2: with r = 60 mm and z = 120 mm; Point 3: with r = 90 mm and z = 180 mm and Point 4: with r = 120 mm and z = 240 mm (center of the log). Thanks to these points coordinates, the impact of the heat fluxes on the temperature distribution in logs during their freezing can be simultaneously determined in radial and longitudinal directions.

For log freezing in accordance with the methodology suggested by the authors (Deliiski and Tumbarkova, 2016), a horizontal freezer was used with adjustable temperature range from -1 °C to -30 °C. Each log equipped with temperature sensors was horizontally placed on a special stand in the open freezer at room temperature. After closing the freezer, it was switched on at full power and the temperature of its freezing air medium, t_m, was gradually lowered until reaching approximately -30 °C.

The automatic measurement and recording of temperature and humidity of the air processing medium in the freezer as well as temperature for four points of the logs during the experiments was carried out with the help of Data Logger type HygroLog NT3 produced by the Swiss firm ROTRONIC AG (http:/www.rotronic.com).

Figure 1 presents, as an example, the change in temperature of the processing air medium, $t_{m,}$ and in its humidity, φ_m, as well as the temperature for the four characteristic points of a pine log with basic density of 423 kg/m^3 and moisture content of 0.49 kg/kg during its 30 h freezing. The left coordinate axis in Figure 1 is graduated at % of φ_m, and the left one is graduated at °C of t.

All data were recorded automatically by Data Logger at 5 min intervals. The Data Logger has HW4 software for the graphical presentation of the experimentally obtained data.

3 RESULTS AND DISCUSSION
3. REZULTATI I RASPRAVA

The mathematical descriptions of the latent heat fluxes $q_{LHv\text{-}fw}$ and $q_{LHv\text{-}bw}$ created above and mathematical descriptions of the thermo-physical characteristics of frozen and non-frozen wood suggested earlier (Deliiski, 2004, 2009, 2011, 2013a) are introduced in the mathematical model of the freezing process of logs, which consists of Eq. 1-13. Our study has shown that, for the calculation of the radial and longitudinal transfer coefficients of the logs, α_{wr-fr} and α_{wp-fr}, respectively, in the boundary conditions (3) and (4) of the model, the following equations are most suitable (Telegin *et al.,* 2002):

- in the radial direction on the cylindrical surface of the horizontally mounted logs:

$$\alpha_{\mathrm{wr-fr}} = 2.56\left[T(0,z,\tau) - T_{\mathrm{m-fr}}(\tau)\right]^{E_{\mathrm{fr}}} \quad (14)$$

- in the longitudinal direction on the frontal surface of the logs:

$$\alpha_{\mathrm{wp-fr}} = 1.123\left[T(r,0,\tau) - T_{\mathrm{m-fr}}(\tau)\right]^{E_{\mathrm{fr}}} \quad (15)$$

Where E_{fr} is an exponent, whose values are determined during solving and verification of the model through minimization of the root mean square error (*RMSE*) between the results calculated by the model and experimentally obtained results on the temperature change fields in logs subjected to freezing.

For the numerical solution of the mathematical model, a software program was prepared in the calculation environment of Visual FORTRAN Professional. With the help of the program, computations were made for the determination of the 2D non-stationary change of t in the longitudinal sections of a pine log, whose experimentally determined temperature distribution is shown in Figure 1. Simultaneously, calculations of the change in heat fluxes q_{LHv-fw} and q_{LHv-bw} during the freezing were carried out.

The model was solved with the help of explicit schemes of the finite difference method in a way analogous to the one used and described in (Deliiski, 2009, 2011, 2013c; Deliiski *et al.,* 2015) for the solution of a model of the heating process of prismatic and cylindrical wood materials. For this purpose, the calculation mesh was built on ¼ of the longitudinal section of the log due to the fact that this ¼ was mirror symmetrical to the remaining ¾ of the same section.

The model was solved with step $\Delta r = \Delta z = 0.006$ m along the coordinates r and z and with the same initial and boundary conditions, as used during the experimental research. The interval between the time levels, $\Delta\tau$, (i.e. the step along the time coordinate), was determined by the software according to the condition of stability for explicit schemes of the finite difference method (Deliiski, 2013b) and in our case it was 6 s.

The mathematical description of the thermophysical characteristics of pine wood with fiber saturation point $u_{fsp}^{293.15} = 0.30$ kg·kg^{-1} was used for solving the model (Nikolov and Videlov, 1987; Deliiski and Dzurenda, 2010).

3.1 Mathematical description of the freezer temperature during log freezing
3.1. Matematički model temperature u zamrzivaču tijekom zamrzavanja trupaca

The curvilinear change in the freezing air medium temperature, T_{m-fr}, which is shown in Figure 1, with high accuracy (correlation 0.99 for the studied log and Root mean square error σ = 0.84 ˚C) was approximated with the help of the software package Table Curve 2D available on Internet (http://www.sigmaplot.co.uk/products/tablecurve2d/tablecurve2d.php) using the Eq. 16

$$T_{\mathrm{m-fr}} = \frac{a_{\mathrm{fr}} + c_{\mathrm{fr}}\tau^{0.5}}{1 + b_{\mathrm{fr}}\tau^{0.5} + d_{\mathrm{fr}}\tau} \quad (16)$$

whose coefficients are: a_{fr} = 285.7898447, b_{fr} = 0.0015713223, c_{fr} = 0.123970584, and d_{fr} = -1.5621·10^{-6}.

Equation 16 was introduced in the software for solving Eq. 3 and 4 of the model.

3.2 Computation of 2D temperature distribution in the log during its freezing
3.2. Izračun 2D raspodjele temperature u trupcu tijekom zamrzavanja

The mathematical model of the freezing process of logs was solved with different values of the exponent E_{fr} in Eq. 14 and 15. The temperature change at the four characteristic points of the longitudinal log section calculated by the model with each of the used values of E_{fr} during freezing was compared mathematically with the corresponding experimentally determined change of t in the same points at 5 min interval. The aim of this comparison was to find that the value of E_{fr} can ensure the best qualitative and quantitative compliance between the calculated and experimentally determined temperature field in the log longitudinal section.

The minimum value of *RMSE*, σ_{avg}, was used as a criterion of the best compliance between the compared values of the temperature for all four characteristic points.

A software program in MS Excel 2010 was prepared for the determination of *RMSE*. With the help of *RMSE,* a total of 1200 temperature-time points were covered simultaneously during 30 h of log freezing. During the simulations, the same initial and boundary conditions were used as during the experiment. A minimum value of *RMSE*, equal to σ_{avg} = 1.69 °C was obtained for all 4 characteristic points of the log.

Figure 2 presents the calculated change in t_{m-fr}, log surface temperature t_s, and t of 4 characteristic points in the pine log.

The comparison between analog curves in Figure 1 and 2 show good qualitative and quantitative conformity between the calculated and experimentally determined changes of complex temperature field of the logs during their freezing. During extensive simulations with the mathematical model, we observed good compliance between the computed and experimentally established temperature fields during the freezing of logs of different wood species and different moisture content. The overall *RMSE* for the four characteristic points in the logs does not exceed 5 % of the temperature ranges between the initial and final temperatures of the logs subjected to freezing. Generally, the larger the differences in moisture content in the logs volume, the larger is the obtained value of *RMSE*.

The curves in Figure 1 and 2, placed on the characteristic points of the log inner layers, show the specific almost horizontal sections with temperature retention for a long period of time in the range from 0 °C to -1 °C, while complete freezing of the whole amount of the free water in the log is occurring in these points. Our experiments and simulations showed that the further the point from the log surface and the larger the amount of the free water in the wood, the greater is the extension of these

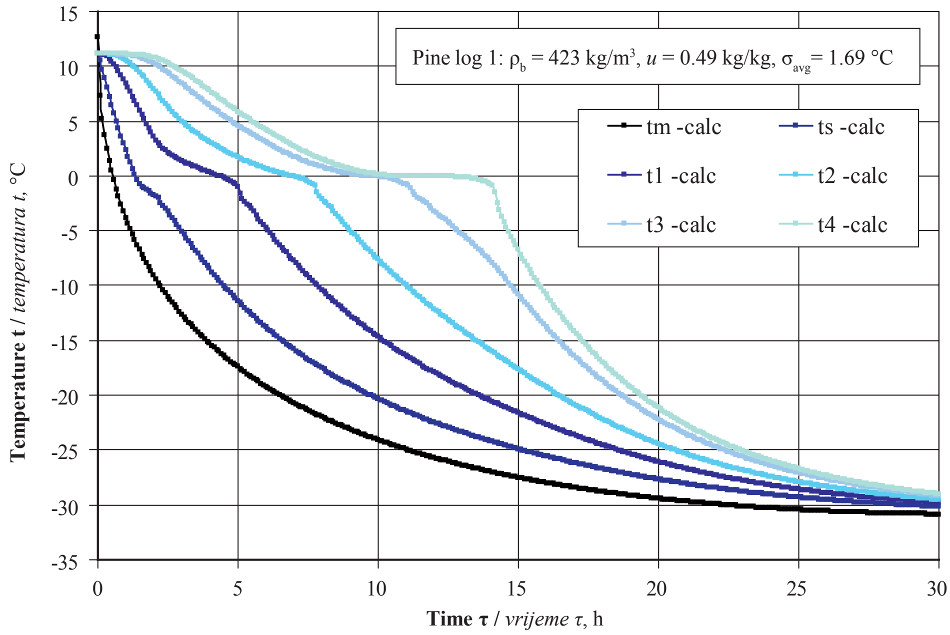

Figure 2 Calculated change in t_m, t_s, and t for four points of the pine log during 30 h of freezing
Slika 2. Izračunana promjena t_m, t_s, i t u četiri točke ispitivanoga borova trupca tijekom 30 sati zamrzavanja

sections with temperature retention. The reason of such a long retention of the wood temperature is the very low temperature conductivity of the wood due to freezing of the free water (Deliiski *et al.,* 2015).

3.3 Change of the current number of knots $N_{ice\text{-}fw}$ and $N_{ice\text{-}bw}$ of the calculation mesh
3.3. Promjena trenutačnog broja čvorova $N_{ice\text{-}fw}$ i $N_{ice\text{-}bw}$ računalne mreže

Figure 3 presents the calculated change in the number of knots $N_{ice\text{-}fw}$ and $N_{ice\text{-}bw}$ during the 30 h freezing of the studied log. The analysis of the results shows that during the first 1.50 h of the log cooling, the temperature decreases in its surface layers from the initial value of 11.1 °C until reaching 0 °C. Only after this 1.50 h, crystallization of free water in the knots begins, starting with free water situated the nearest to the frontal and cylindrical surfaces of the logs. After that, the number of knots $N_{ice\text{-}fw}$ increases almost linearly and reaches a value of $N_{ice\text{-}fw} = N_{total}$ = 800 at 11.50th h from the beginning of the freezing process.

When the temperature in the log surface layers decreases below -1 °C, the increase of the number of knots $N_{ice\text{-}bw}$ from 0 to 800 begins. This occurs from 2.50th h to 14.25th h of the freezing process. The hori-

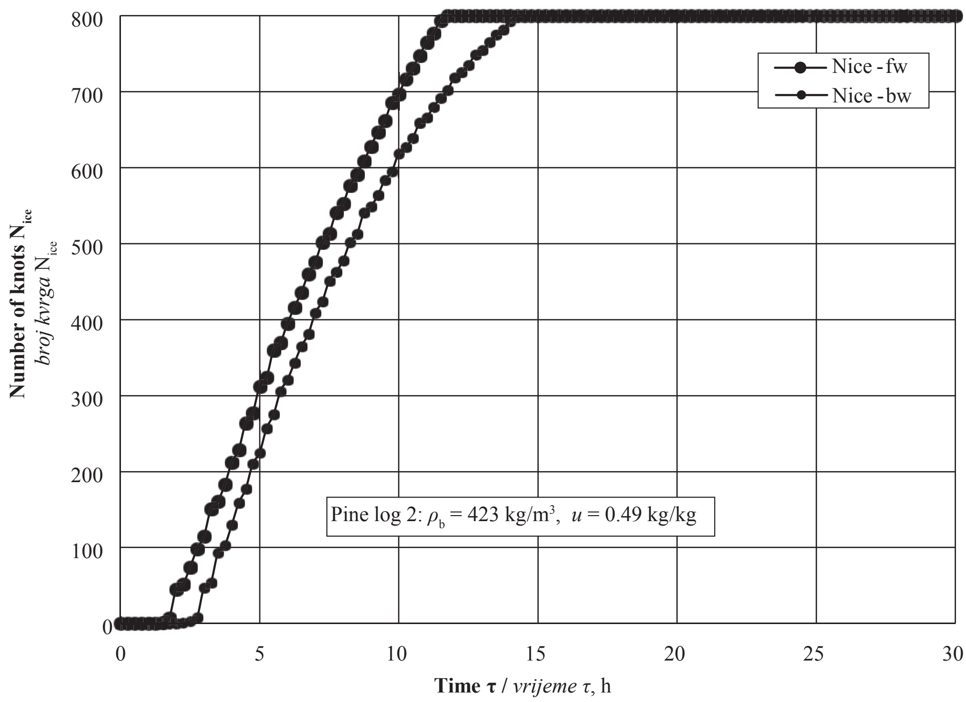

Figure 3 Change in knots $N_{ice\text{-}fw}$ and $N_{ice\text{-}bw}$ during log freezing
Slika 3. Promjene u čvorovima $N_{ice\text{-}fw}$ i $N_{ice\text{-}bw}$ tijekom zamrzavanja istraživanog trupca

zontal distances between the paired graphs of $N_{\text{ice-fw}}$ and $N_{\text{ice-bw}}$ in Figure 3 correspond to the time intervals in which the knots temperature decreases from 0 °C to -1 °C in separate layers of the log. During these intervals, the crystallization of free water in the log occurs.

3.4 Change of log icing degrees $\Psi_{\text{ice-fw}}$ and $\Psi_{\text{ice-bw}}$ and their derivatives
3.4. Promjena stupnja zaleđivanja trupca $\Psi_{\text{ice-fw}}$ i $\Psi_{\text{ice-bw}}$ i njihovih derivata

The icing degree $\Psi_{\text{ice-fw}}$ is calculated simultaneously with the model solving according to the equation

$$\Psi_{\text{ice-fw}}^{n}=\frac{N_{\text{ice-fw}}^{n}}{N_{\text{total}}} \tag{17}$$

During the first 1.50 h of the log cooling process, the whole amount of both free and bound water is in a liquid state and because of that $\Psi_{\text{ice-fw}} = 0$.

From 1.50th h to 11.50th h, the icing degree $\Psi_{\text{ice-fw}}$ increases almost linearly from 0 to 1 and remains 1 until the end of the 30 h log freezing process (Figure 4).

The calculation of the average mass icing degree of the log, $\Psi_{\text{ice-bw}}$, is carried out according to the following equation, obtained in (Deliiski and Tumbarkova, 2017):

$$\psi_{\text{ice-bw}}^{n}=\frac{1}{S_{\text{w}}}\iint_{S_{\text{w}}}\frac{u_{\text{fsp}}^{272.15}-\left\{0.12+(u_{\text{fsp}}^{272.15}-0.12)\cdot\exp[0.0567(T_{i,k}^{n}-272.15)]\right\}}{u_{\text{fsp}}^{272.15}}\mathrm{d}S_{\text{w}} \tag{18}$$

Where $T_{i\,k}^{n}$ is the current temperature in the knot with coordinates *i* along *r* and *k* along *z*, K;

S_{w} – area of ¼ of the longitudinal section of the log subjected to freezing, m^2.

The fiber saturation point at T = 272.15 K, $u_{\text{fsp}}^{272.15}$, used in Eq. 18, can be calculated according to the equation (Stamm, 1964; Deliiski, 2013a)

$$u_{\text{fsp}}^{272.15}=u_{\text{fsp}}^{293.15}+0.021, \tag{19}$$

Where $u_{\text{fsp}}^{293.15}$ is the standardized value of the fiber saturation point of wood species at 293.15 K, i.e. at 20 °C, kg/kg.

The gradual change in $N_{\text{ice-bw}}$ causes a curvilinear increase of $\Psi_{\text{ice-bw}}$ from 0 before 2.50th h until reaching 0.502 at the end of 30 h freezing (Figure 4). This value of $\Psi_{\text{ice-bw}}$ means that 1 - 0.502 = 0.498 relative parts (i.e. 49.8 %) of the bound water in the studied log remains in a liquid state in the cell walls at the end of 30th h of freezing when the calculated average log mass temperature is -29.6 °C.

The change in the derivatives $\mathrm{d}\Psi_{\text{ice-fw}}/\mathrm{d}\tau$ and $\mathrm{d}\Psi_{\text{ice-bw}}/\mathrm{d}\tau$ during 30 h freezing of the log is presented in Figure 5. To reduce the fluctuations in the values of these derivatives, their calculation was done with an interval of 150·Δτ = 900 s = 0.25 h. The fluctuations are caused by the uneven increase of the number of knots $N_{\text{ice-fw}}$ and $N_{\text{ice-bw}}$ during the freezing process of the log, which determines an uneven change in $\mathrm{d}\Psi_{\text{ice-fw}}/\mathrm{d}\tau$ and $\mathrm{d}\Psi_{\text{ice-bw}}/\mathrm{d}\tau$.

When $0 \le \Psi_{\text{ice-fw}} \le 1$, the derivative $\mathrm{d}\Psi_{\text{ice-fw}}/\mathrm{d}\tau$ fluctuates between $0.139\cdot10^{-5}$ s^{-1} and $5.28\cdot10^{-5}$ s^{-1}. When $\Psi_{\text{ice-fw}}$ becomes equal to 1 at 11.50th h, the derivative $\mathrm{d}\Psi_{\text{ice-fw}}/\mathrm{d}\tau$ obtains the value of $2.34\cdot10^{-5}$ s^{-1}. That derivative is equal to 0 when $\Psi_{\text{ice-fw}} = 0$ or $\Psi_{\text{ice-fw}} = 1$.

Before reaching the inflexion point of the curve $\Psi_{\text{ice-bw}} = f(\tau)$, the derivative $\mathrm{d}\Psi_{\text{ice-bw}}/\mathrm{d}\tau$ increases with smaller fluctuations than $\mathrm{d}\Psi_{\text{ice-fw}}/\mathrm{d}\tau$. When the inflexion point of $\Psi_{\text{ice-bw}} = f(\tau)$ reaches 14.50th h, that derivative obtains its maximum value equal to $0.96\cdot10^{-5}$ s^{-1}.

After reaching the maximum value, the derivative $\mathrm{d}\Psi_{\text{ice-bw}}/\mathrm{d}\tau$ decreases gradually and reaches a value

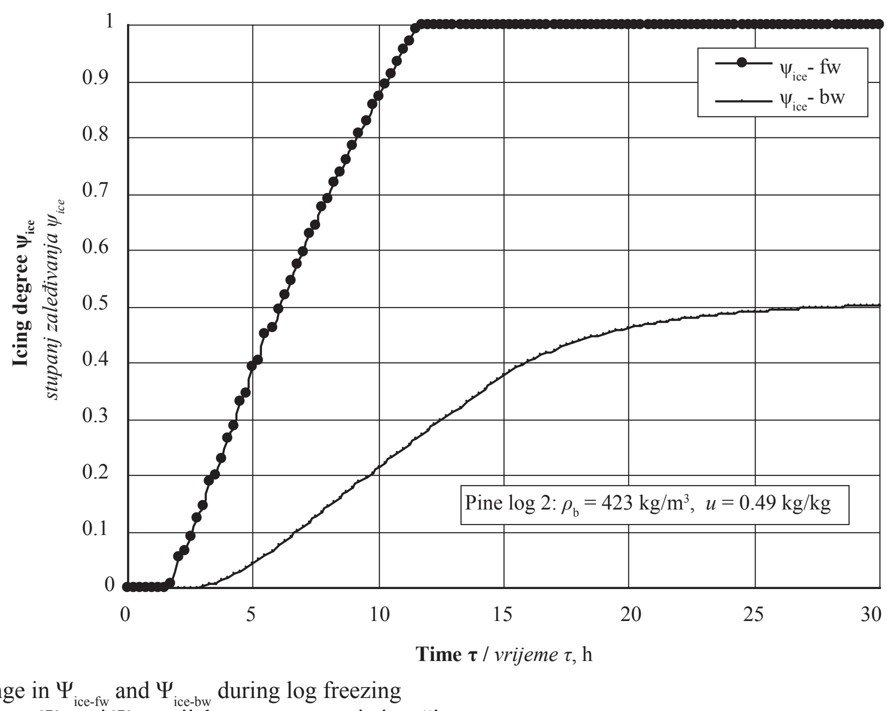

Figure 4 Change in $\Psi_{\text{ice-fw}}$ and $\Psi_{\text{ice-bw}}$ during log freezing
Slika 4. Promjene $\Psi_{\text{ice-fw}}$ i $\Psi_{\text{ice-bw}}$ tijekom zamrzavanja istraživanog trupca

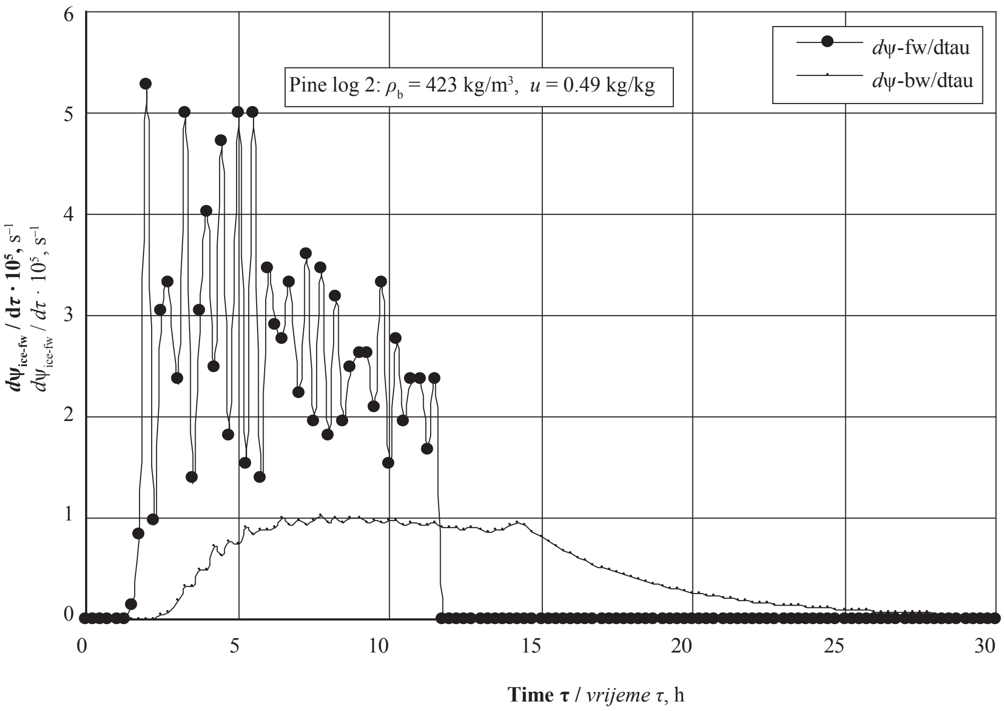

Figure 5 Change in $d\Psi_{ice-fw}$ */dτ and* $d\Psi_{ice-bw}$ /dτ during log freezing
Slika 5. Promjene $d\Psi_{ice-fw}$/dτ i $d\Psi_{ice-bw}$/dτ tijekom zamrzavanja istraživanog trupca

of $0.040 \cdot 10^{-5}$ s^{-1} at the end of 30 h freezing. Derivative $d\Psi_{ice-bw}/d\tau$ is equal to 0 only at the beginning of the freezing process when $N_{ice-bw} = 0$ and $\Psi_{ice-bw} = 0$.

3.5 Change of specific latent heat fluxes q_{LHv-fw} and q_{LHv-bw}
3.5. Promjena specifičnih latentnih tokova topline q_{LHv-fw} i q_{LHv-bw}

Figure 6 presents the change of the specific latent heat fluxes q_{LHv-fw} and q_{LHv-bw} calculated according to Eq. 11 and 12 during 30 h freezing of the studied pine log. It can be seen that, during the log freezing, the heat fluxes q_{LHv-fw} and q_{LHv-bw} change according to three interconnected sections, as follows:

• change of heat flux q_{LHv-fw}:

1. During the first 1.50 h of the log cooling process, the entire quantity of the free and bound water in the log is in a liquid state and because of that $q_{LHv-fw} = 0$ kW/m³.

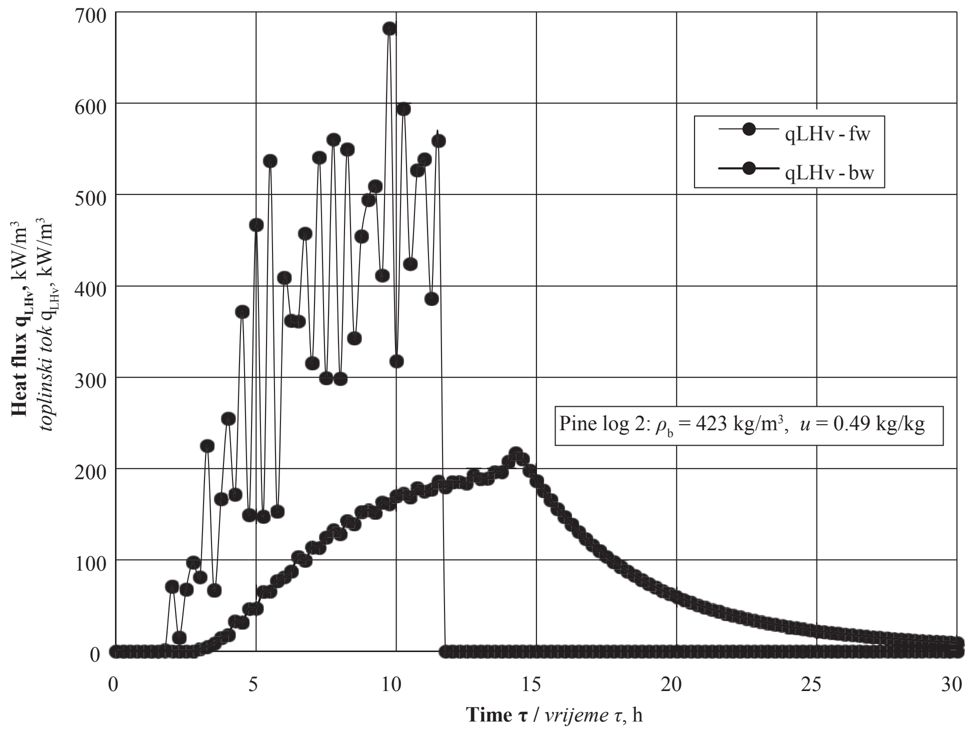

Figure 6 Change in q_{LHv-fw} *and* q_{LHv-bw} during log freezing
Slika 6. Promjene q_{LHv-fw} i q_{LHv-bw} tijekom zamrzavanja istraživanog trupca

2. From 1.50th h of the log freezing process, the free water crystallization starts. Then the heat flux q_{LHv-fw} begins to increase with fluctuations from the value of 0 until reaching the value of 4.934 kW/m^3 at 11.50th h when the number of knots N_{ice-fw} becomes equal to N_{total} = 800. The fluctuations in q_{LHv-fw} are caused by the fluctuations of the derivative $d\Psi_{ice-fw}/d\tau$, used in Eq. 11.

3. After 11.50th h, the icing degree Ψ_{ice-fw} becomes equal to 1. Then the heat flux q_{LHv-fw} obtains a value of 0 kW/m^3, and it remains unchanged until the end of the 30 h freezing.

• change of heat flux q_{LHv-bw}:

1. During the first 2.50 h of the freezing process, the whole amount of the bound water in the log is in a liquid state and because of that q_{LHv-bw} = 0 kW/m^3.

2. From 2.50th h, the crystallization of the bound water in the log starts. Then the heat flux q_{LHv-bw} begins to increase from the value of 0 until reaching its maximum value of 2.012 kW/m^3 at 14.25th h.

3. After 14.25th h, the derivative $d\Psi_{ice-bw}/d\tau$ decreases in comparison with its maximum value and this causes a decrease in the heat flux q_{LHv-bw}. At the end of 30th h of log freezing, when the calculated average log mass temperature is equal to -29.6 °C, the flux q_{LHv-bw} obtains the value of 0.085 kW/m^3.

4 CONCLUSIONS
4. ZAKLJUČAK

This paper presents a methodology for mathematical modeling and research of two interconnected problems: 2D non-stationary temperature distribution in logs subjected to freezing and change in the specific latent heat fluxes of the free and bound water in the logs during the freezing process.

The paper gives mathematical descriptions of the latent heat fluxes q_{LHv-fw} and q_{LHv-bw}, which are formed in logs during the freezing of the free water in the range from 0 °C to -1 °C, and of the bound water below -1 °C, respectively. These descriptions are introduced in our own 2D non-linear mathematical model of the 2D temperature distribution in logs during their freezing.

A software program for the solution of the model and computation of the specific latent heat fluxes q_{LHv-fw} and q_{LHv-bw} was prepared in FORTRAN, and it was input in the calculation environment of Visual FORTRAN Professional developed by Microsoft.

With the help of the program, computations for the determination of the heat fluxes q_{LHv-fw} and q_{LHv-bw} were completed as an example for the case of a pine log with D = 0.24 m, L = 0.48 m, t_{w0} = 11.1 °C, ρ_b = 423 kg/m^3, and u = 0.49 kg/kg subjected to 30 h freezing in a freezer at approximately -30 °C.

It was determined that the values of the specific latent heat fluxes q_{LHv-fw} and q_{LHv-bw} of the studied log change according to complex relationships, as follows:

– the heat flux q_{LHv-fw}, which is only formed by the freezing of the free water in the wood, changes from 0 to 4.934 kW/m^3 during the time from 1.50th h to 11.50th h of the freezing process;

– the heat flux q_{LHv-bw}, which is formed by the freezing of a portion of the bound water in the wood, changes from 0 to 2.012 kW/m^3 during the time from 2.50th h to 14.25th h of the freezing process. After the 14.25th h, the flux decreases and at the end of 30 h freezing, it reaches a value of 0.085 kW/m^3.

By applying the mathematical descriptions of q_{LHv-fw} and q_{LHv-bw} during our simulations with the mathematical model, we observed good conformity between the calculated and experimentally determined changes in the temperature field during the freezing of logs of different wood species with different moisture contents. The overall *RMSE* for the studied four characteristic points in the logs does not exceed 5 % of the temperature ranges between the initial and final temperatures of the logs subjected to freezing. This proves the suitability of the model as well as the correctness of the suggested mathematical descriptions of the studied latent heat fluxes.

Accuracy of the model and good correlation with the results of our comprehensive experimental studies provide the basis for using the model for carrying out various calculations related to the non-stationary temperature distribution and energy characteristics of logs of different wood species during their freezing. The mathematical model, after being connected with other models of the log defrosting process, could be input into the software of programmable controllers for providing optimized model-based automatic control (Deliiski, 2004; Deliiski and Dzurenda, 2010; Hadjiski *et al*., 2018) of thermal treatment of frozen logs in the production of veneer.

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Symbols:

c – specific heat capacity, J/(kgK)
D – diameter, m
E – exponent
L – specific latent heat, J/kg, or length, m
N – number of knots of the calculation mesh
q – specific heat flux, W/m^3
R – radius, m
r – radial coordinate: $0 \le r \le R$, m
S – area of ¼ of log longitudinal section, m^2
T – temperature, K: $T = t + 273.15$
t – temperature, °C: $t = T - 273.15$
u – moisture content, kg·kg^{-1} = %/100
z – longitudinal coordinate: $0 \le z \le L/2$, m
α – heat transfer coefficients between log surfaces and the surrounding air medium, W/(m^2·K)
λ – thermal conductivity, W/(m·K)
ρ – density, kg/m^3
σ – root mean square error, °C

τ – time, s
Δr – step along the coordinates r and z for solving the model, m
$\Delta\tau$ – step along the time coordinate for solving the model, s
Ψ – relative icing degree of logs or relative degree of solidification of metal

Subscripts:
avg – average (for root mean square error)
b – basic (for wood density)
bw – bound water
cr – crystallization
fr – freezing
fsp – fiber saturation point
fw – free water
ice – ice (for logs' icing degrees or for number of knots of the calculation mesh)
i – point of the calculation mesh in the direction along the logs' radius: i = 1, 2, 3,…, 21
k – point of the calculation mesh in longitudinal direction of the logs: k = 1, 2, 3, …, 41
LH – latent heat
m – medium (for cooling substance)
M – metal
Ms – metal in solid state
Mv – volume of the metal
0 – initial
p – parallel to wood fibers
r – radial direction
total – total (for number of knots of the calculation mesh)
v – volume
w – wood
we – wood effective (for specific heat capacity)
wS – wood containing solid state water (ice)
ψ – relative icing degree of logs

Superscripts:
n – time level during model solving: n = 0, 1, 2, …

272.15 – at 272.15 K, i.e. at -1 °C
293.15 – at 293.15 K, i.e. at 20 °C

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