Automatika
Journal for Control, Measurement, Electronics, Computing and Communications

ISSN: 0005-1144 (Print) 1848-3380 (Online) Journal homepage: https://www.tandfonline.com/loi/taut20

Using the second-order information for reconfigurability analysis and design in the fault tolerant framework

Brian Manuel González-Contreras, G. L. Osorio-Gordillo, F. R. López-Estrada, G. Valencia-Palomo & C. M. Astorga-Zaragoza

To cite this article: Brian Manuel González-Contreras, G. L. Osorio-Gordillo, F. R. López-Estrada, G. Valencia-Palomo & C. M. Astorga-Zaragoza (2018) Using the second-order information for reconfigurability analysis and design in the fault tolerant framework, Automatika, 59:1, 51-62, DOI: 10.1080/00051144.2018.1498208

To link to this article: https://doi.org/10.1080/00051144.2018.1498208

Published online: 22 Jul 2018.

AUTOMATIKA
2018, VOL. 59, NO. 1, 51–62
https://doi.org/10.1080/00051144.2018.1498208

REGULAR PAPER

Using the second-order information for reconfigurability analysis and design in the fault tolerant framework

Brian Manuel González-Contreras [a], G. L. Osorio-Gordillo [b], F. R. López-Estrada [c], G. Valencia-Palomo [d] and C. M. Astorga-Zaragoza [b]

[a]Faculty of Engineering and Technology, Universidad Autónoma de Tlaxcala (UATx), Apizaco, Tlaxcala, México; [b]Tecnológico Nacional de México-Centro Nacional de Investigación y Desarrollo Tecnológico (CENIDET), Cuernavaca, Morelos, México; [c]Tecnológico Nacional de México-Instituto Tecnológico de Tuxtla Gutiérrez, Tuxtla Gutiérrez, Chiapas, México; [d]Tecnológico Nacional de México-Instituto Tecnológico de Hermosillo, Sonora, México

ABSTRACT

The control reconfigurability measure defines the capability of a control system to allow recovery of performance when faults occur; therefore, it has been intended to be a tool for designing and synthesizing approaches in the fault tolerant control context. Reconfigurability depends on the controllability gramian, also known as the second-order information (SOI) in a broad sense. This paper proposes the assignation, by feedback, of the deterministic SOI in order to set the control reconfigurability of a given linear system. The theory concerned with this assignation is reviewed, then constructive theorems are given for finding constant feedback gains that approximate a required control reconfigurability for ease implementation. Also an unification of the reconfigurability measures proposed in the fault tolerance literature is given. Once the SOI is assigned by feedback, it can be computed online by using an identification method, which uses process input/output data. Results from simulation of the three tanks hydraulic benchmark, show that this approach can provide information about the system performance for fault tolerant purposes, thus online control reconfigurability computation and fault accommodation are considered. The approach presented in the paper gives an alternative for supervision taking into account the reconfigurability assigned by design.

ARTICLE HISTORY
Received 15 September 2016
Accepted 18 June 2018

KEYWORDS
Second-order information; fault tolerant control; actuator fault; fault accommodation; controllability gramian; observability gramian; feedback control; identification; three tank system

1. Introduction

Fault tolerance of control systems is a requirement becoming indispensable in processes involving the risk to human beings, process components (process itself) and environment. This is a consequence of the development of expanding process automation, which involves an increasing demand on the process performance with independence from human operators in monotonic but precise tasks. The fault tolerant control (FTC) systems are intended to provide the capability of accommodating faults or reconfiguring the control system in order to ensure stability, to guarantee performance and to reduce risks [1, 2].

In the FTC framework, control *reconfigurability* has been defined as a metric of the capability of a control system to perform either fault accommodation or system reconfiguration related to the treated system, i.e. the control system quality/capacity to respond to faults allowing restoration/recovery of control objectives when faults occur [3]. A treated system is called control reconfigurable if the controllability property of the nominal system is kept in a faulty situation [4]. However, controllability matrix can be evaluated in a binary way [5]. Moreover, whatever the faulty cases, a necessary condition is that the system remains stabilizable and detectable [1, 3, 4].

For analysis, evaluation and synthesis in relation to FTC system design, few measures of fault tolerance have been proposed. In [6], a quotient between the controllability gramian norm in the nominal situation and the faulty one was proposed as a measure of how well the system still can be excited after a fault occurrence. In [7], a ratio between the reconfigurability in the worst case and the nominal one is used in order to compare different combinations of sensors, the best one is used in order to design the FTC system. In [3], the smallest second-order modes of a system, proposed by [8], were used in order to establish the potentiality of a process to maintain a certain performance under faulty circumstances. This notion was employed to define the *reconfigurability* concept, currently used in the FTC domain. In [9], in a similar interpretation to [6], the admissible values for reconfigurability are used in order to consider an adequate operation of the system under

CONTACT Brian M. González-Contreras brian.m.g@ieee.org

loss of actuators, the metric is viewed as the control system quality under given faulty conditions, and each faulty case (combination of lost actuators) is evaluated in terms of reconfigurability. Consequently, both approaches represent the performance in terms of system capability to maintain/recover original or admissible requirements. In fact, all the mentioned metrics representing the control reconfigurability can be viewed as a *reconfigurability-based performance* or as an *intrinsic reconfigurability property*, as stated in [10].

However, all these metrics must be done in the predesign phase, i.e. before the system begins to operate and therefore they are evaluated offline. Moreover, in the design phase of the control law, reconfigurability is not known beforehand and it may not be considered as a design criterion. For online purposes, the proposed measures could be more useful for applications such as redesign, remodelling and supervision of a process [2, 3], or simply in order to help in supervisory tasks besides the fault detection and isolation (FDI) tasks [7, 11]. By using the above-mentioned reconfigurability metrics, upper and lower bounds can be established in order to evaluate the FTC system in the design phase and then to predict and to know its potentiality against faults. An index based on the control reconfigurability is proposed in this paper in order to establish the admissible operation conditions of the treated system, under possible partial loss of effectiveness type faults. In this way, redesign and even remodelling are considered for the possible faulty cases.

As it can be noted, the base of the metrics mentioned above comes from the gramian concept. In [8], it is stated that the structure of the controllability gramian, or its singular values and vectors, characterizes how fast and in which directions the associated system dissipates energy. Taking into account this energetic viewpoint, in this paper, the theory for assigning the controllability gramian, known in a broad framework as second-order information (SOI) [12], is proposed in order to establish a closed-loop controllability gramian, and at the same time, the control reconfigurability. This is the main objective of the present paper.

The SOI controllers [12] are intended to assign a specified controllability gramian by state feedback for linear systems. This assignment was first addressed in [13] for stochastic systems. Subsequently, the results have been extended to dynamic controllers of any order in the continuous [14] and discrete [15] cases, and then for deterministic systems [16]. The discrete deterministic case is considered in this paper. Note that setting a prescribed reconfigurability lead to designing and knowing in advance the closed-loop admissible characteristics of a control system.

From identification point of view, a similar interpretation of the gramian is employed on the method known as eigensystem realization algorithm (ERA) [17]. ERA seems to be a good choice in the context of this paper, because it is based on the fact that different lengths of the ellipsoid axes indicate that some directions are more excited from the perspective of an external input excitation [18]. Hence that reconfigurability can be evaluated from the system input/output data. ERA method is different to other realization theory-based methods for state- space identification, such as subspace methods [19], because it directly solves for the system Markov parameters, namely the response to a unit sample input. Finally, by combining the ERA technique for gramian identification to compute an index based on the control reconfigurability is another aim of this paper. An ideal FDI module is considered in order to provide the fault characteristics after fault occurrence.

Taking into account the above background and compared to the relevant already reported literature, the main contributions of this paper are:

- It presents an FTC design where the SOI is assigned not only for control purposes but also to prescribe reconfigurability for a given linear plant, in order to monitoring online its capability for fault tolerance.
- Both reconfigurability measures proposed in literature are studied in order to set their relationship based on the same energetic basis. Algorithms for their calculation and their comparison are given.
- It integrates reconfigurability design and analysis online, the former through the use of the ERA/OKID method, which is founded on the same gramian basis. An algorithm for its calculation is also given.

The rest of the paper is structured as follows. Section 2 presents the notation and faults considered in the paper. In Section 3, an index based on reconfigurability is proposed. Section 4 is devoted to the presentation of the SOI controllers for assigning, by feedback, a prescribed controllability gramian and therefore reconfigurability. The basic setup for identification based on the ERA/OKID method is briefly discussed in Section 5. Finally, in Section 6, the application of the proposed theoretical results is illustrated by the case study of the three tanks benchmark subject to actuator faults. The approach is tested through simulation.

2. Modelling and fault types

Consider the discrete linear-time invariant (LTI) fault-free system with dependence of a regular sampling period h expressed by the linear state-space model:

$$\Upsilon : \begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k), \end{cases} \tag{1}$$

with the state vector $x \in \mathbb{R}^n$, initial state $x(0) = x_0$ at initial time $k = 0$, output $y \in \mathbb{R}^m$, control signal $u \in \mathbb{R}^r$, and matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{m\times n}$.

2.1. Lyapunov equation solutions

In the following, the notation $\mathrm{Lyap}(X_1, X_2)$ is considered, in order to denote the positive semidefinite unique solution Q of the Lyapunov Equation (2) affecting the pair of compatible matrices (X_1, X_2):

$$X_1 Q X_1^T + X_2 X_2^T = Q, \tag{2}$$

where^T stands for transpose of a matrix. Then the controllability and the observability gramians are, respectively, $W_c = \mathrm{Lyap}(A, B)$ and $W_o = \mathrm{Lyap}(A^T, C^T)$. Gramians represent input–output relationships from energetic viewpoint [3, 5, 8]. The physical interpretation of the controllability gramian is that it can be viewed as a map from the state space into itself, where the image of a hyperellipsoid under this map represents the set of all points in the state space reachable from the zero state with a unit-norm input.

2.2. Control effectiveness loss as actuator faults

Actuator faults can be grouped in four categories [2, 4, 20]: lock-in-place, float, hard-over and loss of effectiveness. The faults considered in this work are partial loss of control (actuator) effectiveness [20]. These are described by the control effectiveness factors γ_i, $i = 1, \ldots, r$, which form an effectiveness vector $\gamma = \left[\gamma_1, \ldots \gamma_r\right]^T \in \Omega$, where Ω is the impairment parameter space, i.e. the Euclidean space of all parameters that change their values as the result of some fault occurrence [2, 3]. Each control effectiveness factor is bounded $-1 \leq \gamma_i < 0$, $i = 1, \ldots, r$, where that the ith actuator represents partial loss of control effectiveness. Then the faulty matrix B_f is described by

$$B_f = B(I_r + \Gamma), \quad \Gamma = \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_r \end{bmatrix}. \tag{3}$$

Note that if $\gamma_i = 0$ then the actuator is healthy. In the case $\gamma_i = -1$, the actuator is completely lost. This last case is not considered in this paper.

Under this setup, the solution of the Lyapunov equation with dependence of effectiveness factors is

$$W_c^f = W_c(\gamma) = \mathrm{Lyap}(A, B_f). \tag{4}$$

In the following section, the measures of reconfigurability proposed in the literature are analysed in order to show their relationship.

3. Reconfigurability metrics and equivalences

In a general fashion and independent from the type of metric employed, reconfigurability is expressed by the following Definition 3.1.

Definition 3.1 (Reconfigurability): Reconfigurability of a system is its capability, with the remaining of controllability and observability, to allow restoration of admissible performance under faulty conditions.

3.1. Metrics and their relationship

The criterion employed in [9] defining the reconfigurability ρ is given under these terms as:

$$\rho = \max_{\gamma \in \Omega, i} \{\lambda_i (W_c(\gamma)^{-1})\}, \quad i = 1, \ldots, n, \tag{5}$$

where $W_c(\gamma)$ is the controllability gramian, dependent of the control effectiveness vector (γ), and λ_i is the ith eigenvalue of a square matrix.

On the other hand, the reconfigurability proposed in [3] requires the *Hankel singular values* (Hsv), each one denoted as σ_i, to be evaluated. It can be done using Algorithm 1 [21] which gets the Hsv matrix, denoted as $\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$. It is calculated from singular value decomposition (SVD), and also gets a regular matrix transformation T_b that leads to a balanced representation of the system. Algorithm 2 takes into account the *second-order modes* (Som) [8] of the system.

Algorithm 1 Calculating Hsv

Require: (A, B, C), $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{r\times n}$, $C \in \mathbb{R}^{m\times n}$

Ensure: Hankel singular values matrix Σ, regular matrix T_b, balanced gramians W_c^b, W_o^b

$W_c \leftarrow \mathrm{Lyap}(A, B)$, $W_o \leftarrow \mathrm{Lyap}(A^T, C^T)$
Obtain Cholesky's factors matrix $R : W_c = R^T R$
By using SVD calculate Σ according to equation $R W_o R^T = U \Sigma^2 U^T$, with $UU^T = I$
$T_b = \Sigma^{\frac{1}{2}} U^T (R^T)^{-1}$
$A^b = T_b A T_b^{-1}, B^b = T_b B, C^b = C T_b^{-1}$
$W_c^b \leftarrow \mathrm{Lyap}(A^b, B^b)$, $W_o^b \leftarrow Lyap\left((A^b)^T, (C^b)^T\right)$
return T_b, Σ, W_c^b, W_o^b

Algorithm 2 Calculating Som

Require: (A, B, C), $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{r\times n}$, $C \in \mathbb{R}^{m\times n}$

Ensure: Second order modes σ_i^2, $i = 1, \ldots, n$

$W_c \leftarrow \mathrm{Lyap}(A, B)$, $W_o \leftarrow \mathrm{Lyap}(A^T, C^T)$
Calculate $\mathcal{R} = W_c W_o$
Obtain the eigenvalues λ from $\mathcal{R}$ as $\sigma^2 = \lambda(\mathcal{R})$
return σ^2 vector

Note that Som are positive numbers $\sigma_1^2 \geq \sigma_2^2 \geq \cdots \sigma_n^2$, $i = 1, \ldots n$ related to Hsv in the following form

$$\sigma_i^2 = \lambda_i (W_c(\gamma) W_o), \quad i = 1, \ldots, n, \tag{6}$$

where dependence of the control effectiveness vector (γ) is also considered. In [3], the reconfigurability ϱ is expressed in terms of the Hsv as:

$$\varrho = \min_{\gamma \in \Omega, i} \{\sigma_i(\gamma)\}. \tag{7}$$

Using Hsv (or Som), a balanced representation is considered. This means controllability and observability gramians are diagonal and equal by using the unique regular matrix T_b affecting the original system matrices. As shown Algorithms 1 and 2, the balanced controllability W_c^b and observability W_o^b gramians are related by $W_c^b = W_o^b = \Sigma$.

With the criterion proposed in [9, 22], the reconfigurability ρ in terms of a balanced gramian (either) is:

$$\begin{aligned}\rho &= \max_{\gamma \in \Omega, i} \{\lambda_i(W_c^b(\gamma)^{-1})\} \\ &= \max_{\gamma \in \Omega, i} \{\lambda_i(W_o^b(\gamma)^{-1})\}, \quad i = 1, \ldots, n.\end{aligned} \tag{8}$$

In view of the relationship between maximal and minimal eigenvalues for a positive definite matrix [21], (8) is equivalent to:

$$\rho = \max_{\gamma \in \Omega, i} \{\lambda_i(W_c^b(\gamma)^{-1})\} = \left[\min_{\gamma \in \Omega, i} \{\lambda_i(W_c^b(\gamma))\}\right]^{-1} \tag{9}$$

In fact for a real symmetric positive definite matrix as that of the controllability gramian, the absolute values of its eigenvalues are equal to its singular values [21, 23]. This leads to make a comparison between metrics proposed in [9] and [3], by comparing (7) with (9), i.e. that under a balanced representation the reconfigurability measures ρ and ϱ are inversely proportional:

$$\rho = \frac{1}{\varrho}. \tag{10}$$

Hence that the same fundamental notion already explained for the controllability gramian is considered in both principal reconfigurability measures proposed in the literature.

Advantage of the reconfigurability ϱ of (7) is that values of the Hsv and Som can be obtained in a direct form using Algorithm 2, with no need of calculating matrix T_b as in Algorithm 1. However, the controllability gramian used directly in the reconfigurability concept (that of [9]) is considered in this paper, because this original notion is clearer and well suited in the context of this paper. In what follows, reconfigurability with dependence of the control effectiveness vector will be explicitly represented by $\rho(\gamma)$.

3.2. Reconfigurability-based index

The reconfigurability measures presented above have been considered for open-loop systems. However, they can be applied to closed-loop systems. The following presentation can be applied to open-loop systems as long as they be stable. Hereafter closed-loop is considered in order to establish the reconfigurability.

In a general framework, the problem of controlling the system (1), known as the standard control problem [4], is established in terms of the following set:

$$\{\psi,\ \Upsilon,\ U\}, \tag{11}$$

where Υ denotes the constraints (components and interconnections) as functional relations describing the behaviour of the controlled system through difference equations, in this case represented by Equation (1). ψ is the objective what the system is expected to achieve by using a set of control laws U given in this paper by:

$$u(k) = Gx(k), \tag{12}$$

where $G \in \mathbb{R}^{r\times n}$ is the feedback control gain, also $v \in \mathbb{R}^q$ is considered being an external signal (the reference signal), and $D_p \in \mathbb{R}^{n\times q}$ is a static feedforward gain matrix. The closed-loop representation is thus given by

$$\Upsilon : \begin{cases} x(k+1) = (A+BG)x(k) + D_p v(k) \\ y(k) = Cx(k). \end{cases} \tag{13}$$

which satisfies the Lyapunov equation denoted by

$$X = \text{Lyap}(A+BG, D_p), \tag{14}$$

where X is the closed-loop controllability gramian. The representation in terms of the set (11) for the closed-loop system with control law (12) in faulty situation is:

$$\{\psi,\ \Upsilon_f,\ U_f\}. \tag{15}$$

In order to accommodate the faults defined in Section 2.2, a new control law $u_f \in U_f$ must be defined and then to achieve the objective ψ.

Concerning manipulation of reconfigurability for control purposes, in this work as SOI controller is suggested as the method to obtain the control law driving the system and then to shape the reconfigurability of the closed-loop system.

The criterion (5), without loss of generality, will be useful to measure the reconfigurability once the systems be in operation. Based on (5), a relative index evaluating the controller quality is proposed under a normalization form. Q_ρ is an *index based on reconfigurability* established as:

$$Q_\rho = \frac{\rho_M - \rho_f}{\rho_M - \rho_n} (\times 100\%), \tag{16}$$

where ρ_M is the upper value for ρ in the worst case in terms of faults, ρ_n is the nominal value for ρ, and ρ_f is the value for ρ ranging between ρ_n and ρ_M, namely $\rho(\gamma)$, the value under faulty conditions. Note that values of Q_ρ will fall in the range $[0,\ 100]\%$ due to the normalization, where 100% is equivalent to the best value and 0 is the lowest value.

Based on the work of [22], the following definition of *admissibility* is considered in order to establish limits of system operation using the proposed index.

Definition 3.2 (Admissibility): A solution to the control problem in a faulty (15) is admissible with respect to a control objective ψ if

$$Q_\rho \geqslant Q_{\rm ad}, \tag{17}$$

where Q_{ad} is a predefined threshold which represents the maximal loss of efficiency that can be admitted when a control solution $u \in U_f$ is used, provided that this solution achieves ψ under a faulty situation Υ_f.

Based on $Q_{\rm ad}$, admissible values for the solutions or admissible operation conditions with this value are established in order to limit the worst system operation under the occurrence of faults. Taking into account the importance of the SOI used in the control reconfigurability computation, this information is considered to manipulate it in closed-loop. The aim is to establish a predefined reconfigurability value expected to be computed from input/output data by using an identification technique.

4. SOI by state feedback for discrete systems

The aim of this section is the design of feedback compensators based on the SOI assignment [12, 16], taking into account that, as previously indicated, the SOI represents the controllability gramian. The most important results in SOI assignment by feedback are presented, also, some theorems are reviewed in order to introduce results for achievement of suitable SOI design even if all conditions for assignment are not satisfied. These results will lead to shape the reconfigurability of a given system in order to obtain online estimates of the assigned value in fault free and faulty situations.

For the system (1), the pair (A, B) is supposed to be stabilizing and the pair (A, D_p) in (13) controllable. In addition, it is assumed that

$$\mathrm{rank}(B) \subseteq \mathrm{rank}(D_p). \tag{18}$$

The problem of the SOI is to find the stabilizing state feedback law (12) such that, for the closed-loop system (13) the Lyapunov equation

$$X = (A + BG)X(A + BG)^T + D_0, \tag{19}$$

with $D_0 = D_p D_p^T + x_0^T x_0$, is satisfied. In other words, to find the conditions to assign $X > 0$ and the set of gains assigning this X. Thus, a definite positive matrix X known as SOI (controllability gramian), defined as follows, can be assigned by feedback.

Definition 4.1 (Assignable SOI): Yasuda and Skelton [16] An SOI matrix $X > 0$ is called assignable to the closed-loop system (13), if X satisfies Lyapunov Equation (19) for some G in (12).

The SOI assignment problem is used to find the set of SOI matrices $X > 0$ such that the Lyapunov equation is satisfied, i.e. the conditions to assign X and the set of gains assigning this X. Note that not all systems can achieve a specified SOI. It is necessary to satisfy some conditions to achieve this assignment.

The system (1) is said completely controllable by SOI (hereafter CCSOI) if exist a control law (12) that assigns any specified SOI $X > 0$. Then, the following theorem establish the conditions to have a system CCSOI.

Theorem 4.2 ([13]): *The discrete LTI system* (1) *is CCSOI if and only if the following equivalent conditions fulfil:*

(*i*) $BB^+A = A$,
(*ii*) $BB^+D_p = D_p$,
(*iii*) $B = D_p$ *and* $BB^+A = A$,

Remark 4.1: Note from Theorem 4.2 that D_p identifies the states that can be affected by the SOI controllers. It follows that D_p is directly connected with B by condition (18). Clearly, actuator location have an important role in the assignment of the SOI and even in the states affected by this assignment, in a similar way as that for tracking the reference input $v(k)$. This fact will be considered after.

Because most systems are not CCSOI and in order to assign a required matrix X to system (1), the conditions of the following theorem are necessary and sufficient.

Theorem 4.3 ([24, 25]): *An SOI represented by a matrix X is assignable by feedback to the discrete LTI system* (1) *if and only if:*

(*i*) $X \geq D_0$.
(*ii*) *X satisfies*

$$(I - BB^+)(AXA^T - X + D_0)(I - BB^+) = 0. \tag{20}$$

where $[\cdot]^+$ *represents the Moore–Penrose inverse.*

Remark 4.2: Note that condition (i) from Theorem 4.3 is very restrictive for practical implementation. This will be relaxed in the following by taking null initial conditions.

The following square factors will be useful for the controller set up:

$$Q = X - D_0 = LL^T, \quad X = MM^T, \tag{21}$$

where the following SVDs are also useful:

$$(I - BB^+)L = E\Sigma F_1^T \tag{22}$$

$$(I - BB^{+})AM = E\Sigma F_2^T. \tag{23}$$

The next theorem is used to obtain the state feedback gain.

Theorem 4.4 ([24, 26]): *Suppose the given X is assignable, i.e. Theorem 4.3 is satisfied, then the set of controllers that assign this X to the system* (1) *by feedback is given by:*

$$G = B^{+}\left(LF_1\begin{bmatrix} I_r & 0 \\ 0 & U_F \end{bmatrix} F_2^T M^{-1} - A\right), \tag{24}$$

where $U_F \in \mathbb{R}^{\alpha\times\alpha}$ *with* $\alpha = n - r$ *is an arbitrary orthonormal matrix* $(U_F U_F^T = U_F^T U_F = I_\alpha)$, I_r *represents an identity matrix with dimension r.*

Lemma 4.1 provides a relationship between the choice of the SOI matrix and the pole placement in closed loop.

Lemma 4.1: *The real part Re*[·] *of the closed-loop poles of* $A_{cl} = A + BG$, *i.e.* $Re(\lambda_{cl})$, *is bounded by the eigenvalues of the product* D_0X^{-1}, *and it is placed inside the unit circle under the following criterion:*

$$Re(\lambda^L) \le Re(\lambda_{cl}) \le 1 \tag{25}$$

where

$$\lambda^L = \sqrt{1 - \max_i\{\lambda_i(D_0X^{-1})\}}. \tag{26}$$

Note that the SOI controller assigns a required controllability gramian, in this way, it is possible to set an *a priori* reconfigurability in terms of the criterion (5). The idea is to assign the reconfigurability through the matrix X assigned by feedback. Condition (20) is very restrictive ; however, the following theorem deals with the flexibility given to the assignment of SOI. It ensures obtaining an SOI matrix close to that required by design.

Theorem 4.5: *If the required matrix X is not completely assignable, i.e. condition* (20) *is not satisfied, then a matrix* X_a *can be assigned by feedback using the gain G from* (24), *which is the optimal solution to the following minimization problem:*

$$V = \min_G \|X_a - (A + BG)X_a(A + BG)^T - D_pD_p^T\|_F^2. \tag{27}$$

i.e. the Frobenius norm, $\|\cdot\|_F^2 = tr(\cdot)^T(\cdot)$, *of the discrete Lyapunov equation.*

It follows from Theorem 4.5 that the set of controllers generated by this control gain generates also a set of matrices X_a that are close to matrix X, which is the matrix originally required. This can be viewed as finding the closest matrix representing the SOI required by design and in this research, that required to set the closed-loop control reconfigurability. Now an expected reconfigurability can be established for a given system with regard to pole placement in order to set a required dynamic response.

5. Data-driven computation

5.1. Markov parameters from input/output data

The ERA [17] is proposed here to find a triplet (A,B,C) representing the system (1). ERA employs the largest singular values representing the input/output relationships, which also characterize the system reconfigurability. The Markov Parameters (MP) are used in order to capture the system dynamics and to obtain a representation given by the excited states. The identification technique known as *Observer/Kalman Filter Identification* (OKID) [17], assumes an observer structure of the treated system and then the system MP can be obtained for further use in the ERA technique. Details of OKID can be found in [17] and [18].

5.2. Computation using the ERA

A Hankel matrix [21] is formed using the MP from OKID [17, 18], i.e Y_i, $i = 1, 2, \ldots l$, as follows:

$$H(i-1) = \begin{bmatrix} Y_i & Y_{i+1} & \cdots & Y_{i+s-1} \\ Y_{i+1} & Y_{i+2} & \cdots & Y_{i+s} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{i+q-1} & Y_{i+q} & \cdots & Y_{i+q+s-2} \end{bmatrix}, \tag{28}$$

where s, $q > n$ are positive integers chosen arbitrary, therefore $H(i-1) \in \mathbb{R}^{mq\times rs}$. Estimation of system matrices is done using Algorithm 3 Tsai et al. [18].

Once the triplet $(\tilde{A}, \tilde{B}, \tilde{C})$ is computed, a system representation close to the balanced representation (gramians are equal and diagonal) is obtained, but this is not always the case, except matrix $H(i-1)$ be of large dimension (s,q too large) or system state energy be also balanced.

Since the nominal system is *supposed to be known* (model-based), the dynamics is known and hence a regular matrix T can be obtained, which is the transformation towards a common representation. Consequently, the controllability gramian obtained through ERA is represented in the nominal state coordinates and then compared to the true expected controllability gramian. Likewise, the index derived from the controllability gramian obtained from measures is compared to that obtained analytically. The procedure to obtain the matrix T follows the transformation presented in [5, 21]. The system obtained using ERA, represented by the triplet $(\tilde{A}, \tilde{B}, \tilde{C})$, is placed in the same coordinates of the original one (A,B,C) by means of:

$$A_e = T\tilde{A}T^{-1}, \quad B_e = T\tilde{B}, \quad C_e = \tilde{C}T^{-1}, \tag{31}$$

Algorithm 3 ERA

Require: Y_i, $i = 0, \ldots, l-1$, order system n

Ensure: Identified system matrices $\tilde{A}, \tilde{B}, \tilde{C}$

1: $H(0) \leftarrow Y_i$, from (28) with $i = 1$

2: $S_n \leftarrow H(0) = USV^T$, $S = \begin{bmatrix} S_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$ (using SVD)

3: Form $\bar{H}(0) = U_n S_n V_n^T$, U_n and V_n come from the first n columns of matrices U and V of previous step.

4: $H(1) \leftarrow Y_i$, from (28) with $i = 2$

5: Form selection matrices $E_m^T \in \mathbb{R}^{m \times mq}$, $E_r^T \in \mathbb{R}^{r \times rs}$

$$E_m = \begin{bmatrix} I_m & 0_m & \cdots & 0_m \end{bmatrix} \quad (29a)$$

$$E_r = \begin{bmatrix} I_r & 0_r & \cdots & 0_r \end{bmatrix}. \quad (29b)$$

6: Calculate system matrices

$$\tilde{A} = S_n^{-1/2} U_n^T H(1) V_n S_n^{-1/2}, \quad (30a)$$

$$\tilde{B} = S_n^{1/2} V_n^T E_r, \quad (30b)$$

$$\tilde{C} = E_m^T U_n S_n^{1/2}, \quad (30c)$$

7: **return** $\tilde{A}, \tilde{B}, \tilde{C}$

where (A_e, B_e, C_e) is the *equivalent* triplet obtained from input/output data in the same original coordinate system (A,B,C) with

$$T = \tilde{\mathcal{C}}\mathcal{C}^T(\mathcal{C}\mathcal{C}^T)^{-1}, \quad (32)$$

or

$$T = (\mathcal{O}^T\tilde{\mathcal{O}})^{-1}\mathcal{O}^T\mathcal{O}, \quad (33)$$

where $\mathcal{O}$ ($\tilde{\mathcal{O}}$) represents the observability matrix and $\mathcal{C}$ ($\tilde{\mathcal{C}}$) the controllability matrix of the original (estimated) system. If matrix C is full rank, then a simplification is done by taking from (31) $C_e = C$, therefore

$$T = C^{-1}\tilde{C} = \tilde{C}. \quad (34)$$

Consequently, both systems are in the same coordinates and gramians can be compared by using (2) applied to (13), i.e. W_c, and that obtained by using:

$$W_c^e = \text{Lyap}(A_e, B_e), \quad (35)$$

where W_c^e represents the equivalent controllability gramian obtained from online input/output data. Then indexes can also be evaluated and compared by using the respective controllability gramians. Figure 1 illustrates the procedure. The excitation signal $u_{exc}(k)$ can be introduced as indicated, or even, a setpoint change in $v(k)$ suffices. The identification block is invoked if the FDI module detects a faults, then the former provides an estimated equivalent model that is used to compute the index. This information is used by the

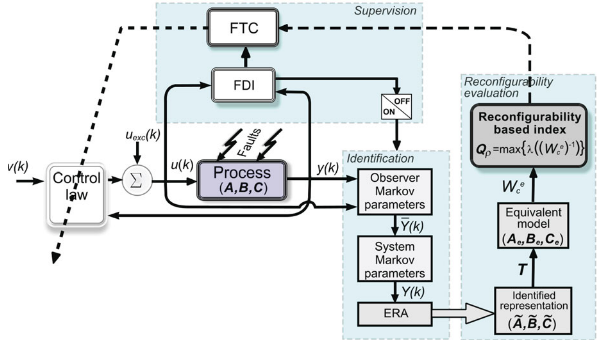


Figure 1. Functional blocks representing the methodology for calculation and comparison of indexes.

FTC module (which also has the information about the fault) in order to set the accommodation of the fault or, depending on the index value and supervision criteria, to switch to other control law.

Remark 5.1: Doublets-type input sequences are proposed as input excitation signals in order to perform the identification [19] and further evaluation of the index based on reconfigurability. Note that we consider the problem from the control viewpoint and we do not try to identify the system for an infinite set of possibilities, but to consider the values of the input between a range of valid control values.

Remark 5.2: Following the previous remark, this type of online identification allows evaluating Q_ρ on demand, i.e. no calculation at each time step using persistent excitations is required, only until it is really needed.

6. Three tanks hydraulic benchmark

In order to illustrate the ideas proposed, this section considers the three tanks hydraulic system shown in Figure 2, which represents a well-understood academic example widely used for teaching and research purposes [27].

For the faulty cases with accommodation of faults, the principle of the pseudo-inverse method [4, 11, 28] is considered in order to recalculate the new feedback and

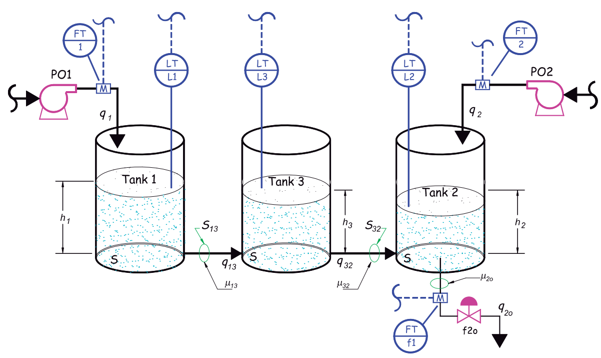


Figure 2. Three tanks system diagram.

feedforward gains under loss of control effectiveness, provided FDI module is *ideal* in this investigation.

6.1. Process description

The plant is composed of three interconnected cylindrical tanks with identical cross-section S, equipped with level sensors $L1$, $L2$, $L3$ in order to measure the liquid level (h_1, h_2, h_3), which are coupled by connecting cylindrical pipes with cross-section S_{13}, S_{32}, and pipe outflow coefficients μ_{13}, μ_{32}. Note that tank 3 is between tank 1 and tank 2. The nominal outflow q_{2o} is located at tank 2 (right in the Figure 2), with circular pipe with cross-section S_{2o} and pipe outflow coefficient μ_{2o}. Two pumps ($PO1$ and $PO2$), driven by DC motors, provide the inflow rates q_1 and q_2, measured by two flow-meters.

The main objective of the control system is to keep prescribed levels h_1, h_2, i.e. a regulatory one of maintaining the level in the tanks at a constant set point. A SOI controller (P-like) with feedforward was implemented in order to show the reconfigurability assignation and estimation, as the control performance was not of prime interest in this work. The idea also is to verify the accommodation performance under certain actuator faults and to check acceptable estimated reconfigurability values. No saturation is given to actuators in order to verify the capability of the reconfigurability estimation under fault accommodation.

6.2. Model

The benchmark is similar to the one presented in [29]. There, the model has been validated with real simulations. The system dynamics can be represented conveniently by the following relationship by using the Torricelli's rule and mass balance equations:

$$S\dot{h}_1(t) = q_1(t) - q_{13}(t) \tag{36}$$

$$S\dot{h}_2(t) = q_2(t) + q_{32}(t) - q_{2o}(t) \tag{37}$$

$$S\dot{h}_3(t) = q_{13}(t) - q_{32}(t), \tag{38}$$

where q_{ab} represents the water flow rate from tank a to b:

$$q_{ab}(t) = \mu_{ab} S_n \, sign(h_a(t) - h_b(t)) \sqrt{2g|h_a(t) - h_b(t)|}. \tag{39}$$

The constraint $h_1 > h_3 > h_2$ is considered in order to keep the following relationship:

$$q_{ab}(t) = \alpha \sqrt{h_a(t) - h_b(t)}, \tag{40}$$

where it has also been considered $\alpha = \mu_{ab} S_n \sqrt{2g}$ and $\mu = \mu_{13} = \mu_{32}$. Table 1 presents the parametric values used in the simulation, upper cases represent the steady-state value. The linearized model is obtained and described by system matrices (41) of a discrete linear state-space representation (1), with a sampling period $h = 1\,s$. Further details about modelling and linearization of this system can be found in [29]. Note that $x = [h_1\ h_2\ h_3]^T$ and $u = [q_1\ q_2]^T$.

Table 1. Parameter values of the system.

Symbol [unit]	Parameter	Value
Q_{10} [$m^3\,s^{-1}$]	Input flow tank 1	3.5×10^{-5}
Q_{20} [$m^3\,s^{-1}$]	Input flow tank 2	2.45×10^{-5}
$q_{1,max}, q_{2,max}$ [$m^3\,s^{-1}$]	Maximal flow from pumps	1.8×10^{-3}
H_{10} [m]	Ref. level tank 1	0.45
H_{20} [m]	Ref. level tank 2	0.225
$H_{n,max}$ [m]	Max. level tank ($n = 1,2,3$)	0.70
S [m^2]	Tanks cross sect.	1.54×10^{-2}
S_{13}, S_{32} [m^2]	Pipe cross-section	5.0×10^{-5}
S_{2o} [m^2]	Pipe cross section	8.0×10^{-5}
μ_{13}, μ_{32}	Pipe coefficient	0.5
μ_{2o}	Pipe coefficient	0.6
ν [m]	Sensor noise ($\mathcal{N}(0, r_b^2)$)	$r_b = 5 \times 10^{-4}$

$$A = \begin{bmatrix} 0.9890 & 0.0001 & 0.0109 \\ 0.0001 & 0.9790 & 0.0114 \\ 0.0109 & 0.0114 & 0.9776 \end{bmatrix}, \tag{41}$$

$$B = \begin{bmatrix} 64.5775 & 0.0014 \\ 0.0014 & 64.2495 \\ 0.3562 & 0.3732 \end{bmatrix}, \quad C = I. \tag{42}$$

In order to test in simulation the online reconfigurability estimation performance, sensor noise is considered to be gaussian with variance 5×10^{-4} added to all sensors.

6.3. Controller design

Taking into account the previous considerations, the design of the controller considers three goals:

- Assigning an SOI in order to calculate from the input/output data an *a priori* value.
- Ensuring a pole placement inside a specific region in order to meet dynamic performance.
- Setting a tracking response with respect to an input reference signal.

6.3.1. Feedback gain

Taking into account condition $rank(B) \subseteq rank(D_p)$, matrix D_p has the value:

$$D_p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{43}$$

The SOI matrix that will be synthesized is represented as:

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{12} & x_{22} & x_{23} \\ x_{13} & x_{23} & x_{33} \end{bmatrix}, \tag{44}$$

and using corollary 4.1 the required poles will be between 0.9 and 1 inside the unit circle. This will ensure a damped dynamics with no overshooting for the closed-loop system [21]. The reconfigurability of the closed-loop system and consequently the controllability gramian will be related by the control synthesis developed.

The upper bound $\lambda^U = 1$ is obtained directly from (25), thus the lower bound λ^L will be found from the values $x_{11}, x_{22}, x_{13}, x_{23}, x_{33}$. Using (43) and (44) in (25), it can be found that the maximal eigenvalue to find λ^L depends on x_{11} and x_{22}. Taking into account the Schur complement, the condition $x_{11} > x_{22}$ should be satisfied. From this product, it can be noted that actuators have no influence on the third output x_{33}. The value $x_{22} = 6.5$ is chosen and in order to have closest eigenvalues $x_{11} = 10$. Because x_{33} has no direct influence in the SOI assignment, a value between x_{11} and x_{22} is chosen, i.e. $x_{33} = 8$. For simplicity, $x_{12} = 0$ and the resting x_{13}, x_{23} variables are chosen smaller in order to satisfy the Schur complement. The imposed limit λ^L is 0.9 by using (26). This value also set the nominal closed-loop reconfigurability value to be ≤ 0.20. Then the final SOI matrix is:

$$X = \begin{bmatrix} 10 & 0 & 0.5 \\ 0 & 6.5 & 0.5 \\ 0.5 & 0.5 & 8 \end{bmatrix}. \tag{45}$$

With this matrix, the closed-loop poles are, using (25) from Corollary 4.1, inside $[0.9195, 1.0]$ and they are $(0.9824, 0.9401, 0.9221)$. However, this matrix cannot be assigned because the condition (ii) of Theorem 4.3 is not satisfied. Instead, Theorem 4.5 is considered to approximate the required matrix to that finally obtained in the Frobenius sense. Thus, if X is the required matrix then X_a will be the final assigned matrix through G, matrix that approximates the first one in the Frobenius norm sens. Using (24), the control gain is:

$$G = \begin{bmatrix} -0.6376 & -0.2149 & 0.0227 \\ 0.1165 & -0.9309 & -0.0683 \end{bmatrix} \times 10^{-3} \tag{46}$$

and the final assigned matrix X_a is:

$$X_a = \begin{bmatrix} 10.1709 & 0.0719 & 1.4417 \\ 0.0719 & 6.5285 & 0.8490 \\ 1.4417 & 0.8490 & 1.1391 \end{bmatrix} \tag{47}$$

which is close to the original required matrix X with poles placed at $(0.9824, 0.9401, 0.9221)$. The Frobenius norm difference between X and X_a is $\|X - X_a\|_F = 7.0092$ introduced by the third state that cannot be driven either by the actuators neither by the external input $v(k)$. This value introduces a reconfigurability value that has not been chosen. Considering the reconfigurability values for each matrix X and X_a (by using Algorithm 2 and (9)):

$$\rho_X = 0.1577, \quad \rho_{X_a} = 1.2579. \tag{48}$$

where the latter is related to the entry x_{33} in X_a. To avoid this, a partition of matrix X_a is considered in order to discriminate the values related to the third entry x_{33}:

$$X_a = \left[\begin{array}{cc|c} 10.1709 & 0.0719 & 1.4417 \\ 0.0719 & 6.5285 & 0.8490 \\ 1.4417 & 0.8490 & 1.1391 \end{array}\right]$$

$$\Rightarrow X'_a = \begin{bmatrix} 10.1709 & 0.0719 \\ 0.0719 & 6.5285 \end{bmatrix}. \tag{49}$$

In a similar way, the following equation is obtained from (45):

$$X' = \begin{bmatrix} 10.0 & 0.0 \\ 0.0 & 6.5 \end{bmatrix}. \tag{50}$$

With the new submatrices, the Frobenius difference is now

$$\|X' - X'_a\|_F = 0.2009, \tag{51}$$

which has been minimized. In addition, if the reconfigurability of the submatrix X' is $\sigma_{X'}$ and the reconfigurability of the submatrix X'_a is $\sigma_{X'_a}$, then for each one the following values are obtained:

$$\rho_{X'} = 0.1538, \quad \rho_{X'_a} = 0.1536, \tag{52}$$

and the difference is small because only the states directly affected by the actuators are considered. Now the assigned reconfigurability corresponds to that required by design.

6.3.2. Feedforward gain

Determination of feedforward gain requires computation of the steady-state value. In order to track the input reference, the number of outputs that can track a reference input vector must be less than or equal to the number of independent inputs. If p denotes the number of output signal to control then it is required that $p \leq r$ be satisfied. Then output matrix C is split as follows

$$C = \begin{bmatrix} C_p \\ C_{n-p} \end{bmatrix} \tag{53}$$

where $C_p \in \mathbb{R}^{p \times n}$ is the matrix associated to the regulated output vector $y_p(k)$ and $C_{n-p} \in \mathbb{R}^{(n-p)\times n}$ is the matrix associated to nonregulated output vector $y_{n-p}(k)$. Then

$$C_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{54}$$

i.e. levels h_1 and h_2 are controlled.

The outputs steady-state response using the control gain (46) is then used. The steady-state gain matrix K_{dc} is used to set the feedforward control gain, and it is calculated using the following transfer function between reference input $v(k)$ and output $y_p(k)$ in steady state:

$$K_{dc} = \lim_{z \to 1} C_p(zI - A_{bf})^{-1} D_p, \tag{55}$$

where $A_{bf} = A + BG$ is the closed-loop matrix. The closed-loop system with feedforward controller is now

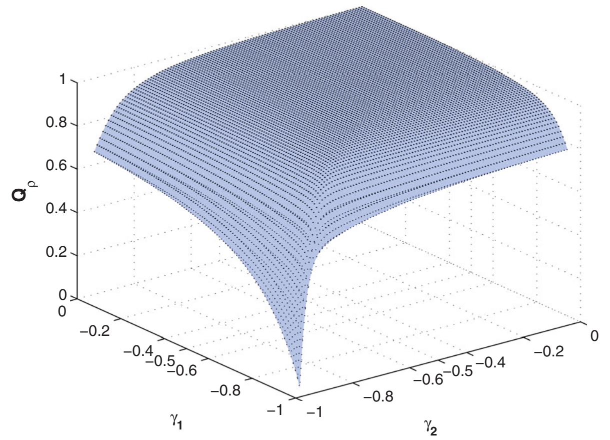


Figure 3. Assigned reconfigurability-based index.

expressed, with $r(t) = K_{dc}^{-1} v(k)$, as:

$$x(k+1) = A_{bf} x(k) + D_p r(t) \qquad (56)$$
$$y_p(k) = C_p x(k),$$

viewed $r(t)$ as the reference signal. Computed values are

$$K_{dc} = \begin{bmatrix} 21.1377 & -2.1148 \\ 2.9806 & 12.6244 \end{bmatrix}. \qquad (57)$$

6.4. Simulation results

In the following, the results from the controllers design applied to the three tanks benchmark are presented, the nominal, faulty and accommodated scenarios are considered. Several fault cases has been considered, however, it is only presented the case where actuator 1 has 70% of loss of control effectiveness, whereas for actuator 2 is 40%.

6.4.1. Fault free case

Figure 3 presents the offline reconfigurability-based index obtained by using the state feedback controller (46) assigning the partitioned SOI (49). Matrix B is viewed as $B = [b_1\ b_2]$, and each of this vectors is affected by the loss of effectiveness, i.e. different values of γ_i are used to evaluate ρ using (5). The nominal value is $\rho_n = 0.1536$ and in the worst case $\rho_M = 12.0223$. After that Q_ρ is computed using (16). Axes x,y show the variation for each γ_i, whereas z axis represents the Q_ρ values. In this way, all possible combinations of actuator faults for this systems can be obtained in terms of Q_ρ and then evolution of reconfigurability from faults can be analysed.

The curve shows that in the case of independent faults affecting only one actuator at a time, the system is more susceptible to faults affecting the actuator 2. However, if faults occur in both actuators at the same time, then the system is more susceptible to faults affecting actuator 1. Whatever the case, the reconfigurability is regular for both actuators.

Nominal system response is shown in Figure 4 where step responses with respect to set-point $v(k)$ are simulated. The dynamic responses (Figure 4(a)) demonstrate that tracking is correctly synthesized. Figure 4(b) shows the corresponding control inputs (inflow to tanks 1 and 2) for step changes in the reference inputs. The online evaluation of the controllability gramian, and therefore Q_ρ, considers to apply ERA with observer at a defined time depending on the faulty conditions or the required evaluation given by the supervision system. In both cases, computation considers set point variations, viewed as the required excitation in order to perform the identification. The nominal reference values for each output (h_1, h_2) are 0.45 m and 0.225 m as indicated by dashed lines in Figure 4(a). For identification purposes, these reference values change between ±10% (doublets) from their nominal value at time 1000 *s*, with a duration of 200 s. This last represents the time window used to get data and then to compute $(\tilde{A}, \tilde{B}, \tilde{C})$ and therefore $\tilde{X}$ (the identified SOI) or $\tilde{W}_c$.

The variations are also reflected on the control signals, i.e. for each input channel (q_1 and q_2), as shown in Figure 4(b). The considered number of MP is 15 (see Section 5.2), and the integers q,s to form the Hankel matrix in (28), are chosen to be $q = 8$ and $s = 2q$. After computation of the triplet $(\tilde{A}, \tilde{B}, \tilde{C})$, Equation (31) is used in order to compare indexes, where matrix T has been calculated from (34).

Using the values obtained from the simulation presented in Figure 4, matrix $\tilde{X}$ is also partitioned in a similar way to (49) in order to obtain the following estimated reconfigurability value:

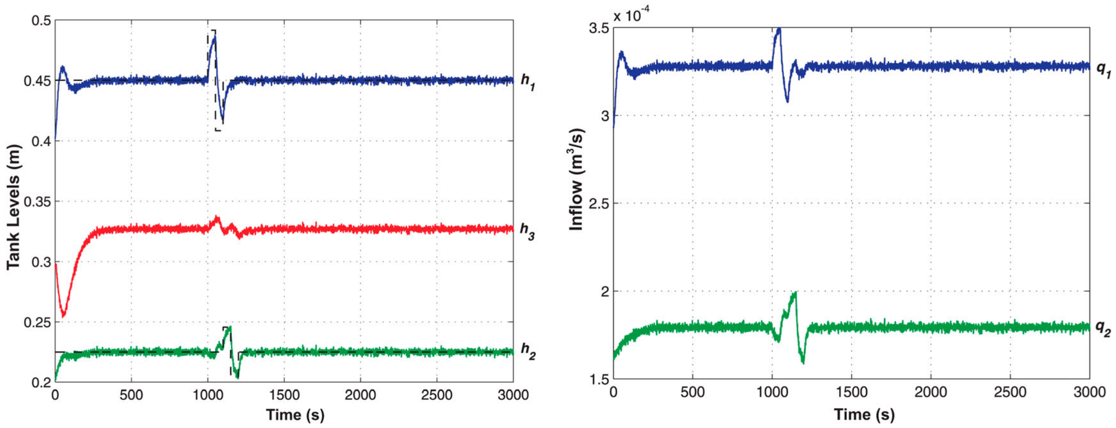
$$\rho_{\tilde{X}} = 0.1535, \qquad (58)$$

which effectively corresponds to that assigned by feedback using the SOI controller (compare with (52)).

6.4.2. Faulty case

The presented faulty case considers fault in pumps $PO1$ and $PO2$ with 70% and 40% s of loss of control effectiveness, respectively. For illustrative purposes, both faults occur at time 1500 s. As shown in Figure 5, the levels in all tanks h_1, h_2, h_3 (labeled as *faulty*) decay according to the effect of loss of control effectiveness in both pumps. Same effect can be seen for the control signals, presented in Figure 6 with signals labelled also as *faulty*. The fault effect on the output performance with respect to the tracking response is noticeable from the figure. Likewise, the fault affects the actuator signals, as it can be noted.

In this faulty case, the online reconfigurability-based index Q_ρ^{on}, is computed at time 2500 s using (5) and then (16), but with identified system matrices obtained from input/output data by using the ERA/OKID technique (same input excitation and parameters as that used for the fault-free case). At this point, $Q_\rho^{on} =$



(a) (b)

Figure 4. Nominal system response with set-point $[0.45,\ 0.225]^T$ m. Doublets applied at 1000 s. (a) Output signals (level tanks) and (b) Control signals (tank inflows).

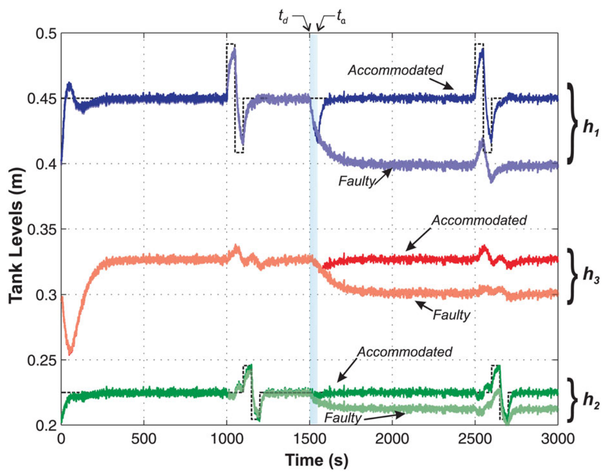


Figure 5. Outputs in faulty and accommodated cases.

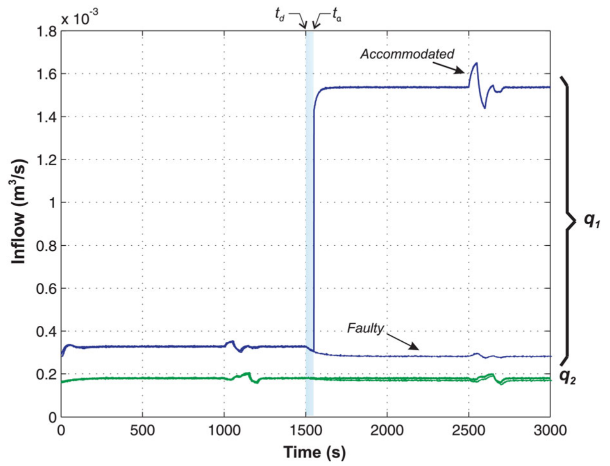


Figure 6. Control signals in faulty and accommodated cases.

95.35%, whereas the assigned offline value was $Q_\rho^{off} = 97.526\%$.

Consider now fault accommodation. In the same Figures 5 and 6 and for comparison, the signals in the fault accommodation case are also shown, they are labelled as *accommodated*. It has been supposed a time delay of 50 s between fault detection time (t_d) and fault accommodation time (t_a), these values are also depicted. In this case, the output performance with respect to the tracking is recovered, as shown in Figure 5. As can be expected, increasing the control signals is required to compensate the loss of actuation, as shown in Figure 6. Afterward, the identification algorithm is invoked again at time 2500 s in order to compute the online reconfigurability-based index $Q_\rho^{on}(\mathrm{acc}) = 91.74\%$.

Under accommodation condition, the computed index is not the same because, in part, noise affects the ERA method due to actuator signal is reduced by the degradation or loss of effectiveness, then noise hides the signal from inputs to outputs, becoming computation less reliable. In addition, due to the compensation used for accommodating the fault, noise in the closed-loop is also amplified by the actuation effect. This is not surprising if it is considered the energetic reasons explained in Section 3.1: Directions with small signal-to-noise ratio have less significant controllability.

7. Conclusion

This paper has proposed a method for the assignation and evaluation of the SOI from input/output data in order to estimate the control reconfigurability of a linear system affected by actuator loss of effectiveness type faults. Founded on the same bases of the reconfigurability concept, the ERA identification technique is proposed in order to compute the SOI. A few assignation basic theorems have also been revisited in order to properly setup an SOI feedback controller. The assignment allowed to shape the reconfigurability expected to be measured using the proposed identification technique. In this way, the online computation of the index based on reconfigurability can further compared to the nominal one obtained offline. The proposed index can be used to determine the capability of the system under fault occurrence and then to set admissible bounds in order to state conditions of operation. The index, in terms of the closed-loop system output response with known inputs, can be useful for analysis and redesign of the FTC system already established for a process.

The good practical adequateness of both approaches, control synthesis (in nominal and faulty cases) and identification of the SOI, were demonstrated by simulation on a hydraulic system of coupled tanks. Simulation results shown the applicability of the algorithm for online computation of reconfigurability after actuator fault occurrence.

Acknowledgements

The first author would like to express his sincere gratitude to Pr. Didier Theilliol and Pr. Dominique Sauter from Lorraine University, at Nancy, France, by the support given in the development of this research.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Brian M. González-Contreras http://orcid.org/0000-0003-1556-8435
G. L. Osorio-Gordillo http://orcid.org/0000-0002-3445-8867
F. R. López-Estrada http://orcid.org/0000-0002-8724-335X
G. Valencia-Palomo http://orcid.org/0000-0002-3382-8213
C. M. Astorga-Zaragoza http://orcid.org/0000-0002-4678-9006

References

[1] Zhang Y, Jiang J. Bibliographical review on reconfigurable fault-tolerant control systems. Annu Rev Control. 2008 Dec;32:229–252.
[2] Lan J, Patton RJ. A new strategy for integration of fault estimation within fault-tolerant control. Automatica. 2016 July;69:48–59.
[3] Wu NE, Zhou K, Salomon G. Control reconfigurability of linear time-invariant systems. Automatica. 2000 Nov;36:1767–1771.
[4] Blanke M, Kinnaert M, Lunze J, et al. *Diagnosis and Fault-Tolerant Control*. 3rd ed. Berlin (Germany): Springer; 2016. (Control Systems Series).
[5] Nestorovic T, Trajkov M. Optimal actuator and sensor placement based on balanced reduced models. Mech Syst Signal Process. 2013;36:271–289.
[6] Frei C, Kraus F, Blanke M. Recoverability viewed as a system property. Proceedings of the European Control Conference, ECC'99. Karlsruhe; June 1999.
[7] Wu NE, Thavamani S, Blanke M. Sensor fault masking of a ship propulsion system. Control Eng Pract. 2006 Nov;14:1337–1345.
[8] Moore B. Principal component analysis in linear systems: controllability, observability, and model reduction. IEEE Trans Automat Contr. 1981 Feb;26:17–32.
[9] Staroswiecki M. On reconfigurability with respect to actuator failures. In: Proceedings of the 15th Triennial World Congress IFAC 2002. Barcelona; July 2002. p. 775–780.
[10] Yang Z. Reconfigurability analysis for a class of linear hybrid systems. In: Zhang H-Y, editor. Proceedings of the 6th IFAC SafeProcess'06. Beijing, China; Aug. 30–Sept. 1 2006. p. 1033–1038.
[11] Jiang J, Yu X. Fault-tolerant control systems: a comparative study between active and passive approaches. Annu Rev Control. 2012 Dec;36:60–72.
[12] Yasuda K, Skelton RE, Grigoriadis KM. Covariance controllers: a new parametrization of the class of all stabilizing controllers. Automatica. 1993;29(3):785–788.
[13] Hotz A, Skelton R. Covariance control theory. Int J Control. 1987;46(1):13–32.
[14] Skelton RE, Ikeda M. Covariance controllers for linear continuous-time systems. Int J Control. 1989;49(5):1773–1785.
[15] Hsieh C, Skelton R. All covariance controllers for linear discrete-time systems. IEEE Trans Automat Contr. 1990 Aug;35:908–915.
[16] Yasuda K, Skelton RE. Assigning controllability and observability gramian in feedback control. AIAA J Guid Control Dyn. 1991;14(5):878–885.
[17] Juang J-N, Phan M, Horta L, Longman R. Identification of observer/kalman filter markov parameters: theory and experiments. AIAA J Guid Control Dyn. 1993 Mar–Apr;16:320–329.
[18] Tsai J-H, Ebrahimzadeh F, Hsu W, et al. Modeling and tracker for unknown nonlinear stochastic delay systems with positive input constraints. Appl Math Model. 2016 Dec;40:10447–10479.
[19] Verhaegen M, Verdult V. *Filtering and System Identification*. Cambridge: Cambridge University Press; 2007.
[20] Yu X, Jiang J. Hybrid fault-tolerant flight control system design against partial actuator failures. IEEE Trans Control Syst Technol. 2012 Jul;20:871–886.
[21] Zhou K, Doyle J, Glover K. *Robust and optimal control*. Englewood Cliffs, NJ: Prentice-Hall; 1996.
[22] Staroswiecki M. Actuator faults and the linear quadratic control problem. In: Proceedings of the Conference on Decision and Control, CDC'03. Honolulu, HI, USA; Dec. 2003. p. 959–965.
[23] Bernstein S. *Matrix Mathematics*. 2nd ed. Princeton, NJ: Princeton University Press; July 2009.
[24] Collins E, Skelton R. A theory of state covariance assignment for discrete systems. IEEE Trans Automat Contr. 1987 Jan;AC-32:35–41.
[25] Xu J-H, Skelton R. An improved covariance assignment theory for discrete systems. IEEE Trans Automat Contr. 1992 Oct;37:1588–1591.
[26] Grigoriadis KM, Skelton R. Minimum-energy covariance controllers. Automatica. 1997;33(4):569–578.
[27] Costa-Castelló R, Puig V, Blesa J. On teaching model-based fault diagnosis in engineering curricula. IEEE Contr Syst Mag. 2016 Feb;8:3–11.
[28] Johansen T, Fossen T. Control allocation: a survey. Automatica. 2013;49(5):1087–1103.
[29] Noura H, Theilliol D, Sauter D. Actuator fault-tolerant control design: demonstration on a three-tank-system. Int J Syst Sci. 2000 Jan;31:1143–1155.