Srečko Devjak
Bank of Slovenia
Banking supervision department
Slovenska 35
1505 Ljubljana
srecko.devjak@bsi.si

Andraž Grum
Triglav Fund Management Company, LLC
Slovenska 54
SI-1000 Ljubljana
Head of fund management sector
andraz.grum@triglav-du.si

MARKET RISK CONTROL IN STABLE PARETIAN MARKETS

UDK / UDC: 336.7(368.025.6)
JEL klasifikacija / JEL classification: G14, G24
Izvorni znanstveni rad / Original scientific paper
Primljeno / Received: 11. studenog 2005. / November 11, 2005
Prihvaćeno za tisak / Accepted for publishing: 27. veljače 2006. / February 27, 2006

Abstract

In risk management process banks as financial investors should consider properties of yield probability distribution for each asset in the trading subportfolio in order to properly measure and control risk to which they are exposed to. Coherent risk control system for positions in a trading subportfolio requires also limits for loss limitation, what is a burdensome responsibility for a risk management in a market, where yields of assets are not normally distributed. As an investor is assumed to aggregate all his different attitudes towards risk of an investment into his utility function, we assume banks being risk averse as investors with hyperbolic risk averse utility function and calculate the fourth central moment of yield probability distribution for selected assets in Slovenian capital market. Based on calculated kurtosis of yield probability distributions, impact of kurtosis on risk management in a commercial bank shall be revealed in terms of limits setting.

***Key words**: risk management, bank, yield probability distribution function, utility function, risk control.*

INTRODUCTION

Instruments on financial markets are exposed to extreme price movements. Empirically documented fact can in practice be identified as fat tails or leptokurtosis of a yield distribution function. Stress testing must be a part of a risk management process in a commercial bank. If yield probability distribution function is leptokurtic, than stress testing is important part of risk management process. Stress testing is even more important in emerging financial markets where, because of low liquidity and lack of institutional investors, the extreme price movements are even more common. To ensure full integration of stress testing into the risk management process in a commercial bank, a bank should define stress test limits based on risk appetite (suffering extreme losses).

Kurtosis is a measure of the fatness of tails and peakedness of the center of a probability distribution relative to the tails and center of a normal distribution. If a value of a probability density function in tails of a yield probability distribution function is higher relative to the normal distribution, then significant price movements are more probable to occur. In this case the yield probability distribution function is said to be leptokurtic. Financial investors dislike leptokurtosis (positive excess kurtosis) of a yield probability distribution function of an asset, as positive excess kurtosis results from exceptional values (significant price movements). If a yield probability distribution of an asset is leptokurtic (positive excess kurtosis), then larger price movements occur more frequently. Platykurtic probability distribution has thin tails as values close to the mean are more probable for a larger surroundings of mean than there would be in a normal distribution, and tails are thinner than there would be in a normal distribution (Culp, 2001). It has also been proven that the risk, measured with standard deviation only, is underestimated if the portfolio yield probability distribution function has an excess kurtosis (Favre and Galeano, 2002).

The presence of leptokurtosis in yield distributions has been shown by Mandelbrot (1963). Mandelbrot hypothesis consequently states that empirical yield distributions should have longer tails than does the normal distribution and therefore should better conform to stable Paretian distributions with characteristic exponents less than 2 than to the normal distribution. The statistical implications of Mandelbrot hypothesis follow mostly from the absence of a finite variance for stable Paretian distributions with characteristic exponents $a < 2$ as it was explored in Fama (1965). Infinite variance means that the variance and the standard deviation of a stable Paretian process with $a < 2$ will show extremely erratic behaviour even for large samples. Because of very erratic behaviour, the sample variance and standard deviation are therefore not meaningful measures of the variability inherent in a stable Paretian process with $a < 2$ (Fama, 1965).

THE KURTOSIS TEST

Assume *n* is the number of time periods of market data for selected assets. Let *x* be a random variable and let x_i be individual value of a random variable *x*. Let also $\mu_{k=1}$ or μ be first central moment of a probability distribution and let $\mu_{k=2} = \sigma^2$ be second central moment of a probability distribution. In general, the *k*-th moment of a distribution $E\left[x^k\right]$ is the expectation value of the variable to the *k*-th power and can be expressed with equation

$$\mu_k = E\left[x^k\right] = \int \left(\frac{x-\mu_{k=1}}{\sigma_x}\right)^k f(x)dx = \frac{1}{\mu_{k=2}^{\left(\frac{k}{2}\right)}} \int (x-\mu_{k=1})^k f(x)dx .$$

KURTosis β_2 (Abramowitz and Stegun, 1972) or α_4 (Kenney and Keeping, 1962) as fourth central moment of a probability distribution of a random variable *x* can then be defined with the following general equation

$$\beta_2 = \alpha_4 = \frac{\mu_{k=4}}{\mu_{k=2}^2} = \int \left(\frac{x-\mu_{k=1}}{\sigma_x}\right)^4 f(x)dx = \frac{E\left[(x-\mu)^4\right]}{\left(\sigma^2\right)^2} = \frac{E\left[(x-\mu)^4\right]}{\sigma^4}.$$

Normal distribution has a kurtosis of $\beta_2 = 3$ (irrespective of their mean or standard deviation). If a kurtosis of a probability distribution is greater than 3 and inequality $\beta_2 > 3$ holds, it is said to be leptokurtic, and if a kurtosis of a probability distribution is less than 3 as $\beta_2 < 3$, it is said to be platykurtic. Leptokurtosis is associated with distributions that simultaneously are peaked and have fat tails. Platykurtosis is associated with distributions that simultaneously are less peaked and have thinner tails.

The estimation of a population probability distribution kurtosis using sample data shall be defined with equation (Campbell, Lo, MacKinlay, 1997):

$$b_2 = \frac{1}{(n-1)} \cdot \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^4}\right)^4 = \frac{1}{(n-1)\cdot\sigma^4} \cdot \sum_{i=1}^{n} (x_i - \mu)^4 .$$

Let γ_2 be excess kurtosis and defined with the following equation

$$\gamma_2 = \frac{\mu_{k=4}}{\mu_{k=2}^2} - 3.$$

Bearing this definition in mind can a population probability distribution excess kurtosis c_2 be estimated as

$$c_2 = \left(\frac{1}{(n-1)} \cdot \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^4} \right)^4 \right) - 3 = \left(\frac{1}{(n-1) \cdot \sigma^4} \cdot \sum_{i=1}^{n} (x_i - \mu)^4 \right) - 3.$$

Let *p* be a price of financial instrument and let $x_i = ln\left(\frac{p_i}{p_{i-1}} \right)$ be a daily yield of financial instrument.

In this paper we will understand a commercial bank as a financial investor. Commercial banks have two possibilities in order to calculate capital charges for market risk they are being exposed to. The first approach is standardised approach, which has to be used by banks in case they do not have an internal model. The standardized approach is based on capital decree legislated by Bank of Slovenia. Alternatively commercial bank can apply internal model for risk management purposes and it can use several risk measures in order to measure risk. Each risk measure has its strengths and its weaknesses. Consequently, the volume of risk calculated using a specific risk measure will vary among risk measures.

The goal of this paper is to calculate kurtosis of yield probability distribution functions for selected financial instruments and consequently to determine, whether the difference between kurtosis of a selected market variable distribution function is significantly different from 3 as it holds for mesokurtic distributions. For this reason, the sampling theory will be applied. Let *N* be a set of all financial data of daily yields available (*n* data for *Y* financial instruments). Then, if a standard sample set includes *n* daily yields, $\binom{N}{n}$ samples with no repetition can be determined. We would like to test the assumption of a yield distribution mesokurtosis for a selected market variable on a set of all market variables, included in the financial analysis. The test will be made not only for the selected time period population data set of *N* data, but also for the set of all samples of *n* data. That assumption allows a general conclusion about the kurtosis of a yield probability distribution function for each asset on a selected set of financial instruments in the financial analysis.

To determine the significance level, the kurtosis test will be applied (Lewis, 2003). Therefore, zero assumption can be defined as $H_0 : c_2 = 0$. For setting alternative assumption we shall use two tailed test and therefore $H_1 : c_2 \neq 0$ as the distribution can be leptokurtic in case when $c_2 > 3$ or platykurtic in case when $c_2 < 3$.

Let c_2 be an excess kurtosis. The test statistic for the kurtosis test is given by (Lewis, 2003):

$$z = \frac{c_2}{\sqrt{\frac{24}{n}}} \sim N(0,\, 1)$$

MARKET VARIABLES IN THE FINANCIAL ANALYSIS

In the financial analysis several financial instruments from Slovenian capital and money market were included. Chosen instruments are the one that are the most frequent to appear in Slovenian commercial bank trading book portfolio. Consequently we were considering yield probability distribution functions of treasury bills issued by Ministry of finance in Slovenia as representatives of money market instruments, selected securities listed on Ljubljana stock exchange, selected mutual fund equities and selected currency pair in the time period from 2nd December 2003 to 30th November 2004. This is in line with quantitative requirement for use of internal models in risk management process as banks have to use one year time series of daily market data. In this case we have time series of 252 daily yields of selected financial instruments. This requirement is stated in a decree of capital adequacy of banks and savings banks legislated by Bank of Slovenia.

Ministry of finance issues treasury bills with 1M, 3M, 6M and 1Y original maturity. In year 2001 OTC – DVP (delivery versus payment) secondary market of Slovenian treasury bills was established. Until then selected commercial banks have been functioning as market makers. Because of the high secondary market turnover, treasury bills are important instruments for balancing liquidity of commercial bank. For each day market makers are obligated to quote bid and ask prices for 16 non-matured treasury bills with different original and remaining maturities. The quoted prices on a specific day were chosen to calculate the yield to maturity curve on the money market segment and the process was repeated for each day by applying Nelson-Siegel (1985) model. When the yield to maturity curve was estimated we used the interpolation and extrapolation method to evaluate the appropriate prices of treasury bills for standardized maturities for each day. The estimation of a yield probability distribution kurtosis has been calculated for treasury bills with remaining maturity of 1D, 1W, 2W, 1M, 3M, 6M, 9M and 1Y. The return rates for predetermined remaining maturities has been calculated form daily quotations.

From capital market several equities and selected bonds from Ljubljana stock exchange were included in the financial analysis. Chosen equities and bonds have the greatest turnover and are therefore best representatives for price movements and yield probability distributions. In the financial analysis were included equities of Krka (KRKG), Mercator (MELR), Petrol (PETG), Gorenje (GRVG), Merkur (MER), Istrabenz (ITBG), Sava (SAVA), Etol (ETOG), Juteks (JTKG), Lesnina (MILG), Helios (HDOG), Zvon ena holding (ZVHR), Infond holding (IFFR) and Maksima holding (MAHR). Besides corporate equities also equities of selected investment companies were included in financial analysis. Investment companies are a result of Slovenian voucher privatization and are very popular between investors because they offer portfolio diversification with relatively low transaction cost. Their investment policy is very similar to the policy of exchange-traded funds. NFD investment company (NF1N) is the biggest closed mutual fund on Slovenian capital market in terms of net asset value and also the most liquid one. Additionally, also Infond ID (IFIR), KD ID (KDIN) and Krona Senior (SN0N) equities were included as investment companies equities. Mutual funds equities were also considered as an investment possibility in the financial analysis. Here we were analysing yield probability distributions of Sova (Sova), Galileo (Galileo), MP Global (Global). Among all listed bonds only bonds issued by Slovenian indemnification company (SOS2E) and governmental bonds RS39 (RS39) were included in the financial analyses.

THE KURTOSIS TEST RESULTS

Table 1.

Descriptive statistics and test values for market variables in the analysis

Statistics

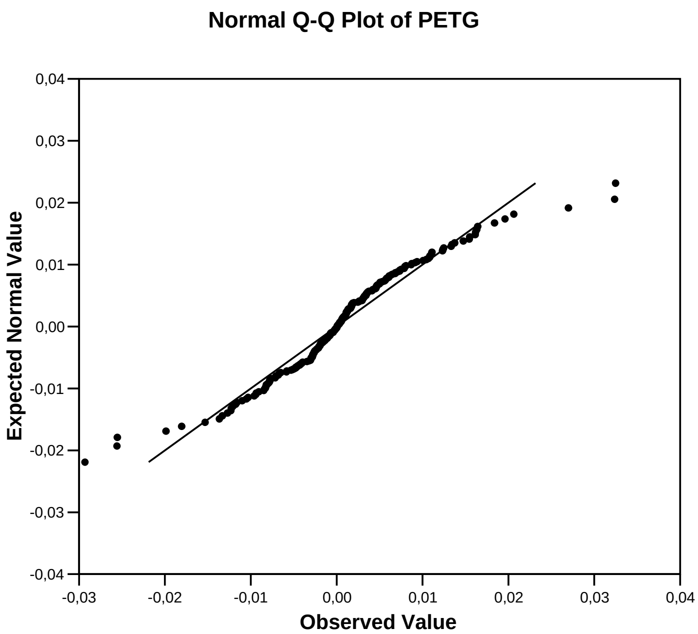
	N	Mean	Std. Deviation	z	Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error
KRKG	252	,0019	,0085	-1,55	2,521	,306
MELR	252	,0011	,0083	-5,67	1,251	,306
PETG	252	,0006	,0080	,02	3,008	,306
GRVG	252	,0009	,0096	6,24	4,924	,306
MER	252	,0014	,0109	1,65	3,509	,306
ITBG	252	-,0004	,0144	6,14	4,895	,306
SAVA	252	,0012	,0078	20,82	9,426	,306
ETOG	252	,0011	,0138	4,72	4,458	,306
JTKG	252	,0006	,0122	-6,45	1,010	,306
MILG	252	,0013	,0115	-6,15	1,101	,306
HDOG	252	,0023	,0148	20,22	9,241	,306
ZVHR	252	,0009	,0130	-1,20	2,629	,306
IFFR	252	,0000	,0120	1,33	3,411	,306
MAHR	252	-,0005	,0142	-6,43	1,016	,306
NFD1	252	,0016	,0077	-,32	2,901	,306
IFIR	252	,0011	,0094	-1,04	2,678	,306
KDIN	252	,0014	,0090	,78	3,242	,306
SN0N	252	,0015	,0104	-7,29	,750	,306
SOS2E	252	,0001	,0014	-1,47	2,547	,306
RS39	252	,0001	,0013	5,91	4,825	,306
Sova	252	,0002	,0004	26,22	11,093	,306
Galileo	252	,0007	,0033	-,25	2,921	,306
Global	252	,0002	,0071	-8,99	,225	,306
EUR	250	,0001	,0001	6,03	4,860	,307
1D	252	-,0009	,0133	-4,40	1,641	,306
1W	252	-,0012	,0101	5,78	4,783	,306
2W	252	-,0013	,0083	3,81	4,175	,306
1M	252	-,0014	,0070	6,25	4,929	,306
2M	252	-,0016	,0063	26,66	11,228	,306
3M	252	-,0016	,0065	41,29	15,743	,306
6M	252	-,0017	,0075	48,98	18,116	,306
9M	252	-,0018	,0084	45,00	16,888	,306
1Y	252	-,0019	,0091	40,90	15,623	,306
Valid N (listwise)	250					

Source: own calculation.

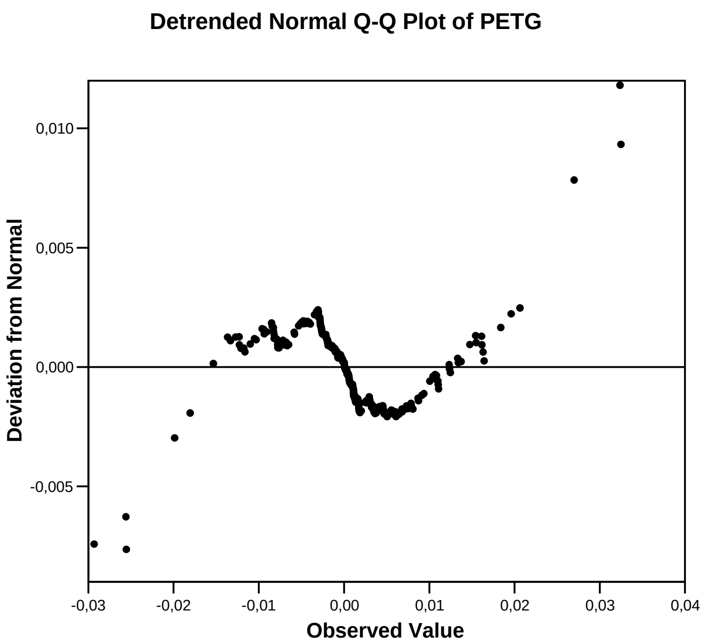
Among foreign currencies, banks in Slovenia have most of their positions denominated in EUR, USD, GBP, CHF and HRK currencies. In the composition of currency portfolio EUR fraction is dominant. Therefore EUR/SIT exchange rate as currency risk factor was included in financial analysis. There are several sources of data available for EUR/SIT exchange rate. To calculate kurtosis of yield probability distribution function for EUR/SIT exchange rate, middle exchange rates of Bank of Slovenia were used.

As we tested zero assumption $H_0 : c_2 = 0$ against alternative assumption $H_1 : c_2 \neq 0$ excess kurtosis can be shown in all cases where $|z| > z_{\alpha/2=0,025} = 1,96$. Consequently, zero assumption $H_0 : c_2 = 0$ cannot be rejected if the value of z fulfils the condition $z_{\alpha/2=0,025} < z < z_{1-\alpha/2=0,975}$. In line with the calculated kurtosis test results excess kurtosis can be shown for all yield probability distribution functions, except for KRKG, PETG, MER, ZVHR, IFFR, NFD1, IFIR, KDIN, SOS2E and Galileo. In all this cases zero assumption cannot be rejected, but we can reject zero assumption and therefore accept alternative assumption in all other cases with significance level of $\alpha < 0,05$. Kurtosis test also shows, that yield probability distribution functions of all governmental securities are leptokurtic, except for treasury bills with one day standardised original maturity. This is very important finding of this financial analysis. Bond SOS2E is a quasi governmental bond as government guarantees for interest and principal payments, but the kurtosis test here shows yield probability distribution to be mesokurtic. Here the calculated z value fulfils the condition $|z| > z_{\alpha/2=0,025} = 1,96$.

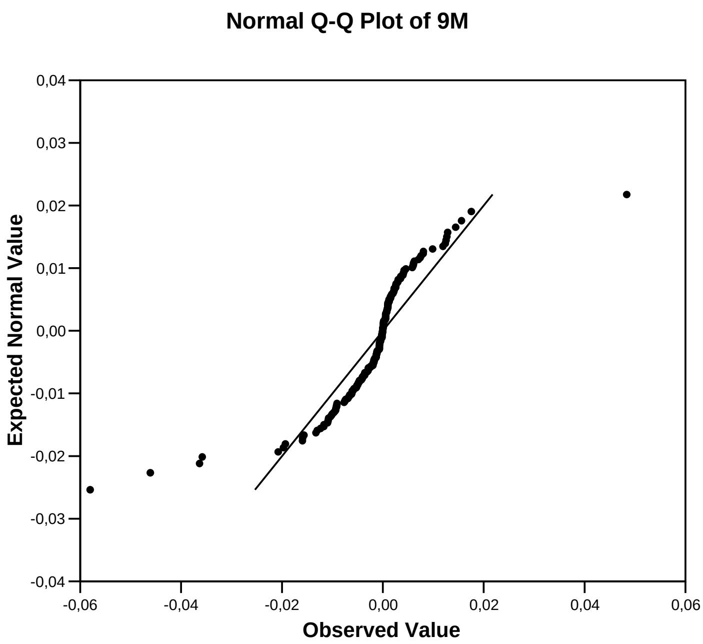
Leptokurtosis or platykurtosis of a yield probability distribution of an asset can be tested also with several alternative approaches. We shall also use graphical approach in order to identify fat tails and peakedness of a probability distribution. For this reason we shall plot quantiles of yield probability distribution of a selected asset against quantiles of a standard normal distribution. Here standard normal distribution is used as a test distribution. When quantiles of a yield probability distribution of an asset matches quantiles of a test distribution, the points cluster around a straight line. For the described purpose, Q-Q plot and Detrended normal Q-Q plot will be applied. Among all assets, PETG equities and 9M treasury bills were selected.



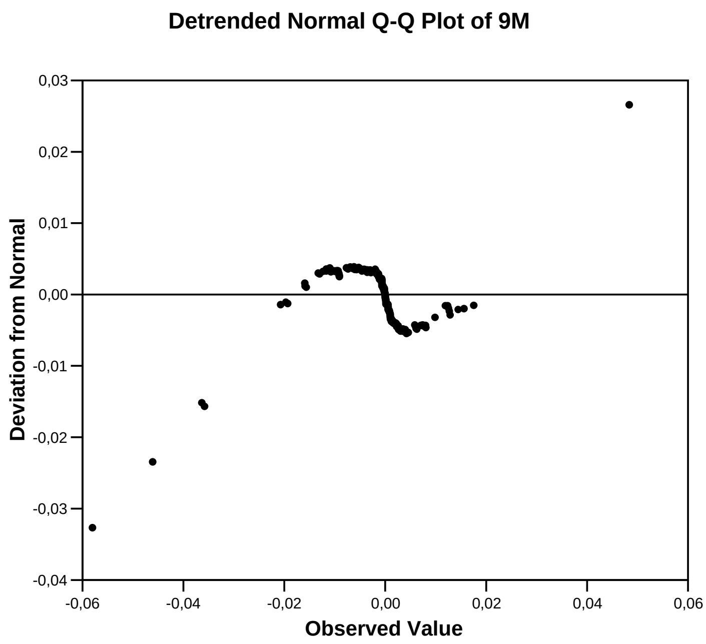
Source: own calculation.



Source: own calculation.



*Source: o*wn calculation.



Source: own calculation.

YIELD VARIABILITY AND RISK MANAGEMENT PROCESS

The important difference between a market dominated by a stable Paretian process with characteristic exponent $a < 2$ and a market dominated by a Gaussian process is that the path of the price level of a given asset in a Gaussian market will be fairly continuous and in a stable Paretian market with $a < 2$ it will usually be discontinuous. In other words, in a stable Paretian market with $a < 2$, the price of an asset will often tend to jump up or down by very large amounts during very short time periods (Fama, 1965).

In an efficient market prices of assets at every point in time represent best estimates of their intristic values. Therefore, when an intristic value changes, the actual price will change instantaneously, where instantaneously also means, that the ctual price wil initially overshoot the new intristic value as often as it will undershoot it. In case of a stable Paretian distribution with $a < 2$ intristic values often change by large amountsduring very short periods of time, what is a situation quite consistent with a dynamic economy in a world of uncertainty (Fama, 1965).

The discontinuous price changes in a stable Paretian market has important practical implications for market risk management. Commercial bank as an investor is obliged to perform risk management. Risk management includes also risk control. For proper risk control function commercial bank has to set limits. The complexity of limit system in a commercial bank depends upon risk exposure of a bank.

The fact that there are a large number of abrupt changes in a stable Paretian market supports the hypothesis that such a market is inherently more risky than a Gaussian market. The variability of a given expected yield is higher in a stable Paretian market than it would be in a Gaussian market, and the probability of large losses is higher (Fama, 1965).

If the probability of large price changes is higher (fat tails of yield distribution) and if therefore also the probability of large losses is higher then in a stable Paretian market with $a < 2$ investors cannot protect themselves from high loses with "stop loss" orders. If the price of an asset is going to fall very much, the total decline will probably be accomplished very rapidly, so it might be impossible to carry out many "stop loss" orders at insignificant time intervals at intermediate prices (Fama, 1965).

As commercial banks should calculate profit and loss for its trading subportfolios. In a stable Paretian markets commercial banks therefore are exposed to higher risk than in a Gaussian market. Leptokurtic yield distributions consequently burdens effective risk management in a commercial bank. As in a stable Paretian markets exist high probability that because of large price downfall and its very rapid accomplishment it would be impossible to carry out "stop loss" orders at insignificant time intervals at intermediate prices, risk management in a commercial bank should think about limits for loss limiting (stop loss limits). Therefore in a stable Paretian environment, risk management should set stop loss limits first. When stop loss limits are set for each position separately, risk management should calculate position limits which have to be within preliminary set total exposure credit limits. Because of high price variability of assets in a stable Paretian markets with $a < 2$ this requirement should always be fulfilled. For trading book items in a commercial bank, stop loss limits should comprise general variability. Therefore risk management in a commercial bank should set a stop loss limit, which is a maximum allowable loss limit. Then, bearing in mind general market variability, risk management should set position limits.

The higher is the positive excess kurtosis, the higher is price variability and the risk is of higher price movements is higher. Risk controlling in terms of limits setting should never combine lower stop loss limits with higher positions as such stop loss limits will not comprise general market variability.

This process of position limits setting will assure that preliminary set stop loss limits will not be exceeded even though prices of assets can change by a very large amount during very short time periods, stop loss limit excession should occur with negligible probability. Because the path of the price level of a given asset in a stable Paretian market with $a < 2$ will usually be discrete, it causes also profit and loss distribution of an assets to be discreet. Moreover, a sound risk management in a stable Paretian market with $a < 2$ requires several stop loss limits with infinitesimal tenors having a status of trigger limits. All these limits form a set. Supremum of this set is a stop loss limit, which was defined as a maximum allowable loss. With this approach, generally low probability of maximum allowable limit excession will even be lower, and such limit exseesion should therefore never occur.

CONCLUSION

Significant price movements of assets are more probable to occur in smaller and emerging financial markets. As Slovenian financial market is smaller financial market, kurtosis of a set of selected assets has been tested using kurtosis test. Within this financial analyses leptokurtosis of yield probability distribution has been shown for all assets, but for KRKG, PETG, MER, ZVHR, IFFR, NFD1, IFIR and KDIN equities, for SOS2E bond and for Galileo mutual fund with a significance level of $\alpha < 0{,}05$. Kurtosis test also shows, that yield probability distribution functions of all governmental securities are leptokurtic.

Therefore, commercial bank as financial investor should consider fourth moment in risk management process and should set a proper system of limits for loss limiting (stop loss limits). When setting stop loss limits, risk management should define a maximum allowable loss as a supremum of a stop loss limits set.

All stop loss limits in the set should have very short tenor and should have a status of trigger limits. Risk management should use maximum allowable loss limit and consequently define position limit. With this approach, generally low probability of maximum allowable limit excession will even be lower, and such limit exseesion should therefore never occur.

LITERATURE

Abramowitz M., Stegun I. A. (Eds.): Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 9th printing. New York: Dover, 1972.

Andersen Torben G., Bollerslev Tim, Diebold Francis X, Ebens Heiko: The distribution of stock return volatility. National Bureau of Economic Research, Working Paper 7933, 2000.

Campbell John Y., Lo Andrew W., MacKinlay Craig A.: The econometrics of financial markets. Princeton: Princeton University Press, 1997.

Culp C. L.: The risk management process. New York: John Wiley&Sons, 2001.

Devjak S.: Optimisation of the securities portfolio as a part of risk management process. Management Vol. 9. No. 1. (2004), Ekonomski fakultet, Split.

Fama F. Eugene: The behavior of stock-market prices. The Journal of Business, 1965 (38), p. 34-106.

Favre L., Galeano J. A.: Mean modified Value-at-Risk optimization With Hedge Funds. Journal of Alternative Investment, 2002 (5).

Gujarati N. Damodar: Basic econometrics. New York: McGraw-Hill, 1995.

Jorion Philippe: Value at risk: The new benchmark for controlling market risk. Second Edition. New York: McGraw-Hill, 2001.

Kenney, J. F. and Keeping, E. S.: Kurtosis. §7.12 in Mathematics of Statistics, Pt. 1, 3rd ed. Princeton, NJ: Van Nostrand, 1962.

Lewis N. Da Costa: Market risk modelling. Applied statistical methods for practitioners. London: Risk books, 2003.

Poon Ser-Iluang, Granger Clive: Forecastin volatility in financial markets: A review. Journal of Economic Literature, Vol. 41, 2 (2002), 478-539.

Srečko Devjak
Slovenska banka
Odjel kontrole
Slovenska 35
1505 Ljubljana
srecko.devjak@bsi.si

Andraž Grum
Triglav –društvo za upravljanje novčanim sredstvima
Slovenska 54
SI-1000 Ljubljana
Rukovoditelj Sektora za upravljanje novčanim sredstvima
andraz.grum@triglav-du.si

KONTROLA TRŽIŠNOG RIZIKA NA STABILNIM PARETOVIM TRŽIŠTIMA

UDK / UDC: 336.7(368.025.6)
JEL klasifikacija / JEL classification: G14, G24
Izvorni znanstveni rad / Original scientific paper
Primljeno / Received: 11. studenog 2005. / November 11, 2005
Prihvaćeno za tisak / Accepted for publishing: 27. veljače 2006. / February 27, 2006

Sažetak

U postupku upravljanja rizikom banke kao financijski ulagači moraju razmotriti svojstva vjerojatnosti raspodjele prinosa za svaku vrijednosnicu u trgovačkom portfelju kako bi se pravilno odredio i kontrolirao rizik kojemu su izloženi. Koherentni sustav kontrole rizika za pozicioniranje u portfelju također zahtjeva limite u određivanju gubitka, što predstavlja veliku odgovornost u upravljanju rizikom na tržištu, gdje se prinosi od vrijednosnica ne distribuiraju normalno. Budući da se od ulagača očekuje da sva svoja različita gledišta vezana za ulagački rizik ujedini u funkciju korisnosti, pretpostavljamo da banke odbacuju rizik kao ulagači s hiperboličnom funkcijom korisnosti i izračunavaju četvrti moment vjerojatnosti raspodjele prinosa za određena sredstva na slovenskom tržištu kapitala. Na osnovi izračunate statističke distribucije vjerojatnosti prinosa, utjecaj statističke distribucije na upravljanje rizikom u komercijalnoj banci treba biti otkriven u smislu uspostavljanja limita.

Ključne riječi: upravljanje rizikom, banka, funkcija vjerojatnosti distribucije prinosa, funkcija korisnosti, kontrola rizika.

JEL classification: G14, G24