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ANAXAGORAS, THE THOROUGHGOING INFINITIST: THE RELATION BETWEEN HIS TEACHINGS ON MULTITUDE AND ON HETEROGENEITY

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ABSTRACT

In the analysis of Anaxagoras' physics in view of the relation between his teachings on multitude and heterogeneity, two central questions emerge: 1) How can the structure of the universe considered purely mereo-topologically help us explain that at the first cosmic stage no qualitative difference is manifest in spite of the fact that the entire qualitative heterogeneity is supposedly already present there? 2) How can heterogeneity become manifest at the second stage, resulting from the noûs intervention, if according to fragment B 6 such a possibility requires the existence of "the smallest", while according to the general principle stated in fragment B 3 there is not "the smallest" but always only "a smaller"? This paper showcases the perplexity of these two questions but deals only with the former. The answer follows from Anaxagoras' being a thoroughgoing infinitist in the way in which no Greek physicist was: the principle of space isotropy operative in geometry is extended to physics as well. So any two parts of the original mixture are similar to each other not only in view of the smaller-larger relation but also because each contains everything that the other one contains. This in effect means that at the stage of maximal possible heterogeneity each part of any part contains infinitely many heterogeneous parts of any kind whatsoever. So, neither can there be homogeneous parts in view of any qualitative *property, nor can there be predominance in quantity of parts of any kind that would make some property manifest.*

Keywords*: *Anaxagoras, infinitism, mereo-topology, gunk, cosmogony, singular cosmic event, fractal universe, double world order

1. Introduction

The relation between Anaxagoras' cosmology and contemporary analytic philosophy is twofold. On one hand, there are authors who mention Anaxagoras as somebody whose ideas can be viewed as a kind of anticipation of certain notions, such as the notion of *gunk*, of *the fractal universe* or of *the singular cosmic event*, which have been introduced and discussed in contemporary analytic metaphysics and physics. On the other hand, there are those who try to clarify Anaxagoras' doctrine by using the method and conceptual apparatus of analytic philosophy. The approach of this paper is closer to that of the latter group, for we shall focus on Anaxagoras' teachings on multitude and on heterogeneity in order to present his cosmology in a consistent manner and to connect the two teachings by filling up gaps in the often only implicit argumentation that can be found in the doxography of ancient philosophy. Hopefully, the resulting interpretation might be also of help in contemporary metaphysical debates such as those concerning the structure of physical continua in general and variety of cosmological models in particular.

We shall start with Anaxagoras' teaching on multitude, because there are statements and arguments that can be understood in purely mereo-topological terms and which as such suggest what the structure of the universe looks like in view of how its parts are related regardless of what those parts are specifically. After elucidating this point, the first of the two central questions will arise: How can such a structure afford the explanation of Anaxagoras' claim that no qualitative difference could be manifest (ἔνδηλος) in the original mixture of everything with everything? This question is rendered particularly perplexing when we take into consideration an additional claim of Anaxagoras, namely that the entire qualitative heterogeneity, which is to be manifest only after the intervention of *noûs*, has been actually present in the original mixture from eternity (ἐξ αἰῶνος).
Giving the answer to the above question will complete the main task of the paper. But, at the end, we shall also address the second central question, complementary to the first one, and mention difficulties related to it.

Namely, given the way in which the teaching on multitude provides the explanation of *why* in the original mixture no qualitative difference can be manifest, it is not easy to give an account consistent with various statements of Anaxagoras about heterogeneity, which he claims *may* become manifest due to the motion caused by *noûs*. However, the answer to this question will be postponed for another occasion.

2. Interlude: Classical Scholarship meets Analytic Philosophy[*]

In the course of almost century and a half[1] of intense scholarly work, Anaxagoras has been interpreted in radically different, mutually incompatible and divergent ways, probably more so than any other Presocratic. This diagnosis of the state of affairs of Anaxagorean scholarship has been stated already in the 1950s by J. E. Raven (1954, 123) who managed to detect a tendency towards "undue complication" common to all competing reconstructions formulated *up to then*. Interestingly, this has become the general opinion applicable also to almost all reconstructions formulated *since then* (as evidenced in McKirahan 2010, 229) and it is characteristic of both types of authors mentioned in the Introduction. The situation up to the '50s can be characterised by the prevalence of the "old-fashioned nothing-but-philologist" approach (Cleve 1973, x), detached from (what were then its contemporary) goings-on in philosophy, so that Anaxagoras was reserved for the classicists. However, a paradigm shift in classical scholarship due primarily to Gregory Vlastos[2] opened up new vistas of research: *analytic ancient philosophy* was born through the application of the tools of logic and analytic metaphysics *alongside* the tools of classical philology in the study of ancient texts. All the prerequisites for a *philosophical reconstruction* (in the sense of Cleve)

[*] *Note*. In what follows, the text is divided into two levels represented by differently sized fonts. The first, "main level" contains all and only those elements which are essential for understanding what we consider to be the accurate reconstruction of Anaxagoras' teaching on the relation between multitude and heterogeneity. For this reason, we have made it as free as possible of all but the most relevant references to the original texts of the fragments and ancient doxographical reports. We introduced the second level (written in smaller font) in order to provide detailed references to and critical discussions of previous attempts at articulating Anaxagoras' metaphysics. Nonetheless, the main level can be read independently of the second.

[1] It is safe to claim that interest in Anaxagoras' theory began to grow rapidly after the publication of Tannery's classical exposition in 1887.

[2] For details about the ground-breaking novelties of Vlastos' approach see, e.g. Burnyeat (1992), Mourelatos (1993), and Graham's introduction in Vlastos (1995).

of Anaxagoras were thus made available. It might be claimed that Felix M. Cleve was anticipating the developments in the '50s since his original publication concerning Anaxagoras appeared in 1917.[3] The present reconstruction can be seen as a continuation of the tradition which he inaugurated.

As to the authors of the first type mentioned in the *Introduction*, i.e. contemporary metaphysicians, they acknowledge not only that Anaxagoras deserves a rightful place in the history of mereology (see, e.g. Mann and Varzi 2006, 593) but also that his style of mereology (details of which are worked out in this paper) represents a relevant contender in various ongoing mereological debates (Rosen and Dorr 2002, 165–6), primarily owing to the fact that his conception can (and, as we believe, should) be seen as a form of *gunkology*, i.e. an *ante litteram* articulation of what came to be known as gunk (following Lewis 1991, 20 et passim). The idea that Anaxagoras was a gunk-theorist is not new. Sider (1993), Markosian (2004 and 2005), Nolan (2006), and Hudson (2007) all credit Anaxagoras' metaphysics with the notion of gunk.

Some authors have also suggested using tools of Mandelbrot's fractal geometry (Mandelbrot 1983) and topology as a means by which we might arrive at an adequate model of the Anaxagorean universe (see, e.g. Graham 1994, 109, Graham 2006, 213 and Drozdek 2005, 173ff.). Probably the most elaborate of such attempts can be found in the works of Petar Grujić (Grujić 2001, 2002, 2006). Section **4.5.** of the present paper presents a novel approach to Anaxagorean fractals.

3. Multitude from a Merely Mereo-Topological Point of View

3.1. The Universe as a Gunk

Citing Anaxagoras, Simplicius in *Phys*. 166.15–16 says that "neither of the small is there the smallest, but always a smaller (οὔτε τοῦ σμικροῦ ἐστι τοὐλάχιστον ἀλλὰ ἔλασσον ἀεί)", adding that "nor is there the largest" (οὔτε τὸ μέγιστον). Immediately after this, Simplicius cites Theophrastus, according to whom Anaxagoras' statement that "everything is in everything" (πάντα ἐν παντί) is "based" (διότι) on the fact that in view of everything large and small there are "infinitely many larger and smaller" (ἐνμεγέθει καὶ σμικρότητι ἄπειρα).

The aforementioned quotations from Simplicius constitute Anaxagoras' fragment DK 59 B 3. But what he says there might, on the first reading, be

[3] *Die Philosophie des Anaxagoras: Versuch einer Rekonstruktion*, Vienna, 1917; the first English translation appeared in 1949.

seen as contradicting what he said previously in B 1 (“in the beginning of his *Physics*”, as Simplicius informs us), namely that “air and aether *covered* all things (πάντα γὰρ ἀήρ τε καὶ αἰθὴρ κατεῖχεν), both being unlimited, for these are *the largest* (μέγιστα) among all things both in quantity and in magnitude (πλήθει καὶ μεγέθει)” (*emphasis added*). How can air and aether be largest, if there is no largest? This apparent contradiction can easily be explained away by taking into account what Anaxagoras himself says in B 2. What he says in B 1 holds *only after* “air and aether were separated off (ἀποκρίνονται) from the all-encompassing multitude (ἀπὸ τοῦ πολλοῦ τοῦ περιέχοντος)”. Simply put, at that stage air and aether are *the only two* differentiated manifest things (χρήματα)—hence, by default, largest—and as such they cover all other non-yet-manifest things. The point is that the cosmogonical process is *gradual*: separating-off happens in successive stages (ἀποκρίνεσθαι κατὰ τάξιν), as Simplicius says *in Phys* 460.30. Therefore, what Anaxagoras says in B 3 holds *globally*, for the entire universe as a whole (τὸ ὅλον), which as such contains both the manifest and the not-yet-manifest things, as well as *locally*, for any of the things which are becoming manifest.

In view of the previous explanation of the fact that B 3 holds for all things in the universe, one is naturally led to the question about *what concretely* these things supposedly are, of which it is said that there are always smaller and larger ones. The answer to this question varies from one interpreter to another.

> This question is usually construed as the task of listing the basic or non-basic ingredients of Anaxagoras’ ontology which essentially amounts to finding (some or all of) the referents of the often-repeated Anaxagoras’ technical general term χρήματα, i.e. “things” or “stuffs”. According to a classificatory scheme due to Patricia Curd (Curd 2007, Essay 2), the scholars can be classified depending on how permissive they take Anaxagoras to be in his conception of “things”. The views fall into three groups, ascribing Anaxagoras’ an *austere*, a *moderate* or an *expansive* ontology.
>
> Authors who adhere to the first option tend to advocate a *reductive* reading of the extant texts (based upon what Anaxagoras himself says in B 15), which results in limiting the list of basic ingredients to the *opposites* (i.e. the hot and the cold, the wet and the dry, etc.).[4] Contrary to them, the “expansionists” favour a *non-discriminative* reading on which all the stuffs (πάντα χρήματα) are treated as being ontologically on a par and for this reason it maximally expands the list of ingredients so as to include all the

[4] See, e.g. Tannery (1886) and (1887), Burnet (1975), Cornford (1975), Vlastos (1975), Schofield (1980), Inwood (1986), Marmodoro (2015, 2017).

stuffs it could possibly include, namely the opposites, the elements (fire, water, earth, and air), the seeds (σπέρματα), homoeomerous material things such as meat and gold, human beings, plants, etc. [5] Finally, "moderationists" tend to be less inclusive than expansionists whilst at the same time being less exclusive than "reductionists" (see, e.g. Curd 2007).

Curd's classificatory scheme can be nuanced even further if we raise the question what sort of stuffs Anaxagoras has in mind. For instance, reductionists typically treat the ingredients as being primarily *qualitative* in nature, i.e. they subscribe to a broadly *non-hyletic* reading of Anaxagoras' ontology. Namely, they interpret the opposites as immaterial yet nonetheless physical substance-like "quality-things" (Cornford 1975, 305) or *tropes*, i.e. instantiated properties (Marmodoro 2017, 3-4). On such an interpretation, Anaxagoras turns out to be a *bundle-theorist*: the ontologically secondary stuffs are nothing over and above mere bundles of (adequately co-located) properties. In Marmodoro's account (which can be seen as an elaboration of Vlastos' thesis (Vlastos 1950, 329, notes 39 and 61)), opposites become causally efficient physical *powers* (δυνάμεις) (Marmodoro 2017, 31-45). On the other hand, both the expansionists and the moderationists are committed to some version of a broadly *materialistic* reading of Anaxagoras' ontology. We thus find interpretations of Anaxagorean material stuffs as either (i) *particulate* in structure with each of these particles being either *infinitely divisible* [6] or *infinitely small* (infinitesimal) (Sorabji 1988), or (ii) akin to *chemical compounds*, i.e. quasi-molecular in structure.[7] According to (i) stuffs turn out to be grainy and resembling sifted powders whilst according to (ii) they blend like liquids or pastes.

For our purposes, it is important to note that practically all of these interpretations focus on the mereological aspects of Anaxagoras' theory, i.e. on the way in which he explains the mutual relations of μοῖραι (usually rendered as "parts", "portions", or "shares" depending on the translation) of his (material or immaterial) stuffs. Hence "large/r" and "small/er" refer to magnitudes (μέγεθος) of parts of Anaxagorean stuffs.

We take fragment B 3 as central to our mereo-topological interpretation of Anaxagoras' notions of small and large. Some interpreters seem to disagree and think that Anaxagoras, at least in certain contexts, takes "small" and "large" to refer to relations of an ingredient of a mixture to the mixture itself. Typically—and we are somewhat simplifying things here for the sake of exposition—they emphasise the fragment B 4b wherein Anaxagoras is reported to have said that, in the original mixture, nothing

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[5] See, e.g. Guthrie (1965), Peck (1926), Barnes (1979), Mourelatos (1986), Furth (1991), Graham (1994).

[6] See, e.g. Guthrie (1965), Kerferd (1969), and the discussion in Essay 3 of Curd (2007).

[7] See, e.g. Barnes (1979) and Inwood (1986).

> was manifest "for the mixture of all things prevented it." Now once the claim from B 1 that "nothing was manifest on account of smallness" is taken into consideration, a case can be made that smallness and the state of ingredients' being mixed are co-referential. So proposals are put forth according to which "small" refers to an ingredient's being *submerged into* the mixture so as not to be manifest, and "large" to its being *emergent from* the mixture so as to be manifest (Curd 2007, 35, 183–7); similar proposals can be found in (Inwood 1986) where "small" and "large" are rendered, respectively, as "being mixed" and "being separated out", and in (Furth 1991), where "latency" and "manifestness" are used. We cannot fully engage with these proposals on this occasion. It is worth noting though that even these interpretations cannot fully avoid understanding "large" and "small" in a mereological way, at least insofar as properties such as *being submerged* and *being mixed* seem bound to be understood in terms of the relation of *being included into* a mixture.

However the aforementioned answers to what *concretely* that which is smaller and that which is larger differ amongst themselves, it seems hardly contestable that "a smaller" and "a larger" can *generally* be understood as meaning "a smaller part" and "a larger part" of something that exists. After all, Anaxagoras himself uses "parts" (μοῖραι) when he says that "everything contains parts of everything (πάντα παντὸς μοῖραν μετέχει)". So, the Anaxagorean universe (τὸ ὅλον) becomes a *gunk* in the sense of David Lewis (1991, 20 et passim), because *gunk* is defined as that of which each part has a proper part. Moreover, since in the above quotations it is said of each part that there is always a smaller as well as a greater part, it is not only *the gunkness axiom* but also *its inverse* that is applicable to the universe as everything that exists: each part *has* a proper part and *is* a proper part of some other part (Arsenijević and Adžić 2014, 141-141).

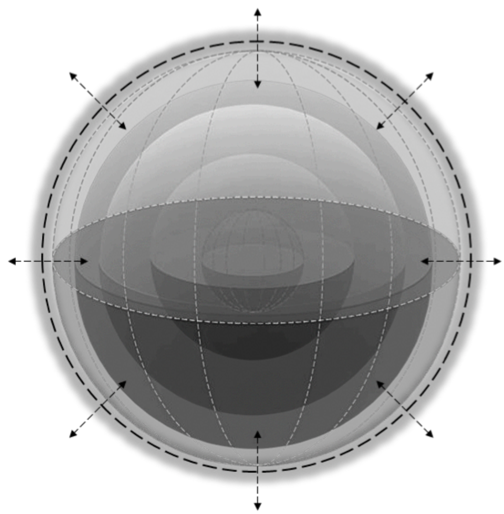

Diagram 1

Now, given the above understanding of the relation between parts, whatever they may be, the structure of the Anaxagorean universe—or at least its first approximation—formulated in purely mereo-topological terms may be represented in the following way: the universe consists of an infinite number of nested spheres (*diagram 1*)—or regions topologically homeomorphic to them—ordered by the *inclusion relation*, where every sphere contains infinitely many spheres as its proper parts and is contained in infinitely many different spheres included into each other, and where between any two spheres there is a sphere larger than one of the two and smaller than the other.

3.2. Ἀπειράκις ἄπειρα: An Infinite Number of Endless Series of Nested Parts

The above mereo-topological representation of the structure of the Anaxagorean universe turns out to be incomplete, since it represents just one endless series of parts, while *in Phys*. 460.4ff. Simplicius says that, according to Aristotle's account, Anaxagoras holds that the universe (τὸ ὅλον), as well as each of its parts (μοῖραι), contains "infinitely many [such] unlimiteds" (ἀπειράκις ἄπειρα). This additional characterization is of crucial importance because without it we could speak only of parts *included* into each other but not of parts that *lie apart* from each other or of parts that *overlap*. In *diagram 2* three different endless series are represented, where each of them contains parts that lie apart from some of the parts of the other two (like the endless series SL', SM' and SR') as well as parts that overlap with some parts of the other two (like the endless series SL" and SM", and SM" and SR"). At the same time, all represented parts of the three endless series are included into a fourth endless series (like SE), but it should be noticed that through the broadening of each of the three endless series by more parts into which the represented parts are included, the three respective points will finally be reached, after which parts of the three series start to overlap with parts of the endless series to which SE belongs.

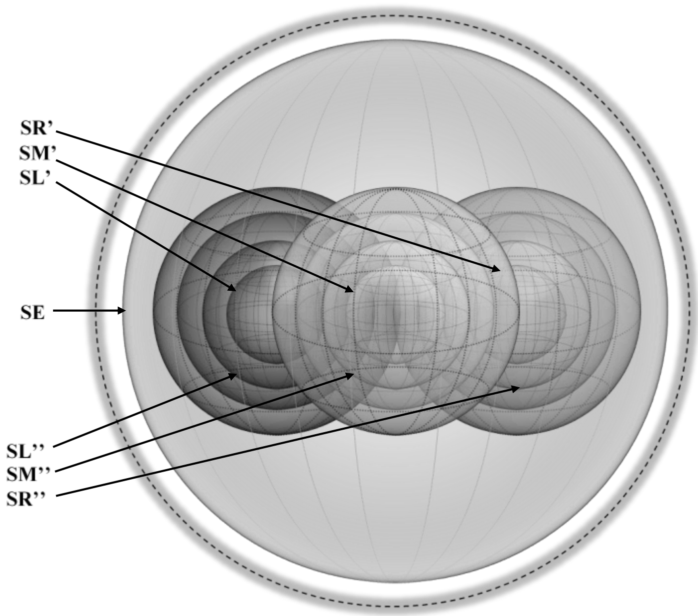

Diagram 2

What the last, completed representation in effect shows is that the whole infinite three-dimensional space is covered by the parts of the universe, for by starting from *any* part whatsoever, in *any* direction *outwards* there is an endless series of parts into which the given part is included, just as in *any* direction *inwards* there is an endless series of parts which are included in the given part. In other words, the Anaxagorean universe can be formally represented by means of a region-based system of the infinite three-dimensional continuum.[8] This is justified by the fact that, historically, Anaxagoras' theory can be seen as an anticipation of Aristotle's theory of the continuum (Ehrlich 2005, 490).

3.3. Anaxagoras against Zeno: Multitude without Proper Units

Anaxagoras' mereo-topological account of multitude represents arguably one of the first elaborate reactions to Zeno's argument against plurality. While Leucippus and Democritus used Zeno's arguments in the proof that there must be atoms, for otherwise there could allegedly be no plurality (Arist. *De gen. et corr*. 315 a15ff.), Anaxagoras rejected Zeno's assumption that any multitude whatsoever could exist only if there were proper units of which it would consist.

> The general consensus among the vast majority of scholars is that Zeno's arguments against plurality were the most important external stimulus to Anaxagoras' teachings on multitude. A venerable tradition detects in Anaxagoras an "unmistakable dependence upon Zeno" (to put it in Raven's words).[9] On the other hand, there are authors who are skeptical towards such an attitude and who think that "there is no reason to suspect that Zeno influenced Anaxagoras at all" (as Inwood claims).[10] Finally, there are even those who think that Zeno was answering to Anaxagoras.[11] Seeing how the relevant doxographical and biographical reports are imprecise enough so as not to favour any one of the aforementioned chronological orderings, our decision to side with the authors of the first group in what follows shall be justified on the basis of the internal logic of Anaxagoras' teachings.
>
> As far as the relation of Anaxagoras' and the atomists' teachings is concerned, the ancient accounts are even more uncertain which detracts modern and contemporary scholars alike from taking sides. Similarly to the previous dilemma, we also believe that here the internal logic of Anaxagoras' teachings points (rather unambiguously) to the fact that his theory was originally formulated with the intention of answering not only to Zeno but also to Leucippus and Democritus (whose theory is, again as a matter of general scholarly consensus, considered the first answer to Zeno's arguments[12]).

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[8] For a region-based axiomatization of a three-dimensional continuum see Tarski (1929); for the two-dimensional case, see Arsenijević and Adžić (2014), and Hellman and Shapiro (2018).
[9] See, e.g. Tannery (1887), Zeller (1922), Cornford (1975), Raven (1954), Kirk and Raven (1977), Guthrie (1965), Vlastos (1975), McKirahan (2010).
[10] See, e.g. Furley (1976), Barnes (1979), Schofield (1980), Inwood (1986).
[11] See, e.g. Windelband (1892), Luria (1932), Mau (1957).
[12] The *locus classicus* is Burnet (1975, 334).

In the first branch of his *double reductio ad absurdum* argument against plurality, Zeno has concluded that nothing can consist of entities without magnitude (DK 29 B 2). Since Anaxagoras never mentions such entities, we may take for granted that he agrees or that he would agree with Zeno about this. However, while in the second branch (DK 29 B 1) of his argument Zeno has concluded that the multitude cannot consist of entities having magnitude either, because the infinite divisibility of the continuum (τὸ συνεχές) precludes the existence of proper units (κυρίως ἕν) [13], Anaxagoras rejects that the existence of proper units is a necessary condition for the existence of multitude, since parts need not be taken as constituents that are *ontologically prior to* the whole. There can be multitudes *without simples* (DK 59 B 3; cf. also B 6: "the smallest [i.e. a minimum] does not exist (τοὐλάχιστον μὴ ἔστιν εἶναι)").[14] The universe is such a multitude, since it contains no simples. But is then the universe itself a complex that can be considered as a unity at all?

3.4. Anaxagoras against Anaximander: In what Sense is τὸ ὅλον a Unity?

Since according to the *inverse gunkness axiom* there is no sphere encompassing all the endless series of nested parts, the universe cannot be identified with any one sphere of the infinitely many endless spheres. The question is then in what sense τὸ ὅλον is to be understood at all. The answer to this question will complete our interpretation of Anaxagoras' teaching on the structure of the universe viewed from a purely mereo-topological standpoint. The problem is to find a meaning in which the universe could be said to be unified and in that sense something that is *one*. After all, though we translate τὸ ὅλον as *universe*, it literally means *the whole*, which leaves open the question of the sense in which *the whole* could be said to be *one* at all. This question—which arises naturally in the course of examining the very notion of "Anaxagorean universe"—had not been previously addressed in the literature on Anaxagoras, at least as far as we know. Answering it ought to be considered a desideratum for every reconstruction of Anaxagoras' cosmology which aims to be complete.

The solution can be found in the above explanation of ἀπειράκις ἄπειρα. In spite of the fact that there is not just an infinite number of parts but also an infinite number of endless series of nested parts, *any two* parts, as it is shown above, are *connected* by being contained in a third part. In view of

[13] Zeno famously proclaimed: "If you give me a unit, then I will give you multitude" (DK 29 A 16; cf. also Simpl. *in Phys*. 138.29–33 and 144.15).
[14] This represents a part of what Strang calls the "hard core of Anaxagoras' physics" (Strang 1975, 361).

this fact, all that exists is *interconnected all over the world.* It is this *interconnectedness* that makes the universe a *whole* that can be said to be something that is *one*.

The point can be illustrated by the comparison of the Anaxagorean universe with one of the possible interpretations of Anaximander's many-worlds thesis, according to which his ἄπειρον generates an infinite number of universes, the plurality of which is to be understood only as *the multiverse* and not as *the universe* any longer. So, reporting on Theophrastus' account of Anaximander's originative substance, Simplicius (*in Phys.* 24.13), Hippolytus (*Ref.* I, 6) and Ps.-Plutarch (*Strom.* 2) use the plural forms of *cosmos* and *heaven* (κόσμοι καὶ οὐρανοί), implying clearly that they are not parts of one and the same *universe* (see *diagram 3*), as the parts of the Anaxagorean τὸ ὅλον *are*, in the way in which it is explained above.

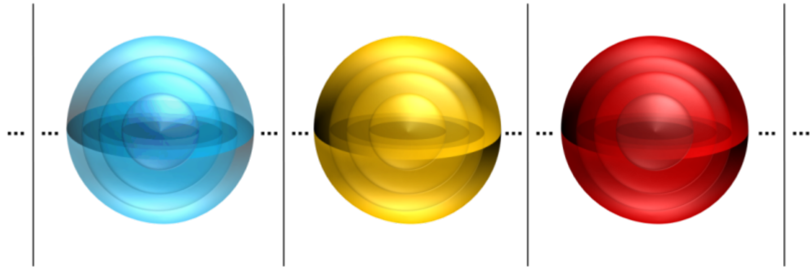

Diagram 3

> Anaximander's "plurality of worlds" thesis has received considerable attention in scholarly literature with three main lines of interpretation having been formulated: Anaximander believed in (i) infinitely many *separate single* worlds *succeeding* one another in time[15], (ii) infinitely many *co-existent yet separate* worlds[16], and (iii) a *single* world.[17] Even though historians have not reached a general consensus on the matter, the majority favour option (i) as being the closest in spirit to what Anaximander possibly could have had in mind. Option (iii) is most difficult to fit with the extant *testimonia* which explicitly speak of "infinite worlds" (ἀπείροι κόσμοι) (Aët. *Placita*, 3, 3, and Pseudo-Plutarch ad loc.). Authors

[15] This is the Zellerian tradition: see, e.g. Zeller (1922), Cornford (1934), Finkelberg (1994).
[16] This is the Burnetian tradition: see, e.g. Burnet (1975), West (1971), McKirahan (2010).
[17] See, e.g. Kahn (1960), Kirk and Raven (1977).

who favour it tend to discredit Theophrastus' account as guilty of "a false and anachronistic attribution" (Kirk and Raven 1977, 123). Namely, they believe that Theophrastus identified what Anaximander was saying with the atomists' thesis—their worlds are also infinite in number and successive (DL IX, 31; cf. also Simpl. *in Phys*. 1121.5)—and accuse the entire doxographical tradition, which relies upon Theophrastus, of being guilty of the same mistake. For the purposes of our illustration, it is not necessary to go into any minute details and take a decisive stance on the matter which of the above interpretations is the right one. For the sake of argument, we consider option (ii), since it provides a striking contrast with Anaxagoras' theory. Similarly to Anaximander's ἄπειρον which is spacious enough so as to encompass (περιέχειν) (Hyp. *Ref.* I, 6) infinitely many co-existent yet separate universes (which then makes it a multiverse), Anaxagoras' universe is as spacious so as to contain infinitely many worlds. However, it could not be said that there actually are many worlds in Anaxagoras' universe since they are *not separated* but are *all interconnected* in the manner explained above.

4. Heterogeneity in View of the Mereo-Topological Structure of the Universe

4.1. Anaxagoras against Anaximander once Again: The Universe Heterogeneity at the Basic Level

Anaximander and Anaxagoras agree that, though the world is obviously heterogeneous at the level of appearance, this represents a fact whose origin one ought to seek by appealing to a more basic level of reality. What they disagree about is that, while Anaximander assumes that the underlying ontological basis (ἀρχή), which he calls τὸ ἄπειρον, is not only *infinite* but also *qualitatively indefinite* (ἀόριστον), so that qualitative opposites (ἐναντία) are only to come into being through the differentiation of it, Anaxagoras endorses the Parmenidean *ex nihilo nihil* principle and claims that, if there is ever to be heterogeneity, it must have been already present in the original stuff from eternity (cf. Galen, *De nat. fac*. I 2, 4).

We arrive at the indefiniteness of Anaximander's ἄπειρον indirectly via Theophrastus' account of Anaximenes (ap. Simpl. *Phys*. 24.26): "Anaximenes […], a companion of Anaximander, also says that the underlying nature is one and *infinite* like him, but not *indefinite* as Anaximander said but definite." (*emphasis added*)

As far as Parmenides' principle is concerned, the wording of the canonical Latin version most people are familiar with is more similar to the principle enunciated by Lucretius in *De Rer. Nat*. 1.156 (*nil posse creari de nihilo*) than it is to fragment B 8 of Parmenides' poem Περὶ φύσεως. There

> Parmenides says that "what is" is uncreated for he does not allow us neither to say nor to think (οὐ γὰρ φατὸν οὐδὲ νοητόν) that it is created "from that which is not" (ἐκ μὴ ἐόντος). The clearest attribution of the *ex nihilo* principle to Anaxagoras is to be found in DK 59 B 10 (Scholium on Gregory of Nazianzus, *Patrologia Graeca* 36 911 Migne): "Anaxagoras discovered the old belief that nothing comes from that which is not in any way whatsoever (Ὁ δὲ Ἀναξαγόρας παλαιὸν εὑρὼν δόγμα ὅτι οὐδὲν ἐκ τοῦ μηδαμῇ γίνεται)."

But then, the question arises whether the heterogeneity at the basic level is present there *in the same way* in which it is present at the level of appearance. For, if it were so, what could the difference between the two levels consist in at all? This is how we come to the first of the two central questions mentioned in the Introduction: How does the mereo-topological structure (explained in section **3.**) help us in explaining the heterogeneity of the original mixture of everything with everything?

4.2. The Meaning of ἐν παντὶ πάντα Principle in accordance with the Mereo-Topological Structure of the Universe: The Maximal Heterogeneity in the Original State of the Universe

According to the *everything in everything* principle (ἐν παντὶ πάντα), one of the main ontological principles of Anaxagoras' cosmology, everything contains parts of everything. When applied to the *original mixture* of everything with everything, it means not only that the mixture contains everything that can ever become manifest (ἔνδηλος), but more than this, it denotes the *maximal possible heterogeneity* of everything with everything. If so, it can be *proved*, on the basis of the purely mereo-topological structure of the universe that in the original mixture (σύμμιξις) there is *no part* that is *homogeneous in itself* in regard to *any qualitative property* whatsoever. But before we turn to this proof, we must consider *why* the *maximal possible heterogeneity* is to be assumed at all.

Proceeding analytically, we may notice that, *without* any further principle in addition to the *ex nihilo nihil* principle, there is *no reason* why *any specific* distribution of heterogeneity would be assumed to be present in the original mixture. But then, in modern terminology, it is the *principle of indifference* that forces us to assume that heterogeneity at the basic level is maximal. More precisely, once it is supposed that we ought to assume nothing else but what is needed for the very existence of heterogeneity, the state of *maximum entropy* suffices, while at the same time any other state would represent some *order* that requires an additional reason or preference. *Principia praeter neccessitatem non sunt multiplicanda*. This shows why the meaning of ἐν παντὶ πάντα principle just explained is required for the proof of the non-existence of homogeneous parts in the original mixture. It should be noted that we take entropy here in the most general sense as indicating a state of *maximal disorder* of original mixture. However, one should be careful not to take this as implying that Anaxagoras' original mixture is a *dynamic system*; quite the contrary, it is a *static system* until *noûs* introduces kinematic factors, i.e. motion into it.[18]

> It is worth noting that the above type of reasoning was not unheard of in ancient Greek philosophy. Aristotle reports (*De caelo* 295 b 10–16) that Anaximander thought the Earth does not move due to its equidistance from the edges of the universe. Being so positioned, there is no reason why it should move in one direction rather than any other and so it remains at rest at the centre of the universe. Similarly, there is no reason why Anaxagoras' original mixture should be heterogeneous in any particular way different from the heterogeneity in the state of maximal entropy; the obtaining of any other heterogeneous state would require there to be some ground for imposing order (however minimal) on the default distribution of entities which comes about solely through the minimal conditions for the existence of heterogeneity.[19]

4.3. The Proof that in the Original Mixture there can be no Homogeneous Parts

Once we have adopted the above explanation of the ἐν παντὶ πάντα principle, it becomes a nice piece of exercise to formulate the proof of the non-existence of homogeneous parts, where it is in accordance with the principle of charity to suppose that Anaxagoras had some such proof in mind.

As in 3.1., where by speaking about "small" and "large" we did not have to decide between different interpretations concerning what concretely that which is small and that which is large are, so now again we do not have to worry about what "everything" (πάντα) may refer to, since Anaxagoras himself explicitly says that "everything contains parts of everything" (πάντα παντὸς μοῖραν μετέχει) (DK 59 B 6 and B 16), so that the proof of the non-existence of homogeneous parts does not depend on how

[18] In other words, it could not be said that Anaxagoras' universe *reached* a state of maximum entropy given the infinite time (ἄπειρον χρόνον) that passed before the intervention of *noûs* (cf. Arist. *Phys*. 250 b 26), during which entropy could have *gradually increased*. Thus, strictly speaking, it would be misleading to describe Anaxagoras' universe as a 'primeval chaos' in the sense of contemporary *chaos theory* as, e.g. Graham seems to do (cf. Graham 1994, 108ff. and Graham 2006, 301).

[19] Rescher (1960) provides a historical overview of occurrences of the problem of options without preference, the first of which is Anaximander's argument concerning Earth's position.

concretely "parts" are conceived, whether as opposites, seeds, tropes, properties of underlying hyletic substances (ὑποκείμενα) or as ontological ἀρχαί of any other kind whatsoever.

A good and reliable example that we shall use in the proof are *colours*, since they are one of Anaxagoras' own examples for the non-existence of homogeneous parts in the original mixture of everything with everything. He says that, though all colours are present in the original mixture, no colour is manifest (οὐδὲ χροιὴ ἔνδηλος ἦν οὐδεμία) (DK 59 B 4b). This might be taken as equivalent to saying that there is no part that is homogeneous in view of any colour whatsoever. However, it is not so, since the statement that no colour is manifest only *implies* that there is no part homogeneous in view of any colour, for if there were such a part, some colour would be manifest in the original mixture. But, as we shall see, an additional step is necessary in order to show that the implication holds in the reverse direction as well. To say that no colour is manifest is more than to say that there are no homogeneous parts in view of any colour.

Let us suppose that there is a part for which it is true that *redness* is present in each of its parts. Wittgenstein would then say that this part is *certainly* homogeneous in regard to its *colour* (Wittgenstein 1929).[20] Not so Anaxagoras! From the mereo-topological point of view, given that each part is infinitely complex, the fact that *redness* is present in each part of the given part does not preclude that some *other* colour is also present in each of the parts. As an analogy, according to Dedekind (1872) and Cantor (1895), there is no segment of the field of positive real numbers represented by a straight line endless on one side in which there are no *rational* numbers, but this does not mean that there is any segment in which there are no *irrational* numbers as well. The analogy is not jeopardised by the fact that in the case of the Dedekind-Cantor axiom the rationals and irrationals are extensionless while Anaxagoras' parts are not. After all, both rationals and irrationals can be represented as *stretches* between rational and irrational numbers respectively. The point-based and the stretch-based systems are mutually obtainable with the use of two sets of suitably chosen translation rules (Arsenijević and Kapetanović 2008).

The analogy between the case of colours and the case of numbers has to do only with the nature of infinity. The infinite complexity makes it possible for there to be *enough room* for an infinite number of red parts and an infinite number of yellow parts to be present in any part of a given part, as it is the case with the overlapping parts of the series of red spheres and the series of yellow spheres in *diagram 4* below. In the same diagram there is

[20] Cf. Ramsey (1923), Schlick (1969), Waismann (1971, 57–58).

also a common part of the red, yellow and blue spheres, in which all the three colours are present in each of its parts. So, if ἐν παντὶ πάντα *principle* implies the *maximal possible heterogeneity*, there can be no part homogeneous in view of any colour. By generalizing the result, we get that there can be no part homogeneous in view of a property of any kind whatsoever. Such a generalization is justified in light of B 10, where it is explicitly stated that what holds in the case of colours holds in the same way (τὸ αὐτὸ) in the case of other properties (e.g. "light" and "heavy").

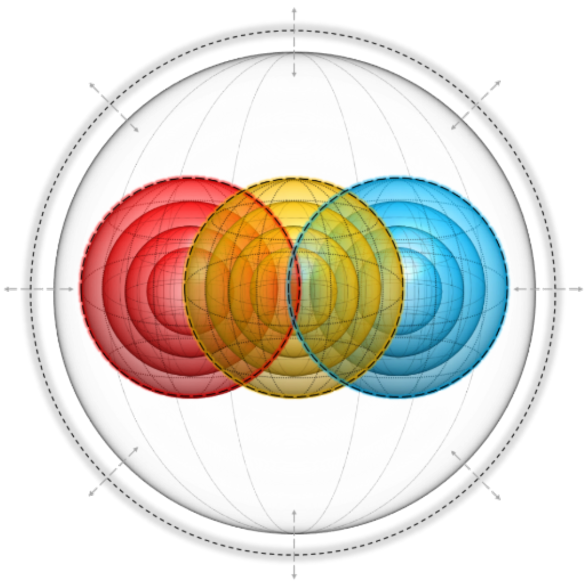

Diagram 4

4.4. The Proof that in the Original Mixture no Colour can be Manifest

As a nice illustration of the difference between Anaximander and Anaxagoras in view of the *explanation* of the fact about which they would agree—that at the basic level of reality no colour is manifest—we may consider the famous Newton's experiment (cf. *Opticks*, Book 1, Part II, Prop. II, Theor. II et passim) in which a narrow beam of sunlight, in which no colour is manifest, passes through a triangular glass prism and, after having been projected on a wall, appears as a rainbow bend of manifest colours (see *diagram 5*).

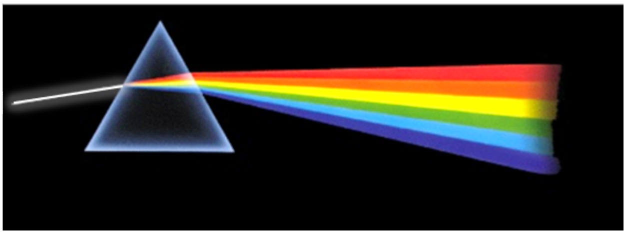

Diagram 5

If we take the beam of sunlight before it passes through a triangular glass prism as representative of the original state in which nothing is manifest and the rainbow bend of colours as representative of the level of appearance, Anaximander would say that the sunbeam originally contains *no colour at all*, while Anaxagoras would say that it contains *all the colours* that are to appear in the rainbow bend of colours. Now, independently of the explanation of *how* and *why* non-manifest colours *become* manifest, Anaxagoras has to explain in the first place *why* colours are *not manifest* in the sunbeam, given that they are presumably present in it. As we have suggested above, the very fact that in the original mixture there are no homogeneous parts whatsoever does not suffice. Namely, one could use the idea of Empedocles' physics and say that one (monochromatic) colour could be *predominant* and as such *manifest* in the beam of sunlight. In order to show that this is not possible according to Anaxagoras' assumptions, we have to compare his physics with the physics of Empedocles.

For our purposes, it is not important to work out precisely and decide definitively whether Empedocles influenced Anaxagoras or vice versa, or which of the two philosophers is older and which younger. These questions are somewhat controversial, especially because of what Aristotle says in *Met*. 984 a 11, namely that "Anaxagoras was prior (πρότερος) to Empedocles in age yet posterior [ὕστερος: literally, later] in his activities". It suffices that Empedocles' theory could have been known to Anaxagoras (and vice versa) without there being any need for assuming any interdependence or interaction between the two theories for our comparison to work. However, since we intend to occasionally compare certain aspects of Anaxagoras' theory with those of Empedocles' theory for illustrative purposes, it is necessary to state the basic tenets of Empedocles' physics in order for such illustrations to function as intended. Here as elsewhere, we do not wish to engage in various scholarly controversies but rather to provide a minimalist account of those aspects of

> Empedocles' theory which are sufficient for elucidating our points about Anaxagoras.
>
> The central part of Empedocles' poem is DK 31 B 17. There we find out that the basic items of Empedocles' ontology are the elemental "roots" (ῥιζώματα)—fire, water, air, and earth—and that they are involved in a continuous and infinite cycle governed by two cosmic powers, Love and Strife (i.e. the attractive and the repulsive force, respectively). The cosmic cycle is divided into four stages, the first being the so-called "triumph of Love", i.e. an ideal limit to the process of gradual mixing and interpenetrating of the roots represented by a sphere, and the last being the "triumph of Strife" where the roots are completely separated as if the sphere were cut apart in four sections. Contrary to Barnes (1979, 242–243), the triumph of Love should not be conceived as the "homogenous sphere"—the actually completed mixing of the roots—but only as a never-completing process of their mutual interpenetration. The other two stages are transitional between these two extremes. It is important to note that even though during the triumph of Love the roots "run through each other" (DK 31 B 21), they nonetheless remain qualitatively distinct no matter how thorough the mixture might be — "*they are always unchanged in a cycle*" (DK 31 B 17, *emphasis added*) in the sense that they can *never* completely interpenetrate so as to become co-located. There are no traces of any other elements in, say, water. In other words, water is not predominantly water but water through and through. Any interpretation that does not take this into account ought to be rejected.

The crucial thing is that Empedocles' cosmology doesn't allow the state of *maximum entropy*. Namely, however fire, water, earth, and air as the heterogeneous "roots" (ῥιζώματα) of everything were mixed, there could be no part of the universe in which they would be co-located. The roots may be mixed *more and more again*, but *never absolutely*, since there where one of them is present, no other can be. However, the mereo-topological structure of the Anaxagorean universe allows the state of *maximum entropy* in which there is no part not containing *everything*. In such a state, there can be *no predominance in quantity* (ἐπικρατεῖν πλήθει) of any property (colour in our case), since (the number of heterogeneous parts being *infinite*) any two sets such that the members of one of them and the members of the other are heterogeneous amongst themselves are *equinumerous*.

> Interpretations according to which predominance is understood as predominance in quantity are not rare. The above discussion suffices to show why such views cannot be satisfactory. Surprisingly, this kind of view can be found even in authors who recognize the gunky nature of Anaxagoras' universe, e.g. in the works of Anna Marmodoro (2015, 2017). Marmodoro tries to show that Anaxagoras' infinitism is not incompatible

with the predominance of quantity with the use of the example of prime numbers whose density is greater at initial segments than it is in the further expansion of the infinite series of natural numbers (Marmodoro 2017, 97). But this example is inadequate in the context of Anaxagoras' cosmology because it concerns the comparison in density between different *segments* that have *finitely many* members, while each part of the Anaxagoras' universe contains only parts which themselves presumably contain an *infinite* number of parts.

4.5. The Thoroughgoing Infinitism: From the Mathematical Principle of Space Isotropy to Anaxagoras' Fractal Structure of the Physical Universe

One of the basic assumptions practically operative during the whole history of Greek geometry can be called the *principle of space isotropy*. Generally, this principle refers to *uniformity of space* in all directions, which is, especially in the case of Greek geometry, essential for the similarity between any two parts of the same dimension in view of divisibility and magnitude, be these parts one-dimensional segments, two-dimensional areas or three-dimensional regions. The principle, in the context of Greek geometry, amounts to the following two tenets. First, all segments, areas and regions are endlessly divisible no matter how division is performed, meaning that there are no indivisible parts of entities of any dimension whatsoever. Second, in spite of the infinite divisibility, there are no parts that are either infinitely smaller or infinitely larger than any given part of some geometrical entity of the same dimension, so that all parts that are of the same dimension belong to one and the same category: there are no infinitely small just as there are no infinitely large parts.[21]

Now, one of the nicest *reductio ad absurdum* arguments in the whole history of Greek philosophy might appear as being directed against the principle of isotropy. This is the proof of Leucippus and/or Democritus in favour of the existence of atoms. The proof is reproduced in detail by Aristotle in *De gen. et corr.* 315 a 15ff. It runs as follows.

Let us suppose, following the principle of isotropy, that a given body is divisible everywhere (σῶμα πάντη διαιρετόν), and also that it is *simultaneously* (ἅμα) divided everywhere where it is divisible. What will remain at the end of such a division? It is impossible that what remains are some entities of a lower dimension, because this would mean that the original body could be recomposed out of them, which is precluded by what Aristotle calls *Zeno's axiom* (*Met.* 1001 b 7). But it is also impossible

[21] The second tenet is codified by what Stolz has called *Archimedes' axiom* (Stolz 1881 and 1883).

that what remains are entities of the same dimension as the original body was before division, since they would then be further divisible, which is contrary to the hypothesis that the original body has been divided everywhere. So, in order to avoid the contradictions along both branches of the argument, we must assume that, contrary to the hypothesis, the body is not divisible everywhere.

Aristotle praises the argument as the attempt to reply to Zeno's proof against plurality by questioning some other, tacit hypothesis instead of the main hypothesis that the plurality exists, but he considers the atomist argument not conclusive either. His own solution is that what ought to be rejected is not the hypothesis that the given body is divisible everywhere but only that it is divisible *simultaneously* (ἅμα) everywhere where it is divisible. In such a way the rejection of the principle of isotropy is avoided.

In his comment of the atomists' argument in *De caelo* 303 a 20 and 306 a 26, Aristotle explicitly accuses them of "coming into conflict with our most exact science, namely mathematics" which could be understood as a criticism directed against their apparent violation of the principle of space isotropy. However, according to the interpretation Vlastos has offered to Furley (in Furley 1984, 513, note 17), and with which we agree, the "conflict" is to be understood as the incongruence between *mathematics* and *the physics of Leucippus and Democritus* rather than as the incompatibility between their understanding of mathematics with one of the basic mathematical principles, namely the principle of space isotropy. After all, Democritus was known as a great mathematician[22], and it is highly unlikely that he wanted to question one of the basic principles of geometry.

We come now to what is our main concern, that is, to what Anaxagoras has to say about the relation between mathematics and physics. What we have said in 3.1. and 3.3. clearly implies that Anaxagoras does not want to question the principle of isotropy in mathematics. So, the question is only how he would react to the above argument of Leucippus and Democritus, independently of whether we assume that he was acquainted with it or not. Given that he often speaks of parts of the universe in the unqualified sense and that in his teaching there is nowhere any trace of atomism, we can take for granted that he would not agree with the conclusion of the atomist *reductio ad absurdum* argument. And then, given that he speaks nowhere of the extensionless entities of any kind whatsoever, the only reasonable option is that he would say that the set of alternatives offered in the

[22] As is evidenced by the list of his mathematical works given in DK 68 A 31.

conclusion is *not exhaustive*. He would simply say that by any division whatsoever one can get nothing else but something that is also divisible.

What we may infer from the above analysis of what Anaxagoras should have to say on the basis of his teaching taken as a whole is that he is, contrary to all other Greek philosophers including Aristotle, a *thoroughgoing infinitist*: in Anaxagoras the validity of the *principle of isotropy* is not restricted to *mathematics*, but it holds in relation to *physics* as well. More elaborately put, all parts of the universe in its original state are similar not only *mathematically*, in the sense of *space isotropy*, but also *physically*, in view of *what* they contain, since each contains everything that any other contains.

> It is strange how many authors tend to classify Anaxagoras among precursors of non-Archimedean mathematics when he is obviously a *thoroughgoing anti-infinitesimalist* (so much so that he has been described as anticipating Bolzano and Cantor (Sinnige 1971, 129–137). However, Raven (1954) and others[23] see him as a revolutionary who introduces the notion of the infinitesimal. Such interpretations are probably motivated by fragment B 1[24] where the words ἄπειρα σμικρότητα and τὸ σμικρὸν ἄπειρον ἦν appear which are usually rendered as "infinite smallness" or "infinitely small". However, this should be read in light of what Anaxagoras says in B 3: the fact that "of the small there is always a smaller" does not entail that there is actually anything which would be infinitely small. The "infinitely small" is to be understood in the sense of containing infinitely many smaller parts (since "there is always a smaller"), and not as being itself infinitely small, i.e. infinitesimal.
>
> Interestingly, even though Vlastos originally[25] endorsed the non-Archimedean reading of Anaxagoras, he later came to endorse the opposite view, which we share. We reproduce the remarks from the revised version of his paper *in extenso* (Vlastos 1975, 341, note 1): "I have made no substantive changes in the text, with one exception: I have eliminated references to 'the infinitesimal' and even to 'the infinitely small' in Anaxagoras. As I have since come to see (in the course of trying to thread my way through Zeno's paradoxes) the notion of 'the infinitesimal' is a confused one, and even the expression 'infinitely small' is misleading. There is some excuse for using the latter, since Anaxagoras himself said practically the same thing in such a phrase as τὸ σμικρὸν ἄπειρον ἦν. There is none whatever for using the former, for there is absolutely no basis in the

[23] Cf. also Kirk and Raven (1977), Guthrie (1965), Sorabji (1988), Gershenson and Greenberg (1964).
[24] It is interesting to note how reading of B 1 *in isolation* causes similar problems to those we resolved in 3.1.
[25] "The Physical Theory of Anaxagoras" appeared in 1950 and was included in Allen and Furley's (1975) collection.

> fragments for thinking that Anaxagoras was guilty of the confusions epitomized by that term. *In B 3 he gives us an admirably precise statement of what he means*." (*emphasis added*) In addition to this, most historians of ancient Greek mathematics would agree that it was Archimedean in that it contained no references whatsoever to infinitesimals.

Now, given that *any* part of the universe contains infinitely many parts not only in the purely mereo-topological sense but also if these parts are taken as containing the physical heterogeneity of the universe as the whole, then given the principle of *maximal heterogeneity* that holds for the *original mixture of everything with everything*, the universe in its original state is *fractal* in the sense that *any two parts* are *similar to each other* not only in view of the *smaller-larger* relation that holds between the parts contained in them—"the parts of the large and the small are equal in quantity" (ἴσα πλῆθος ἐντοῖσι μείζοσί τε καὶ ἐλάσσοσι)—but also in view of the fact that *every* part is similar to *any* other part in regard to *what* they contain.

This conclusion can be confirmed by a direct quotation from Simplicius (*in Phys*. 460.4ff. = DK 59 A 45), where he speaks of Aristotle's account of *homoeomeries*, i.e. of "parts similar amongst themselves". It is explicitly said there that each of the homoeomeries (ἑκάστην ὁμοιομέρειαν) is similar to the whole (ὁμοίως τῷ ὅλῳ) in that it contains everything within it (πάντα ἔχουσαν ἐνυπάρχοντα). This is represented in *diagram 6*:

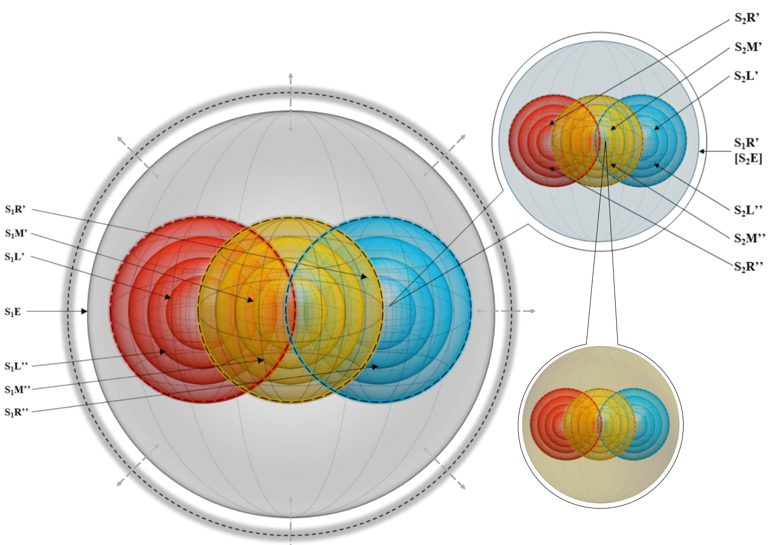

Diagram 6

Even though both Aristotle (see, e.g. *Phys*. 203 a 19–33) and the entire doxographical tradition unanimously ascribe some kind of "homoemerism" to Anaxagoras and repeat it (*ad nauseam*) as a defining feature of his theory, many modern authors deny that there can be "Anaxagorean homoeomerism". These include, among others, Peck (1926), Guthrie (1965), Furley (1967), Graham (1994), Mathewson (1958) and Curd (2007). Others, such as Barnes (1979) and Teodorsson (1982), just deny it the role of a fundamental principle of Anaxagoras' physics. While it is true that the term ὁμοιομερές and its cognates do not appear in the extant fragments (B-fragments), and that it was probably coined by Aristotle, this does not mean however that Anaxagoras could not have been an *ante litteram* "homoeomereologist" (the term is Lanza's (1966, 50)): he "could have articulated the concept of homoeomereity without having used Aristotle's terminology" (Sisko 2009, 92).

It is important to note that homoeomerism is primarily a mereo-topological notion, since it deals with the like-partedness in terms of the larger-smaller, the parthood and inclusion relations, as well as in terms of spatial partitioning, i.e. infinite divisibility.[26] As such, it can also be formalised by means of some region-based theory as discussed in 3.2. Interestingly, by basing spatial regions upon spheres, the above representation (*diagram 6*) also agrees with Simons' formal account of homoeomeries (cf. Simons 2003, 220). Such a characterisation of homoeomerism would be purely *quantitative*.

However, there is also a *qualitative* aspect to homoeomerism, since the like-partedness also has to do with the likeness-in-kind of the parts and the whole to which they belong. In other words, Anaxagoras' homoeomerism demands that parts and wholes be similar in view of non-quantitative properties as well (this explains the appearance of colours on *diagram 6* above[27]). In light of the everything-in-everything principle, this means that the universe as a whole and any of its spatial sub-regions (parts) are homeoemerous, since they are *exactly alike* in view of *all* quantitative and qualitative properties. This also shows why translating ὁμοιομέρειαι as "homogenous parts" or "homogenous stuffs" is *wrong*[28] — namely, just as the original mixture is *maximally possibly heterogeneous*, so are all of its parts as well.

It is interesting to note that some authors, like Anna Marmodoro, consider the previously described structural complexity of Anaxagorean universe in its original state as "defying representation", "incomprehensible" or "unintelligible" (Marmodoro 2017, 112–113). However, in light of Sextus

[26] This has been emphasized already in Sisko (2009) and in Sharvy (1983).
[27] Here, as well as in 4.4., colours are taken as an illustration standing for all other properties.
[28] As is done in, e.g. Curd (2007). It could be said that the mixture and its parts are *quasi-homogenous* since they are thoroughly mixed.

> Empiricus' distinction (which can be taken as *locus classicus*: *Adv. Phys*. 390–392) between objects that are *perceptible* (αἰσθητά), *imaginable* (φαντασιωτά[29]) and *intelligible* (νοητά), it is not clear why the structure of Anaxagoras' universe and its parts would not be said to be intelligible in spite of the fact that it is neither perceptible nor imaginable. Namely, it could be stated to be unintelligible only after it were proved that the very notion of such a structure is self-contradictory. But, given that region-based mereo-topology and fractal geometry, which serve as *mathematical* models of Anaxagoras' physics, are not inconsistent, being as such intelligible, there is no reason why the same would not hold for Anaxagoras' *physics* as well, i.e. for his theory of the fractal and homoeomeric physical universe, in its original state at least.

So, if the universe were counterfactually broken into whatever number of parts, they would all be completely similar amongst themselves. In *that* sense the universe can be said to be *fractal*. The principle of *fractality of the physical universe* is in *congruence* with the principle of *space isotropy*, and in that sense Anaxagoras is the only Greek physicist who, due to his thoroughgoing infinitism, has made physics completely congruent with mathematics.

The only aspect in which the parts obtained by a counterfactual breaking-apart of the universe were not similar to the original whole consists in the fact that for them the *inverse gunkness axiom* would not hold any longer. But this follows analytically from the fact that they are *proper parts* of the universe, while the universe as a *whole* (τὸ ὅλον) is not a proper part of anything. However, for any part being a *proper part of the universe* and not as something obtained by a counterfactual breaking-apart of the universe, the *inverse gunkness axiom* does hold, since there is no largest sphere encompassing either a part of the universe or the universe as a whole.

> Similarly to the case of Anaximander discussed in 3.4. above, there are authors who attribute to Anaxagoras the "plurality of worlds" thesis on account of DK 58 B 4a, with the usual interpretation viewing Anaxagorean universe as containing multiple separate yet co-existent worlds (see, e.g. Burnet 1975 or Barnes 1979). However, such a reading seems to flatly contradict both what Anaxagoras himself emphasises in fragment B 8—"in *the one* cosmos (ἐν τῷ ἑνὶ[30] κόσμῳ)" (*emphasis added*)—and what Aristotle and Simplicius attribute to him, namely that he only believes and, consequently, speaks of *a single* cosmos only (ἕνα τὸν κόσμον) (cf. Arist. *Phys*. 250 b 18ff. and Simpl. *in Phys*. 178.25). So, even Simplicius who, as

[29] From the verb φαντασιόω employed by Sextus himself ad loc.
[30] ἑνὶ can here be taken as indicating either *uniqueness* or *internal unity* of a given cosmos. We see no reason not to take it in the first sense.

Gregory (2007, 109) rightfully notices, tended to find more many-worlds theorists than there actually were, nowhere classifies Anaxagoras as one of them. The fundamental obstacle to any many-worlds interpretation of Anaxagoras, which constitutes a s*ufficient* reason for rejecting it, is the aforementioned fact that the statement about the existence of separate "worlds" would violate the *inverse gunkness axiom*, which guarantees the interconnectedness of all parts of the universe taken as a whole (τὸ ὅλον) (see 3.4. above). In other words, one cannot maintain the many-worlds interpretation without thereby sacrificing a basic tenet of Anaxagoras' teaching on multitude (B 3). An additional reason for rejecting such a reading would be that Anaxagoras simply could not *individuate* multiple co-existent worlds within the universe as a whole (τὸ ὅλον) if these worlds ought to be *all exactly alike* (Vlastos 1975, 359).

However, there is another line of interpretation which allows us to speak—albeit *only metaphorically*—about many worlds in Anaxagoras without thereby contradicting any of the points which we previously established. This so-called "Leibnizian reading" of Anaxagoras' many-worlds thesis has been recently advocated by John E. Sisko (2003). The basic idea is that Anaxagoras can be seen as endorsing an early version of the Leibnizian monadological thesis according to which there exist "worlds within worlds to infinity" (*mundi in mundis ad infinitum*) (cf. Leibniz A VI, 2, 226). Such an interpretation essentially depends on the *fractality* of the universe in the above explained sense. The following explanation should be taken as holding at least for the original mixture without thereby suggesting anything about the state after the intervention of *noûs*.

As Strang has justly emphasised (by focusing especially on fragment B 6), complexity for Anaxagoras is not a function of size (Strang 1975, 366). Put into more technical terms, this basically corresponds to an important feature of fractals, namely *invariance under scaling* (i.e. transformation of scale). This gets us to the most important feature of fractals — *self-similarity*: fractals which are invariant under ordinary geometric similarity are called self-similar (Mandelbrot 1982, 18).[31] For Anaxagoras, structural complexity is *recursive* all the way up and all the way down. The universe as a whole and all of its parts are structured in exactly the same manner. In effect, the notion of self-similarity in this sense also corresponds to the previously explained notion of homeomereity: if we were to zoom into any part of the universe with a theoretical microscope (illustrated in *diagram 6* above) we could observe that it is exactly alike in every respect—that is, not only in view of all structural and quantitative but also in view of all qualitative properties—to the universe as a whole. However, this should not be taken as suggesting that there is ever more than one world in the Anaxagorean universe. Every part of the universe is a "world" only metaphorically in virtue of the universe as a whole being self-replicating

[31] Interestingly, Leibniz was probably the first to study self-similarity.

and everywhere self-similar. And, finally, as Grujić rightfully notes, fractality is congruent with isotropy (Grujić 2002, 51).

5. Necessity of Existence of two Different Successive Stages of the Universe: The Intervention of *Noûs* as the Singular Cosmic Event

By generalising the example concerning colours, we have concluded in 4.4. that in the state of maximum entropy of the original mixture there is no property of any kind whatsoever that could be manifest, which is the consequence of the *mereo-topological structure* of the universe *and* the *maximal possible heterogeneity* assumption. This in effect means that, if different "things" (χρήματα) present in the original mixture, however concretely specified, are to become manifest, this can happen only in a state of the universe which is *radically different* from the state of maximum entropy. To explain in which way the difference between the two states is to be understood exactly, and how it is brought about through the intervention of *noûs*, represents a big task which lies outside the scope of this paper. But, without going into detail, we may put in general terms in what sense the difference must be *radical* and why the two states must be *chronologically successive*, thus vindicating speaking about them as different *stages* of the universe. The comparison with Empedocles' cosmology may be of use again.

As we have seen in 4.4., Empedocles' cosmology doesn't allow the state of the *maximal possible* heterogeneity within the parts of the universe, because it is impossible for any two "roots" of everything to be completely co-located in one and the same part. Any complex part actually contains some finite number of strictly separated parts, each of them occupied by a single "root", and though this number *can always* be greater than it actually is, it can *never* become infinite. So, the infinity related to the number of parts is only *potential* in Aristotle's sense. Consequently, any difference between any two states—no matter how close one of them is to the triumph of Love in view of the greatness of the number of heterogeneous parts present in the mixture of "roots" in any of the parts of the universe—must always be a *matter of degree*. Contrary to this, in Anaxagoras' cosmology the original mixture is *the single state* in which entropy is maximal and from which any other state differs *radically* and not only in degree. Due to this radical difference, any state of the universe that is not the original state may occur *only after* the original state, belonging as such to the *second stage* of the universe viewed chronologically.

The transition from the first stage, which was the state of the universe from eternity (ἐξ αἰῶνος), to the second stage, in which what was undistinguishable in the original mixture is to become manifest (ἔνδηλος), is caused by *noûs*, whose intervention *as the singular event* in the Anaxagorean cosmogony is in that respect similar to the *big bang* in modern cosmology.

> What makes Anaxagoras' cosmogony *unique* in the history of ancient Greek cosmogony is exactly the postulating of such a singular event. This is evidenced by both Aristotle (*Phys*. 187 a 21ff.) and Simplicius (*in Phys*. 154.30) who agree that for Anaxagoras the "cosmos was born *only once* (ἅπαξ γενόμενος ὁ κόσμος)" (*emphasis added*), i.e. it *began* at some instant (νῦν). Interestingly, Aristotle even criticises Anaxagoras on this account in *Phys*. 252 a 15ff., claiming that such a singular event is "no longer to be considered as a work of nature (οὐκέτι φύσεως ἔργον)"; in other words, it is non-natural and inexplicable. Thus, Anaxagoras might be seen as the first proponent and Aristotle as the first opponent of Big Bang type of theories (cf. Gregory 2007, 172). This salient feature of Anaxagorean cosmogony is usually not sufficiently emphasised in the relevant literature but rather only incidentally touched upon (case in point being Gregory 2007). However, Cleve proposed a reading similar to the one developed in this paper already in 1917 — to him it was "evident" that "cosmogony had to start from one point" and that Anaxagoras in thinking that was "alone, almost[32], among the philosophers of ancient Greece" (cf. Cleve 1973, 45, 132ff.).[33] Cleve explicitly says that "the 'beginning of cosmopoeia' (ἀρχὴ τῆς κοσμοποιίας) *must have been meant as a true beginning in time*." (Cleve 1973, 134; *emphasis added*)

The above results obtained analytically from Anaxagoras' teachings on multitude and on heterogeneity taken as a whole should be faced with the following passage from Simplicius (*in Phys*. 461.10–16):

> […] Anaxagoras […] gave a riddled double account of the [world] order: the one general (ἡνωμένην), intelligible (νοητὴν), always present and time-independent (οὐ χρόνῳ), (for it does not change in time), subsistent both in view of what is (οὐσίας) and in view of what can be (δυνάμεως); the other distinguished from the former (διακεκριμένην ἀπὸ ταύτης) but in accordance with it (κατὰ ταύτην), which comes into being due to the demiurgic *noûs* (ὑπὸ τοῦ δημιουργικοῦ νοῦ).

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[32] This restriction is due to the fact that Simplicius also mentions Archelaos and Metrodoros of Chios as advancing similar theses to Anaxagoras' in regard to cosmogony (cf. *in Phys*. 1121.21).
[33] Interestingly enough, Cleve had no modern cosmological model such as the Big Bang that could have motivated his interpretation. Lemaître proposed it in 1927 and the very term "big bang" appeared only in 1949.

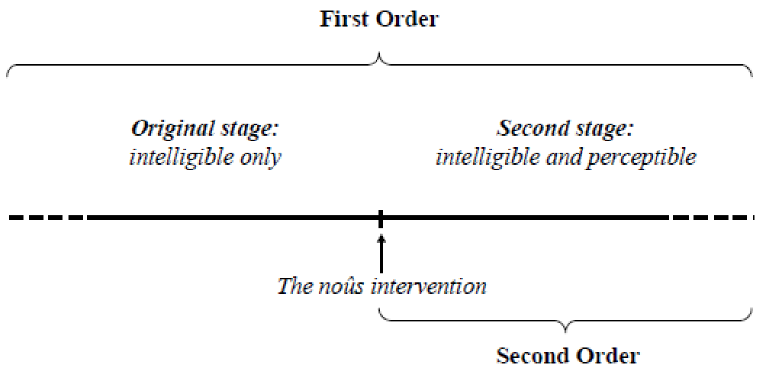

Diagram 7

In view of our reconstruction, the explanation of the "riddled double account of the world order" is straightforward (see *diagram 7*). The "first order" is "general" (or "uniform") because it concerns the purely mereo-topological structure of the universe, which is "always present and time-independent" due to the fact that the relation between the universe as a whole and its parts as well as the relations between its parts remain *the same* independently of how the cosmic stuff is distributed or redistributed in view of non-manifest or manifest heterogeneity. This explains, at the same time, why the "second order" is "in accordance with the first one" in spite of the fact that it concerns the second stage of the universe at which what was undistinguishable has become manifest, and which makes the second order "distinguished" from the first one. And then, the "generality" of the first order along with the relation between the two orders explains why the "first order" is "intelligible" also in regard to the first stage of the universe where no heterogeneity is manifest, since its intelligibility does not depend on perceptual distinguishability. And finally, the fact that the difference between two orders is not a matter of degree, there must be a singular event that, due to the "demiurgic *noûs*", separates the two successive stages of the universe.

Simplicius ascribes double world order to Anaxagoras also in *in Phys.* 157.17 and *in Cael*. 608.32. Some of Simplicius' remarks in the surrounding text have been taken by some interpreters, like Curd and Schofield, as indicative of his Neoplatonic interpretation of Anaxagoras, which they themselves consider unacceptable (Curd 2007, 214) and "hopelessly ahistorical" (Schofield 1996, 5). In Simplicius' differentiation between what is *noetic* and what is *perceptible* Curd finds speaking of two different "ontological levels" (Curd 2007, 212, 214). In similar vein, Schofield finds in *in Phys.* 34.18ff. a picture on which Anaxagoras posits an original *ur*-condition of unity, a "purely intelligible *kosmos*" that ensues from the original *ur*-condition, and finally our perceptible *kosmos* as a derivation of the intelligible *kosmos* (Schofield 1996, 4).

However, there is no good reason to take Simplicius' speaking of "the ordering that is [only] intelligible" and "that which is perceptible" as relating to two different *ontological levels*, since he himself *explains* this rather *epistemological* than ontological difference by claiming that in the former "all things were together" while in the latter "they have been made separate from that unification by demiurgic *noûs*" (*in Cael*. 608.32ff.). This can be completely understood with our explanation given above and illustrated in *diagram 7*, where the difference in question is represented "*horizontally*", as the difference between chronologically successive stages and not "*vertically*", as the difference between ontological levels. After all, the alleged Neoplatonic rendering of Anaxagoras does not fit well with what Simplicius states elsewhere about Anaxagoras' cosmology. For instance, as we have seen, in B 2 he ascribes to Anaxagoras the claim that "air and aether were separated off (ἀποκρίνονται) from the all-encompassing multitude (ἀπὸ τοῦ πολλοῦ τοῦ περιέχοντος)". Separation of air and aether appears to be the initial separation from the original mixture (according to B 2) so that air and aether "covered all things" (according to B 1 and B 2 taken together), and the text suggests that such a separation was an eminently *hyletic* affair and not some Neoplatonic emanation from an intelligible realm.

And finally, if we wanted to reject the Neoplatonic interpretation and yet at the same time avoid the suggested interpretation according to which there must be the *singular event* in Anaxagoras' cosmology—something that is indeed incongruent with the "spirit" of the whole Greek philosophy—we might take the account of the "original state" as the *counterfactual description* of what the universe *would look like if* all the things were mixed together, from which *de facto* any state of the universe is always more or less different.[34] But, no matter how ingenious and exciting this interpretation may make Anaxagoras appear to today's analytic philosophers, it is of course highly unlikely that Anaxagoras actually made such a proposal. On the other hand, as for those who have an affinity for finding in Anaxagoras' teaching early anticipations of significant notions and ideas of contemporary physics, they can be said to be right when taking the idea of the *noûs* intervention as *the singular event* as a precursor of the Big Bang theory.

[34] Such an interpretation has been advanced in Fränkel (1955) and Vlastos (1959).

6. Concluding Remark

In the present article we have tried to give a consistent and historically authentic answer to the first of the two central questions concerning the relation between Anaxagoras' teachings on multitude and on heterogeneity, which explains why in the original mixture nothing was manifest in spite of the fact that everything that has become manifest at the second stage of the universe, resulting from the intervention of *noûs*, must have been already present there from eternity.

The second central question is complementary to the first one. How *the manifest heterogeneity* is possible at all, given that the first cosmic order is general, remaining the same forever? The answer to this question is tricky and requires an insightful philological in addition to an inventive philosophical analysis, because Anaxagoras is explicit in B 6 that "*if it is not possible* that there is the smallest, *it would not be possible* to be separated (χωρισθῆναι) or to come into being by itself, but just as at the beginning (ὅπωσπερ ἀρχήν) so also now (καὶ νῦν), everything *would be* together (πάντα ὁμοῦ)". A way must be found to reconcile this claim with the general principle stated in B 3, that "of the small there is not the smallest, but always a smaller". Postponing the answer to this difficult question to some later occasion, we may only note that, if Anaxagoras is to be interpreted in a consistent manner, then either we are wrong when claiming that B 3 states the principle of the general order or there is some way to explain in what sense the necessity of "the smallest" at the level of appearance, stated in B 6, does not contradict the general principle stated in B 3.

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