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MATRIX CALCULATIONS OF CROATIAN NATIONAL ACCOUNTS

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Summary

Prediction of amounts for Croatian National Accounts in this paper was made based on Indirect Rates of Growth and clearly shows that GDP is going to be the leader. The second largest amount will be Import as a National Account. This data has to be warning for the Croatian government, because Croatia will not be able to balance her deficit in international trade. Personal Consumption is the third largest amount and it means that Import will be higher than Personal Consumption. The fourth largest amount is Export, which will be more than 24 thousands B of Kunas behind the Import. Currently the foreign debt in Croatia came to \$25 B, and a further increase of foreign debt is not a very positive sign. The only positive sign is coverage of Import with Export will be increased from 84% in 2003 to 88% in 2006. Gross Fixed Capital will not make extraordinary results and Government Consumption will be a very small amount.

Key words: matrix calculation; Croatian National Accounts; Indirect Rates of Growth

1. PREFACE

In economic analysis economists usually operate with rates of growth for every economic indicator separately. These rates are known as Direct Rates of Growth (DRG), because the focus is on the change of some indicator separately. One of the most important observations regarding rates of growth can be performed as Indirect Rates of Growth (IRG).

Calculation of IRG for National Accounts (i.e., there are two National Accounts "A" and "B") means to test development of changes even though they

are not linked and have no impact on each other.

IRG calculations use the matrix equations system, and it provides the indirect and direct rates of growth at the same time for all searched National Accounts.

When direct and indirect Matrix Rates of Growth are analyzed at the same time then it is possible to follow up with the intensity of growth and structural changes between analyzed National Accounts.

The main target is to present all structural changes between National Accounts, and then to create dynamic system of movements for national Accounts in the period "Z" on the bases of "Z_1" period.

The important characteristic of the Matrix Rates of Growth is the possibility to be analyzed and make comparison with each other in different periods. This enables the possibility to get a complete picture of changes that have occurred.

It is also important to emphasize the possibility to calculate the average Matrix Rates of Growth for longer periods. These Average Matrix Rates of Growth are very important for planning of future economic activities and creation of national economic policy. AMRG¹ can get information about highest and lowest limits of interval for planning and projection rates of growth for future economic development initiatives.

¹ AMRG - Average Matrix Rates of Growth

1. Model of Indirect Rates of Growth

Make an assumption about mutual dependence between National Accounts of any country.

The value of some NA^2 as G_{it} and $G_{i,}$ t-1, (i = 1,2,...n), value of accounts I in period t and t -1. The value growth of accounts I in the period (t -1, t) is:

(1)
$$\Delta G = G_{it} - G_{it-1}$$
 $i = 1,2,...n$

Indirect Rate of Growth between two accounts can be explained as the relation between the growth of i account, ΔY_{it} , and value of accounts j in the period t, otherwise:

Indirect Rates of Growth can be expressed like Matrix Rates of Growth.

(3)
$$Z_{11} \dots Z_{1n} \dots Z_{1n}$$

The numbers of the main diagonal represent direct rates of growth, (i = j), and all others $(i \neq j)$ indirect rates of growth.

Indirect Rates of Growth z_{ijt} show the growth of "i" national account compared to the value "j" national account in some "t" period of time. In other words, Indirect Rates of Growth show how much will be the growth of "i" national account if some "j" national account would grow by \$1.

If you refer to formula (2), then the statement Indirect Rates of Growth are strictly in relation with G_{jt} for period t. The link between the rates of the growth made in relation with G_{jt} and G_{j} , t-1 has been expressed:

(4)
$$zij = z'_{ijt} / 1 + z_{jjt}$$
; $z'_{ijt} = z_{ijt} / 1 - z_{jjt}$; $i, j = 1, 2, ... n$ and $t = 1, 2, ... T$

² NA - National Account

There is:

(5)
$$z'_{ij} = \Delta G_{it} / G_{it}, \dots n$$
, $(t = 1, 2, \dots T)$.

The Matrix of the Rate of Growth is:

(6)
$$Z_{11t}$$
 Z_{1t} Z_{nt} Z_{nt} Z_{nnt} Z_{nnt}

The Matrix of the Rate of Growth "Zt" can be definite as outside multiply calculation of vector from the growth of national account and vector of reciprocal value $(1/G_t) = (1/G_t, ..., G/G_{nt})$, or in the other way:

$$\Delta G_{t} \qquad \Delta G_{t}/G_{1t} \dots \Delta G_{1t}/G_{nt}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$(7) \quad Z_{t} = \Delta G_{t} (1/G_{t}) = \qquad (1/\Delta G_{t}, \dots 1/GY_{nt}) = \qquad \vdots$$

$$\Delta G_{nt} \qquad \Delta G_{nt}/G_{1t} \dots \Delta G_{nt}/G_{nt}$$

There is ΔG_t column vector, line (n,1) and (1/ G_t) row vector line (1, n). Now it is possible to be seen that outside multiplicities calculation of vectors give direct and indirect rates of growth and relationship between themselves.

The Matrix of Average Rates of Growth can also be used to calculate longer periods. The elements of Matrix of Average Rates of Growth " R_a ", can be calculated as weight average value belonging to succession elements of Matrix for every period from t = 1, 2, ... T.

(8)
$$z_{(a)ij} = \sum_{t=1}^{T} z_{(a)ijt} G_{jt} / \sum_{t=1}^{T} G_{jt}$$
 (i,j = 1,2, ... n)

2. Theory of Matrix Model

Any kind of change regarding the status in National Accounts indicates the way the national economy is moving in the future. For the purpose of analyses, all national accounts from the above will be marked from Y_1 , to Y_7 . Relative change in one of the national accounts is calculated as follows:

$$z_{ij} = \Delta G_i / G_{jt}$$
 $i,j = 1,2,...n; i \neq j$

If the portion of change ΔY , would be put in relation with Y_{jn} then the Indirect Growth Rate is obtained.

Direct and Indirect Rates of Growth can be represented as the Matrix of Growth, and elements on the main diagonal (i=j) represent Direct Rates of Growth. The remaining elements (i \neq j) represent Indirect Rates of Growth. This statement can be expressed as follows:

$$Z_{11}$$
 Z_{1n} Z_{1n} Z_{nn} ... Z_{nn} ... Z_{nn} ... Z_{nn} ... Z_{nn}

The columns and rows of this Matrix can be treated like vectors of growth. In that case, column "j" is column of growth:

$$z_j = (z_{1j}, z_{2j}, ... z_{nj}); j = 1,2, ... n$$

This is represent vector of growth for researched elements in relation with "j" value.

The same like in the previous case, "i" type of matrix

$$Z_{l} = (z_{l1}, z_{l1}, ... z_{ln});$$
 $z = 1,2,...n$

This is representing the vector of growth "i" value in relation with other values.

Whenever the vectors (columns and rows) are proportional, it means the matrix equation is singular type. For the purpose of this research work, has been used only singular type of matrix equation. The vectors in this case are National Accounts.

These National Accounts have mutual relations and make impact on economic growth. Mutual impacts of National Account between each other are very changeable in the same time. It means the growth of GDP in relation with Gross

Fixed Capital, Personal Consumption, Government Consumption, Export and Import, in that case the assemblage of these Indirect relations represent the Row Vector of Growth of GDP. Column Vector of Growth is a vector where on the one side we assemble the National Account in relation with only one Account.

Creation of the Model of Matrix of Growth has the following two assumptions:

- a) Linear relations and
- b) Indirect rates of growth are constant.

Indirect Rates of Growth can be constant only in terms of average mean value. That is why the model can be expressed in the form of:

$$G_{it} - 1/n-1 \sum_{j \neq i=1}^{n} k_{ij} G_{jt} = G_{i,t-1} \quad i = 1,2,...n$$

The Matrix form can be expressed as the following:

$$(I - 1/n-1 K) G_t = G_{t-1}$$

The solution of matrix system has to be expressed as the following:

$$G_t = (I - 1/n-1 K)^{-1} G_{t-1}$$

Where is "I" can be expressed by:

and "Kij" can be expressed as following:

$$\begin{vmatrix} 0 & k_{12} & \dots & k_{1n} \\ k_{21} & 0 & \dots & K_{2n} \\ \dots & \dots & 0 & \dots \\ k_{n1} & k_{n2} & \dots & 0 \end{vmatrix} = K_{ij}$$

The Matrix "G" means the average value of IRG.

3. Applied Matrix Model on Croatian National Account

For the purpose of this research the data about Canadian National Accounts from the International Financial Statistics report will be used. The data from the national accounts is the following:

- 1. Gross Fixed Capital,
- 2. Gross Domestic Product,
- 3. Personal Consumption,
- Government Consumption,
- 5. Export and
- 6. Import.

Table 1. Amounts at the National Accounts of Croatia³

National Account	Selected Years					
	1998	1999	2000	2001	2002	2003
GDP	137604	141579	152519	162909	176429	189883
GC	36642	39341	39816	38496	38280	40359
PC	81067	81545	89637	97768	106027	111918
GFC	32066	33025	33281	37252	43674	52637
Export	54546	57920	71899	80015	81136	90654
Import	67700	69731	79693	89056	96748	108235

On the bases of the data from Table 1, Matrix Z_{1-6} , can be calculated as follows:

$$Z_1 \ = \ \begin{array}{c} 0.028 & 0.101 & 0.049 & 0.120 & 0.069 & 0.057 \\ 0.019 & 0.069 & 0.033 & 0.082 & 0.047 & 0.039 \\ 0.003 & 0.012 & 0.006 & 0.014 & 0.008 & 0.007 \\ 0.007 & 0.024 & 0.012 & 0.029 & 0.016 & 0.014 \\ 0.024 & 0.086 & 0.041 & 0.102 & 0.058 & 0.048 \\ 0.014 & 0.052 & 0.025 & 0.061 & 0.035 & 0.029 \\ \hline \\ Z_2 \ = \ \begin{array}{c} 0.072 & 0.275 & 0.122 & 0.329 & 0.152 & 0.137 \\ 0.003 & 0.012 & 0.005 & 0.014 & 0.007 & 0.006 \\ 0.053 & 0.203 & 0.090 & 0.243 & 0.113 & 0.101 \\ 0.002 & 0.006 & 0.006 & 0.008 & 0.004 & 0.003 \\ 0.092 & 0.351 & 0.156 & 0.420 & 0.194 & 0.175 \\ 0.065 & 0.250 & 0.111 & 0.299 & 0.138 & 0.125 \\ \hline \end{array}$$

³ International Financial Statistics, New York, USA, February 2004, page 309

On the bases of all these Matrix calculations, the Average Matrix Model can be created as follows:

K =
$$\begin{vmatrix} 0.062 & 0.266 & 0.105 & 0.237 & 0.133 & 0.115 \\ 0.005 & 0.018 & 0.003 & 0.019 & 0.012 & 0.009 \\ 0.037 & 0.158 & 0.062 & 0.155 & 0.078 & 0.068 \\ 0.023 & 0.090 & 0.040 & 0.092 & 0.050 & 0.422 \\ 0.044 & 0.180 & 0.075 & 0.189 & 0.094 & 0.083 \\ 0.048 & 0.206 & 0.081 & 0.201 & 0.102 & 0.089 \\ \end{vmatrix}$$

On the bases of Matrix "K" it is possible to make a calculation of IGR for 5 years in advance. According to our Model, the final solution for our calculation is as follows:

$$G_t = (I - 1/n-1 K)^{-1} G_{t-1}$$
 (where is n=6)

and has to be expressed as:

$$G_{2008} = (I - 1/5 K)^{-1} G_{2003}$$

After all the multiplications with Matrix K, this "new" Matrix can be expressed as K_c and that Matrix K_c has to be multiplied by the value vector of National

Accounts from the year 2002 signed as G_{2002} . The value of K_c and G_{2002} are as follows:

$$K_c \ = \ \ \, \begin{vmatrix} 1.041 & 0.440 & 0.167 & 0.384 & 0.208 & 0.314 \\ 0.008 & 1.015 & 0.008 & 0.031 & 0.018 & 0.025 \\ 0.060 & 0.265 & 1.042 & 0.243 & 0.124 & 0.193 \\ 0.063 & 0.269 & 0.106 & 1.165 & 0.131 & 0.520 \\ 0.070 & 0.301 & 0.118 & 0.287 & 1.060 & 0.228 \\ 0.076 & 0.336 & 0.127 & 0.308 & 0.158 & 1.164 \\ \end{tabular}$$

$$\mathbf{G}_{2003} = \begin{bmatrix} 189883 \\ 40359 \\ 111918 \\ 52637 \\ 90654 \\ 108235 \end{bmatrix}$$

The final result is $G_{2007} = K_c \cdot G_{2002}$ and can be represented as follows:

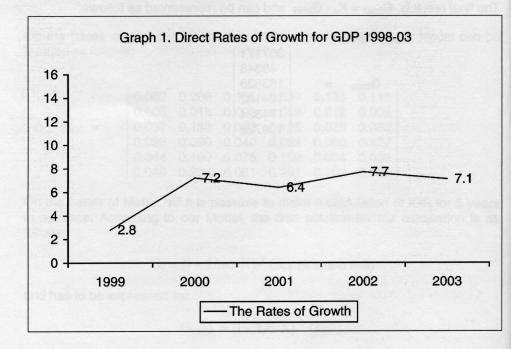
4. Interpretation of Calculated Results

Analysis of the results from the matrix calculations has to be made through every National Account separately. The differences in changes between National Accounts are significant and for that reason have to be analyzed individually.

It is also very important to that one assumption that was made is regarding the calculated expectation of amount for every National Account in five years. Those amounts can be expected only in the case that nothing is going to change in the next five years!

4.1. Gross Domestic Product

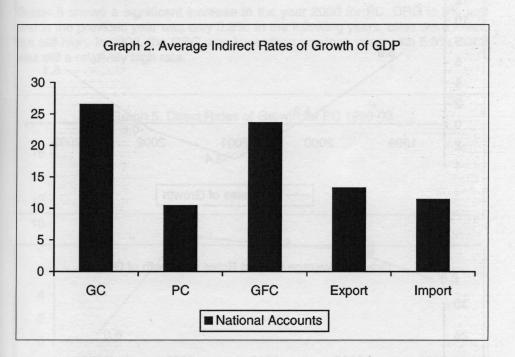
The data regarding GDP is significantly more important than any other National Account for every country in the world. In this case, the average direct rate of growth for GDP was 6.2%. Gross Domestic Product in five years can reach the level of 307,171Kn B, if no significant changes occur.



Graph 1 shows the movement of Direct rates. The lower rate of growth occurred in 1999 with only 2.8%. In all other years the rates of growth were very high and moving around 7%. The year 2001 had a lower rate of growth of 6.4%, but this

rate is still relatively high. The highest rate of growth was made in 2002 and it was 7.7%.

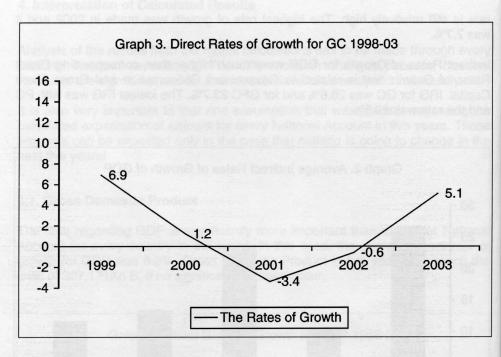
Indirect Rates of Growth for GDP were much higher than corresponding Direct Rates of Growth and is related to Government Consumption and Gross Fixed Capital. IRG for GC was 26.6% and for GFC 23.7%. The lowest IRG was with PC and the rate was 10.5%.

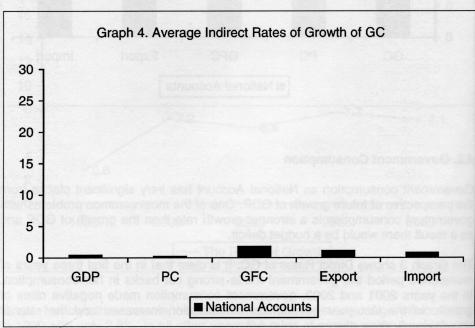


4.2. Government Consumption

Government consumption as National Account has very significant places from the perspective of future growth of GDP. One of the most common problems with government consumption is a stronger growth rate than the growth of GDP and as a result there would be a budget deficit.

The Graph 3 shows Direct Rates of GC. It is clear that in the first three years of researched period the government made strong cut backs in her consumption. In the years 2001 and 2002, government consumption made negative rates of growth. In the last year government consumption increased and the rate of growth was 5.1%.

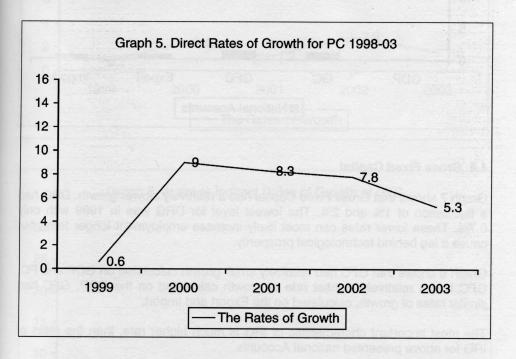




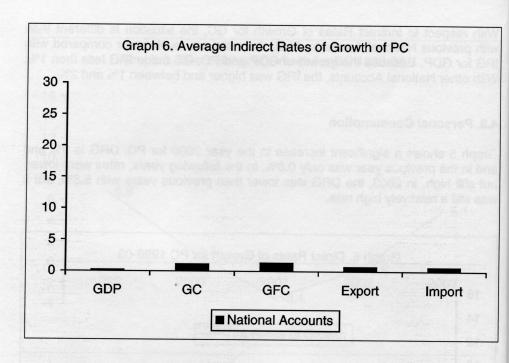
With respect to Indirect Rates of Growth for GC, the situation is different than with previous National Accounts. First, the rates are much lower compared with IRG for GDP. Because the growth of GDP and PC, GC made IRG less then 1%. With other National Accounts, the IRG was higher and between 1% and 2%.

4.3. Personal Consumption

Graph 5 shows a significant increase in the year 2000 for PC. DRG is 9% and and in the previous year was only 0.6%. In the following years, rates were lower, but still high. In 2003, the DRG was lower than previous years with 5.3%, but it was still a relatively high rate.



The situation with IRG is similar with GC and can be found in Graph 6. All rates were lower than 2% and this is a main characteristic. The highest IRG were made for GFC and GC and that was higher than 1%.

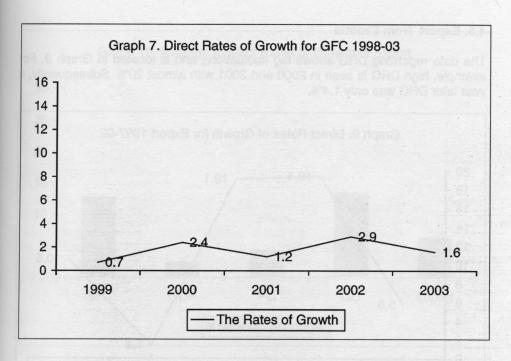


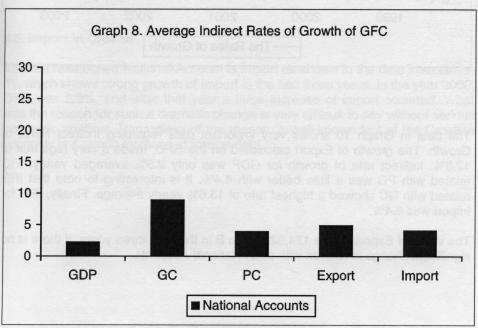
4.4. Gross Fixed Capital

Graph 7 shows that Gross Fixed Capital had a relatively slower growth. DRG had a fluctuation of 1% and 2%. The lowest level for DRG was in 1999 with only 0.7%. These lower rates can most likely increase employment longer term, but cause a lag behind technological prosperity.

Graph 8 shows that GFC had relatively small growth calculated on GDP and PC. GFC had a relatively highest rate of growth calculated on the GDP. GFC had similar rates of growth, calculated on the Export and Import.

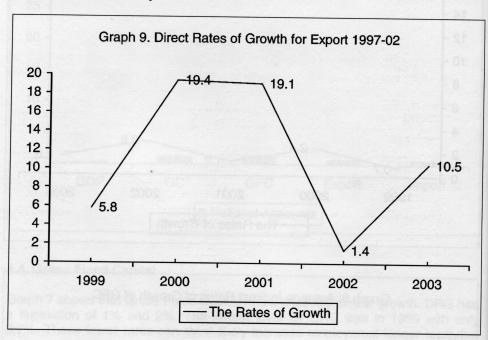
The most important characteristic of IRG is much higher rate, than the rates of IRG for above presented national Accounts.





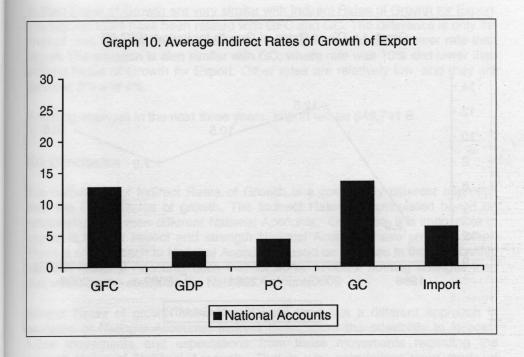
4.5. Export from Croatia

The data regarding DRG shows big fluctuations and is located in Graph 9. For example, high DRG is seen in 2000 and 2001 with almost 20%. Subsequently, a year later DRG was only 1.4%.



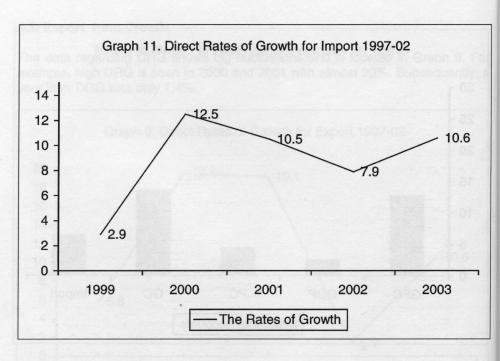
The data in Graph 10 shows very important data regarding Indirect Rates of Growth. The growth of Export calculated on the GFC, made a very high rate of 12.8%. Indirect rate of growth for GDP was only 2.5% averaged yearly. IRG related with PC was a little better with 4.4%. It is interesting to note that IRG related with GC showed a highest rate of 13.6% yearly average. Finally, IRG for Import was 6.4%.

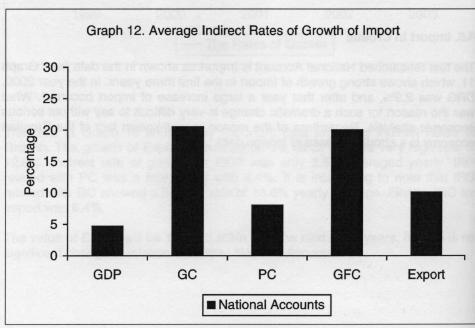
The value of Export will be 174,523.83Kn B in the next three years, if there is no significant change in relation with other National Accounts.



4.6. Import in Croatia

The last researched National Account is Import as shown in the data from Graph 11, which shows strong growth of Import in the first three years. In the year 2000, DRG was 2.9%, and after that year a large increase of import occurred. What was the reason for such a dramatic change is very difficult to say without serious economic analysis. Regardless of the reason, a well-known fact of the Croatian economy is a strong increase of foreign debt.





Indirect Rates of Growth are very similar with Indirect Rates of Growth for Export. The highest rates have been related with GFC and GC. The difference is only the level of rate. Indirect Rate of Growth for GFC is 9.5% and it is a lower rate than export. The situation is also similar with GC, where rate was 10% and lower than Indirect Rates of Growth for Export. Other rates are relatively low, and they are between 2% and 4%.

If nothing changes in the next three years, Import will be \$49,741 B.

5.0. Conclusion

The calculation of Indirect Rates of Growth is a completely different approach from the Direct Rates of growth. The Indirect Rates are calculated based on relationships between different National Accounts. Of course, it is impossible to say what kind of impact and strength National Accounts have on each other. There is an approach to National Accounts, based on changes in the relationship between National Accounts such that for some period if nothing changes then that would be the status on the National Accounts.

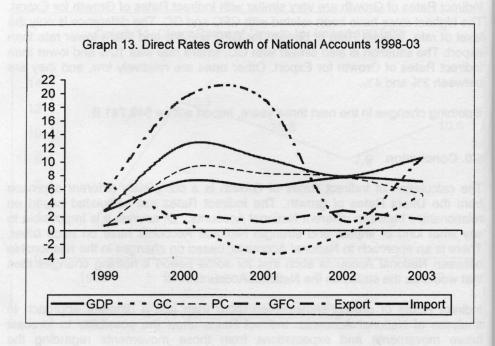
Indirect Rates of growth shows even more than just a different approach in analyses of National Accounts. Indirect Rates show the possibility to forecast future movements and expectations from those movements regarding the amount status of National Accounts. That is why calculations were made of expected amounts of National Accounts in three years for every separate National Account.

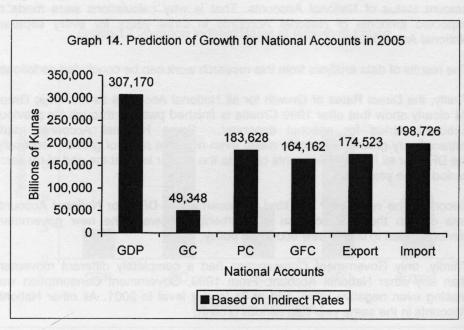
The results of data analysis from this research work can be concluded as follows:

Firstly, the Direct Rates of Growth for all National Accounts as shown in Graph 14 clearly show that after 1999 Croatia is finished passing through it's previous turbulent period for national economy. Some National Accounts made extraordinary growth, and some made even negative rates of growth! Generally, the DRG for all National Accounts came to the similar level at the end of research period in the year 2003.

Secondly, the reason for this kind of movement of DRG for National Accounts was due to the new coalition government in power. The new government obviously tried to create new economic policy.

Thirdly, only Government Consumption had a completely different movement than any other National Account. From 1999, Government Consumption was making even negative rates, and at its lowest level in 2001. All other National Accounts in the same year had serious growth.





Prediction of amounts for National Accounts was made based on Indirect Rates of Growth and clearly shows that GDP is going to be the leader. The second largest amount will be Import as a National Account. This data has to be warning for the Croatian government, because Croatia will not be able to balance her deficit in international trade. Personal Consumption is the third largest amount and it means that Import will be higher than Personal Consumption. The fourth largest amount is Export, which will be more than 24 thousands B of Kunas behind the Import. Currently the foreign debt in Croatia came to \$25 B, and a further increase of foreign debt is not a very positive sign. The only positive sign is coverage of Import with Export will be increased from 84% in 2003 to 88% in 2006. Gross Fixed Capital will not make extraordinary results and Government Consumption will be a very small amount.

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MATRIČNE KALKULACIJE HRVATSKIH NACIONALNIH RAČUNA

Sažetak

Predviđanje iznosa hrvatskoga nacionalog računa u ovome je radu doneseno na osnovi neizravnih stopa rasta i jasno pokazuje da će BDP imati vodeću ulogu. Drugi najveći iznos odnosi se na uvoz kao nacionalni račun. Ovi podaci bi za Hrvatsku vladu trebali biti upozoravajući jer Hrvatska neće moći uravnotežiti svoj deficit u međunarodnoj trgovini. Osobna potrošnja predstavlja treći najveći iznos, a to znači da će uvoz biti viši od osobne potrošnje. Četvrti najveći iznos je izvoz koji će više od 24 tisuće milijardi kuna zaostajati za uvozom. Hrvatski vanjski dug trenutačno iznosi 25 milijardi dolara, a daljni rast vanjskog duga nije dobar znak. Jedini pozitivan znak je taj to će izvoz koji će pokriti uvoz porasti s 84% u 2003. na 88% u 2006. Čvrsti bruto kapital neće dati neke posebne rezultate, a vladina potrošnja će biti niska.

Ključne riječi: matrična kalkulacija, hrvatski nacionalni računi, neizravne stope rasta

JEL klasifikacija: C82, E66