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Overview and analysis of methods for assessing ride comfort on tram tracks

Authors:



Assist.Prof. **Ivo Haladin**, PhD. CE
University of Zagreb
Faculty of Civil Engineering
Department of Transportation Engineering
ihaladin@grad.hr



Prof. **Stjepan Lakušić**, PhD. CE
University of Zagreb
Faculty of Civil Engineering
Department of Transportation Engineering
laki@grad.hr



Marijan Bogut, MSc. EE
Končar – Institut za elektrotehniku d.d.
bogut@koncar-institut.hr

Subject review

Ivo Haladin, Stjepan Lakušić, Marijan Bogut

Overview and analysis of methods for assessing ride comfort on tram tracks

An overview of methods for evaluating comfort on tram tracks is given in this paper. Tramway systems differ in many aspects from standard railway tracks and, therefore, standards developed for standard rail lines have to be analysed and evaluated. Equivalent level of vibrations, Sperling ride index, and several methods proposed in EN 12299 are described, and these methods are compared on two trial tram sections 21 km in total length. Conclusions and recommendations for measuring ride comfort on tram tracks are presented.

Key words:

tram tracks, vibration, tramway, Sperling index, ride comfort, EN 12299, ISO 2631

Pregledni rad

Ivo Haladin, Stjepan Lakušić, Marijan Bogut

Pregled i analiza metoda za ocjenu udobnosti vožnje na tramvajskim kolosijecima

U radu je dan pregled metoda za ocjenjivanje udobnosti vožnje na tramvajskim kolosijecima. Tramvajski se sustavi u brojnim segmentima razlikuju od standardnih željezničkih sustava, pa se stoga standardi razvijeni za standardne željeznice trebaju analizirati i evaluirati. Opisuju se metode ekvivalentne razine vibracija, Sperlingovog indeksa udobnosti vožnje te niz metoda predloženih normom EN 12299. Metode se uspoređuju na dvije probne tramvajske dionice ukupne dužine 21 km. Predložene su i preporuke za mjerenje udobnosti vožnje na tramvajskim kolosijecima.

Ključne riječi:

tramvajski kolosijek, vibracije, tramvaj, Sperlingov indeks, udobnost vožnje, EN 12299, ISO 2631

Übersichtsarbeit

Ivo Haladin, Stjepan Lakušić, Marijan Bogut

Überblick über und Analyse von Methoden zur Bewertung des Fahrkomforts auf Straßenbahnschienen

Diese Arbeit bietet einen Überblick über Methoden zur Bewertung des Fahrkomforts auf Straßenbahnschienen. Straßenbahnsysteme unterscheiden sich in vielen Segmenten von Standardbahnsystemen, weshalb Normen, die für Standardschienen entwickelt wurden, analysiert und bewertet werden müssen. Die äquivalenten Vibrationswerte, der Sperling-Fahrindex und verschiedene in EN 12299 vorgeschlagene Methoden werden beschrieben und mit zwei Testabschnitten der Straßenbahn mit einer Gesamtlänge von 21 km und 19 Unterabschnitten verglichen. Es werden Schlussfolgerungen und Empfehlungen zur Messung des Straßenbahnfahrkomforts sowie Schritte zur weiteren Verbesserung der Analysemethoden für Straßenbahnsysteme gegeben.

Schlüsselwörter:

tramway lines, Vibration, Straßenbahn, Sperling-Index, Fahrkomfort, EN 12299, ISO 2631

1. Introduction

Tram and light rail systems constitute a backbone of modern urban transport networks of many European cities. Millions of tram journeys are completed every day, and each passenger onboard a tram vehicle is exposed to a certain level of vibrations, noise, temperature, etc. To remain competitive in comparison with other modes of transport, and to attract more passengers and maintain good level of service, it is important to monitor and improve ride quality onboard tram vehicles. Ride quality in combination with other factors such as travel time, congestion, and price, can have an important role in choosing a favoured means of transport. Research conducted with regard to ride quality is a complex endeavour because it involves an objective component – actual level of vibration exposure, as well as a subjective passenger component – sensitivity of passengers exposed to a certain level of vibrations.

Light rail vehicles, such as tramway, have a specific ride pattern that differs from standard railway tracks because the running surface is often shared with road vehicles. Tramway ride can be a comfortable city connection on an open track, but it can also be a slow start-stop ride in traffic shared with road vehicles, with winding tight curves. These different types of ride comfort should be evaluated and investigated by tramway service providers in order to provide the best possible comfort to their passengers.

The track structure on trams and light rail vehicles is often quite different from the standard ballasted railway track, and this difference is mostly manifested in the use of grooved rails, continuous reinforced concrete slabs as a base, embedding the track in the road running surface, continuously supported track, shallow groove at switches and crossings, etc. These substantial differences when compared to a classic ballasted track reduce the ability to effectively use standard measuring equipment or standard maintenance procedures. The size of networks often cannot justify purchase of costly measurement vehicles for infrastructure monitoring. In such situations, a simple measurement setup proposed in this article can provide an appropriate overview of ride comfort data based on track condition.

Methods and standards used in this paper for vibration analysis are frequently applied for vehicles operating on standard railway tracks that have different structural features, different operating speeds, and longer travel distances, compared to an average tram network. Because of these characteristics, tram vehicle operation is specific in vibration analysis, especially with regard to ride comfort parameter. Tram journey is characterised by relatively short distances between tram stops, lower speed, complex track curvature that often follows the route of city roads, intensive mixed road/rail traffic along the route, different travel motivations and conditions along the journey.

This paper provides an insight into three methods for evaluating ride comfort on a tramway network, and discusses their applicability, practicality, and accuracy on tram and light rail systems. The Faculty of Civil Engineering of the University of Zagreb has been involved in the track evaluation process on the network scale in the Croatian cities of Osijek (30 km of track in 2016) [1] and Zagreb (120 km of track in 2018/2019) [2], where various tram infrastructure parameters were considered in order to make an elaborate evaluation of each track segment along the entire tram network. Important parameters for this evaluation were the ride quality and ride comfort assessment methods based on bogie and car body vibrations recorded using an instrumented in-service tram vehicle. This paper describes experience gained during these studies and discusses further approaches for the evaluation of ride comfort.

Vibration data gathered from in-service rail vehicles can be interpreted and applied in a variety of ways including:

- evaluation of ride quality (evaluation of vehicle performance and evaluation of infrastructure performance) [3]
- detection of faults on the track (welds, corrugation, rail breaks, track geometry) [4]
- detection of faults on vehicles [5]
- detection of faults on the overhead line [6]
- evaluation of passenger ride comfort in vehicles [7].

Evaluation of ride quality has been a topic of many research papers, but this issue has mainly been considered in relation to standard railway tracks. The focus of ride quality is the study of wheel-rail interaction and track condition that acts upon the running rail vehicle. The ride quality can be defined as the capacity of a vehicle to fulfil transport requirements from the perspective of level of exposure to vibrations, depending on the vehicle type, number of trips, goods transported, and the engine operating staff. As to the ride comfort, its evaluation must take into account the effect of mechanical vibrations on human body. [3] Usually, parameters measured for this purpose are accelerations of the rail bogie and car body which are affected by irregularities on the wheel and the track, as well as the roll velocity and lateral jerk that can be used to evaluate various track geometry features such as alignment and twist.

Passenger comfort presents a different take on vibration analysis in that it interprets human response to vibrations. Human perception of vibrations is dependent on many factors that can be divided into physical (amplitude, duration, frequency range) and psychological (type of population, gender, age, level of expectation, level of awareness) [8]. Research has shown that long-term exposure to high frequency vibrations of small amplitudes can induce problems with concentration, while short term exposure to low frequency vibrations of great amplitudes can cause damage to muscles and internal organs [9]. The ISO 2631 set is the basic worldwide standard that assesses human

response to vibrations [10]. It was amended by the ISO 2631-4 set for assessing influence of vibrations on passenger and crew comfort in fixed-guideway transport systems [11].

The perception of vibrations in the frequency range of 1 to 10 Hz is proportional to accelerations, while in the frequency range of 10 to 100 Hz it is proportional to oscillation velocity. The limit of vibration perception presented as acceleration is 0.001 m/s^2 and it increases up to 0.1 mm/s^2 at 100 Hz frequency. If vibrations are expressed in terms of velocity, the perception limit is between 0.1 and 0.3 mm/s for the vibration frequency range of 1 to 100 Hz. [12] If vibrations are observed in terms of displacement, human body can detect vibration amplitude of 0.001 mm, while human fingers can be up to 50 times more sensitive. People can be annoyed by oscillations of 0.2 mm amplitude at 5 Hz, which do not inflict any structural damage [13]. Since human response to vibrations is highly subjective and is different for every individual, the problem of influence of vibrations on people is often addressed in terms of statistical parameters such as percentage of people who have perceived vibrations in a certain way.

Ride comfort and quality of ride in railway vehicles are research topics that have been studied by many railway authorities. It is a complex area of track-vehicle interaction, vehicle response and, most importantly, human response to exposure to vibrations during a train journey. The study of this issue is quite complex due to human perception of vibrations and human interpretation of comfort. In fact, comfort is not only influenced by a certain level of vibrations, but also by the length of the journey and by nature of the journey (commute, business, tourist, ...). For example, vibrations of standard railway vehicles were evaluated on three types of standard passenger trains in Sweden: Inter-City (IC), Regional X50 and Double Decker X40, with simultaneous measurement of vibrations, and with the conduct of passenger opinion survey for passengers that used laptop computers. Vibrations were measured using the Sperling index and ISO 2631 [14]. The research revealed that even though vibration levels measured using the Sperling and ISO methods defined the ride as comfortable, the passengers were experiencing some difficulties in performing sedentary activities such as typing on laptop computers.

Another study shows that the track quality index obtained by direct measurements of track geometry cannot directly be correlated to the Sperling ride quality index [15].

Research conducted so far on tramway networks in Croatia includes the Veski study involving the Sperling ride index calculation along Zvonimirova street in Zagreb [16], where the ride quality and ride comfort were evaluated on three different tram track sub-sections along Zvonimirova street, at different operating speeds.

In the scope of detailed evaluation of the entire tramway network in the City of Osijek, conducted in 2016 by the Faculty of Civil Engineering of the University of Zagreb [1], a

significant attention was given to the ride comfort and ride quality, measured from an in-service vehicle traveling at an average speed of 30 km/h. In this study, the equivalent level of vibrations method was used to evaluate ride quality and ride comfort. A similar study, conducted by the same team of researchers, was undertaken in the City of Zagreb in 2018/2019 [2] where the Sperling index was additionally determined on the total of 120 km of tramway tracks with an in-service vehicle traveling at 20 km/h.

2. Methods for analysis of ride comfort

This paper describes three methods that are used for evaluating ride comfort based on vibration data recorded using an in-service tram vehicle running on a tramway track:

- Equivalent level of vibrations (L_{aeq})
- Ride Index W_z
- EN 12299 (based on ISO 2631)

In this study, the same data were used for the above three methods. The data were collected on the Osijek and Zagreb networks using in-service tram vehicles. Test sections will be considered in greater detail in Section 3.1.

The meaning of several important concepts used in this work are explained below according to HRN ISO 5805:2016 Mechanical vibration and shock - Human exposure - Vocabulary [17]:

Ride: measurable motion environment (including vibration shock, translational and rotational accelerations) as experienced by people in or on a vehicle,

Ride quality: degree to which the whole subjective experience (including the motion environment and associated factors) of a journey or ensemble of journeys by vehicle is perceived and rated as favourable or unfavourable by passengers or operators,

Comfort: subjective state of well-being or absence of mechanical disturbance in relation to the induced environment (concerning mechanical vibration or repetitive shock).

Comfort is defined according to EN 12299 [18] as follows:

Ride comfort: Complex sensation produced during the application of oscillations and/or inertia forces, via whole-body transmission caused by the railway vehicle body motions.

One of early but still applicable methods for the assessment of ride quality and comfort based on ride index method is the Sperling Ride Index W_z . After development of standards for the whole body vibration assessment, ISO 2631, special attention was paid to fixed-guideway systems' influence on whole-body vibrations [11]. Latest standard related to passenger comfort, EN 12299, was introduced by CEN/TC256/WG7 and accepted as European standard in 2009 [18].

2.1. Equivalent level of vibrations (L_{aeq})

This procedure was established based on previous professional and scientific work of the Chair for Railways at the Faculty of Civil Engineering of the University of Zagreb, as a practical procedure for evaluating different tram vehicle passes over a certain point on the track [19, 20] and as an effective evaluation parameter for comparing different track segments along the tram network for vibrations measured off from an in-service tram vehicle [1]. Determination of the equivalent level of vibration is a fairly simple process for evaluating different events in acceleration time signal record. In this case, when mounted on an in-service vehicle, accelerometer can be used at various positions in vertical and lateral directions for expressing ride quality (if mounted on tram bogie) or ride comfort (if mounted in tram passenger compartment).

Acceleration signals measured in vertical and lateral directions are filtered in the 2 Hz to 200 Hz range, and vibration levels are calculated for 1-second intervals according to the following expression (1):

$$L_a = 20 \cdot \log_{10} \left[\frac{a}{a_0} \right] \tag{1}$$

where L_a is the level of acceleration in dB, a is the effective value of acceleration (m/s^2), and a_0 is the referent acceleration of $10^{-6} m/s^2$. The equivalent level is calculated for some tramway track sections based on 30s segments, taking into account the energy average level, Eq. (2):

$$L_{aeq} = 10 \cdot \log_{10} \left[\frac{1}{N} \left(10^{\frac{L_{a1}}{10}} + 10^{\frac{L_{a2}}{10}} + 10^{\frac{L_{a3}}{10}} + \dots + 10^{\frac{L_{aN}}{10}} \right) \right] \tag{2}$$

where L_{aeq} is the equivalent level of vibrations for a segment along the line expressed in dB, N is the number of 1-second intervals that is taken into account (usually 30 s of constant speed interval), $L_{a1} - L_{aN}$ is the 1 second level of vibrations at any point on the track.

Taking into account the average L_{aeq} from all the measured signals responsible for ride quality or ride comfort, the indices I_c and I_q are calculated by ranking the observed segments of the line on a scale from 0 to 1, based on the entire network rating (0 being the network section with least vibrations and 1 being the network section with highest vibrations).

2.2. Sperling's ride index Wz

The Wz (Wertzungzahl) ride index method [21], introduced by Sperling, is used to evaluate the ride quality and comfort of railway vehicles. In estimating ride quality, the vehicle itself is considered. Ride comfort implies that the vehicle is to be assessed according to the effect mechanical vibrations have on the occupants. If applied at constant speed with the same vehicle, it can also be used to indicate the ride quality and

comfort at various track sections [15, 16]. The advantage and clarity of the Sperling Ride index Wz method arise from the fact that its implementation leads to a number with an accurate significance, enabling easy interpretation. Since it is determined as a function of the level of vehicle vibrations, the ride index Wz supplies information on dynamic behaviour of the vehicle, which enables identification of solutions for improving dynamic performance of the vehicle in terms of the ride quality and ride comfort [3]. This method is often used in practice on standard railway tracks [3, 22, 23], as well as on light rail networks such as tramway tracks [15, 16].

In this method, clear evaluation scales have been introduced for different ride quality (Table 1) and ride comfort (Table 2) scenarios. These scales are based on vibration tests with people and on other test results [21].

Table 1. Ride quality evaluation scales

Ride indeks Wz	Ride quality
1	Very good
2	Good
3	Satisfactory
4	Acceptable for running
4.5	Not acceptable for running
5	Dangerous

Table 2. Ride comfort evaluation scales

Ride indeks Wz	Comfort (vibration sensitivity)
1	Just noticeable
2	Clearly noticeable
2.5	More pronounced but not unpleasant
3	Strong, irregular, but still tolerable
3.25	Very irregular
3.5	Extremely irregular, unpleasant, annoying; prolonged exposure intolerable
4	Extremely unpleasant; prolonged exposure harmful

This vibration analysis method require data processing of accelerations up to 30 Hz. The value of the method is it's simplicity and ability to continuously monitor Wz index along the track therefore pointing out irregularities on the track. Ride index Wz is weighted on the frequency range based on the following expressions (3) and (4):

$$Wz = \sqrt[10]{a^3 \cdot B^3} \text{ (mirnoća hoda)} \tag{3}$$

$$Wz = \sqrt[6.67]{a^2 \cdot B^2} \text{ (udobnost vožnje)} \tag{4}$$

where B represents a weighting factor and is calculated, for ride quality, according to the following expression (5):

$$B(f) = 1.14 \cdot \sqrt{\frac{\left[(1 - 0.056f^2)^2 + (0.645f)^2 \cdot (3.55f^2) \right]}{\left[(1 - 0.252f^2)^2 + (1.547f - 0.00444f^3)^2 \right] \cdot (1 + 3.55f^2)}} \quad (5)$$

The ride comfort weighting factor B is calculated according to expression (6):

$$B(f) = k \cdot \sqrt{\frac{1.911f^2 + (0.25f^2)^2}{(1 - 0.277f^2)^2 + (1.563f - 0.0368f^3)^2}} \quad (6)$$

where $k = 0.588$ for vertical vibrations (B_v), and $k = 0.737$ for lateral vibrations (B_w).

Values of weighting curves are plotted in the 0.5 Hz to 30 Hz frequency range in 1/3 octave bands (Figure 1), for further signal processing of peak acceleration values a_{peak} [cm/s²].

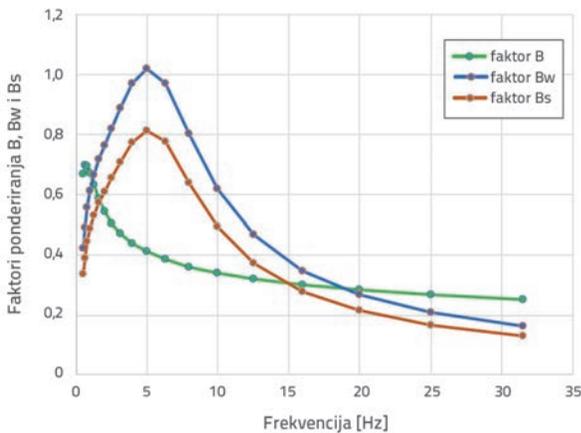


Figure 1. B , B_s and B_w weighting curves for Sperling's ride index W_z

According to [3], where a numerical model was developed and Sperling's ride index simulated along the car body, the ride index W_z is smaller at the carbody centre and higher against the two bogies and towards its ends. This shows that it is important to pick a representative location for measuring the ride quality index.

2.3. Ride comfort assessment according to EN 12299

The work of the European Committee for Standardization CEN/TC256/WG7 is concerned with the passenger ride comfort. The European pre-standard ENV 12299 was published in 1999. The standard for the ride comfort measurement and evaluation was revised and finally accepted as European standard in 2009 [18, 24]. Two methods are proposed as a basis for determining the ride comfort:

- Indirect measurement: measurement of motion environment by different motion quantities, such as acceleration or roll velocity

- Direct measurement: Measurement of actual passenger reactions, for example by asking passengers to fill in a questionnaire.

While there are some guidelines for direct measurements, this standard mostly describes methods based on indirect measurements of vehicle acceleration data. Motions of a vehicle are mostly measured by accelerometers and gyros fitted to the vehicle body at certain positions. Direct tests based on test subjects are not defined in EN 12299 [4], but some guidelines are given in an informative annex.

The R.M.S.-based evaluation method, specified in ISO 2631-1 *Evaluation of human exposure to whole-body vibration* [10], extended for fixed guideway systems in ISO 2631-4 [25], is essentially the base method (equivalent to Continuous Comfort Method) for further statistical methods developed in EN 12299. During a train ride, occupants can experience large fluctuations in both acceleration and frequency levels. ISO 2631-1 is suitable for evaluation of motion environment with small variations in levels, while the statistical method introduced in EN 12299 addresses the fluctuations and variation associated with passengers. The statistical method also avoids sensitivity to artefactual extremes. Thus, compared with ISO 2631, EN 12299 is considered to be more accurate and has been adopted by most countries for evaluation of ride comfort levels [6, 15]. Three distinct comfort evaluation methods are defined in EN 12299:

- Continuous comfort C_{CX} , C_{CY} and C_{CZ}
- Mean comfort standard method N_{MV}
- Mean comfort complete methods N_{VA} and N_{VD}
- Comfort on discrete events P_{DE}
- Comfort on curved transitions P_{CT}

Vehicle condition, accelerometer position, test speed, test sections, relevant time intervals etc., are defined for each method. Acceleration signals recorded are filtered through band-pass or low-pass filters and weighted against W_c and W_d curves for lateral and longitudinal motion (curves are the same as in ISO 2631-1 [10]), while the low-pass filter W_p (used in P_{CT} and P_{DE} methods) and the weighting curve W_b for vertical direction, are specially designed for railway applications [18]. Additionally, post-processing is defined for each of the methods. It involves sliding windows calculations, RMS calculations, averaging, statistical analysis, etc.

Different measuring procedures require different positions of accelerometers and measuring scenarios. For an overview, a list of positions is given in [18], Table 3.

The standard also suggests different applications that the measured data could be used for, other than for the ride comfort evaluation, Table 4. This information is very valuable since the main purpose of this paper is to evaluate different track segments and the effect of state of infrastructure

Table 3. Motion quantities and measurement positions according to [18]

	Mean comfort standard method	Mean comfort complete method		Continuous comfort	Comfort on curve transitions	Comfort on discrete events
Comfort index	N_{MV}	N_{VA}	N_{VD}	$C_{Cx}/C_{Cy}/C_{Cz}$	P_{CT}	P_{DE}
Motion quantities	Accelerations in three directions	Accelerations in three directions		Accelerations in three directions	Lateral acceleration, lateral jerk, roll velocity	Lateral acceleration
Measuring position	Floor	Floor	Floor and interfaces	Floor	Floor	Floor

Table 4. Guidance for the use of different methods for alternative applications [18]

	Mean comfort standard method	Mean comfort complete method	Continuous comfort	Comfort on curve transitions	Comfort on discrete events
Comfort index	N_{MV}	N_{VA} i N_{VD}	$C_{Cx}/C_{Cy}/C_{Cz}$	P_{CT}	P_{DE}
Track geometry				+	
Maintenance track	+		+		+
Maintenance vehicle	+		+		

and maintenance on ride comfort. This method, if applied to different vehicle types, can also be used to evaluate the influence of vehicles on ride comfort, which will also be investigated in scope of this work. Therefore, the Mean Comfort Standard Method and Continuous Comfort methods will be used in scope of this paper.

2.4. Continuous comfort method (C_{Cx} , C_{Cy} , C_{Cz}), EN ISO 2631 equivalent

This method, together with evaluation scales, is equivalent to the method described in EN ISO 2631-1 and EN ISO 2631-4, and includes the newly proposed weighting curve W_b that is different from the curve W_k that was presented in older versions of the standard [26]. The standard [4] suggests that time for averaging the vibration amplitude is 5 seconds. 5-second a_{rms} values are used in the continuous comfort evaluation. This method considers all 5 second a_{rms} values that are calculated from the initial vibration signal filtered up to 100 Hz and weighted with W_d and W_b curves depending on the direction of the coordinate system.

$$C_{Cx} = a_{POX}^{W_d}(t) \tag{7}$$

$$C_{Cy} = a_{POY}^{W_d}(t) \tag{8}$$

$$C_{Cz} = a_{POZ}^{W_d}(t) \tag{9}$$

Measured continuous comfort values can be compared to the scale presented in Table 5.

Such a method seems appropriate for assessing the comfort in short-length journeys on a tram network. 5 second RMS, weighted against the weighting curves and exact limits for different comfort levels, can serve as a guide to infrastructure maintenance and repairs along the track because of more

accurate location determination (around 50 – 70 m, depending on tram speed) and clear thresholds for the assessed comfort level.

Table 5. Continuous comfort scale for CCy and CCz

Condition	Comfort
$C_{Cy}(t), C_{Cz}(t) < 0.20 \text{ m/s}^2$	Very comfortable
$0.20 \text{ m/s}^2 < C_{Cy}(t), C_{Cz}(t) < 0.30 \text{ m/s}^2$	Comfortable
$0.30 \text{ m/s}^2 < C_{Cy}(t), C_{Cz}(t) < 0.40 \text{ m/s}^2$	Medium
$C_{Cy}(t), C_{Cz}(t) > 0.40 \text{ m/s}^2$	Less comfortable

2.5. Mean comfort standard method N_{MV}

Comfort during a continuous 5-minute run for seated passengers is evaluated in this method. Weighting curves W_b and W_d are used, extracting vibrations in the frequency range from 0.4 Hz to 100 Hz. The method is suitable for fairly straight routes since it neglects quasi-static acceleration resulting from track curvature.

The acceleration is measured in longitudinal (x), lateral (y) and vertical (z) directions. After frequency weighting, 60 continuous 5-second weighted RMS accelerations are calculated for each direction. From the 60 RMS values, the 95th percentile (4th highest value) is used for further processing. 95th percentiles of weighted accelerations in 3 directions ($a_{XP95}^{W_d}, a_{YP95}^{W_d}, a_{ZP95}^{W_d}$) are combined with the root-sum-square calculation according to the following equation (10), valid for a 5-minute period.

$$N_{MV} = 6\sqrt{(a_{XP95}^{W_d})^2 + (a_{YP95}^{W_d})^2 + (a_{ZP95}^{W_d})^2} \tag{10}$$

The following expressions can be used for individual assessment of ride comfort in each direction:

$$N_{MVx} = 6 \cdot a_{XP95}^{W_d} \quad (11)$$

$$N_{MVy} = 6 \cdot a_{YP95}^{W_d} \quad (12)$$

$$N_{MVz} = 6 \cdot a_{ZP95}^{W_d} \quad (13)$$

The resulting N_{MV} can be interpreted according to limit values for continuous comfort defined in the standard, Table 6.

Table 6. Mean ride comfort N_{MV} index scale according to [4]

Condition	Comfort
$N_{MV} < 1.5$	Very comfortable
$1.5 < N_{MV} < 2.5$	Comfortable
$2.5 < N_{MV} < 3.5$	Medium
$3.5 < N_{MV} < 4.5$	Uncomfortable
$N_{MV} > 4.5$	Very uncomfortable

The N_{MV} method has many similarities with the traditional vibration analysis according to ISO 2631-1. The controversial part is the use of the 95th percentile where only the 4th highest value is considered. Different sections could have the same apparent level of comfort even though values of all other than the 4th highest 5-second interval are different. It is also hard to locate the exceeding values along the 5-minute section because of such averaging. For tramway networks, a 5-minute interval seems impractical since sections and distances are much smaller in urban rail systems compared to the ones used on standard railway tracks. Although curvature also plays an important role in tramway traffic, it is neglected when this method is used.

2.6. Mean comfort complete methods N_{VA} and N_{VD}

Mean comfort complete methods evaluate 5-minute continuous comfort similar to the Mean Comfort standard method (N_{MV}). The N_{VA} comfort index is used for seated passengers and is based on the 95th percentile as well. N_{VA} is not only based on accelerometers mounted on vehicle floor but also on a seat pan (for lateral and vertical directions) and seat back (for longitudinal direction). This makes the method more difficult to use in experiments that previously described the method. The following expression (14) is used to calculate the comfort of seated passengers:

$$N_{VA} = 4 \cdot (a_{ZP95}^{W_b}) + 2 \cdot \sqrt{(a_{YA95}^{W_b})^2 + (a_{ZA95}^{W_b})^2} + 4 \cdot (a_{XD95}^{W_c}) \quad (14)$$

where ZP, YA, ZA, XD denote different positions and directions of accelerometers on the floor, seat pan, and seat back according to [18].

N_{VD} method is validated for standing passengers. Accelerations are measured at the floor only. The N_{VD} comfort index is

based on median values (50th percentile) of accelerations measured in all three directions and on the 95th percentile of accelerations measured in the lateral direction. The methods both have a disadvantage similar to that of the mean comfort standard method because 55 of 60 5-second RMS values are dismissed and detailed location of events along the line cannot be determined.

$$N_{VD} = 3 \cdot \sqrt{(a_{XP50}^{W_d})^2 + (a_{YP50}^{W_d})^2 + (a_{ZP50}^{W_b})^2} + 5 \cdot (a_{YP95}^{W_d}) \quad (15)$$

2.7. Comfort on discrete events P_{DE} and comfort on curve transitions P_{CT}

The standard provides the following two methods for evaluating comfort based on research conducted at British Rail Research (BRR) [27]. They use the mean lateral acceleration and the peak to peak lateral acceleration as two main variables of discomfort. It has been adopted and slightly modified but it is designed to be used for high-speed railway lines. Therefore, this method will not be further explained since tramway application does not present challenges such as high speed tilting, and long transition curves.

3. Measurement of ride comfort

In the scope of this work, the ride comfort on tramway network was investigated on two test sections. The test sections were selected to conform to different driving regimes, different track alignment configurations, various track types, track covering and fastening solutions, as well as to different types of tram vehicles.

3.1. Test section characterisation

Test sections were selected along two tram networks in Croatia: Osijek tram network and Zagreb tram network. Each of test sections was divided into a series of subsections based on the type of permanent way structure, type of paving, and date of last reconstruction.

3.1.1. Test section 1 - City of Osijek

Test section 1 was selected on a tram network of the City of Osijek. Osijek has a 30 km long narrow-gauge (1000 mm) tram network with infrastructure segments ranging from 2 to 29 years since last major reconstruction [1]. Track configuration ranges from track sections in designated tram corridors, with the speed of up to 50 km/h, to track sections paved with concrete blocks in semi-designated corridors (for trams and service vehicles), and to closed city centre track sections featuring tight curves, track paved with concrete and asphalt wearing course,



Figure 2. Different types of tramway tracks in Osijek



Figure 3. Test section 1 with subsections D07 to D15

surface shared with road vehicles, with an average speed of 10 km/h (Figure 2).

A representative section of the entire network was selected for the purpose of this investigation of ride comfort. This selection was made based on comprehensive evaluation of the entire

tram network as conducted in Osijek in 2016, with a detailed analysis of track geometry, noise, vibrations, and comfort [1]. The total length of Test Section 1 is 9,5 km. This section was divided into subsections based on tram track properties, as shown in Figure 3 and Table 7.

Table 7. Characteristics of subsections along Test section 1

Subsection	Descriptive section name	Exploitation period [year]	Permanent way structure	Length [m]
D07	A. Starčevića square – Rokova	2	B	553
D08	Rokova - Kanižlićeva	13	E	673
D09	Kanižlićeva - Svilajska	15	A	1960
D10	Svilajska - Višnjevac	12	E	324
D11	Strossmayerova 347 - Bana Jelačića / I.G. Kovačića Intersection	3	C+D	2452
D12	Višnjevac – Svilajska	19	E	334
D13	Svilajska - Kanižlićeva	15	A	1955
D14	Kanižlićeva – Rokova	16	E	660
D15	Rokova - A. Starčević Square	17	B	542
Total length of Section 1				9463

Permanent way structure legend:

A - Base: RC slab (25 cm); Fastening: DEPP; Paving: stone aggregate 16-31.5 mm. B - Base: RC slab (25 cm); Fastening: ZG-2; Paving: concrete/asphalt. C - Base: RC slab (30 cm); Fastening: DEPP; Paving: concrete blocks. D - Base: RC slab (30 cm); Fastening: DEPP; Paving: concrete/asphalt wearing course. E - Base: crushed stone (25 cm); Fastening: ballast 30-60 mm (20 cm); Paving: stone aggregate 16-31.5 mm.

As to its alignment, the track mostly runs in a straight line or presents a generally low curvature of $R > 500$ m, albeit with some exceptions such as $R_{min} = 19$ m at the turning point and $R = 33$ m (D11) and $R = 50$ m (D07 and D 15). The longitudinal slope is negligible and amounts to less than 1 ‰.

3.1.2. Test section 2 – City of Zagreb

Zagreb tram network is more comprehensive, consisting of a total of 116 km of operational tram tracks (not including shunting yards and service tracks), with 1000 mm gauge that spreads through most of the Zagreb urban area. Track configuration is similar to the one in Osijek, and ranges from open track sections in designated tram corridors with speeds of up to 50 km/h, to tracks paved with concrete blocks in semi-designated corridor (for trams and service vehicles) and, finally, to closed city centre sections featuring tight curves, track paved with concrete and asphalt wearing course or RC blocks, surface shared with road vehicles, with an average speed of 10 km/h. The selected tram track section, Section 2, is a part of tram track network that is representative of

most of the tram network in Zagreb, with regard to different track and paving configurations as well as track exploitation periods. Section 2 stretches from its starting point at Ribnjak Street (intersection with Vlaška Street) to Dolje turning point. It consists of 20 subsections and measures 11.5 km in total length.

A comprehensive evaluation of tram track was performed on this section in the scope of the *Study on the Tram Traffic Development in Zagreb* prepared by the Faculty of Civil Engineering, University of Zagreb, for the Zagreb tram operator – ZET [2].

Subsections measuring over 50 m in length were considered in this evaluation as most of the parameters cannot be calculated accurately for shorter sections (e.g. 5-second continuous comfort). The layout of the examined subsections is provided in Figure 4, while construction details are available in Table 8.

Track alignment mostly follows a straight line or presents a generally low curvature of $R > 300$ m. There are however some exceptions such as $R_{min} = 50$ m on D-30-01 and D-30-02. Track grade is up to 20 ‰ at the section from Ribnjak to Mihaljevac, and up to 60 ‰ at the Mihaljevac – Dolje section (D-35-01 and D-35-02).

Table 8. Construction properties of subsections on Test Section 2

Subsection	Descriptive section name	Exploitation period [year]	Permanent way structure	Length [m]
D-30-01	Šoštarićeva - Ribnjak (Grškovićeva - Vlaška)	22	A	679
D-30-02	Šoštarićeva - Ribnjak (Vlaška - Grškovićeva)	22	A	670
D-31-01	Medveščak (Grškovićeva - Ribnjak)	22	B	101
D-31-02	Medveščak (Ribnjak - Grškovićeva)	22	B	101
D-31-03	Medveščak (Gupčeva Zvijezda - Grškovićeva)	11	C	930
D-31-04	Medveščak (Grškovićeva - Gupčeva Zvijezda)	11	C	928
D-36-03	Ksaver (Jandrićeva - Gupčeva Zvijezda)	14	C	620
D-36-04	Ksaver (Gupčeva Zvijezda - Jandrićeva)	14	C	617
D-36-07	Ksaver (Jandrićeva sjever - Jandrićeva jug)	22	D	242
D-36-08	Ksaver (Jandrićeva jug - Jandrićeva sjever)	22	D	242
D-36-09	Ksaver (prijelaz Jandrićeva sjever, Mihaljevac - Gupčeva Zvijezda)	22	A	114
D-36-10	Ksaver (prijelaz Jandrićeva sjever, Gupčeva Zvijezda - Mihaljevac)	22	A	114
D-36-11	Ksaver (Ksaverska jug - Jandrićeva sjever)	22	D	250
D-36-12	Ksaver (Jandrićeva sjever - Ksaverska jug)	22	D	251
D-36-13	Ksaver (prijelaz Ksaverska jug, Mihaljevac - Gupčeva Zvijezda)	22	A	77
D-36-14	Ksaver (prijelaz Ksaverska jug, Gupčeva Zvijezda - Mihaljevac)	22	A	77
D-36-15	Ksaver (Ksaverska sjever - Ksaverska jug)	22	D	138
D-36-16	Ksaver (Ksaverska jug - Ksaverska sjever)	22	D	139
D-35-01	Mihaljevac - Dolje (Dolje - Mihaljevac)	1	E	2482
D-35-02	Mihaljevac - Dolje (Mihaljevac - Dolje)	1	E	2532
Total length of Section 2				11471

Permanent way structure legend:
A - Base: RC slab (25 cm); Fastening: ZG-3/2; Paving: RC blocks. B - Base: RC slab (25 cm); Fastening: ZG-3/2; Paving: stone aggregate 16-31.5 mm. C - Base: RC slab (25 cm); Fastening: DEPP; Paving: stone aggregate 16-31.5 mm. D - Base: crushed stone (25 cm); Fastening: ballast 30-60 mm (20 cm); Paving: stone aggregate 16-31.5 mm. E - Base: RC slab (25 cm); Fastening: PPE; Paving: stone aggregate 16-31.5 mm.

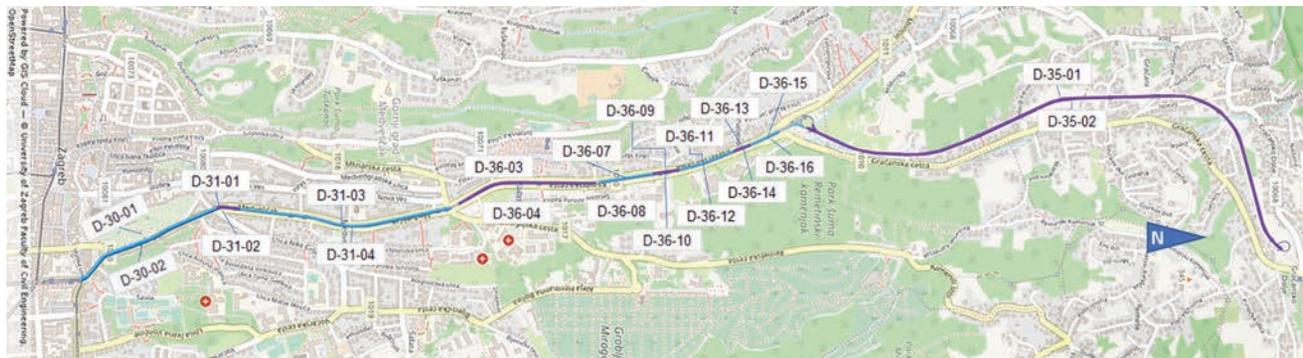


Figure 4. Section 2 with its subsections

3.2. Measurement setup

An in-service tram vehicle was equipped with measurement equipment for vibration recording to conduct measurements on the described test sections. A laptop and a multichannel analyser (Brüel & Kjær, type Pulse 3560C) were positioned in passenger compartment, and 4 accelerometers were mounted on the bogie (Brüel & Kjær type 4508), while 3 were placed in passenger compartment (Brüel & Kjær type 4507). The recording frequency was set to 4096 Hz at Test Section 1 and to 16384 Hz at Test Section 2. Both vehicles were also fitted with the GPS receiver and a Full HD video camera (for spatial reference, speed, and distance tracking).

On the bogie, accelerometers are positioned as close as possible to the wheel (axle) on the left side and right side of the bogie, in vertical and lateral directions. The influence of both rails could thus be evaluated. The measurement setup for standing passenger ride comfort was fitted in the passenger compartment, with 3 axis accelerometer at the floor of the vehicle for longitudinal, lateral and vertical directions, and with two single-axis accelerometers at top handlebar for vertical and lateral directions.

Similar setup was used on both test sections, but using different tram types typical for the network concerned, i.e. T3PVO garage no. 0717 in Osijek, Figure 6, and TMK2200, garage no. 2284 in Zagreb, Figure 7.

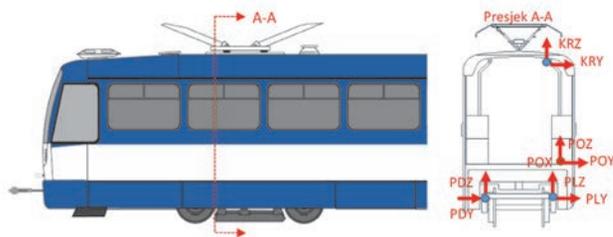


Figure 6. T3PVO (ČKD Praha) vehicle layout with accelerometer positions used at Test Section 1

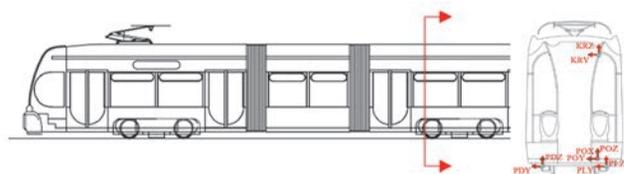


Figure 7. TMK2200 (Končar KEV) vehicle layout with accelerometer positions used on Test Section 2

A constant speed driving regime was imposed (30 km/h at Test Section 1 and 20 km/h at Test Section 2) in order to compare different track segments and the influence of infrastructure condition on driving comfort. All measurements were conducted in night period, without interfering tram traffic and very little road traffic, when continuous driving with minimal stopping intervals can be achieved.



Figure 5. Paving structure along Test Section 2 - RC paving blocks (left), asphalt (centre), stone aggregate (right)

3.3. Ride comfort results

For the purpose of this paper, only the accelerometer at tram floor will be considered (POX, POY and POZ) in order to evaluate ride comfort according to previously described procedures, Figure 8.

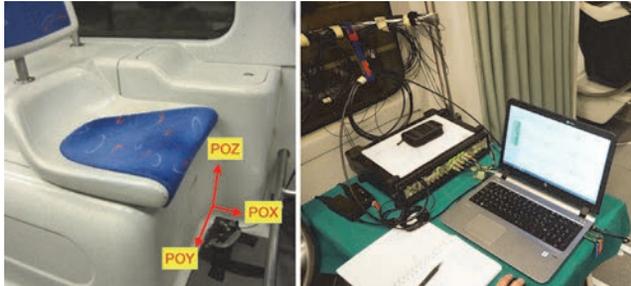


Figure 8. Position of 3 axial accelerometer on tram floor (left), data acquisition (right)

3.3.1. Equivalent level of vibrations (L_{aeq})

Equivalent acceleration level was calculated according to an average vibration level for each sub section (D7 to D15), based on measured acceleration for longitudinal, lateral and vertical directions (Figure 9).

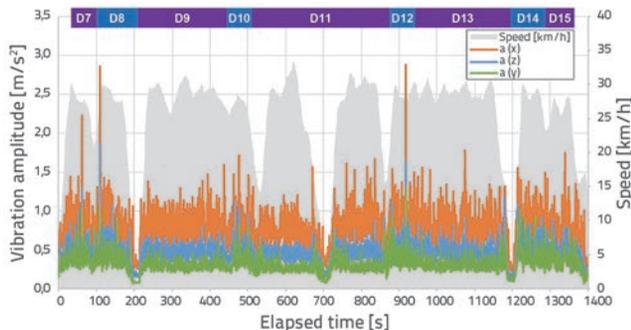


Figure 9. Vibration amplitude at floor level in 3 directions on Test section 1 with indicated subsections

Table 9. Equivalent levels of vibrations on Test section 1 and calculated ride comfort index (I_c)

Subsection	$L_{aeq(z)}$ [dB]	$L_{aeq(y)}$ [dB]	$L_{aeq(x)}$ [dB]	Ride comfort index (I_c)
D07	114.3	110.8	118.8	0.65
D08	115.1	111.0	119.3	0.75
D09	113.0	108.3	118.1	0.26
D10	114.3	109.9	118.8	0.57
D11	112.9	107.6	118.2	0.10
D12	114.6	110.5	118.8	0.74
D13	113.6	109.1	118.4	0.43
D14	114.1	109.7	118.3	0.55
D15	114.5	110.7	119.3	0.66

Further index for ride comfort (I_c) was calculated based on all three directions of the calculated equivalent level of vibrations (L_{aeq}), Table 9.

The ride comfort index I_c expressed in Table 9 is a relative measure of comfort on a scale from 0 to 1 (where 0 represents best comfort and 1 represents worst comfort) in a tram, on each subsection, based on results of the entire tram network.

The same principle for determining the equivalent level of vibrations is used on Test Section 2 – City of Zagreb, with time dependent vibration amplitudes as plotted in Figure 10 (East track) and in Figure 11 (West track).

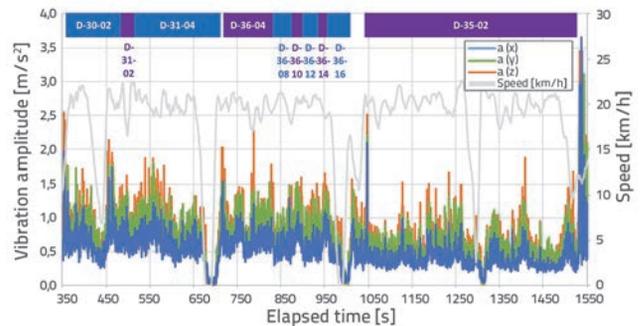


Figure 10. Vibration levels at Test Section 2, east track

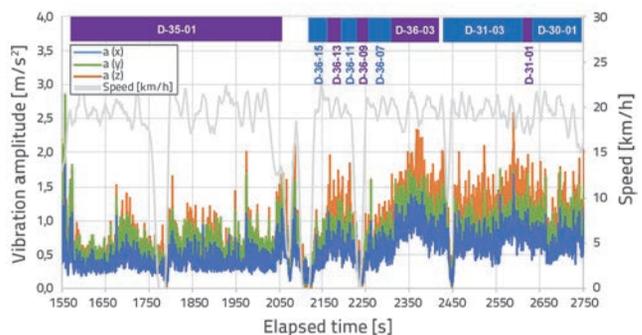


Figure 11. Vibration levels at Test section 2, west track

Results of L_{aeq} for each direction is given for each subsection together with the ride comfort index I_c generated based on the entire tram network in the city of Zagreb, where 0 represents the lowest average level of L_{aeq} and 1 the highest average value of L_{aeq} for each subsection.

The method, while being simple for implementation, does give a general overview of the state of comfort on the entire tram network [2]. The main disadvantage is the lack of limiting values describing level of comfort of passengers. The main purpose of the method is to identify sections with highest I_c where low level of comfort is detected, and where further investigation into track geometry and state of infrastructure must be conducted.

3.3.2. Sperling's Ride Index W_z

The Spring ride index was calculated in two directions (vertical and lateral) from 1-second amplitudes of peak accelerations $a_{15,peak}$ [cm/s²], in 19 1/3 octave frequency bands from 0.5 to

Table 10. Equivalent levels of vibrations at Test Section 2 and calculated ride comfort index (I_c)

East track					West track				
Subsection	L _{aeq(z)} [dB]	L _{aeq(y)} [dB]	L _{aeq(x)} [dB]	Indeks (I _c)	Subsection	L _{aeq(z)} [dB]	L _{aeq(y)} [dB]	L _{aeq(x)} [dB]	Indeks (I _c)
D-30-02	118.3	120.7	121.4	0.4	D-30-01	117.0	119.6	120.6	0.4
D-31-02	115.8	118.8	118.8	0.3	D-31-01	118.5	121.1	122.8	0.4
D-31-04	115.7	118.2	118.2	0.3	D-31-03	116.3	118.7	121.2	0.4
D-36-04	116.6	119.5	120.2	0.4	D-36-03	118.0	120.8	122.4	0.4
D-36-08	114.9	117.5	117.6	0.2	D-36-07	115.2	117.6	118.6	0.3
D-36-10	118.1	120.9	121.2	0.3	D-36-09	113.6	116.3	116.9	0.1
D-36-12	114.8	118.1	118.6	0.2	D-36-11	115.6	117.9	120.3	0.3
D-36-14	117.2	119.7	120.5	0.3	D-36-13	116.0	118.5	120.2	0.2
D-36-16	112.9	116.5	116.8	0.2	D-36-15	113.3	116.6	116.8	0.2
D-35-02	110.6	114.1	114.2	0.0	D-35-01	111.7	115.0	115.3	0.1

31.50 Hz. The frequency weighting was performed for these bands in lateral and vertical directions. The weighting factors are presented in Table 11.

Wz was first calculated according to expressions (3) and (4) and Wz_{tot,1s} was expressed for the entire frequency range as shown in the following expressions (16) and (17).

$$Wz_{tot,1s} = \sqrt[10]{Wz_1^{10} + Wz_2^{10} + \dots + Wz_{19}^{10}} \quad (\text{ride quality}) \quad (16)$$

$$Wz_{tot,1s} = \sqrt[6.67]{Wz_1^{6.67} + Wz_2^{6.67} + \dots + Wz_{19}^{6.67}} \quad (\text{ride comfort}) \quad (17)$$

Table 11. Frequency range of 1/3 octave bands and weighting factors for calculation of Wz

Index	Lower band limit [Hz]	Center frequency (fc) [Hz]	Upper band limit [Hz]	Vertical weighting factor B _v	Lateral weighting factor B _w
1	0.447	0.500	0.562	0.337	0.422
2	0.562	0.630	0.708	0.390	0.489
3	0.708	0.800	0.891	0.444	0.556
4	0.891	1.000	1.120	0.489	0.613
5	1.12	1.25	1.41	0.53	0.66
6	1.41	1.60	1.78	0.57	0.72
7	1.78	2.00	2.24	0.61	0.76
8	2.24	2.50	2.82	0.65	0.82
9	2.82	3.15	3.55	0.71	0.89
10	3.55	4.00	4.47	0.77	0.97
11	4.47	5.00	5.62	0.81	1.02
12	5.62	6.30	7.08	0.78	0.97
13	7.08	8.00	8.91	0.64	0.80
14	8.91	10.00	11.22	0.49	0.62
15	11.22	12.50	14.13	0.37	0.47
16	14.13	16.00	17.78	0.28	0.35
17	17.78	20.00	22.39	0.21	0.27
18	22.39	25.00	28.18	0.17	0.21
19	28.18	31.50	35.48	0.13	0.16

Wz_{tot,1s} are plotted against time signal to show the Sperling ride index for Test Section 1, where vibrations were recorded at an average running speed of 30 km/h. Only ride comfort values for vertical and lateral directions are presented, since the procedure does not account for longitudinal vibrations, Figure 12.

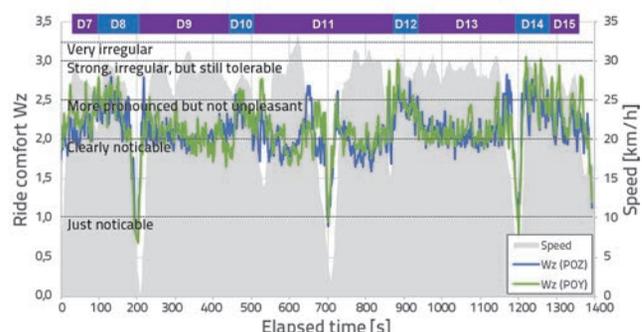


Figure 12. Sperling ride comfort Wz index for 1-second intervals on Test Section 1

Short events that surpass a certain scale of comfort according to Table 2 can clearly be distinguished from the time graph of the Sperling ride comfort index Wz. By overlapping such results with tram track geometry, or GPS data, certain points along the line can clearly be identified and further examined. For example, at the start and end of D11 there is a turning point on the track and the curvature is high, and so, in spite of lower speed, the ride comfort surpasses the 2.5 limit, even reaching the level of 3.0 (strong, irregular, but still tolerable) with lateral comfort being much more pronounced, which can be expected at turning points.

It is however not easy to estimate the overall performance of a certain test section on the line. Therefore a histogram approach was used to determine the percentage of values that surpass certain limit at each test section, Figure 13 and Figure 14. From the histogram analysis shown in figures 13 and 14, it is apparent that most subsections fall into 2-2.5 scale of

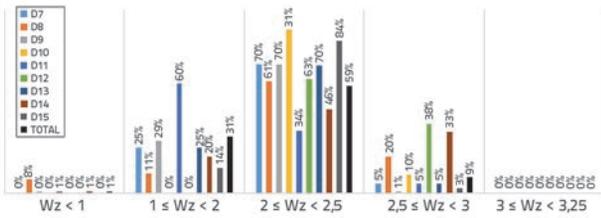


Figure 13. Distribution of Spurling ride comfort index Wz at Test Section

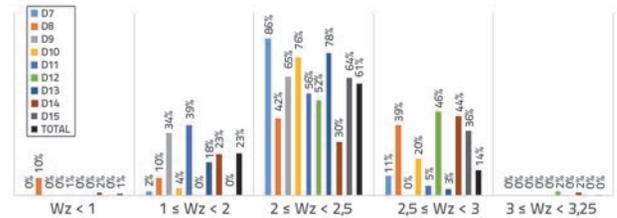


Figure 14. Distribution of Spurling ride comfort index Wz at Test Section 1 in lateral direction

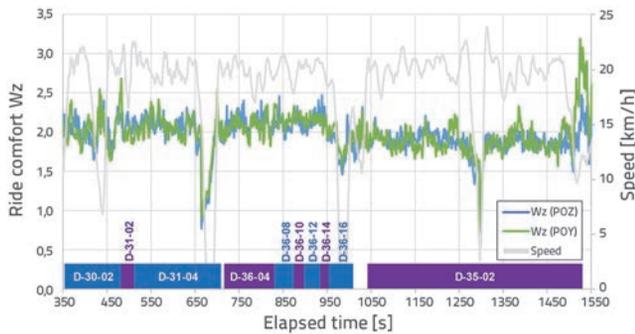


Figure 15. Spurling ride comfort index Wz for Test Section 2, east track, vertical and lateral directions

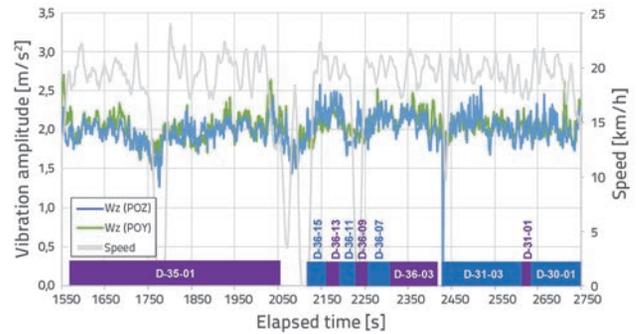


Figure 16. Spurling ride comfort index Wz for Test Section 2, west track, vertical and lateral directions

the Spurling ride comfort index. This describes the feeling of vibrations as more pronounced but not unpleasant. Subsection D11 clearly stands out with most of the values in 1-2 range (just noticeable to clearly noticeable) and is therefore categorized as the best section as to comfort in both vertical and lateral direction. There are, however, track subsections, such as D 12 and D 14, where almost half of recorded values fall into category 2.5-3, which can be described as strong, irregular, but still tolerable vibrations, with peak values reaching 3-3.25 at these sections in lateral direction (very irregular vibrations). A similar procedure for determining the Spurling ride comfort Wz index was used at Test Section 2. It is important to notice

that, for this measurement, the average speed of tram was 20 km/h, Figure 15 and Figure 16.

It can be seen from figures 15 and 16 that Wz is dependent on speed and that it is significantly lower at Test Section 2 due to the average speed of 20 km/h. Therefore, the average speed test is valuable for comparing individual sections. However, real in-service driving conditions should be ensured to properly determine the level of comfort. There is distinction in overall level between sections D-35-01, D-35-02 and other sections, other being mostly over the 2.0 threshold. For detailed insight into distribution of values, further analysis was performed, as presented in figures 17 to 20.

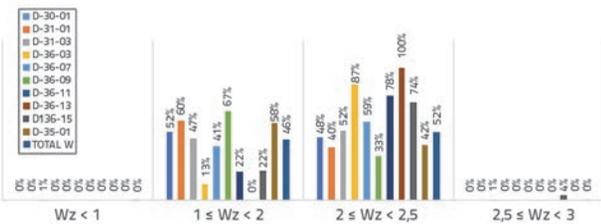


Figure 17. Spurling ride comfort index distribution Wz on Test Section 2 in vertical direction, west track

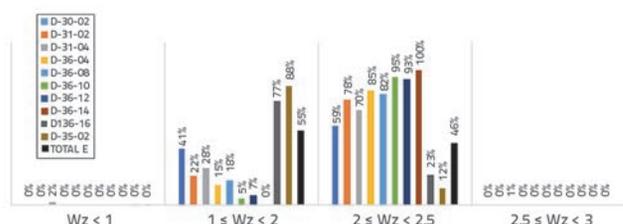


Figure 18. Spurling ride comfort index distribution Wz on Test Section 2 in vertical direction, east track

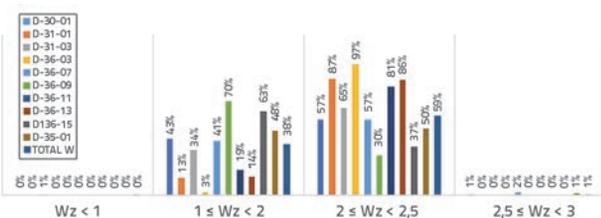


Figure 19. Spurling ride comfort index distribution Wz on Test Section 2 in lateral direction, west track

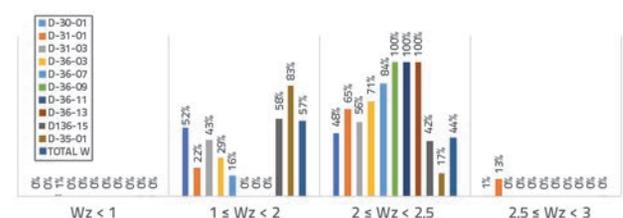


Figure 19. Spurling ride comfort index distribution Wz on Test Section 2 in lateral direction, west track

3.3.3. EN 12299

The mean comfort standard method (N_{MV}) and the continuous comfort method (C_{Cx} , C_{Cy} , C_{Cz}) were used on tram tracks to determine their suitability for the urban areas under study. Vibrations were processed in frequency domain using the weighting curves prescribed in the standard [18]. Weighting curve Wb (for vertical direction) and Wd (for longitudinal and lateral directions) are proposed. Wd is based on ISO 2631-1 [10], while Wb curve is slightly modified with regard to the one proposed in ISO 2631-4 [11] and defined in EN 12299 [18], Figure 21.

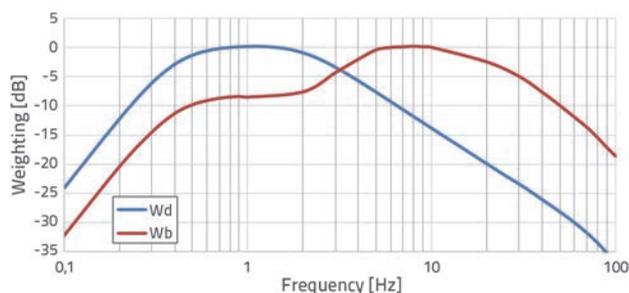


Figure 21. Weighting curves for longitudinal, lateral (Wd) and vertical directions (W_b) [18]

To calculate the continuous comfort (C_{Cx} , C_{Cy} , C_{Cz}), accelerations in 5-second intervals (a_{POX}^{Wd} , a_{POY}^{Wd} , a_{POZ}^{Wb} [m/s^2]) have to be calculated according to the following equation (18):

$$a_{X_i}^{W_x} = \left[\frac{1}{T} \int_{t-T}^t (a_{W_i}(t))^2 dt \right]^{0.5}, T = 5s \tag{18}$$

Values $a_{X_i}^{W_x}$, used according to expressions a_{pOX} (longitudinal), a_{pOY} (lateral) and a_{pOZ} (vertical), are denoted according to (8), (9) and (10) as C_{Cx} . For Test Section 1, these values are plotted in the time graph as shown in Figure 22.

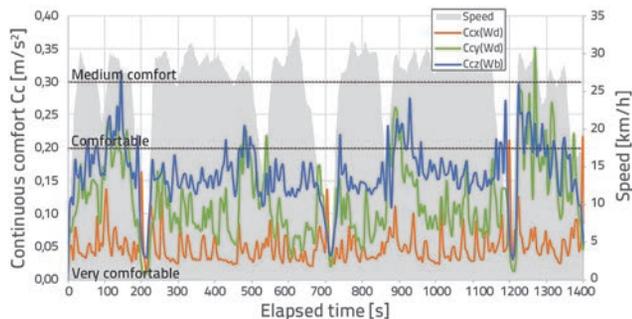


Figure 22. Continuous comfort (C_{Cx} , C_{Cy} , C_{Cz}) at Test Section 1

It can clearly be seen that comfort in X direction, although small in general, rises at each vehicle braking, which is a result of longitudinal accelerations. Most pronounced vibrations are those in vertical direction but, generally, according to evaluation

scales given in Table 5, the ride can be evaluated as very comfortable (under $0.2 m/s^2$) with some values in the zone of 0.2 to 0.3, which is defined as comfortable. Only a few values are over the 0.3 limit in vertical and lateral directions (medium comfort). However, it can clearly be seen that D8, D10, D12 and D14 are characterized by the least level of comfort, while D9, D11 and D13 are the most comfortable.

Histograms of continuous comfort based on sub-sections was calculated and presented for Test Section 1, as shown in Figure 23.

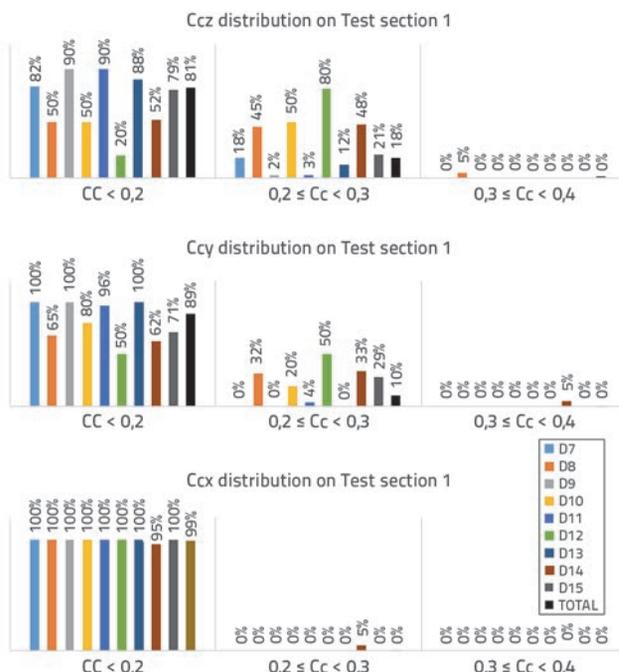


Figure 23. Distribution of continuous comfort values for individual sub-sections on Test Section 1

It can be seen that most of the values are distributed in the “very comfortable” range (Cc up to $0.2 m/s^2$), with vertical comfort being more pronounced in the “comfortable” range, with only a few values exceeding $0.3 m/s^2$ which belongs to the “medium comfort” range. Longitudinal direction presents the least problems with comfort, which is logical as the constant speed of $30 km/h$ with minimal acceleration and braking was imposed for the purposes of this measurement. Section D12 is the least comfortable one, while sections D9 and D11 are the most comfortable.

Based on continuous comfort values, the mean comfort standard method was used to calculate N_{MV} values as the 95th percentile of 5-minute sections according to eq. (7), while the mean comfort complete method N_{VD} was used for standing passengers based on the 50th percentile of the same values according to eq. (15), Table 12.

According to evaluation scales defined in Table 6, it can be seen that 4 out of 5 5-minute intervals fall into the “comfortable” category and the last section in the “medium” comfort category.

Table 12. Mean comfort standard method N_{MV} and complete method N_{VD} at Test Section 1

Time span [s]	$a_{POZ95}^{W_b}$	$a_{POY95}^{W_d}$	$a_{POX95}^{W_d}$	$N_{MV,POZ}$	$N_{MV,POY}$	$N_{MV,POX}$	N_{MV}	$a_{POZ50}^{W_b}$	$a_{POY50}^{W_d}$	$a_{POX50}^{W_d}$	N_{VD}
0-299	0.25	0.23	0.13	1.48	1.35	0.78	2.15	0.16	0.12	0.04	2.17
300-599	0.21	0.20	0.08	1.27	1.21	0.51	1.83	0.15	0.10	0.04	1.90
600-899	0.22	0.24	0.08	1.31	1.44	0.48	2.00	0.14	0.08	0.04	1.99
900-1199	0.24	0.18	0.10	1.41	1.10	0.59	1.88	0.16	0.11	0.04	1.88
1200-1400	0.25	0.29	0.20	1.52	1.72	1.19	2.59	0.19	0.16	0.06	2.73

N_{VD} does not greatly differ from N_{MV} for the 5 intervals at Test Section 1.

It can be seen from the graphs that the comfort on Test Section 2 varies from very comfortable to comfortable. The best comfort was recorded at sections D-35-01 and D-35-02, which have been rehabilitated most recently.

Histograms of continuous comfort based on sub-sections were calculated and presented for Test Section 1, Figure 26. These histograms clearly show to what extent does the comfort fall into the category "very comfortable" (up to 0.2 m/s²) or "comfortable" (0.2 to 0.3 m/s²). The values reach second scale "comfortable" mostly for vertical direction, with section D-36-11 being the least comfortable

with more than 50% of values in scale "comfortable". The mean comfort standard method N_{MV} and the mean comfort complete method N_{VD} (standing passengers) were applied and the corresponding results for Test Section 2 are presented in Table 13.

According to evaluation scales, both methods show that all but one 6-minute intervals fall into the "comfortable" category according to Table 6. One section (1200 s – 1499 s) falls into the "very comfortable" category. This interval covers most of the sub-section D-35,-02 which has been rehabilitated very recently. Generally, it can be seen that N_{MV} gives slightly higher results than N_{VD} , but it is not the case for all the intervals.

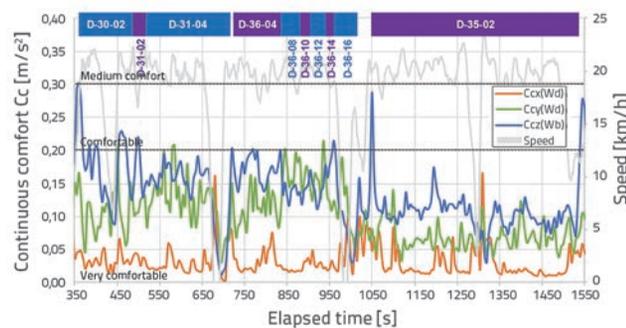


Figure 24. Continuous comfort (C_{cx} , C_{cy} , C_{cz}) at Test Section 2, east track

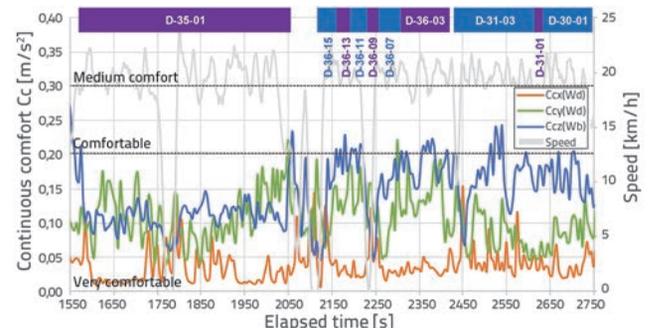


Figure 25. Continuous comfort (C_{cx} , C_{cy} , C_{cz}) at Test Section 2, west track

Table 13. Mean comfort standard method N_{MV} and complete method N_{VD} at Test Section 2

Time span [s]	$a_{POZ95}^{W_b}$	$a_{POY95}^{W_d}$	$a_{POX95}^{W_d}$	$N_{MV,POZ}$	$N_{MV,POY}$	$N_{MV,POX}$	N_{MV}	$a_{POZ50}^{W_b}$	$a_{POY50}^{W_d}$	$a_{POX50}^{W_d}$	N_{VD}
0-299	0.28	0.17	0.09	1.66	1.02	0.52	2.02	0.16	0.10	0.04	1.74
300-599	0.30	0.20	0.06	1.81	1.22	0.37	2.21	0.16	0.12	0.03	1.98
600-899	0.20	0.19	0.08	1.19	1.12	0.47	1.70	0.15	0.12	0.02	1.83
900-1199	0.19	0.17	0.09	1.15	1.05	0.52	1.64	0.11	0.08	0.03	1.56
1200-1499	0.13	0.11	0.07	0.80	0.64	0.41	1.10	0.10	0.06	0.02	1.06
1500-1799	0.23	0.16	0.09	1.35	0.97	0.53	1.75	0.11	0.10	0.03	1.56
1800-2099	0.19	0.18	0.08	1.17	1.07	0.48	1.66	0.12	0.10	0.03	1.67
2100-1399	0.22	0.19	0.12	1.32	1.15	0.73	1.90	0.17	0.13	0.03	1.96
2400-2699	0.23	0.14	0.09	1.35	0.85	0.54	1.69	0.17	0.08	0.04	1.53
2700-3000	0.25	0.18	0.08	1.47	1.11	0.51	1.91	0.17	0.11	0.04	1.87

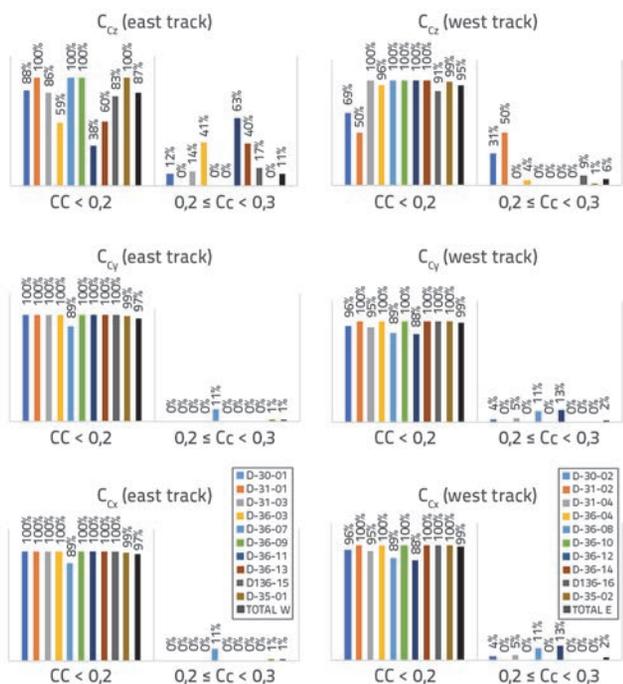


Figure 26. Histograms of continuous comfort scales for individual subsections at Test Section 2

It is obvious that these methods, based on 6-minute intervals, are not a substantial solution for analysis on tramway networks because of network length. Analysis of subsections cannot be made based on NVD and NMV. Only a general overview of comfort can be given.

4. Evaluation of ride comfort methods on tramway lines

Based on comprehensive data analysis according to three methods, ride comfort was described on two test sections forming part of 1000-mm gauge tram networks. Various comfort measurement procedures were evaluated for this purpose. A number of parameters and criteria were compared in order to find the solution that is most suitable for the urban railway environment.

4.1. Frequency range, weighting and direction

In terms of frequency range, three described methods can be differentiated:

equivalent level of vibrations uses the 2 Hz to 200 Hz range, which is selected because of a need to compare different track segments and their properties such as rail corrugation, rail welds, and their effect on comfort.

Sperling’s ride comfort index Wz ranges from 0.5 Hz to 30 Hz in frequency and disregards higher frequencies, which are known to affect human perception of vibrations as shown in a

number of studies [28]. For example, the frequency of bad weld at a tramway track has a dominant frequency response at 50 Hz. Such events and their effects on comfort are disregarded when the Sperling method is applied [29].

EN 12299 is based on ISO 2631 procedure where a frequency range of 0.4 Hz to 100 Hz is considered substantial, although some studies indicate that higher frequency range, even up to 315 Hz, should be considered [30].

As for weighting curves, different weighting is used for both vertical and horizontal directions for Sperling Wz ride comfort and EN 12299. This comparison is shown in Figure 27.

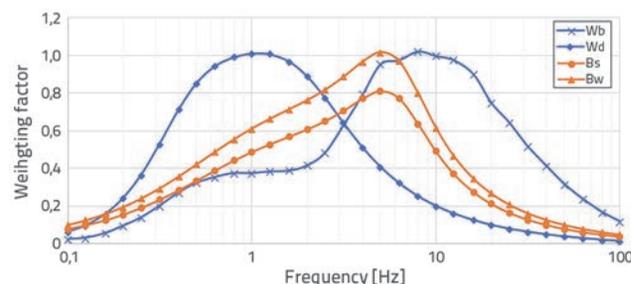


Figure 27. Weighting factors according to Sperling (orange) and EN 12299 (blue)

Different frequency weighting is applied for each direction in each method. The Wb curve used for vertical direction in EN 12299 gives greatest weight to the 2 – 40 Hz range, with the secondary band of 0.3 to 3 Hz. The Wd curve for lateral and longitudinal directions is in the 0.3 – 4 Hz range, with a very low weight for high frequency vibrations. Sperling’s weighting curves Bs (vertical) and Bw (horizontal) have the same characteristic, but vertical direction is more pronounced and so $B_w(f) = 1.25 B_s(f)$. They both peak at 5 Hz with a very steep decline towards higher frequencies.

As explained earlier, for some very common events on a tram track, such as poor geometry of rail weld, the dominant frequency falls into the 40-60 Hz range at the speed of 20-30 km/h, which would be disregarded by the Sperling’s Bs weighting curve for vertical direction, Figure 28.

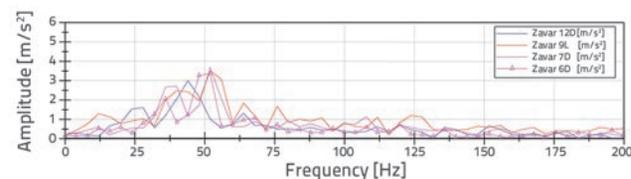


Figure 28. FFT analysis of acceleration induced by 4 rail welds at Test Section 2 [29]

The Sperling index is also not used for longitudinal vibrations, which can have a significant impact on drive comfort in the start-stop riding regime on surfaces shared with road traffic. In

Table 14. Comparison of comfort evaluation scales based on results obtained at Test Section 1

Sperling Wz (5s)	POZ [%]	POY [%]	POZ [%]	POY [%]	EN 12299 Cc
Scale					Scale
Just noticeable	1 %	1 %	83 %	83 %	Very comfortable
Clearly noticeable	28 %	23 %	17 %	17 %	Comfortable
More pronounced but not unpleasant	65 %	64 %	0 %	0 %	Medium comfort
Strong, irregular, but still tolerable	6 %	13 %			

this study, the driving regime involved a constant speed with minimum acceleration/braking sequences, because different track subsections had to be compared and analysed. However, in real in-service conditions on a busy tram network, longitudinal vibrations would have a significant impact on drive comfort. This can clearly be seen in figures presenting continuous comfort against speed, where POX direction (longitudinal) is pronounced on acceleration and deceleration sections along the track, Figure 22.

4.2. Scales for evaluating comfort

Both the Sperling ride index method and EN 12299 provide scales for evaluating human perception of comfort based on research conducted with human subjects. In the following comparison, where Sperling index is averaged over 5-second intervals in order to be comparable to the continuous comfort method by EN 12299, the measured levels of comfort using the two methods are analysed on Test Section 1, Figure 29.

As shown in previous figure, in respect of Sperling index scales, EN 12299 tends to underestimate comfort, especially in lateral direction (POY). Most of the values fall into the *very comfortable* category, while for the Sperling Wz index, most of the values are categorized as *more pronounced but not unpleasant* with 65% of values falling into the *strong, irregular, but still tolerable* category, Table 14. This behaviour was also described in research on standard railway lines presented in [31].

4.3. Sub-section evaluation and ranking

As shown in Table 4, the state of infrastructure can be evaluated according to EN 12299 continuous comfort and N_{MV} . According to the approach used in this paper, the same in-service vehicle travelling at constant speed was used for all subsections on each test section, so that the state of infrastructure at different test sections can be compared. This has, however, imposed a significant challenge due to different characteristics of the track and subsections. Subsections were identified according

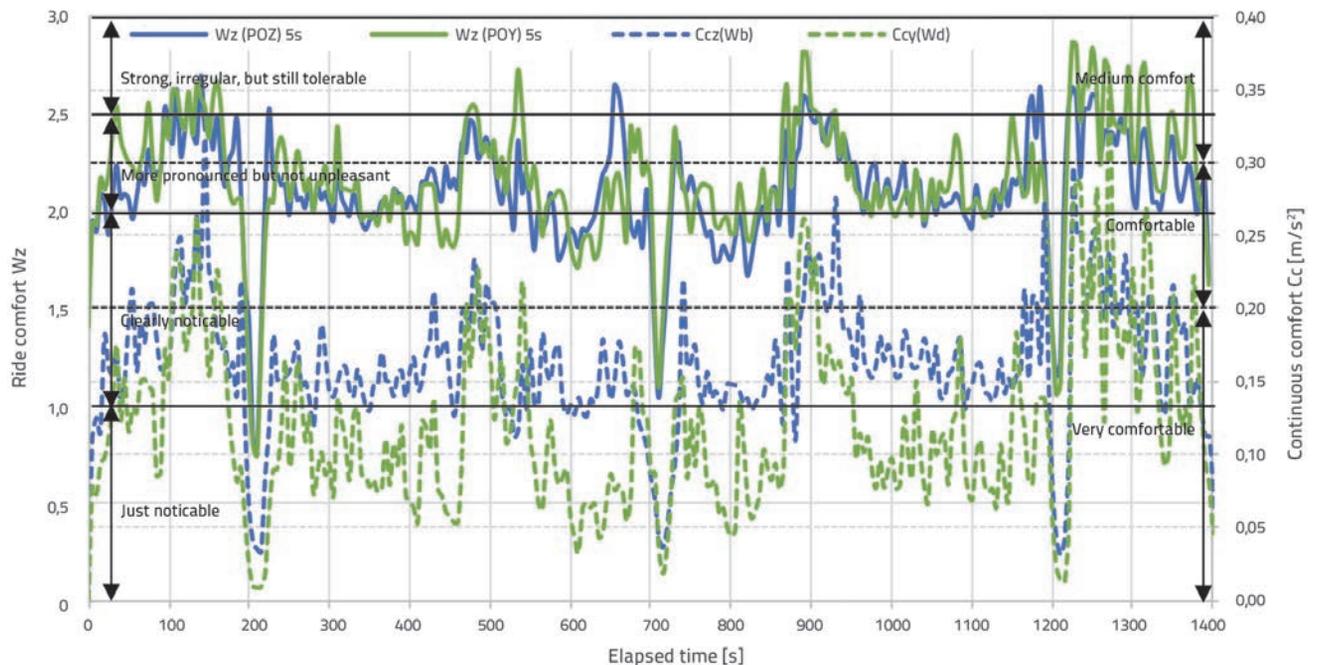


Figure 29. Comparison of Sperling ride comfort index Wz (5s) and EN 12299 continuous comfort method with evaluation scales at Test Section 1

Table 15. Calculation of total weighted Wz for subsection D7 at Test Section 1

Direction	POZ			POX			POY		
	Cc < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4
Scale	Cc < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4
Wz histogram [%]	82 %	18 %	0 %	100 %	0 %	0 %	100 %	0 %	0 %
Weight	1	2	3	1	2	3	1	2	3
Weighted Wz	0.82	0.35	0.00	1.00	0.00	0.00	1.00	0.00	0.00
Weighted Wz average	0.20			0.17			0.17		
Total weighted Wz	0.54								

Table 16. Calculation of total weighted Cc for subsection D7 at Test Section 1

Direction	POZ			POX			POY		
	Cc < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4
Scale	Cc < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4	CC < 0.2	0.2 ≤ Cc < 0.3	0.3 ≤ Cc < 0.4
Wz histogram [%]	82 %	18 %	0 %	100 %	0 %	0 %	100 %	0 %	0 %
Weight	1	2	3	1	2	3	1	2	3
Weighted Wz	0.82	0.35	0.00	1.00	0.00	0.00	1.00	0.00	0.00
Weighted Wz average	0.20			0.17			0.17		
Total weighted Wz	0.54								

to type of superstructure and year of last rehabilitation (Table 7 and Table 8), but other parameters, such as the curvature, section length, surface shared with road vehicles, etc. also vary for each section. Constant speed could also not be reached over the entire length of each section due to stops at red light, slowdowns in tight curves, etc.

Therefore, the equivalent level of vibrations was imposed to conveniently rank the sub-sections on entire tram networks and compare them to one another.

The Sperling method provides an equation to calculate the Wz index for an arbitrary interval of measurement data, but can not be used to evaluate all directions together in a single equation. Therefore, a histogram approach was used to determine the level of comfort on each sub-section, Figure 13, Figure 14, Figure 17, Figure 18, Figure 19, and Figure 20.

EN 12299 provides a single-value of comfort as N_{MV} or N_{VD} , but it does not provide a calculation for an arbitrary length of test sections. N_{MV} and N_{VD} are calculated on the 5-minute basis which is an excessively long period for the evaluation of sub-sections

on tram networks. It can therefore be used to rank individual sub-sections by means of histograms (Figure 22, Figure 24, and Figure 25).

The ranking was made based on weights given to each scale of comfort for Sperling method and EN 12299 method. The smallest scale of comfort was given the weight of 1 and every consecutive scale had the weight of 1+n, as presented in the following example, Table 15 and Table 16.

The final for each test section is based on the total weighted Wz for Sperling method and on the total weighted Cc for EN 12299 method. The ranking for equivalent level of vibrations method is based on the ride comfort index I_c , Table 9 and Table 10.

According to ranking based on three different methods, there is a very good correlation between Wz and Cc methods on most sub-sections of Test Section 1, as shown in Figure 31. At Test Section 2, Figure 31, most sections according to EN 12299 fall into scale 1, *very good comfort*, because of lower speed of tram and high quality test vehicle, tram TMK 2200, and are therefore are ranked very high, which is not the case with the Sperling

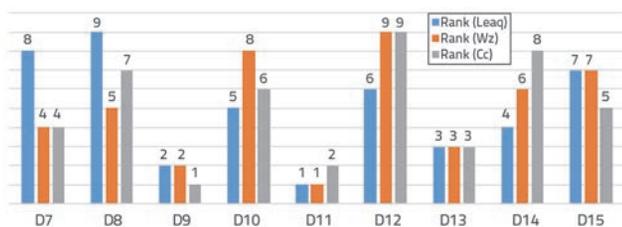


Figure 30. Ranking of subsections at Test Section 1 based on 3 different methods, Laeq, Wz, Cc

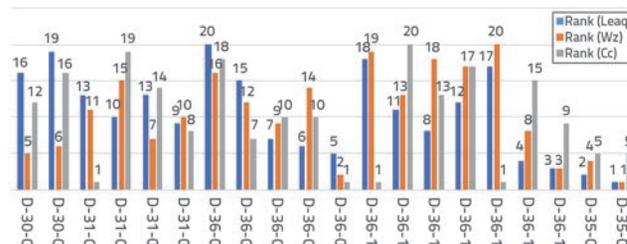


Figure 31. Ranking of subsections at Test Section 2 based on 3 different methods, Laeq, Wz, Cc

index or the equivalent level of vibrations method because different scaling levels are applied. Due to that fact, a better correlation is obtained between the Laeq and Wz method.

5. Conclusion

This paper provides an extensive overview of methods for evaluating ride comfort in railway vehicles. It is conceived around the idea of evaluating comfort on tram networks, where the track structure, driving conditions, speed, and many other parameters, greatly differ from the ones applicable to classic railway networks.

Test sections were selected among more than 150 km of tramway tracks in Zagreb and Osijek and vibration-related data were recorded from in-service vehicles [1, 2]. Analysed parameters were accelerations on tram floor in 3 directions (vertical, lateral, and longitudinal), speed, and GPS position. These parameters were used to calculate ride comfort according to the Spering Ride Index (Wz) method, equivalent level of vibrations (L_{aeq}) method, and methods described in EN 12299 (based on EN ISO 2631), i.e. the continuous comfort method (C_{cx} , C_{cy} , C_{cz}), mean comfort standard method (N_{MV}), and complete (N_{VD}) method. All methods were calculated for the same set of data measured on tramway network in Osijek in 2016 (Test Section 1) and tramway network in Zagreb in 2018 (Test Section 2). The driving regime involved driving at continuous speed (30 km/h in Osijek and 20 km/h in Zagreb), in order to evaluate and compare different sub-sections to one another from a controlled vehicle at constant speed.

Various methods for the evaluation of ride comfort were examined and compared against each other to find an optimum procedure for tramway comfort evaluation.

It can be concluded that the equivalent level of vibrations method (L_{aeq}) constitutes a very powerful tool for comparing tramway track sub-sections against each other. It provides a clear overview of the state of comfort along the entire network and can be used to conveniently pinpoint track sections that need special attention or intervention to improve the level of comfort. Since accelerations are not weighted and the frequency range is broad (up to 200 Hz), the method is more sensitive to higher frequency vibrations. Unweighted signals can then be used to identify underlying vehicle-based problems or problems in tram infrastructure by means of the FFT analysis of the section, as shown in Figure 28. According to [30], these vibrations can also be considered relevant for drive comfort, especially for standing passengers who constitute the majority of passengers in tram networks under study, especially during rush hours. The method does not provide evaluation scales for the level of comfort that is measured according to human perception of vibrations.

The Spering ride index Wz method is very convenient for the assessment of tramway infrastructure. The method provides very elaborate scales for the determination of comfort. Besides comfort, the method is also used to determine ride quality.

Ranking of sub-sections based on histogram values for different comfort scales is also possible. The method can not be used for measuring comfort in longitudinal direction. For the evaluation of comfort at constant speed, as described in this paper, vertical and lateral directions are sufficient. However, if comfort is evaluated for in-service driving regime on tramway lines where start-stop driving occurs regularly, especially on sections with surface shared with road vehicles, longitudinal direction is of great importance for ride comfort. Another limitation is that it uses a low frequency range (up to 30 Hz) thus eliminating vibrations from certain track irregularities such as rail welds (described in Section 4), which greatly affects the comfort especially on tramway networks where weld geometry is not as strictly controlled during construction, and weld grinding is inadequate.

The methods described in EN 12299 are well documented and sufficient for evaluating ride comfort on railway lines. The N_{MV} and N_{VD} methods use 5-minute samples for evaluating comfort. These two methods are the only ones that provide an integral formula for calculating an overall level of comfort for all 3 directions, equations (10) and (15). Although these methods provide the tools and scales to calculate the overall level of comfort, they are nevertheless burdened with some drawbacks. The method N_{MV} uses the 95th percentile, which disregards all but top 5 values of comfort on a 5-minute long section. This can lead to misinterpretation of sections as explained in [31] and [24]. Also, the sample size is too long for practical application on tramway lines, since stretches of track with similar properties are usually shorter than the equivalent of ~1,5 km, which is an average distance covered by tramway in 5 minutes (travelling at 30 km/h). Therefore, these methods, although practical for application on conventional railway lines, do not provide enough detail to be usable on networks such as the ones described in this paper.

On the other hand, the continuous comfort method described in EN 12299 (equivalent to the method proposed in ISO 2631) provides sufficient 5-second averaged level of vibrations in all 3 directions. It provides evaluation scales for vertical and lateral directions. It is practical for use on tramway lines especially because of broader frequency range compared to the Spering method (up to 100 Hz).

The evaluation of comfort using the provided scale, compared to Spering method, underestimates the level of comfort on tramway lines. This is more obvious on Test Section 2 because of lower average speed of vehicles during data collection (20 km/h), where 95% of all values fall into the "very comfortable" zone. A similar observation is made in [31] where an analogous comparison of methods is discussed on standard railway lines. This makes the method impractical for comparing test sections to each other, as most of the sub-sections fall into the same scale of comfort. For this method to be completely useful for tramway applications, a study on scales of comfort by mean of passenger survey should be conducted in order to establish actual scales for such applications.

The paper proposes a method for ranking individual subsections based on histogram approach where various comfort scale limits are used to count the number of 5-second intervals that fall into each comfort scale. Based on such approach, the Sperling ride index method and the continuous comfort method can be used to evaluate overall level of comfort of subsections.

Passenger comfort should be a very important parameter for railway/tramway operators. It can help attract more passengers, generate higher revenues, and point to some defects along the

track infrastructure or on rail vehicles. A number of parameters affect human perception of comfort, and it is therefore quite difficult to measure this perception in an objective manner. Some methods provide the tools and scales for determining comfort in railway vehicles. For tramway applications, the continuous comfort method proposed in EN 12299 constitutes a decent tool for evaluating passenger comfort. However, additional surveys should be conducted on these types of networks to provide for better understanding and improved categorisation of comfort on such networks.

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