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Advanced backstepping control based on ADR for non-affine non-strict feedback nonlinear systems

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ABSTRACT
Few results are made on non-affine non-strict feedback nonlinear systems, which is a challenging problem in the control theory. In this paper, a novel control method based on an advanced backstepping and auto disturbance rejection is presented for a class of non-affine non-strict nonlinear feedback systems. The proposed advanced backstepping controller consists of differentiator and extended state observer, which are respectively used to approach the derivative of the virtual control and estimate the unknown part of the system. The framework of the proposed controller is both systematic and simple, and the assumptions have been relaxed. Moreover, the input to state stability analysis shows that the system states can asymptotically converge to an arbitrarily small region of equilibrium point. The simulation studies proved the effectiveness of the proposed design scheme.

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Non-affine non-strict feedback; auto disturbance rejection (ADR); extended state observer (ESO); track differentiator (TD); advanced backstepping; input to state stability (ISS)

1. Introduction

In the past decade, the problem of control design for complex nonlinear systems has received considerable attention and lots of powerful control approaches have been proposed for the affine system, such as feedback linearizing control design based on differential geometric [1], adaptive backstepping control design [2] and so on. However, relatively fewer results are available for the control of non-affine nonlinear systems, which have non-affine appearance of the control, more representative than strict feedback systems [3]. Many practical systems are of the non-affine form, such as biochemical process, aircrafts control system and so on [4,5].

Until now, no systematic control design approach has been formed for the non-affine system, of which the control is still a challenging and meaningful issue in the nonlinear area. Based on the diffeomorphism transform and implicit function theorem, a direct adaptive multi-layer neural network control scheme has been developed for the single-input, single-output, non-affine, nonlinear dynamical systems with strong relative degree [6]. With the high gain observer, the method in [4] is extended to the non-affine nonlinear system with zero dynamics [7]. Then, a direct adaptive neural network control is proposed for the strict feedback system with non-affine input and unknown saturation, in which a disturbance observer is developed to estimate the unknown compounded disturbance [8]. A novel adaptive critic controller based on the robust neural network is proposed for the strict feedback system with non-affine input [9]. In the control design for non-affine nonlinear system, the fuzzy system parameter adaptation laws are tuned by the projection algorithm to ensure that the parameter matrix is bounded away from singularity and prevent parameter drift [10,11]. More specifically, it is worth mentioning that for the above approaches, the adjustable parameters of the fuzzy or neural network systems are updated by an adaptive law, which depended on the Lyapunov stability analysis. Different from others, the adjustable parameters in the fuzzy system are updated by using a gradient descent adaptation algorithm, meanwhile, the direct adaptive fuzzy control scheme for chain integration system with non-affine input is proved effective [12,13]. From the above papers, it can be noticed that the design schemes for the non-affine pure feedback nonlinear systems belong to the direct adaptive neural network control, in which the implicit function theorem is applied to illustrate the existence of an ideal controller that can achieve the control objective and neural networks are applied to construct this unknown ideal implicit controller. However, adaptive neural networks controllers based on the backstepping design method have some drawbacks. First, the inputs of neural networks or fuzzy systems used to construct the unknown ideal implicit controllers include the partial derivatives of virtual control signals, which contain neural or fuzzy basis function or sign function leading to the partial differential tedious and complex. Second, the complexity exhibits an exponential increase as the order of the controlled system grows.

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In [14,15], by utilizing the mean value theorem, the original non-affine system is transformed to a new system in which virtual and actual control variables appear in an affine form, then, the indirect adaptive neural network control incorporating the dynamic surface control technique is proposed, avoiding the problem of explosion of complexity, but it needs to know the upper bound of the gain function in advance. In [3], the Taylor series expansion method is adopted to transform the non-affine input system into the affine input system. Subsequently, the indirect adaptive Gaussian radial basis function neural network sliding mode control approach is employed. The indirect adaptive fuzzy control laws for the chain integration system with non-affine input proposed in [16,17] are composed of three terms: a linear term specifying the desired closed-loop dynamics, an adaptive fuzzy term used to approximately construct an ideal uncertainty compensator and a robustifying term applied to compensate for disturbances and approximation errors. The approach proposed in [16,17] is applied to the strict feedback nonlinear system with non-affine input [18] and a chaotic system with non-affine input [19]. More specifically, the basic idea in [16–19] that change the non-affine form into the affine form is adding and subtracting a linear term of control variable to non-affine input terms. This basic idea is equivalent to the auto disturbance rejection controller (ADRC) design idea proposed in [20,21]. The necessity of a paradigm shift in the feedback control system, the basic idea of ADRC, is argued in [22]. Moreover, the ADRC design idea is successfully applied in many fields [23,24]. In [25–29], the ADRC design idea is proposed for non-affine nonlinear pure feedback systems.

However, the object models in the above references are non-affine strict feedback or pure feedback nonlinear systems. Until now, few results about the non-affine, non-strict feedback form have been reported in the existing literature, since the traditional backstepping method is applicable to the strict feedback form. To deal with the non-strict feedback form, an extended backstepping method is proposed in [26]; however, the system studied has the affine form. In this paper, we consider the non-affine, non-strict feedback nonlinear system and put forward an ADRC design scheme based on the advanced backstepping, in which auto disturbance rejection (ADR) design is embedded in each step. Moreover, differentiator and extended state observer (ESO) are respectively used to estimate the virtual control of the derivative and the unknown part of the system, avoiding the partial differential tedious of virtual control. What is more, the input to state stability (ISS) analysis is used to show that the system states can asymptotically converge to an arbitrarily small region of equilibrium point.

The rest of the paper is organized as follows: a class of single-input, single- output (SISO), non-affine, non-strict feedback, nonlinear systems and control objective as well as some preliminary results are described in Section 2. The proposed ADRC scheme is presented in Section 3 with its stability analysis. In Section 4, the proposed control algorithm is used to control a simple, non-affine, non-strict feedback, nonlinear system, meanwhile, the comparison with the approach mentioned in [25] is made. Conclusions are drawn in Section 5.

2. Problem formulation and preliminaries

2.1. Problem statement

Consider the SISO, non-affine, non-strict feedback, nonlinear systems in the following form :

$$\begin{aligned} \dot{x}_1 &= f_1\left(x_1, x_2, x_3\right) \\ &\vdots \\ \dot{x}_i &= f_i\left(x_1, \ldots, x_i, x_{i+1}, x_{i+2}\right) \\ &\vdots \\ \dot{x}_{n-1} &= f_{n-1}\left(x_1, \ldots, x_{n-1}, x_n\right) \\ \dot{x}_n &= f_n\left(x_1, \ldots, x_n, u\right), \end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in R^n$ are the state vectors and $u \in R$ is the control input. The unknown nonlinear functions $f_i(\cdot)$ are sufficiently smooth. In this paper, the control objective is to design a controller for the systems so that (i) all the signals in the closed-loop remain uniformly ultimately bounded and (ii) the states $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ are stabilized near the equilibrium point.

Assumption 2.1: For a compact set Ω, let $g_i(\bar{\boldsymbol{x}}_{i+1}, x_{i+2}) = (\partial f_i(\bar{\boldsymbol{x}}_{i+1}, x_{i+2}))/(\partial x_{i+1}) \neq 0, i = 1, \ldots, n-1,\quad g_n(\bar{\boldsymbol{x}}_n, u) = (\partial f_n(\bar{\boldsymbol{x}}_n, u))/(\partial u) \neq 0$, where $x_{n+1} = u$, $\bar{\boldsymbol{x}}_{i+1} = [x_1, \ldots, x_{i+1}]^{\mathrm{T}}, i = 1, \ldots, n-1$. Without loss of generality, assume that there exists a positive constant b such that $0 < b \leq g_i(\cdot) < \infty, \forall(\bar{\boldsymbol{x}}_{i+1}, x_{i+2}) \in \Omega, i = 1, \ldots, n-1$.

Assumption 2.2: If and only if $\bar{\boldsymbol{x}}_{i+1} = 0, i = 1, \ldots, n-1$, then $f_i(\bar{\boldsymbol{x}}_{i+1}, x_{i+2}) = 0$. In other words, the equilibrium point of system(1) is the origin point.

Assumption 2.3: All states are available.

Remark 2.1: $f_i(\cdot)$ is sufficiently smooth, then $\forall(\bar{\boldsymbol{x}}_{i+1}, x_{i+2}) \in \Omega$, $g_i(\cdot)$ is also continuously smooth. Assumption 2.1 guarantees the controllability of the system(1) and Assumption 2.3 is the prerequisite for states feedback control and ESO design.

2.2. ESO

Consider the following system:

$$\dot{z} = H(t) + BU, \tag{2}$$

where $H(t)$ is the total uncertainty and U is the input. Then, we add an extended state z_0 as the uncertainty $H(t)$, and system (2) can be written as

$$\begin{aligned} \dot{z} &= z_0 + BU, \\ \dot{z}_0 &= G(t), \end{aligned} \tag{3}$$

where the function $G(t)$ is the derivative of the uncertain term $H(t)$, which is uncertain as well. Then the ESO for system (3) is proposed in the following form such as mentioned in [23]:

$$\begin{aligned} E_1 &= Z_1 - z, \\ \dot{Z}_1 &= Z_2 - \beta_{01} f_{c1}(E_1) + BU, \\ \dot{Z}_2 &= -\beta_{02} f_{c2}(E_1), \end{aligned} \tag{4}$$

where E_1 is the estimation error of the ESO, Z_1, Z_2 are the observer states, and $\beta_{01} > 0$, $\beta_{02} > 0$ are the observer gains. The function $f_{ci}(\cdot)$ is a nonlinear function or linear, which satisfies that $ef_{ci}(e) > 0$, $\forall e \neq 0, f_{ci}(0) = 0$, waiting to be properly constructed. For example, we can design $f_{ci}(\cdot)$ as the following:

$$\begin{aligned} f_{c1}(E_1) &= E_1, \\ f_{c2} &= |E_1|^{\alpha_1} \operatorname{sign}(E_1), \end{aligned} \tag{5}$$

where $0 < \alpha_1 < 1$.

Lemma 2.1: *Consider system* (2) *and ESO* (4), *there exist observer gains* β_{01}, β_{02} *and* $\alpha_1 \in (0,1)$, *such that the estimated states* Z_1, Z_2 *converge into a residual set of the actual states* $z, H(t)$. *In order to improve the estimation accuracy, when choosing the gains, we should have to expand the following inequality* [26]:

$$\frac{1}{4}\beta_{01}^2 > \beta_{02} > |G(t)|. \tag{6}$$

Detailed proof can be found in [23,27].

2.3. Tracking differentiator

To estimate a signal without the mathematical expression or difficult to construct the model, tracking differentiator (TD) is employed to estimate the derivation of the signal [27]. In this paper, to avoid the complicated differential of virtual control, we adopt the second-order system-derived differentiator proposed in [27], such as the following:

$$\begin{aligned} \dot{\upsilon}_1 &= \upsilon_2, \\ \dot{\upsilon}_2 &= -\lambda^2 \operatorname{sign}(\upsilon_1 - r(t))\,|\upsilon_1 - r(t)|^{\alpha} - \lambda \upsilon_2, \end{aligned} \tag{7}$$

where $r(t)$ is the virtual control in the backstepping design. υ_1, υ_2 are the differentiator states. α and λ are parameters to be designed and satisfy the following inequality:

$$0 < \alpha < 1, \quad \lambda > 0. \tag{8}$$

Then $\upsilon_2 \to \dot{r}(t)$, meanwhile, $\upsilon_1 \to r(t)$.

2.4. ISS

Definition 2.1: System [1]

$$\dot{x} = f(t, x, u), \tag{9}$$

where f is piecewise continuous in t and locally Lipschitz in u is said to be ISS if there exist a class KL function β and a class K function γ, such that, for any $x(0)$ and for any input $u(\cdot)$ continuous and bounded on $[0, \infty)$ the solution exists for all $t \geq 0$ and satisfies

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0) + \gamma\left(\sup_{t_0 \leq \tau \leq t} |u(\tau)|\right). \tag{10}$$

For all t_0 and t such that $0 \leq t_0 \leq t$.

Suppose that for system(9), there exist a C^1 function $V : \Re_+ \times \Re^n \to \Re_+$ such that for all $x \in \Re^n$ and $u \in \Re^m$ [1]

$$\begin{aligned} &\gamma_1(|x|) \leq V(t,x) \leq \gamma_2(|x|), \\ &|x| \geq \rho(|u|) \Rightarrow \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x,u) \leq -\gamma_3(|x|), \end{aligned} \tag{11}$$

where γ_1, γ_2 and ρ are class K_∞ functions and γ_3 is a class K function. Then, system (9) is ISS with $\gamma = \gamma_1^{-1}\gamma_2\rho$.

3. Advanced backstepping based on ADR

3.1. ADR concept for non-affine sytems

Consider the following non-affine system:

$$\dot{x} = f(x, u), \tag{12}$$

where $x \in \Omega \subset R, \Omega$ is a compact set, f is a uncertain smooth continuous function, meanwhile, $(\partial f/\partial u) \neq 0$ and x is available. Without loss of generality, let $(\partial f/\partial u) > 0$, then the feedback linearization is performed by rewriting (12) as:

$$\dot{x} = f(x, u) - c_0 u + c_0 u, \tag{13}$$

where $c_0 > 0$ is the parameter to be chosen. Define $F(x,u) = f(x,u) - c_0 u$ as the new uncertain term. According to (4), the ESO for (13) can be constructed as follows:

$$\begin{aligned} e_1 &= z_1 - x, \\ \dot{z}_1 &= z_2 - \beta_1 g_{c1}(e_1) + c_0 u, \\ \dot{z}_2 &= -\beta_2 g_{c2}(e_1), \end{aligned} \tag{14}$$

where ESO's states $z_1 \to x$, $z_2 \to F(x,u)$. $\beta_{01} > 0$, $\beta_{02} > 0$ are the observer gains and the function $g_{ci}(\cdot)$ is a nonlinear function or linear function to be designed. Then, the stabilizing controller for system (12) can be chosen as follows:

$$u(t) = \frac{1}{c_0}(-z_2 - kx), \tag{15}$$

where $k > 0$ is the parameter to be designed.

Lemma 3.1 ([25,29]): *Consider system* (12), *controller* (15) *based on ESO can asymptotically stabilize system*(12)*to a residual set of the origin, whose size is dependent on* β_1, β_2, k *and* $g_{ci}(\cdot)$.

Remark 3.1: When $g_{c1}(\cdot), g_{c2}(\cdot)$ are chosen as nonlinear functions, the parameters are still needed to satisfy that $\beta_1, \beta_2 > 0$ and $\beta_1^2 - 4\beta_2 > 0$. The detailed proof can be found in [28].

3.2. Advanced backstepping design

According to Lemma 2.2 and assumptions, system (1) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1 &= c_2 x_2 + F_1(x_1, x_2, x_3), \\ \dot{x}_2 &= c_3 x_3 + F_2(\bar{\boldsymbol{x}}_2, x_3, x_4), \\ &\vdots \\ \dot{x}_n &= c_{n+1} u + F_n(\bar{\boldsymbol{x}}_n, u), \end{aligned} \tag{16}$$

where $F_i = f_i - c_{i+1}x_{i+1}, i = 1, \ldots, n-1$, $F_n(\bar{\boldsymbol{x}}_n, u) = f_n(\bar{\boldsymbol{x}}_n, u) - c_{n+1}u$ are new uncertain functions, $c_i, i = 2, \ldots, n+1$ are parameters to be chosen, and the sign of $c_{i+1}, i = 1, \ldots, n$ is the same with the sign of $g_i, i = 1, \ldots, n$.

Next, we will discuss the advanced backstepping design based on ADR. First, second-order ESO is constructed for each state to estimate the uncertain term $F_i(\cdot), i = 1, \ldots, n$ in every subsystem, such as follows:

$$\begin{aligned} e_{i,1} &= Z_{i,1} - x_i, \\ \dot{Z}_{i,1} &= Z_{i,2} - \beta_{i,1} g_{i,1}(e_{i,1}) + c_{i+1} x_{i+1}, \\ \dot{Z}_{i,2} &= -\beta_{i,2} g_{i,2}(e_{i,1}), \\ & i = 1, 2, \ldots n, \end{aligned} \tag{17}$$

where $Z_{i,1}, Z_{i,2}$ are states of ESO,$g_{i,1}(\cdot), g_{i,2}(\cdot)$ are nonlinear function to be designed, for example: $g_{i,1}(e_{i,1}) = e_{i,1}$, $g_{i,2}(e_{i,1}) = |e_{i,1}|^{1/2}\mathrm{sgn}(e_{i,1})$, and $Z_{i,1} \to x_i$, $Z_{i,2} \to F_i(\bar{\boldsymbol{x}}_i, x_{i+1})$.

Similarly, we can also design TDs for virtual control $x_{id}, 2 \le i \le n$ defined below. TDs are designed as follows:

$$\begin{aligned} \dot{\upsilon}_{i,1} &= \upsilon_{i,2}, \\ \dot{\upsilon}_{i,2} &= -\lambda^2 \mathrm{sign}\left(\upsilon_{i,1} - x_{id}\right)\left|\upsilon_{i,1} - x_{id}\right|^{\alpha} - \lambda \upsilon_{i,2}, \end{aligned} \tag{18}$$

where $\upsilon_{i,1}, \upsilon_{i,2}$ are states, and α, λ are the parameters to be designed.

Now, to design the controller for non-strict feedback system (16), the advanced backstepping is proposed. The design procedure is as follows:

Step 1: Define $e_1 = x_1$. Its derivative is

$$\dot{e}_1 = F_1(x_1, x_2, x_3) + c_2 x_2. \tag{19}$$

Take x_2 as the virtual control variable. Since F_1 contains x_3, according to the traditional backstepping, F_1 should be countervailed. However, we do not deal with F_1 in this step. F_1 will be countervailed in step 3. So, we choose the following virtual control:

$$x_{2d} = -\frac{k_1}{c_2} e_1. \tag{20}$$

where $k_1 > 1$ is a constant to be chosen.

Choose a Lyapunov function as follows:

$$V_1 = \frac{1}{2} e_1^2. \tag{21}$$

Define $e_2 = x_2 - x_{2d}$, combining with formula (20), the derivative of V_1 along the system trajectory is:

$$\dot{V}_1 = -k_1 e_1^2 + e_1 F_1 + c_2 e_1 e_2. \tag{22}$$

Step 2: The derivative of e_2 is

$$\dot{e}_2 = F_2 + c_3 x_3 - \dot{x}_{2d}. \tag{23}$$

Similar to step 1, F_2 containing x_4 will be countervailed in step 4. So, the virtual control can be designed as follows:

$$x_{3d} = -\frac{1}{c_3}\left(k_2 e_2 + c_2 e_1 - \upsilon_{1,2}\right), \tag{24}$$

where $k_2 > 1$ is a constant to be chosen. $\upsilon_{1,2}$ is the state of TD defined in (18), avoiding the computation of $\dot{x}_{2d}$. Choose a Lyapunov function V_2 as follows:

$$V_2 = V_1 + \frac{1}{2} e_2^2. \tag{25}$$

Define $e_3 = x_3 - x_{3d}$, combining with the virtual control (24), the derivative of V_2 along the system trajectory is:

$$\begin{aligned} \dot{V}_2 &= -k_1 e_1^2 + e_1 F_1 + e_2(c_2 e_1 + \dot{e}_2) \\ &= -k_1 e_1^2 + e_1 F_1 + e_2(c_2 e_1 + c_3 x_3 + F_2 - \dot{x}_{2d}) \\ &= -k_1 e_1^2 - k_2 e_2^2 + e_1 F_1 + c_3 e_2 e_3 \\ &\quad + e_2 F_2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right). \end{aligned} \tag{26}$$

Step 3: The derivative of e_3 is

$$\dot{e}_3 = F_3 + c_4 x_4 - \dot{x}_{3d}. \tag{27}$$

Similar to step 1, F_3 containing x_5 will be countervailed in step 5. And, F_1 contains only x_1, x_2, x_3, so the virtual control should counteract F_1. Therefore, the virtual control can be designed as follows:

$$x_{4d} = \begin{cases} -\dfrac{1}{c_4}\begin{pmatrix} k_3 e_3 + c_3 e_2 \\ -\upsilon_{2,2} + \dfrac{e_3 e_1 Z_{1,2}}{|e_3|^2} \end{pmatrix}, |e_3| \geq \varsigma, \\ 0, \quad |e_3| < \varsigma, \end{cases} \tag{28}$$

where $k_3 > 1$ and $\varsigma > 0$ are constants to be designed. $Z_{1,2}$ is the state of ESO defined in (17), which is used to estimate F_1 in real time. $\upsilon_{2,2}$ is the state of TD defined in (18), avoiding the computation of $\dot{x}_{3d}$, choose a Lyapunov function V_3 as follows:

$$V_3 = V_2 + \frac{1}{2}e_3^2. \tag{29}$$

Define $e_4 = x_4 - x_{4d}$. The derivative of V_3 along the system trajectory is:

$$\begin{aligned} \dot{V}_3 &= -k_1 e_1^2 - k_2 e_2^2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) \\ &\quad + e_3\left(c_3 e_2 + \frac{e_3 e_1 F_1}{|e_3|^2} + \dot{e}_3\right) + e_2 F_2 \\ &= -k_1 e_1^2 - k_2 e_2^2 + e_2 F_2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) \\ &\quad + e_3\begin{pmatrix} c_3 e_2 + \dfrac{e_3 e_1 F_1}{|e_3|^2} + F_3 \\ +c_4 x_{4d} - \dot{x}_{3d} \end{pmatrix} + c_4 e_3 e_4. \end{aligned} \tag{30}$$

Step i: The derivative of e_i is

$$\dot{e}_i = F_i + c_{i+1} x_{i+1} - \dot{x}_{id}. \tag{31}$$

Similar to step 1, F_i containing x_{i+2} will be countervailed in step i+2. It is noticed that F_{i-2} contains $x_1, \ldots, x_i$, so the virtual control should counteract F_{i-2}. Therefore, the virtual control can be designed as follows:

$$x_{(i+1)d} = \begin{cases} -\dfrac{1}{c_{i+1}}\begin{pmatrix} k_i e_i + c_i e_{i-1} - \upsilon_{i-1,2} \\ +\dfrac{e_i e_{i-2} Z_{i-2,2}}{|e_i|^2} \end{pmatrix}, |e_i| \geq \varsigma, \\ 0, \quad |e_i| < \varsigma, \end{cases} \tag{32}$$

where $k_i > 1$ and $\varsigma > 0$ are constants to be designed. $Z_{i-2,2}$ is the state of ESO defined in (17), which is used to estimate F_{i-2} in real time. $\upsilon_{i-1,2}$ is the state of TD defined in (18), avoiding the computation of $\dot{x}_{id}$, choose a Lyapunov function V_i as follows:

$$V_i = V_{i-1} + \frac{1}{2}e_i^2. \tag{33}$$

Define $e_{i+1} = x_{i+1} - x_{i+1d}$. The derivative of V_i along the system trajectory is:

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + e_i \dot{e}_i \\ &= -\sum_{j=1}^{2} k_j e_j^2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) + e_{i-1}F_{i-1} \\ &\quad + e_{i-2}F_{i-2} + c_i e_{i-1} e_i + e_i \dot{e}_i \\ &\quad + \sum_{j=3}^{i-1} e_j \begin{pmatrix} c_j e_{j-1} + \dfrac{e_j e_{j-2} F_{j-2}}{|e_j|^2} \\ +c_{j+1} x_{j+1} - \dot{x}_{jd} \end{pmatrix} \\ &= -\sum_{j=1}^{2} k_j e_j^2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) + e_{i-1}F_{i-1} \\ &\quad + \sum_{j=3}^{i-1} e_j \begin{pmatrix} c_j e_{j-1} + \dfrac{e_j e_{j-2} F_{j-2}}{|e_j|^2} \\ +c_{j+1} x_{j+1} - \dot{x}_{jd} \end{pmatrix} \\ &\quad + e_i\left(c_i e_{i-1} + \frac{e_i e_{i-2} F_{i-2}}{|e_i|^2} + \dot{e}_i\right) \\ &= -\sum_{j=1}^{2} k_j e_j^2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) \\ &\quad + e_i F_i + e_{i-1}F_{i-1} + c_{i+1} e_i e_{i+1} \\ &\quad + \sum_{j=3}^{i} e_j \begin{pmatrix} c_j e_{j-1} + \dfrac{e_j e_{j-2} F_{j-2}}{|e_j|^2} \\ +c_{j+1} x_{(j+1)d} - \dot{x}_{jd} \end{pmatrix}. \end{aligned} \tag{34}$$

Step n–1: The derivative of e_{n-1} is

$$\dot{e}_{n-1} = F_{n-1} + c_n x_n - \dot{x}_{(n-1)d}. \tag{35}$$

Similar to step 1, F_{n-1} containing x_n will be countervailed in step n. It is noticed that F_{n-3} contains $x_1, \ldots, x_{n-1}$, so the virtual control should counteract F_{n-3}. Therefore, the virtual control can be designed as follows:

$$x_{nd} = \begin{cases} -\dfrac{1}{c_n}\begin{pmatrix} k_{n-1} e_{n-1} - \upsilon_{n-2,2} \\ +\dfrac{e_{n-1} e_{n-3} Z_{n-3,2}}{|e_{n-1}|^2} \\ +c_{n-1} e_{n-2} \end{pmatrix}, |e_{n-1}| \geq \varsigma, \\ 0, \quad |e_{n-1}| < \varsigma, \end{cases} \tag{36}$$

where $k_{n-1} > 1$ and $\varsigma > 0$ are the constants to be designed. $Z_{n-3,2}$ is the state of ESO defined in (17), which is used to estimate F_{n-3} in real time. $\upsilon_{n-2,2}$ is the state of TD defined in (18), avoiding the computation of

$\dot{x}_{(n-1)d}$, choose a Lyapunov function V_{n-1} as follows:

$$V_{n-1} = V_{n-2} + \frac{1}{2}e_{n-1}^2. \tag{37}$$

Define $e_n = x_n - x_{nd}$. The derivative of V_{n-1} along the system trajectory is:

$$\begin{aligned}\dot{V}_{n-1} = & -\sum_{j=1}^{2} k_j e_j^2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) \\ & + e_{n-1}F_{n-1} + e_{n-2}F_{n-2} + c_{n+1}e_{n-1}e_n \\ & + \sum_{j=3}^{n-1} e_j \begin{pmatrix} c_j e_{j-1} + \dfrac{e_j e_{j-2} F_{j-2}}{|e_j|^2} \\ + c_{j+1}x_{(j+1)d} - \dot{x}_{jd} \end{pmatrix}.\end{aligned} \tag{38}$$

Step n: The derivative of e_n is

$$\dot{e}_n = F_n + c_{n+1}u - \dot{x}_{nd}. \tag{39}$$

In this step, F_n, F_{n-1}, F_{n-2} should be counteract by control u. Therefore, the control can be designed as follows:

$$u = \begin{cases} -\dfrac{1}{c_{n+1}} \begin{pmatrix} k_n e_n + c_n e_{n-1} \\ + \dfrac{e_n e_{n-2} Z_{n-2,2}}{|e_n|^2} + Z_{n,2} \\ + \dfrac{e_n e_{n-1} Z_{n-1,2}}{|e_n|^2} - \upsilon_{n-1,2} \end{pmatrix}, |e_n| \geq \varsigma, \\ 0, \quad |e_n| < \varsigma, \end{cases} \tag{40}$$

where $k_n > 1$ and $\varsigma > 0$ are the constants to be designed. $Z_{n-2,2}$, $Z_{n-1,2}$,$Z_{n,2}$ are the states of ESO defined in (17), which is used to estimate F_{n-2}, F_{n-1}, F_n respectively in real time. $\upsilon_{n-1,2}$ is the state of TD defined in (18), avoiding the computation of $\dot{x}_{nd}$, choose a Lyapunov function V_n as follows:

$$V_n = V_{n-1} + \frac{1}{2}e_n^2. \tag{41}$$

The derivative of V_n along the system trajectory is:

$$\begin{aligned}\dot{V}_n = & -\sum_{j=1}^{2} k_j e_j^2 + e_2\left(\upsilon_{1,2} - \dot{x}_{2d}\right) \\ & + \sum_{j=3}^{n-1} e_j \begin{pmatrix} c_j e_{j-1} + \dfrac{e_j e_{j-2} F_{j-2}}{|e_j|^2} \\ + c_{j+1}x_{(j+1)d} - \dot{x}_{jd} \end{pmatrix} \\ & + e_n \begin{pmatrix} c_n e_{n-1} + \dfrac{e_n e_{n-1} F_{n-1}}{|e_n|^2} + F_n \\ + \dfrac{e_n e_{n-2} F_{n-2}}{|e_n|^2} + c_{n+1}u - \dot{x}_{nd} \end{pmatrix}.\end{aligned} \tag{42}$$

The above design process can be summarized by the following theorem.

Suppose that Assumptions 2.1–2.3 are satisfied and that the above proposed design procedure is applied to system (1), only if the parameters in TD and ESO design are chosen appropriately, then, the virtual controls (20),(24),(28),(32),(36) and control (40) can make the states of system(1) asymptotically converge to an arbitrarily small region of origin.

Proof: (1) When $\|e_i\| \geq \varsigma, i = 3, \ldots, n$, substitute the virtual controls and control into formula (42), then formula (42) can be rewritten as follows:

$$\begin{aligned}\dot{V}_n = & -\sum_{j=1}^{n-1} k_j e_j^2 + \sum_{j=1}^{n-1} e_j\left(F_j - Z_{j,2}\right) + e_n\dot{e}_n \\ & + \sum_{j=1}^{n-2} e_{j+1}\left(\upsilon_{j,2} - \dot{x}_{(j+1)d}\right) + c_n e_{n-1}e_n \\ = & -\sum_{j=1}^{n} k_j e_j^2 + \sum_{j=1}^{n} e_j\left(F_j - Z_{j,2}\right) \\ & + \sum_{j=1}^{n-1} e_{j+1}\left(\upsilon_{j,2} - \dot{x}_{(j+1)d}\right).\end{aligned} \tag{43}$$

Let

$$\bar{\boldsymbol{e}}_n = [e_1, \ldots, e_n]^{\mathrm{T}} \quad \text{and}$$

$$Q = \begin{bmatrix} F_1 - Z_{1,2} \\ (F_2 - Z_{2,2}) + (\upsilon_{1,2} - \dot{x}_{2d}) \\ \vdots \\ (F_n - Z_{n,2}) + (\upsilon_{n-1,2} - \dot{x}_{nd}) \end{bmatrix},$$

then Q can be viewed as the disturbance input of closed-loop system. So, Equation (43) can be rewritten as follows:

$$\begin{aligned}\dot{V}_n \leq & -\bar{\boldsymbol{e}}_n^{\mathrm{T}} \begin{bmatrix} k_1 & & & \\ & k_2 & & \\ & & \ddots & \\ & & & k_n \end{bmatrix} \bar{\boldsymbol{e}}_n \\ & + \|\bar{\boldsymbol{e}}_n\| \left\| \begin{matrix} F_1 - Z_{1,2} \\ F_2 - Z_{2,2} + \upsilon_{1,2} - \dot{x}_{2d} \\ \vdots \\ F_n - Z_{n,2} + \upsilon_{n-1,2} - \dot{x}_{nd} \end{matrix} \right\|.\end{aligned} \tag{44}$$

According to the ISS theory, only if $\|Q\|$ is bounded, then $\bar{\boldsymbol{e}}_n$ can asymptotically converge to a neighbourhood of the origin which depends on $k_i, i = 1, \ldots, n$. Obviously, it can be seen that $\|Q\|$ is bounded from (4) and (7). Apart from this, the size of neighbourhood is also tied closely to the accurate estimation of the uncertain nonlinear terms and virtual controls by ESOs and TDs, respectively.

(2) When $\|e_i\| < \varsigma, i = 3, \ldots, n$, according to the formulas of the virtual controls and control, we have $x_{id} = 0, i = 3, \ldots, n$ and $u = 0$. Then from $e_i = x_i - x_{id}, i = 3, \ldots, n$, we have that states $x_i, i = 3, \ldots, n$ are

bounded, that is to say that states can asymptotically converge to a ς-neighbourhood of the origin. Until then by formula(43), it also can be derived e_1, e_2 is bounded. $x_{2d} = -(k_1/c_2)e_1$ is bounded too. That is to say x_1, x_2 also converge to a neighbourhood of the origin. Of course when a $\|e_i\| < \varsigma$, the same conclusion can be drawn. Moreover, it is not difficult to obtain that all the signals in the closed-loop system are bounded. ∎

Remark 3.2: The contribution of this paper is that the framework of the control design is both systematic and simple, and the only thing required is the knowledge of the order of the system and few assumptions stated

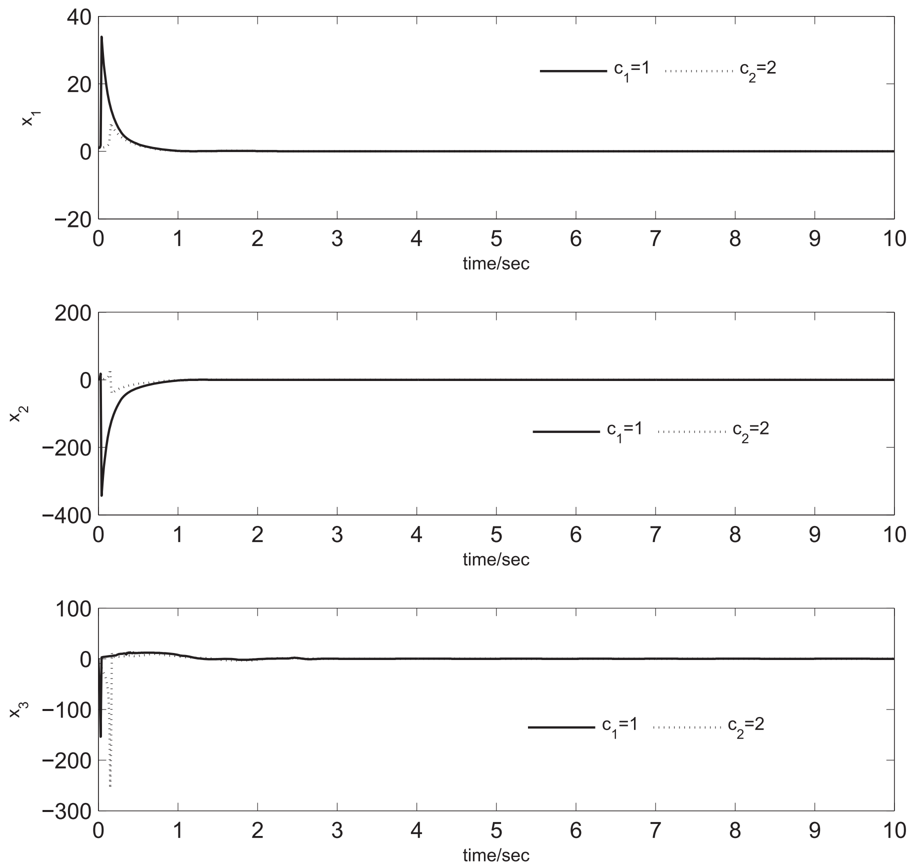

Figure 1. Time response of states.

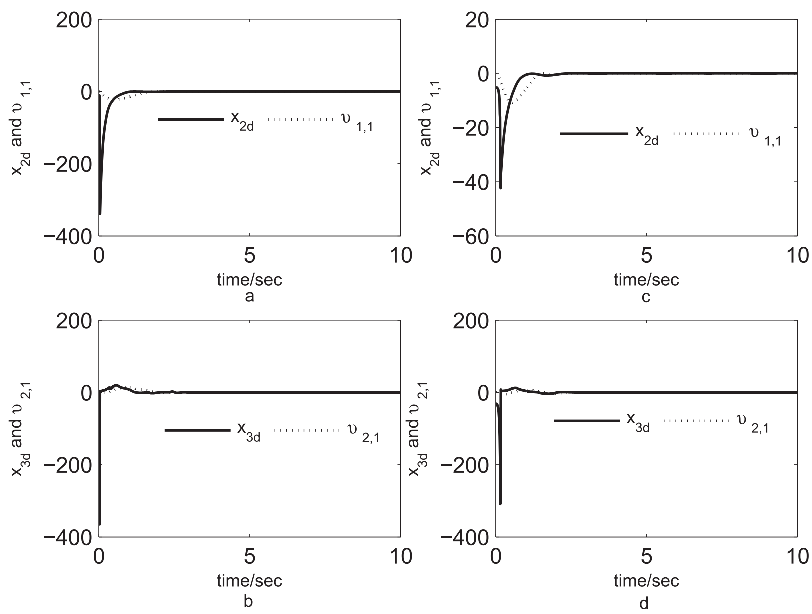

Figure 2. Approximation of virtual control via TD.

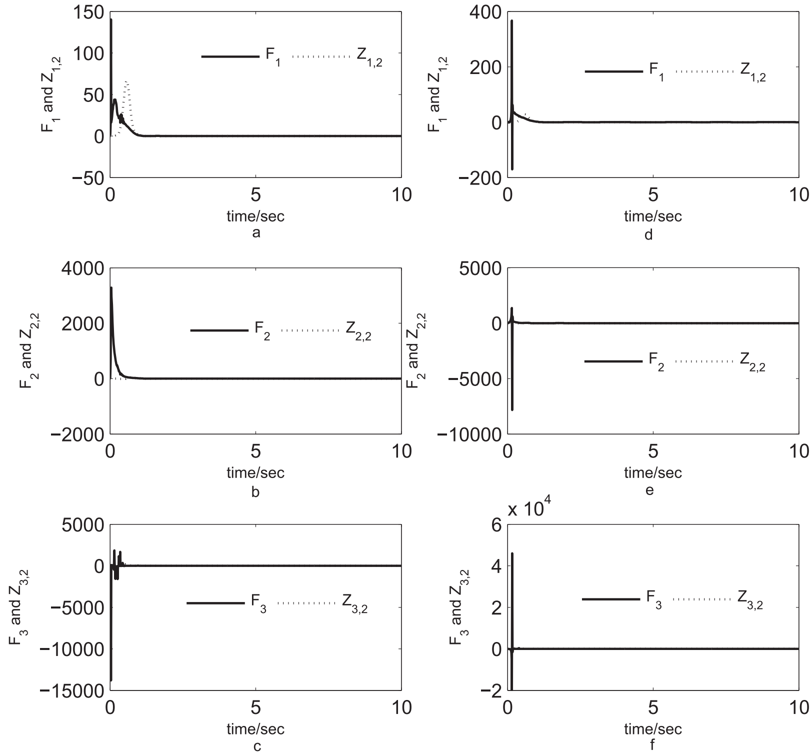

Figure 3. Estimation of uncertainties via ESO.

above. Moreover, this control scheme design procedure can be applied to many other nonlinear plants.

4. Simulation results

In this section, the feasibility and performance of the advanced backstepping based on ADR are illustrated via two examples.

Example 4.1: To illustrate that the proposed method is robust for designing the parameters c_i, the following third-order, non-affine, non-strict feedback, nonlinear system is used for the simulation:

$$\begin{aligned}\dot{x}_1 &= x_2 - 0.05x_2x_3 + x_1\cos x_3,\\ \dot{x}_2 &= (1 + x_1^2)x_3 + x_1\cos x_2 + 0.2x_3\sin 2t + 0.15x_3^2,\\ \dot{x}_3 &= x_2\cos t + 2x_3\cos t + 0.3x_3\sin t\\ &\quad + (1 + x_1^2)u + \sin(0.1u),\end{aligned} \tag{45}$$

where $f_1 = x_2 - 0.05x_2x_3 + x_1\cos x_3$, $f_2 = (1 + x_1^2)x_3 + x_1\cos x_2 + 0.2x_3\sin 2t + 0.15x_3^2$ and $f_3 = x_2\cos t +$

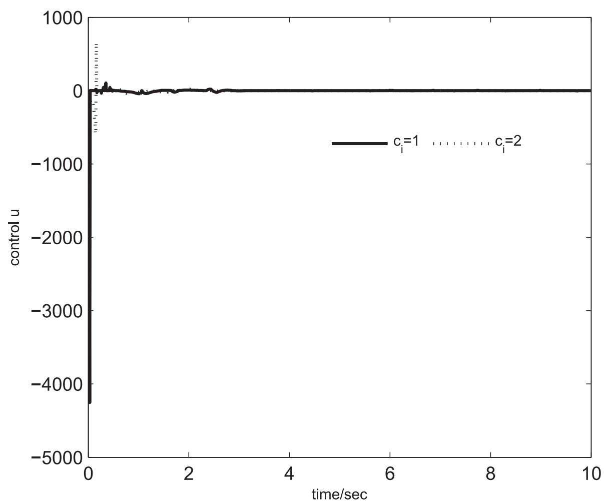

Figure 4. Control input.

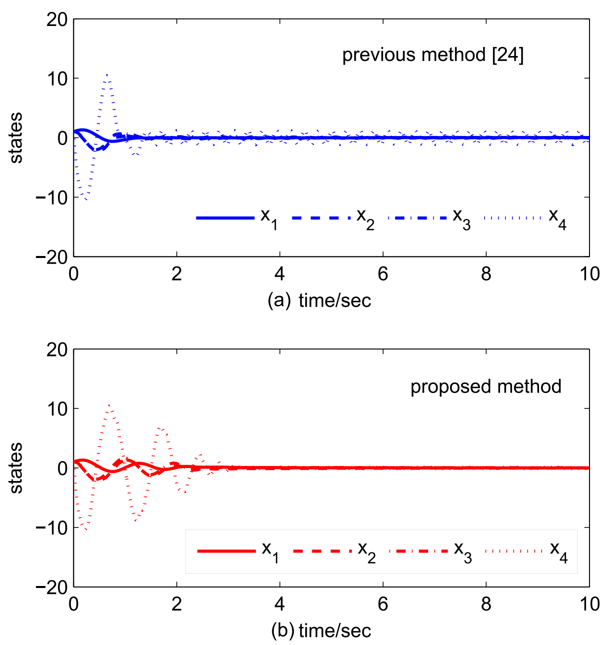

Figure 5. Time response of states.

$2x_3 \cos t + 0.3x_3 \sin t + (1 + x_1^2)u + \sin(0.1u)$ are uncertain terms. Obviously, the origin is the equilibrium point.

In this simulation, the system state's initial conditions are $\boldsymbol{x} = [1, 1, 1]^{\mathrm{T}}$ and the ESOs and TDs state's initial conditions are 0, the design parameters are $\lambda = 6$, $\alpha = 18$, $\beta_{01} = 100$, $\beta_{02} = 1500$, $\alpha_1 = 0.25$, $k_1 = k_2 = 10$, $k_3 = 20$, $\varsigma = 0.1$. The simulation results of case $c_i = 1$ and the case $c_i = 2$ are shown in Figures 1–4.

In accordance with Figure 1, the method proposed here can make states asymptotically converge to a neighbourhood of the origin in both case $c_i = 1$ and the case $c_i = 2$. Figure 2(a, b) represents case $c_i = 1$, while Figure 2(c, d) represents case $c_i = 2$, both cases illustrate the performances of TD estimating the derivative of virtual control, which indicates that the values of parameters λ, α are chosen independent of parameters c_i. Only if parameters λ, α are designed appropriately, each component of the differentiator states $\upsilon_{i,1}$ approximate to the virtual control x_{id}. Figure 3(a–c) represents case $c_i = 1$, while Figure 3(d–f) represents case $c_i = 2$. Both cases show the effectiveness of ESOs, and the parameters $\beta_{01}, \beta_{02}, \alpha_1$ are selected regardless of c_i. Both ESO and TD have the independence in the whole control design procedure. Figure 4 shows that the control input in both case $c_i = 1$ and case $c_i = 2$ is bounded.

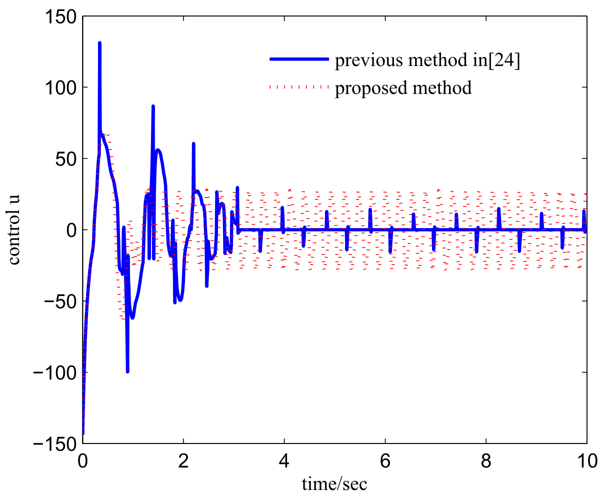

Figure 6. Control input.

Example 4.2: In order to illustrate the control design procedure and performance based on the advanced backstepping, the following fourth-order, non-affine, non-strict feedback nonlinear system is used for the simulation:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_3 + \sin x_3, \\ \dot{x}_2 &= x_3 + x_4 + x_2 \sin x_4, \\ \dot{x}_3 &= x_4 + \sin\left({x_4}^3\right), \\ \dot{x}_4 &= u + \sin u, \end{aligned} \tag{46}$$

where $f_1 = x_2 + x_3 + \sin x_3$, $f_2 = x_3 + x_4 + x_2 \sin x_4$, $f_3 = x_4 + \sin x_4^3$, $f_4 = u + \sin u$ are uncertain terms. Obviously, the origin is the equilibrium point.

In this simulation, the system state's initial conditions are $\boldsymbol{x} = [1, 1, 1, 1]^{\mathrm{T}}$ and the ESOs and TDs state's

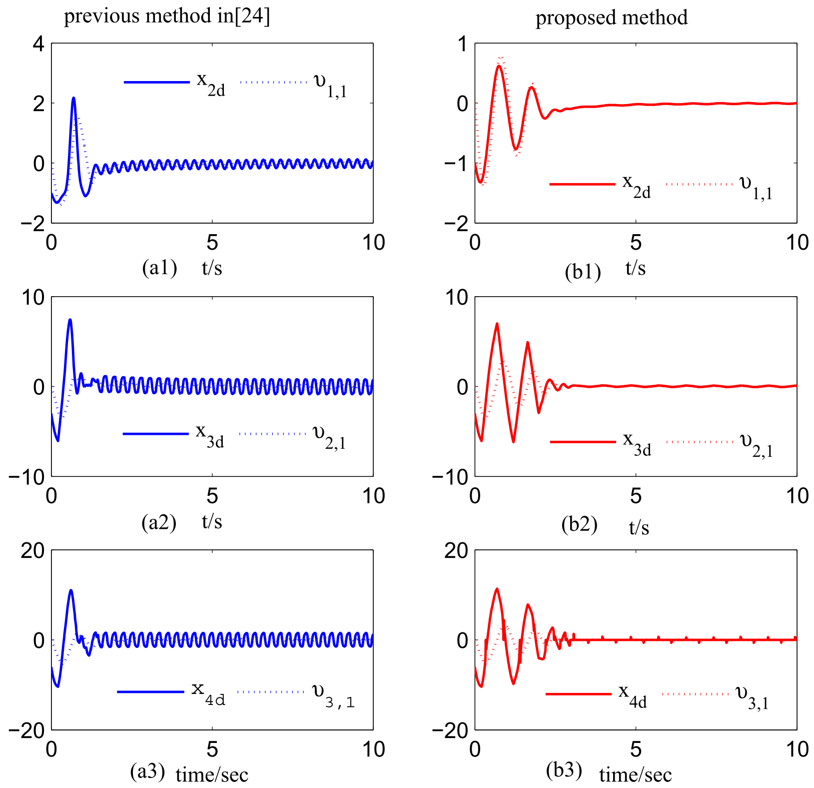

Figure 7. Approximation of virtual control via TD.

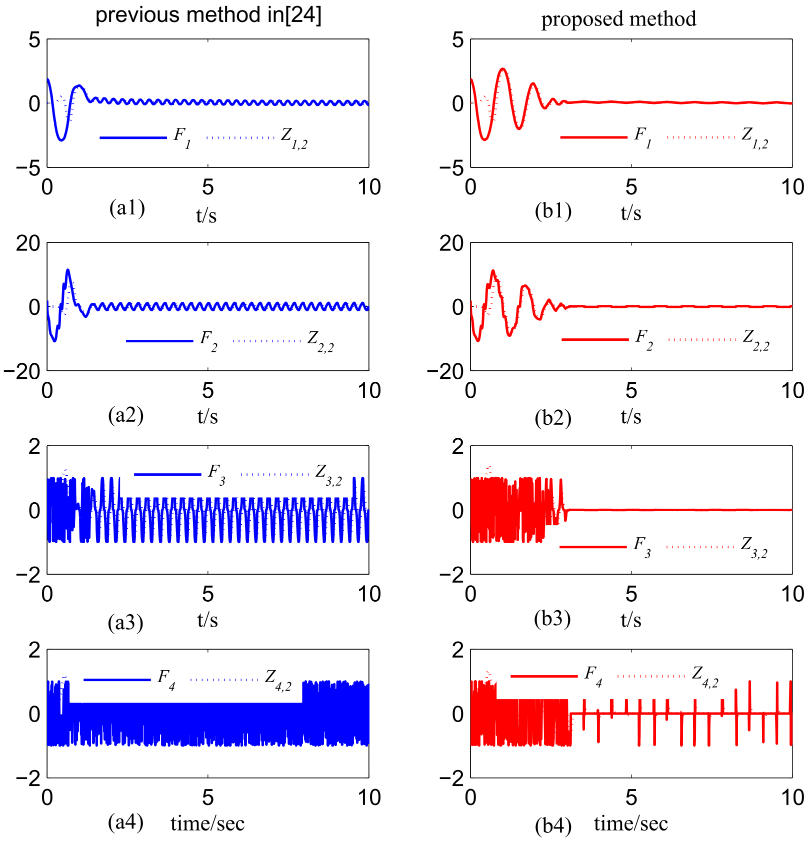

Figure 8. Estimation of uncertainties via ESO.

initial conditions are 0, the design parameters are $c_i = 1$, $\lambda = 6$, $\alpha = 18$, $\beta_{01} = 100$, $\beta_{02} = 1500$, $\alpha_1 = 0.25$, $k_1 = k_2 = k_3 = 1, k_4 = 20, \varsigma = 0.1$.

The simulation results are shown in Figures 5–8. In accordance with Figure 5, both the proposed method here based on the advanced backstepping and the previous method in [25] based on the traditional backstepping can make states asymptotically converge to a neighbourhood of the origin, but we can obviously have that method proposed here is better than the previous method in [25]. Figure 6 shows that the control input is bounded. Figure 7 illustrates the performances of TD estimating the derivative of the virtual control. Only if the values of parameters λ, α are chosen appropriately, each component of the differentiator states $\upsilon_{i,1}$ approximates to the virtual control x_{id}. The performances of ESO observing the uncertainties $F_i, i = 1, 2, 3, 4$ are given in Figure 8. By selecting appropriate values of parameters $\beta_{01}, \beta_{02}, \alpha_1$, each component of the estimated states $Z_{i,2}$ converges to the actual uncertainty component F_i.

5. Conclusion

The main contribution is that the advanced backstepping design based on ADR for non-affine, non-strict feedback, nonlinear systems has been proposed. In the design scheme, each step of the advanced backstepping is combined with the idea of ADR. The scheme consists of an advanced backstepping used to deal with the non-strict feedback form systems, ESOs applied to estimate the uncertain, TDs used to approximate the virtual controls, the ISS theorem illustrating the effectiveness of the control scheme. The advantages of this scheme are as follows: (1) avoiding the tedious derivation of virtual controls; (2) more independence for selecting parameters $c_i, i = 1, \ldots, n$; (3) more independence for designing estimators of uncertain terms, that is to say the choices for parameters in TD and ESO are independent of the closed-loop system stability. The simulation results performed on a simple, non-affine, non-strict feedback, nonlinear system demonstrate the feasibility of the proposed adaptive control scheme.

Disclosure statement

No potential conflict of interest was reported by the authors.

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