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Mathematical Modelling of Natural and Social Context at Preschool Level of Education

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Abstract

This work indicates the importance of mathematical modelling, which results in the adoption of mathematical concepts through development of problem-solving abilities and logical-mathematical thinking. Teaching methodology literature perceives mathematical modelling as a frame for learning mathematics within which other sciences and scopes of life are studied through mathematical contents. For preschool children, it is the world they live in, where each new situation represents a real natural problem. Mathematical modelling is another social experience and novelty in their lives. At an early age, modelling is a revealing activity that entails mathematisation of a real situation and creation of a model. This enables children to focus their attention on different types of presentation of the variable relations. This paper presents the research of preschool teachers' attitudes about values of mathematical modelling, its appropriateness for early childhood, and possible application in preschool mathematics learning. By using the questionnaire on the sample of 197 subjects, the obtained results confirm preschool teachers' positive attitude towards modelling, but also the insufficient knowledge about theoretical and practical trends related to this topic.

Key words: *early childhood; learning in context; logical-mathematical thinking; problem-solving abilities; visual models.*

Introduction

The preschool age is a period characterized by readiness and ability to receive various types of experience and different types of learning. When it comes to formation of mathematical concepts and development of thinking processes, whether these abilities will result in quality or not depends primarily on the approach to mathematical education at preschool age.

Starting from the fact that all initial mathematical concepts occur at the perception level and that children of this age form the notions of elementary mathematical concepts most easily, through contact with the objects from their surrounding reality, special emphasis is placed on the need for learning mathematical contents through the context of realistic situations. Contemporary theories of mathematical education recognize mathematical modelling, by which applicable mathematical knowledge is built, as the most efficient method of learning in context. Works of many authors are evidence that already at an early age children are capable of adopting the model approach to learning mathematical contents.

The book published by the NCTM (the National Council of Teachers of Mathematics), Principles and Standards for School Mathematics (2000), emphasises the need for implementing mathematical modelling in preschool education since children of early age have the ability and potential to learn complex mathematical concepts (Baroodi, 2004; Sarama & Clements, 2009). The research carried out by English and Watters (2005) confirms the ability and readiness of early age children to participate actively in the modelling process. Fox (2006) indicates that although children may not have developed the skill of writing in preschool institutions, they could participate in the development of a model. He notices the connection between mathematical modelling and high-quality learning of mathematics at an early childhood. Bredekamp and Copple (1997) suggest that all the aspects of child development are closely connected, i.e. that development in one sphere influences the development in other spheres. Therefore, the activities of modelling may potentially help the overall development of a child and contribute to the integrated nature of plans and programmes for early childhood. Lingefjard (2006) believes that one of the most important goals of mathematical education is actually children's understanding of the value of mathematical modelling.

It is evident that mathematical modelling may have an important impact on the early mathematical development. That is why it is necessary to provide children with quality access to rich and challenging mathematical experiences, which would help them in future. The experiences of mathematical modelling correspond to current perspectives of learning in early childhood and offer unlimited possibilities for children's mathematical development.

With the aim of highlighting the issues referring to the importance and role of mathematical modelling in early childhood, the possibility and manner of application in forming elementary mathematical concepts, and the need for placing the focus of preschool curricula on mathematical modelling, an empirical research has been conducted. We believe the results will reveal the paths and manners of implementing mathematical modelling in educational activities and give contribution to this, still insufficiently explored, topic when it comes to preschool age.

Mathematical Modelling in Early Childhood

Modelling has been a frequently used term in previous years. Its notion designation, when it comes to education, has been a topic of study for many scientists. Popp (1970) defines modelling as a simplified presentation of a complex and complicated structure with the aim of its understanding and solving the emerging problem. Kvaščev (1978, p. 9) defines modelling as a construction of natural or artificial structures for studying objects of knowledge, which appears (as an object, process, situation, etc.) as analogous to another phenomenon, object, process, situation etc. whose study is difficult or impossible. Greer (1997) sees modelling as a construction of the connection between mathematics as a way of designing our real world and mathematics as a group of abstract, formal structures. According to Petrović (2001, p. 90), modelling entails a mental-theoretical activity of constructing logical and mathematical systems, abstract models of certain objective systems and also construction of corresponding practical realistic analogues, i.e. realistic models of various kinds to these theoretical models. Heymann (2003) defines mathematical modelling as application of mathematics in the real world, emphasizing its relation towards this world and the process of modelling as a simple manner of presenting this relation. Yanagimoto (2005) discusses mathematical modelling not only as a process of solving problems from the natural and social context using mathematics, but also as application of mathematics, which is extremely important and useful in life.

Modelling is a process initiated by a realistic situation, some phenomenon from natural and social context, and ended by understanding and description of this situation. When the understanding of a situation from the context is achieved by the use of mathematics, the process is called mathematical modelling. One of the most comprehensive definitions of mathematical modelling, but also one of those which describe modelling in the best possible way, was provided by Meerschaert (2010, p. 27): "Mathematical modelling is a connection between mathematics and the rest of the world."

Teaching methodology literature observes mathematical modelling as a frame for learning mathematics within which, through mathematical contents, other sciences and scopes of life are studied, too. For children of early age, it is the world they live in where each new situation represents a true natural problem and mathematical modelling a new, life and social experience which enables research and understanding of mathematical dimensions of the environment. At an early age, modelling is a revealing activity which includes the mathematisation of a realistic situation and results in mathematical ideas and new mathematical concepts. Therefore, in preschool mathematics, the focus is on learning mathematics through problem solving and learning problem solving through creation of mathematical models.

With the aim of improving education, modelling originally appeared in engineering sciences and then expanded to other fields. Its purpose was to bridge the gap between mathematical thinking and thinking in problem situations from real life. In the second half of the 20th century, with the development of technology based on mathematical accomplishments, the need for modelling increases. Simultaneously, the need to apply modelling in the mathematical education at all levels arises. In that context, mathematical modelling is developed in two trends: the first one is pragmatic and the other one is scientific-humanistic.

The pragmatic or realistic trend stresses the perspective of modelling focused on utilitarian goals. Henry Pollak (2003), Austrian-American mathematician, is considered a typical representative of this trend. According to Pollak, logical-mathematical activities should provide children with instructions such as, "This is a problem, solve it" or "This is a situation, think about it". His perspective was that mathematics should be learned as a consequence of realistic, natural and social problems. Children are put into a situation in which they solve realistic problems by making the model and thus independently develop mathematical ideas. Pollak was among the first to describe the manner of applying modelling in the educational process of connecting mathematics with "the rest of the world". Within mathematics, Pollak differentiates between theoretical and applied mathematics; "the rest of the world" represents life situations, everyday activities and other sciences. The connection between these fields is mutual and they complete each other. When a problem is solved, the first transition is from "the rest of the world" to theoretical mathematics. Then, within applied mathematics, mathematical methods for solving a realistic problem are developed. The interpretation of the solution represents return to "the rest of the world". In logical-mathematical activities, the focus should be on the modelling process itself and on model formation, less so on its application. In his papers in the field of mathematical education, Pollak highlighted the increasing need for applied modelling in the educational process and positive implications that may emerge from it.

The other scientific-humanistic or epistemological trend is aimed at mathematics as a science in the humanistic ideal of education, with the focus on children's abilities to make the relation between mathematics and reality. The founder of this trend is a Dutch mathematician and didactical scientist Hans Freudenthal (1991) and his early contributions to the methodology of mathematical education. Freudenthal advocated the thesis that mathematics, as a humanistic achievement, must be connected with reality, close and understandable for children, with the importance for social context. According to Freudenthal, mathematics is learned with the aim of developing mathematical thinking and therefore must be connected with reality in which children learn, acquire experience and live. Mathematical concepts are learned through realistic context, which is understandable and close to children, i.e. realistic. Firstly, the problem which is comprehensible and which children can imagine and understand is presented, and then comes its solving, i.e. from informal to formal mathematical language that improves the understanding of the learned.

De Corte, Verschaffel, and Greer (2000) claim that the inclusion of realistic problems into the process of forming elementary mathematical concepts is very important because it shows children when and how to implement their mathematical knowledge in everyday life situations. Carpenter, Moser, and Bebout (1988) point out that children may analyse and solve simple realistic problems which include addition and subtraction prior to inclusion into formal education. They add that children may use their fingers, objects from the immediate environment and sophisticated strategies of counting with the aim of understanding the relations of the given quantities in the problem. Also, they claim that mathematical problems which preschool children may solve are limited by their age level. It means that children do not have to know mathematical symbols (+, –, =), but they can solve certain realistic problems which indicate addition and subtraction. For instance, a child cannot determine the value of expression 2 + 1, but without any difficulties may reply to the problem: Darko had 2 balls. At New Year, he got another one as a present. How many balls does he have in total?

The stronghold for learning mathematical concepts in real context is also found in the definition of mathematical literacy. According to OECD (Organisation for Economic Cooperation and Development), it represents the ability of an individual to recognize and understand the role of mathematics in the world, to make well-founded decisions and apply mathematics in the manners which suit the needs of the life of an individual as constructive, interested and mindful citizen (OECD, 2010, 2014). It is not limited only to knowledge of mathematical terminology, facts or mathematical actions nor only to the execution of certain operations even though it refers to the possession of these competences, too. It is also related to active use and understanding of mathematics in everyday life.

Are children in early childhood capable of modelling and solving certain problems from the natural and social context?

Children's mental potential necessary for approaching, understanding, modelling and solving certain problems from the natural and social context, was the topic of many studies related to preschool mathematical education (Altun, Dönmez, İnan, Taner, & Özdilek, 2001; Carpenter, Hiebert, & Moser, 1983; Carpenter, Franke, Ansell, Fennema, & Weisbeck, 1993; Davis & Pepper, 1992; Manches, O'Malley, & Benford, 2010; Monroe & Panchyshyn, 2005; Patel & Canobi, 2010; Tarim, 2009). They emphasize the importance of presenting the problems to children from the earliest age as fun challenges which encourage them to be intellectually active, because mental activity, even though periodical in the beginning, is the foundation of permanent cognition activity. A preschool child should be both listened to and observed. From the earliest age, it is also very important that a child becomes an active participant in the process of creation, the one who provides answers independently, creates and solves. Children should be constantly "fed" with new suitable contents. Is a preschool child capable of solving certain problems? Of course he/she is – in accordance with their age and psychophysical abilities, which will be developed precisely due to solving various problem situations (Peteh, 2008, p. 58).

The place of mathematical modelling, i.e. mathematical models and methods in the outcomes of logical-mathematical activities executed on the basis of the Programme of Preschool Education of the Republic of Srpska (2007) and Mutual Core of Curricula for Mathematical Area (Agency for Preschool, Primary and High School Education in Bosnia and Herzegovina, 2015) are defined as follows:

– Chooses and combines strategies, methods and operations for solving problems and provides a solution in the context of the problem; estimates the correctness and accuracy of chosen strategies, methods, operations and obtained solutions, discusses the final solution in the context of the problem (mathematical operations).

– Applies mathematical models for the representation and interpretation of quantitative relations (algebra).

– Discusses solutions in the context of problems, provides graphic presentation of the solution (equations, inequalities and their presentation).

– Determines geometric properties in the objects from the real world and models space relations in the course of problem solving; presents the results of composing and disassembling two-dimensional and three-dimensional forms in suitable model situation (geometry and measuring).

– Uses elements of combinatorics (probability elements).

The first step in the process of mathematical modelling is the choice of a problem from the natural or social context. Finding a suitable realistic problem for preschool age is not a simple task due to children's limited life and mathematic experience. We should choose a problem that emerges from the context of early childhood, one that is interesting, challenging and important for children, that suits their needs and level of development, and contains relevant mathematical problem via mathematical concept that we want transferred.

Example 1

Maria's piggy bank

1) How much money has Maria saved?

2) The snack at kindergarten costs 2KM[1]. Maria bought it today for her and her friend. How much money did she spend?

3) How much money does she have now?

4) Maria wants to buy a new toy which costs 10 KM. How much money does she lack?

[1] KM (*Konvertibilna marka* - currency in Bosnia and Herzegovina)

Example 2

Measure, knead, form balls

	Inka	Blaženka
	1	1
	2	1
	2	3
	2	2
	3	1

1) Measure the ingredients Inka used by using a cup. How many cups are there?

2) Measure the ingredients Blaženka used by using a cup. How many cups are there?

3) Knead the dough. According to which recipe did you get less dough? Why?

4) Form the balls. Did you get an equal number of balls from both masses? Why?

5) Whose balls taste better?

In order to solve problems from the natural and social context successfully, it is necessary for preschool children to establish a relationship between the specific and abstract, i.e. to model natural and social context. Taking into consideration that contexts are most frequently complex structures, the processes of their modelling and mathematisation go through various stages. The following scheme (Figure 1) (according to Blum & Leiss, 2007) represents a simplified representation that symbolically shows how the solution is reached and may be applied in the given situation via modelling.

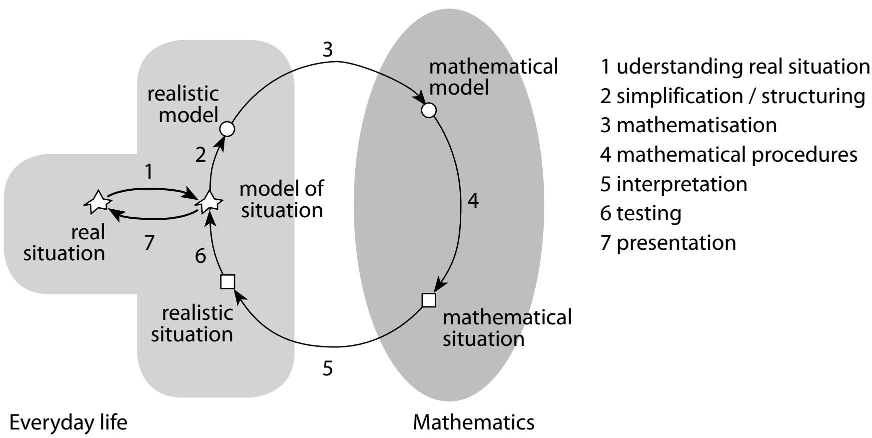

Figure 1. Modelling process (Blum & Leiss, 2007)

It is evident that the starting point in introducing the child to the problem is a realistic context. The modelling of the situation is done at the first stage and it is different from child to child since it reflects comprehension and personal understanding of the actual context. The situation model, which represents the basis for the creation of a realistic model at the second stage, is simplified in accordance with the knowledge and experience of a child; the characteristic features are kept since they are necessary for solving contextual problems. In fact, a realistic model represents a simplified description or an image of a situation model. The mathematisation and formation of a mathematical model follow, which means that visual representations obtain mathematical symbolism, i.e. they are expressed in mathematical language. By choosing adequate strategies and mathematical procedures, the mathematical solution to the problem is formed and its interpretation in realistic situation is performed. Thus, we return to the realistic model which is transformed into a realistic solution, so its testing and presentation in the realistic situation is done (Milinković, 2013).

In order to be successful in modelling activities, children need to have access to various representative media, primarily visual. Which media will children choose to express their model depends on their abilities.

The exploration of the visualization process at preschool level is predominantly oriented to graphic representations in the context of adopting elementary mathematical concepts, modelling and problem solving. The choice of suitable mental images, i.e. models which Terwel, Van Oers, Van Dijk, and Van den Eeden (2009) define as a structural form of representation, are considered crucial for the problem-solving process. Their role is primarily in the perspective of the structure of the problem and in mathematical reasoning during the problem solving process.

Bruner (1971, as cited in Wood, 1995) suggests the model of spiral approach to problem solving with the following steps:

– Presentation of the task with specific objects;
– Use of speech in describing the manner of solving a problem;
– Presentation of the solution using pictures (graphic);
– Symbolic presentation of the task (generalization of a specific experience).

In other words, mathematical concepts should be approached in the manner that will provide a specific experience to children, whenever possible. Then the observed should be described and explained after which it is presented in the form of a picture or scheme and subsequently with adequate symbols (Vlahović-Štetić & Vizek Vidović, 1998).

Methods

The aim of the present research was to examine the attitudes of educators regarding the values of mathematical modelling and its application in the work with preschool children. In the research, a descriptive method and the questionnaire were used as the basic measuring instruments. To examine preschool teachers' attitudes, a 5-point scale of Likert type, which contains the following response categories: 5 – I completely agree, 4 – I mostly agree, 3 – I am indecisive, 2 – I mostly disagree, 1 – I completely disagree was applied. The scale was constructed on the basis of 11 indicators in total, as the following claims: (1) Early childhood represents the most convenient time to introduce children with mathematical modelling; (2) Preschool children are capable and interested in mathematical modelling; (3) Modelling has an important influence on early mathematical development and improvement of children; (4) Modelling shows the relevance of abstract mathematical concepts in real life; (5) Modelling enables the adoption of mathematical concepts via solving the problems from the natural and social context; (6) Mathematical modelling contributes to the development of mathematical abilities and mathematical thinking; (7) I see modelling as the key factor of high-quality mathematics in early childhood and "vital" basis for learning mathematics in the future; (8) Mathematical modelling is not frequently applied despite the quality of such approach; (9) Preschool teachers do not have corresponding mathematical and teaching methodology knowledge in the field of mathematical modelling; (10) Preschool curricula should focus on mathematical modelling; (11) I believe that, with additional education, I would focus mathematical activities on modelling in early childhood. The value of Cronbach's alpha coefficient (0.94) indicates good psychometric characteristics of the instruments and justifies their acceptance. The research was anonymous in order to ensure honesty of the participants and avoid giving desired answers.

The research was carried out in April 2016, on the sample of 197 participants. A hundred and three (52.28%) preschool teachers (T) with various length of work experience and education, who work at kindergartens on the territory of the Republic of Srpska, and 94 (47.72%) students (S) of the final year of preschool education studies participated in the research. For the purposes of statistical analysis, students were included in the research intentionally because teaching methodology exercises begin in the winter semester of the final year, and dealing with the issue of modelling is present more.

Descriptive and comparative statistics were used for data processing.

Results and Discussion

On the basis of the applied research instrument, which expresses the indicators of the attitude about certain aspects of mathematical modelling, it is determined that the participants generally expressed positive attitude about this issue, since the average value exceeded the methodological limit for designating the attitude direction. However, it is visible that its intensity is at the level of moderate positivity (SE=3.54), which clearly shows the complexity of this topic (Table 1).

Regarding the subjects' opinion on the readiness of preschool children for modelling activities, its usefulness and level of application, individual indicators will be considered mutually since it refers to certain key questions of mathematical education in early childhood.

Table 1

Scale values of participants' attitudes on mathematical modelling

Statement	The level of agreement with the statement: I completely agree (f / %)	I mostly agree (f / %)	I am indecisive (f / %)	I mostly disagree (f / %)	I completely disagree (f / %)	I_{SE}	Differences between participant groups (p)
T_1	8 4.06	21 10.66	54 27.41	86 43.65	28 14.22	2.47	.034*
T_2	22 11.17	33 16.75	70 35.53	45 22.84	27 13.71	2.89	.185
T_3	17 8.63	71 36.04	61 30.96	41 20.81	7 3.56	3.43	.056
T_4	33 16.75	79 40.10	55 27.92	28 14.21	2 1.02	3.57	.074
T_5	58 29.44	67 34.01	43 21.83	24 12.18	5 2.54	3.54	.026*
T_6	106 53.80	65 32.99	19 9.64	4 2.05	3 1.52	4.35	.001*
T_7	112 56.85	64 32.49	19 9.64	2 1.02	0 .00	4.44	.089
T_8	71 36.04	73 37.06	34 17.26	13 6.59	6 3.05	3.96	.0001*
T_9	21 10.66	26 13.20	63 31.98	66 33.50	21 10.66	2.80	.0003*
T_{10}	15 7.61	91 46.20	72 36.55	15 7.61	4 2.03	3.49	.959
T_{11}	85 43.15	34 17.26	64 32.49	11 5.58	3 1.52	3.95	.434
			Average I_{SE}			**3.54**	

*The difference is significant at $p<.05$

The first part of the questionnaire (indicators T_1, T_2 and T_3) referred to the adequacy of early childhood, children's readiness and interest in getting familiar with modelling, and its influence on children's early mathematical development and improvement. The results showed that only 14.72% of the subjects see the preschool age as convenient for inclusion in the activities of mathematical modelling, even though 27.92% of them express children's readiness and motivation for the activities of that kind, and 44.67% notice the importance of modelling in mathematical development in early childhood. Even though certain differences in the intensity of the attitude between the groups of participants (T and S) are noticed for all three indicators, a statistically significant difference was found only in the evaluation of early childhood's appropriateness for introducing modelling and model development ($p<.05$). More attitudes that are positive are expressed by students who deal with modelling more intensively at the final year within teaching methodology exercises and teaching practice while preschool teachers express more negative attitudes.

The perception of qualitative aspects of mathematical modelling is an important factor for the subjects' commitment to applying it more intensely in the future. Statements T_4, T_5, T_6 and T_7 indicate crucial values of the model approach to contents which are the basis of the preschool education curriculum and refer to the development of mathematical concepts. The fact that the majority of participants (89.34%) see modelling as a key factor of high quality mathematics in early childhood and "vital" basis for learning mathematics in the future is the evidence of awareness about the aspects of modelling quality. The results showed that 86.79% of the participants agreed with the statement that mathematical modelling contributes to the development of mental abilities, which is one of the dominant goals of mathematical education at all levels. The role of modelling in the acquisition of mathematical concepts through solving problems from authentic context was understood and positively evaluated by 63.45% of participants. Furthermore, the importance of modelling in the indication of relevance of abstract mathematical concepts in real life was perceived by 56.85% of participants. With regards to the opinion about the stated aspects of the quality of mathematical modelling between preschool teachers and students, there is a difference in intensity of attitudes for all indicators. This difference is statistically significant in the evaluation of the role of modelling in the acquisition of mathematical concepts through solving problems from the natural and social context and contribution of modelling to the development of mathematical abilities and mathematical thinking ($p<.05$). Taking into consideration the "fresh" theoretical and practical education, more positive attitudes are expressed by the students of the final year of studies.

The third part of the questionnaire (statements T_8, T_9, T_{10} and T_{11}) refers to the presence of modelling in learning preschool mathematics, competence of preschool teachers in this area, need for professional education and focus of preschool curriculum on mathematical modelling. The research determined that only 9.64% of the participants expressed more frequent application of mathematical modelling, whereas 55.84% did not have or were not certain about their knowledge when it comes to the activities of mathematical modelling. In addition, 53.81% of participants suggested innovations in the preschool curriculum in terms of focusing on mathematical modelling, and 60.41% think that, with additional education, mathematical activities in early childhood could focus on modelling. The students of the final year had less affirmative evaluations for indicators T_8 and T_9 ($p<.05$), the difference being statistically significant. The reason for such result could be the fact that they consider preschool teachers, as their mentors, competent for the job they perform and thus also for mathematical modelling so, in that respect, they apply it more frequently in the formation of mathematical concepts.

It is evident that the participants understand the usefulness of modelling, seeing it as a key factor of high quality mathematics in early childhood and "vital" basis for learning mathematics in the future. Therefore, with additional education, mathematical activities in early childhood would be based on modelling. Do such understandings of current and future preschool teachers indicate modelling as a perspective of learning in early childhood? For this purpose, a more detailed analysis (Table 2) of the results for the stated indicators (T_7 i T_{11}) was made.

Table 2

The participants' attitudes on the understanding of modelling and possibility of more intensive application

			I believe that, with additional education, I would focus mathematical activities to modelling in early childhood			Total
			I agree	I am indecisive	I disagree	
			f %			
I see modelling as a key factor of high quality mathematics in early childhood and "vital" basis for learning mathematics in the future	I agree	f %	119 60.41	52 26.40	5 2.54	176 89.35
	I am indecisive		0 .00	12 6.08	7 3.56	19 9.64
	I disagree		0 .00	0 .00	2 1.01	2 1.01
Total			119 60.41	64 32.48	14 7.11	197 100.00

χ^2=57.357; p<.001

With regards to the participants' opinion on modelling as a key factor of high quality mathematics in early childhood and a solid foundation for learning mathematics at higher levels of education, the greatest number expressed a positive attitude (89.35%). However, when it comes to potential focus of mathematical activities on modelling, with additional education, a smaller number of participants (60.41%) agreed. The obtained Chi square (χ^2=57.357; p<.001) indicates statistically significant differences in relations of the stated indicators. Nevertheless, the obtained results may be considered as positive support to intensification of the implementation of modeling into early mathematical education.

Conclusions

Mathematical modelling plays an important role in solving problems from the natural and social context. It cannot lead to a solution directly, but it enables and indicates the problem solving process. Models represent a certain structural form of representation, i.e. "material" which may express or create mathematical ideas. They determine the process of researching mathematical problems and are the source of meaning of mathematical concepts and their relations. Via applied modelling in early childhood, children learn mathematics by understanding it; they are encouraged to solve realistic problems from their own aspect, and to talk about the problem both among themselves and with the preschool teacher. Learning mathematics by experiencing and revealing it in natural and social environment, in real-life situations, contributes to the promotion of understanding both at conceptual and operational level, since the skill of applying the acquired knowledge and abilities is more required than knowing arithmetic and geometry (Ćurčić, Milinković, & Radivojević, 2017). Children who learn from the perspective of modelling and solving contextual problems have the horizons of logical-mathematical thinking opened and they help in understanding mathematics. Charlesworth and Leali (2012) consider the abilities of modelling and problem solving an important instrument for the evaluation of mathematical thinking of preschool children. Green and Gallagher (2014) claim that introduction of mathematical modelling in early childhood may be very useful for neurological development of children because in this period the brain goes through significant developmental changes which are best stimulated by complex and engaged activities such as modelling.

The research results generally indicate that current and future preschool teachers have positive attitudes toward mathematical modelling (SE=3.54) and are the evidence that the participants recognize the quality of mathematical modelling. There is a suspicion when it comes to the adequacy of early childhood, children's readiness and interest in getting familiar with modelling and its influence on children's early mathematical development and improvement (SE=2.93). On the one hand, that may be an indicator of insufficient knowledge about theoretical and practical trends of this issue because more than a half of the subjects (55.84%) were not sure about their knowledge when it comes to this field. In addition, 60.41% expressed the need for additional education with the aim of focusing mathematical activities on modelling. On the other hand, it may be indicative to the fact that preschool teachers have more trust in the work they are used to because more positive attitudes were expressed by students of the final year of studies. One of these reasons is the fact that 73.10% of the participants firmly stick to the attitude that mathematical modelling is not frequently applied in the educational practice, despite the quality of such approach, while only 9.64% claim the opposite.

The participants expressed firmly determined positive attitudes about the values and qualities of modelling aspects (SE=3.97). They clearly observe the contribution of modelling to the development of mental abilities (SE=4.35), solving realistic problems and learning in context (SE=3.54), and application of abstract mathematical concepts in real life (SE=3.57). With extremely high positivity, the attitude on perceiving modelling as the key factor of high quality mathematics in early childhood and "vital" basis for learning mathematics in the future (SE=4.44) was shown. Moreover, 60.41%

of the subjects responded decisively that, with additional education, mathematical activities in early childhood would focus on modelling. However, 32.48% of the participants were indecisive about this indicator, so conditionally, they might be considered as potential executors of mathematical contents by model approach. Only 7.11% of the participants expressed disagreement with this attitude. In addition, 53.81% of the participants were decidedly of the opinion that preschool curriculum should focus on mathematical modelling, while 36.55% of them potentially expressed this need, and only 9.64% negated it.

The research generated the information that quality and functionality of mathematical modelling, when it comes to early childhood, were more recognized by students of the final year of study because of their "fresh" knowledge about this topic. Additionally, they considered preschool teachers with more experience to be more competent for mathematical modelling than themselves.

The problem of implementation of mathematical modelling into preschool curriculum and inclusion of children in the modelling activities is still insufficiently explored. There are many unjustified and ungrounded attitudes which negate its influence on early mathematical development. The aforementioned supports the need for further research in this context and identifies basic causes due to which preschool teachers have doubts about modelling. The positive attitude toward the introduction of modelling into preschool curriculum and support of majority to the potential concept of its implementation into learning mathematics in early childhood must not be ignored. Basically, these results represent a stable stronghold for further research of this topic.

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Matematičko modeliranje prirodnog i društvenog konteksta u predškolsko doba

Sažetak

U radu se ukazuje na važnost matematičkog modeliranja, koje u osnovi rezultira usvajanjem matematičkih pojmova, razvijanjem sposobnosti rješavanja problema i logičko-matematičkog mišljenja. Metodička literatura promatra matematičko modeliranje kao okvir za učenje matematike, u kojem se putem matematičkih sadržaja proučavaju i druge znanosti i područja života. Za djecu predškolskog uzrasta to je svijet u kome žive, u kome svaka nova situacija predstavlja pravi prirodni problem, a matematičko modeliranje novo, životno i društveno iskustvo. Modeliranje u ranoj dobi otkrivajuća je aktivnost koja podrazumijeva matematizaciju realne situacije i stvaranje modela kojim se pažnja djece usredotočuje na različite vrste prikaza odnosa varijabli. Rad se bavi ispitivanjem stavova odgojitelja o njihovim vrijednostima, primjerenosti ranom djetinjstvu i mogućnosti primjene u učenju predškolske matematike. Anketnim upitnikom na uzorku od 197 ispitanika došlo se do rezultata koji potvrđuju pozitivan stav odgajatelja prema modeliranju, ali i nedovoljnu upućenost u teorijske i praktične spoznaje u vezi s temom.

Ključne riječi: *logičko-matematičko mišljenje; rano djetinjstvo; sposobnost rješavanja problema; učenje u kontekstu; vizualni modeli.*