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Dynamic system with no equilibrium and its chaos anti-synchronization

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ABSTRACT
Recently, systems with chaos and the absence of equilibria have received a great deal of attention. In our work, a simple five-term system and its anti-synchronization are presented. It is special that the system has a hyperbolic sine nonlinearity and no equilibrium. Such a system generates chaotic behaviours, which are verified by phase portraits, positive Lyapunov exponent as well as an electronic circuit. Moreover, the system displays multistable characteristic when changing its initial conditions. By constructing an adaptive control, chaos anti-synchronization of the system with no equilibrium is obtained and illustrated via a numerical example.

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Chaos; hyperbolic sine; equilibrium; multistability; control; anti-synchronization

1. Introduction

The discovery of Lorenz has promoted the investigation of various chaotic systems [1–6]. Numerous studies have attempted to explain chaos synchronization [7–10]. Different schemes have been developed for synchronization of chaos, for example adaptive synchronization scheme [11], active control scheme [12], backstepping control [13], hybrid function synchronization [14], etc. Moreover, applications of chaotic systems have been reported in robust watermarking algorithm [15], cryptography [16–18], S-Box generation [19–21], steganography [22], or modulation scheme for cognitive radio [23].

Numerous chaotic systems with different terms were studied. Butterfly attractor was observed in a 10-term system by Pehlivan et al. [24]. Bao et al. found chaos in a nine-term system with four line equilibria [25]. By using three quadratic nonlinearities, Vaidyanathan constructed an eight-term polynomial chaotic system [26]. A seven-term chaotic system with a single cubic nonlinearity was reported in [27]. A Lorenz system family with six terms was presented by Pehlivan and Uyaroglu [28]. However, there were few chaotic systems with five terms [29,30]. Five-term systems are attractive because of their algebraic simplicity. Especially, a five-term system is one of simple continuous systems which generate chaos [31]. Thus, the purpose of this work is to study a five-term system with hyperbolic sine nonlinearity and its chaos anti-synchronization. Moreover, such a five-term system is a system without equilibrium [32–36].

2. The system with five terms and its chaos

A five-term system is studied in this work. The simple system with five-term is described by

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -a\sinh(x) - yz, \\ \dot{z} &= y^2 - b, \end{aligned} \tag{1}$$

in which a and b are positive parameters $(a,b > 0)$. Specially, there is a hyperbolic sine term in system (1). We set the right-hand side of system (1) to find its equilibrium points:

$$y = 0, \tag{2}$$

$$-a\sinh(x) - yz = 0, \tag{3}$$

$$y^2 - b = 0. \tag{4}$$

By comparing Equations (2) and (4), we confirm that system (1) is a non-equilibrium one due to $b > 0$.

When $a = 1$ and $b = 2$, system (1) displays chaotic behaviour for initial conditions $(x(0), y(0), z(0)) = (0, 0.1, 0)$ as shown in Figure 1. Chaos in system (1) is also verified by the positive Lyapunov exponent $L_1 = 0.0977$. It is noting that few chaotic systems without equilibrium have been found [32–36].

Multistability leads to complex behaviours in a dynamical system [37–39]. Multistability features have been investigated in numerous systems recently [40–43]. Interestingly, we have found that system without equilibrium (1) displays various behaviours when

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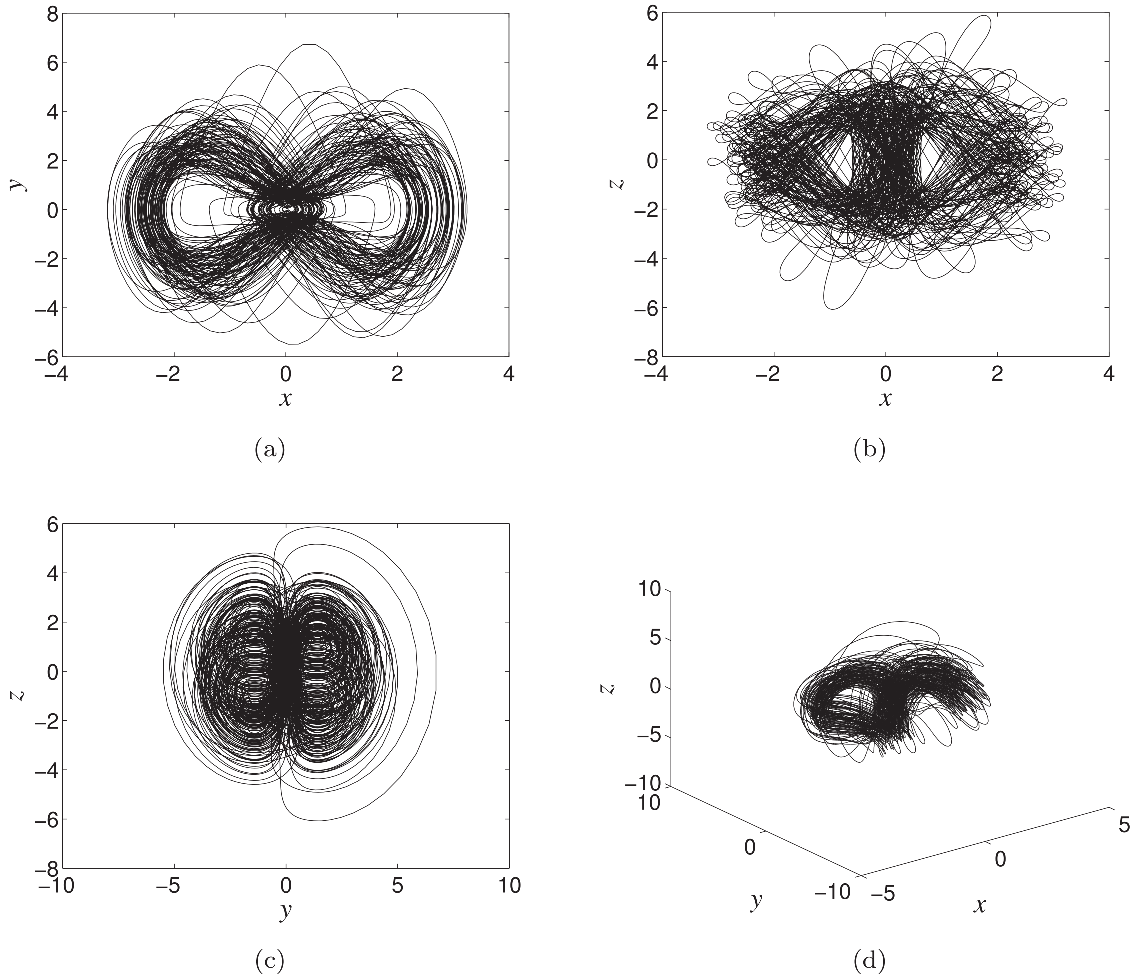

Figure 1. Chaotic behaviour in (a) $x-y$ plane, (b) $x-z$ plane, (c) $y-z$ plane, and (d) $x-y-z$ space. The set of parameters is $a=1, b=2$ while initial conditions are $(x(0), y(0), z(0)) = (0, 0.1, 0)$.

changing the initial conditions. Figure 2 illustrates the multistability property of the five-term system. In addition, the bifurcation diagram versus initial conditions is reported in Figure 3. We set $x(0) = 0$ and $z(0) = 0$ while $y(0)$ is changed from 0.5 to 3.5.

3. Electronic circuit of the five-term system

The dynamics of the simple five-term no-equilibrium system has been investigated in the preceding section by using numerical methods. It is revealed that system (1) exhibits complex dynamical behaviours including chaos and multistability. In this section, we design and implement an electronic circuit in PSpice capable to mimic the dynamics of system (1) in order to validate the numerical results carried out previously. The schematic diagram of the proposed electronic circuit for system (1) is depicted in Figure 4.

The circuit comprises seven resistors, three capacitors, two analogue multipliers chips (AD633JN), a pair of semiconductor diodes (1N4148) and a quadruple operational amplifier (TL084CN). The analogue multipliers and pair of semiconductor diodes connected in anti-parallel are used to implement respectively, the quadratic nonlinearity and the hyperbolic sine term. The current–voltage characteristic $(I-V)$ of the pair of semiconductor diodes (D1 and D2) is defined by the following Shockley diode equation [44]:

$$I_d = I_{D_1} - I_{D_2} = 2I_s \sinh(V_d / \eta V_T), \tag{5}$$

where I_s is the saturation current of the junction, η is an ideality factor $(1 < \eta < 2)$ and V_T is a thermal voltage. By applying Kirchhoff's laws into the circuit of Figure 4, we obtain its mathematical model given by the following set of three coupled first-order differential equations:

$$\begin{aligned} \frac{\mathrm{d}V_x}{\mathrm{d}t} &= \frac{V_y}{RC}, \\ \frac{\mathrm{d}V_y}{\mathrm{d}t} &= -\frac{2I_s \sinh(V_x / \eta V_T)}{C} - \frac{V_y V_z}{10R_1 C}, \\ \frac{\mathrm{d}V_z}{\mathrm{d}t} &= \frac{V_y^2}{10R_2 C} - \frac{V_{DC}}{R_3 C}, \end{aligned} \tag{6}$$

where V_x, V_y and V_z are the output voltages of the operational amplifiers OP_1, OP_2 and OP_3, respectively. System (6) is equivalent to system (1) with the following settings of variables and parameters: $x = V_x/\eta V_T$,

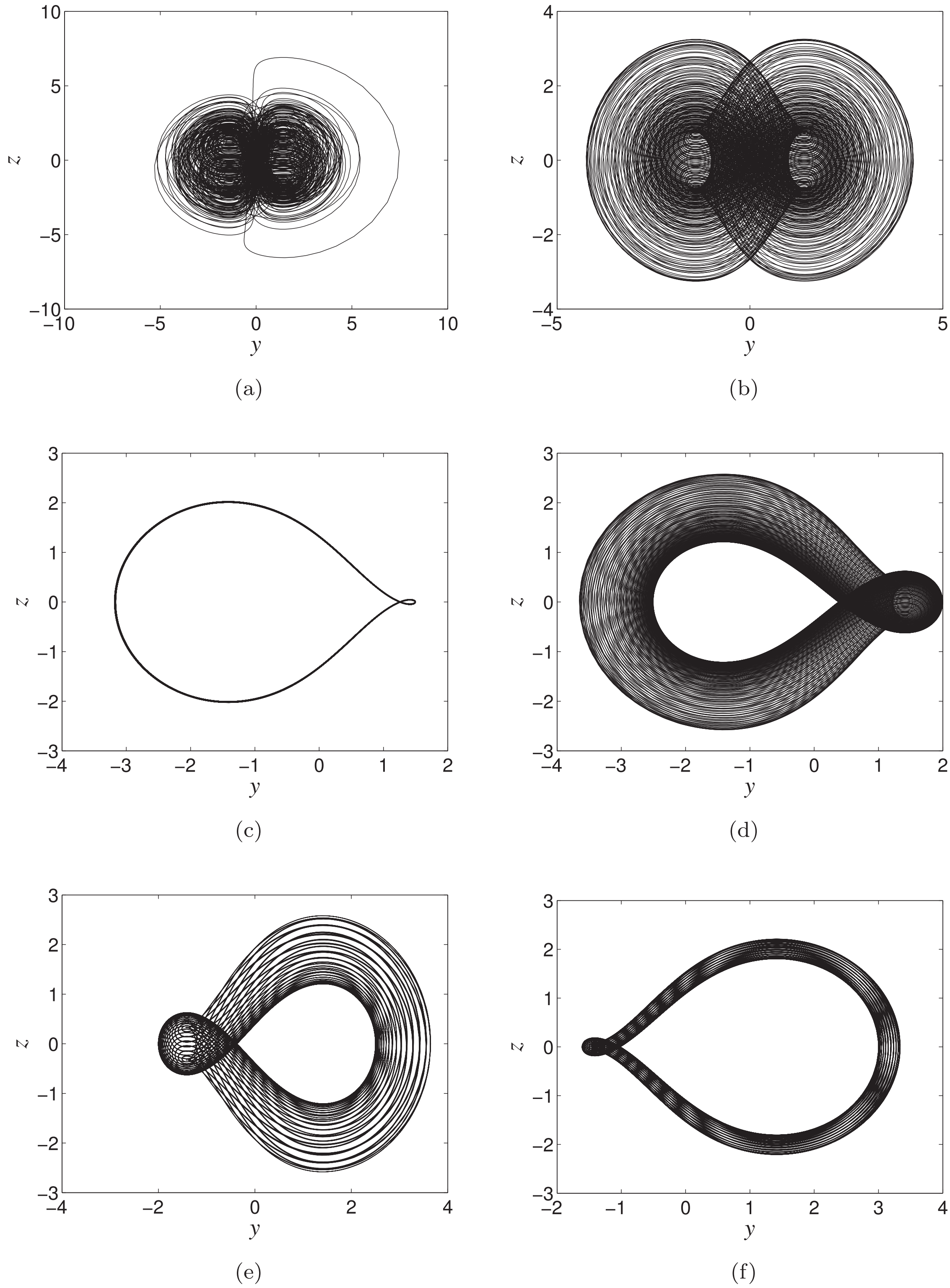

Figure 2. Various behaviours of the five-term system in $y - z$ plane when keeping $a = 1$, and $b = 2$. We have observed the five-term system for different initial conditions: (a) $(x(0), y(0), z(0)) = (0, 0.5, 0)$, (b) $(x(0), y(0), z(0)) = (0, 1, 0)$, (c) $(x(0), y(0), z(0)) = (0, 1.5, 0)$, (d) $(x(0), y(0), z(0)) = (0, 2, 0)$, (e) $(x(0), y(0), z(0)) = (0, 2.5, 0)$, and (f) $(x(0), y(0), z(0)) = (0, 3, 0)$.

$y = V_y/\eta V_T$, $z = V_z/\eta V_T$, $t = \tau RC$, $a = 2RI_s/\eta V_T$ and $b = R/R_3$. For $a = 1$ and $b = 2$, the circuit components have the following values: $V_{DC} = 1\,\text{V}$, $C = 10\,\text{nF}$, $R = 10\,\text{k}\Omega$, $R_1 = R_2 = 1\,\text{k}\Omega$, $R_3 = 5\,\text{k}\Omega$, $I_s = 2.682\,\text{nA}$, $V_T = 26\,\text{mV}$ and $\eta = 1.9$. The power supply is $\pm 15\,\text{V}$. The PSpice chaotic phase portraits of the circuit in (V_x, V_y), $((V_x, V_z)$ and (V_y, V_z) planes are shown in Figure 5.

One can see from Figure 5 that the PSpice results agree with those obtained numerically. These results

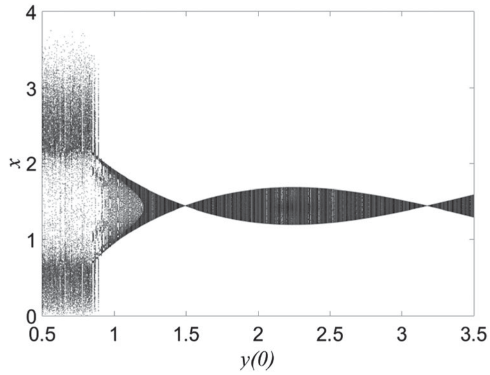

Figure 3. The bifurcation diagram versus initial conditions. We set $x(0) = 0$ and $z(0) = 0$ while $y(0)$ is changed from 0.5 to 3.5.

confirm that the proposed electronic circuit is capable to mimic the dynamical behaviours of system (1).

4. Anti-synchronization of the five-term system

Investigation of chaos synchronization is vital in the theoretical issues and engineering applications [45–48]. Authors proposed numerous kinds of synchronization such as complete synchronization, phase synchronization, anti-phase synchronization, lag synchronization, anticipating synchronization, projective synchronization and anti-synchronization [49–56]. It is noted that anti-synchronization is an attractive scheme, where two dynamical systems are synchronized in amplitude, but with opposite sign [57–59]. Anti-synchronization was applied in different areas, for example temporal pattern recognition [60], memristive neural network [61], multi-degree-of-freedom dynamical system [62], security communication [63] and coupled systems [64,65]. Motivated by the fact that chaos anti-synchronization phenomena are of fundamental importance in the study of dynamical systems, in this section, the anti-synchronization of two non-equilibrium systems (master and slave ones) is reported. The master system with five terms is described by

$$\begin{aligned} \dot{x}_1 &= y_1, \\ \dot{y}_1 &= -a\sinh(x_1) - y_1 z_1, \\ \dot{z}_1 &= y_1^2 - b. \end{aligned} \tag{7}$$

In system (7), a and b are unknown parameters. By using the adaptive control $\mathbf{u} = [u_x, u_y, u_z]^{\mathrm{T}}$, the slave

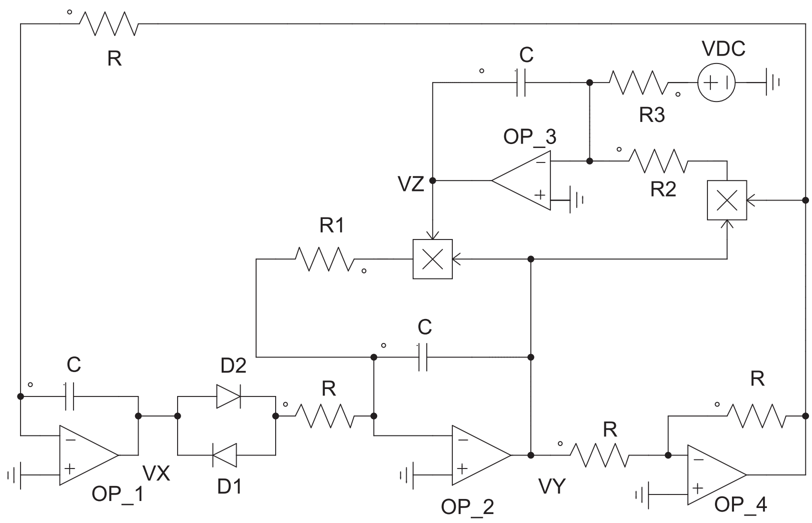

Figure 4. Electronic circuit diagram of the chaotic five-term system (1).

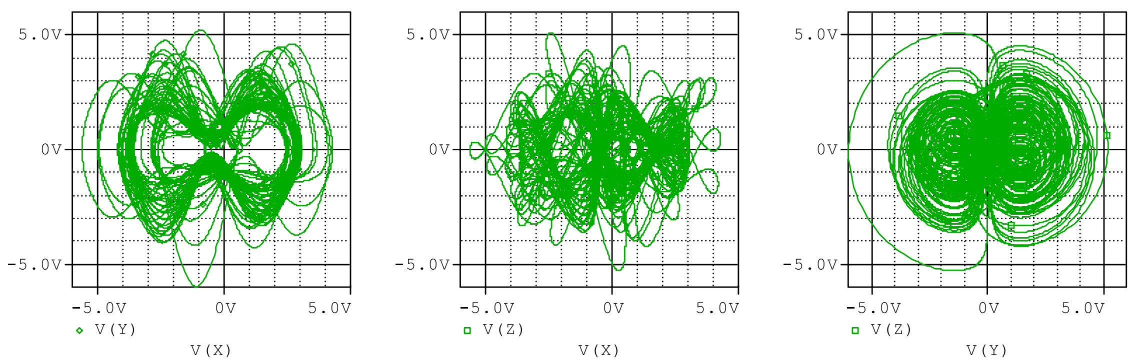

Figure 5. Phase portraits of chaotic attractors in different planes: (V_x, V_y), (V_x, V_z) and (V_y, V_z) using the designed circuit. The values of electronic components are those setting in the text.

system with five terms is

$$\begin{aligned} \dot{x}_2 &= y_2 + u_x, \\ \dot{y}_2 &= -a \sinh(x_2) - y_2 z_2 + u_y, \\ \dot{z}_2 &= y_2^2 - b + u_z. \end{aligned} \tag{8}$$

We calculate the state errors of system (7) and system (8):

$$\begin{aligned} e_x &= x_2 + x_1, \\ e_y &= y_2 + y_1, \\ e_z &= z_2 + z_1. \end{aligned} \tag{9}$$

It is noted that $\hat{a}$ and $\hat{b}$ are the estimations of the unknown parameters (a, b), thus we define the parameter estimation error:

$$\begin{aligned} e_a &= a - \hat{a}, \\ e_b &= b - \hat{b}. \end{aligned} \tag{10}$$

For getting the anti-synchronization $(x_2 = -x_1, y_2 = -y_1, z_2 = -z_1)$, we introduce the following adaptive control:

$$\begin{aligned} u_x &= -e_y - k_x e_x, \\ u_y &= \hat{a}\left(\sinh(x_1) + \sinh(x_2)\right) + y_1 z_1 + y_2 z_2 - k_y e_y, \\ u_z &= -y_1^2 - y_2^2 + 2\hat{b} - k_z e_z, \end{aligned} \tag{11}$$

with positive gain constants $(k_x > 0,\ k_y > 0,\ k_z > 0)$. Moreover, we construct the parameter update law:

$$\begin{aligned} \dot{\hat{a}} &= -e_y\left(\sinh(x_1) + \sinh(x_2)\right), \\ \dot{\hat{b}} &= -2e_z. \end{aligned} \tag{12}$$

We can confirm the anti-synchronization when applying adaptive control law (11) and parameter update law (12) as follows.

The selected Lyapunov function is

$$V\left(e_x, e_y, e_z, e_a, e_b\right) = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2\right). \tag{13}$$

Thus, the differentiation of (13) is

$$\dot{V} = e_x \dot{e}_x + e_y \dot{e}_y + e_z \dot{e}_z + e_a \dot{e}_a + e_b \dot{e}_b. \tag{14}$$

From (9), we get

$$\begin{aligned} \dot{e}_x &= -k_x e_x, \\ \dot{e}_y &= -e_a\left(\sinh(x_1) + \sinh(x_2)\right) - k_y e_y, \\ \dot{e}_z &= -2e_b - k_z e_z. \end{aligned} \tag{15}$$

A simple calculation of (10) gives

$$\begin{aligned} \dot{e}_a &= -\dot{\hat{a}}, \\ \dot{e}_b &= -\dot{\hat{b}}. \end{aligned} \tag{16}$$

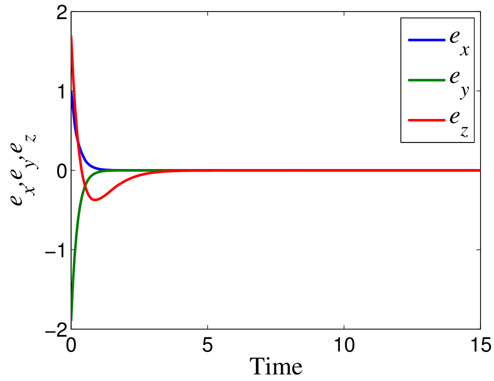

Figure 6. Time history of anti-synchronization errors between master five-term system (7) and slave five-term system (8).

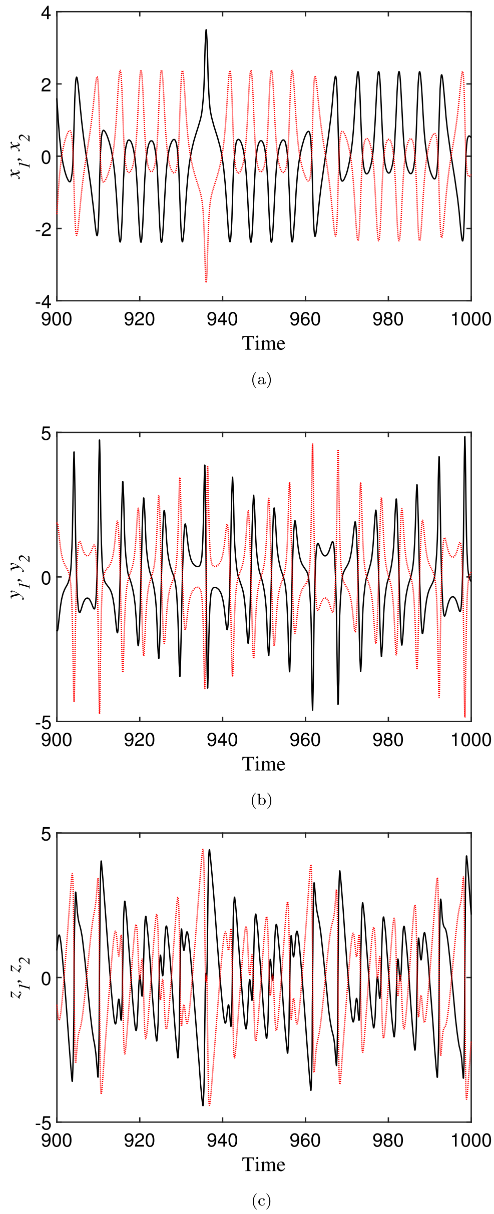

Figure 7. The time series of state variables of the master system (black solid curves) and the slave system (red dotted curves): (a) $x_1(t)$ and $x_2(t)$, (b) $y_1(t)$ and $y_2(t)$, (c) $z_1(t)$ and $z_2(t)$ (colour online).

By combining (15), (16) and (14), it is simple to calculate the differentiation of the Lyapunov function:

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2. \quad (17)$$

As a result, the anti-synchronization is obtained due to $e_x \to 0$, $e_y \to 0$, and $e_z \to 0$ exponentially as $t \to \infty$ [66].

For the numerical example, we fixed the parameter values

$$a = 1, \quad b = 2. \quad (18)$$

We assume that the initial states of master system (7), slave system (8) and the parameter estimate are given by

$$x_1(0) = 0, \quad y_1(0) = 0.1, \quad z_1(0) = 0, \quad (19)$$

$$x_2(0) = 1, \quad y_2(0) = -2, \quad z_2(0) = 1.7, \quad (20)$$

$$\hat{a}(0) = 1.5, \quad \hat{b}(0) = 1.5. \quad (21)$$

For selected gain constants $k_x = 4$, $k_y = 4$, and $k_z = 4$, the time-history of the anti-synchronization errors e_x, e_y, e_z is reported in Figure 6. In addition, the time series of state variables of the master and slave systems are displayed in Figure 7. We observe the time evolution in opposition of states variables of the master and slave systems which is the signature of anti-synchronization process.

5. Conclusions

This work has introduced an attractive chaotic system with five-terms, which include a hyperbolic sine term. The simple system has no equilibrium and displays different behaviours depending on initial conditions. Chaotic behaviour of the system is validated by a circuit, in which the hyperbolic sine term was realized with two diodes. Anti-synchronization of the system has been obtained by designing an adaptive control and illustrated by a numerical example. In our future works, applications of such a system with five terms will be investigated.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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