Economic Research-Ekonomska Istraživanja

ISSN: 1331-677X (Print) 1848-9664 (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

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To cite this article: Aiyuan Tao, Yingjun Zhu & Xiangqing Zou (2019) Efficiency comparison of unit and ad valorem taxation in mixed duopoly, Economic Research-Ekonomska Istraživanja, 32:1, 473-491, DOI: 10.1080/1331677X.2018.1555048

To link to this article: https://doi.org/10.1080/1331677X.2018.1555048

Published online: 13 Jul 2019.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2019, VOL. 32, NO. 1, 473–491
https://doi.org/10.1080/1331677X.2018.1555048

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Efficiency comparison of unit and ad valorem taxation in mixed duopoly

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ABSTRACT
In this paper, we consider the relative efficiency of unit taxation and ad valorem taxation in terms of welfare implications in different imperfectly competitive markets. Under the assumption that these two taxations can produce the same total output in equilibrium, which is also used by Anderson (2001), we show that ad valorem taxation is always welfare superior to unit taxation under full nationalisation, full privatisation, and partial privatisation.

ARTICLE HISTORY
Received 25 July 2016
Accepted 11 May 2018

KEYWORDS
Unit taxation; ad valorem taxation; efficiency; partial privatisation

1. Introduction

Why were public firms so common in the past, even in western countries, and why were many of them privatised during the past few decades? Some recent papers argue that through state ownership the government can avoid the distortions to managerial incentives created by corporate income tax. Amiram, Bauer, and Frank (2013) exploit exogenous changes in a country's shareholder dividend tax policy to examine managerial incentives for corporate tax avoidance. They conclude that corporate tax avoidance significantly increases (decreases) after an exogenous elimination (enhancement) of an imputation system and closely-held firms in countries where shareholder benefits exist (do not exist) engage in more (less) corporate tax avoidance. Koethenbuerger and Stimmelmayr (2014) analyse whether the cost of investment should be tax exempt using an agency model of firm behaviour. They find that an efficient tax system may not allow for a full deduction of the cost of investment and that a switch to an allowance for a corporate equity system can increase investment, but may reduce welfare. When the corporate tax rate is high enough, state ownership may be less inefficient than private ownership. If this argument was right, then the capital intensity of public firms should fall with privatisation (Gordon, 2001). However, the data showed instead that firms lay off a sizable fraction of their work

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force when they are privatised, suggesting that public firms are unusually labor intensive.

Early theoretical interest about the interaction between public and private firms begins in the 1980s (De Fraja and Delbono, 1989). This paper considers n private firms competing with one public firm producing homogenous goods with the same technology involving a fixed cost and increasing marginal costs (and no capacity constraint). Under Cournot-Nash competition and provided that the market is sufficiently competitive (n exceeds some threshold), they show that welfare is improved if the public firm maximises profit instead of welfare. This is a strong argument for privatisation when markets are sufficiently competitive.

The study of taxation in duopolistic settings has drawn much attention recently. However, there have been very few analyses of taxation in the context of mixed duopolies, where private and public firms coexist in the same market and maximise different objective functions.

Also, taxation incidence is concerned with the effect of taxation upon prices and profits. Since perfectly competitive firms earn zero profits, under perfect competition there is only a price effect. However, under imperfect competition, there are both price and profit effects. Since prices are set above marginal cost, an increase in cost due to a change in taxation need not be reflected in an identical increase in price. In addition, in most countries, it is common observe 'mixed oligopolies’ in which state-owned public firms compete against private firms. For example, public and private firms coexist in industries such as public utilities, telecommunication, transportation, postal service, banking, insurance, housing, education, and health care. In these mixed markets, public and private firms compete not only in price or quantity, but in the quality of their services or goods. Although the number of studies on mixed markets has increased in recent years, quality competition in mixed oligopoly has received little interest in the literature. The purpose of this paper is to compare the effects of indirect taxation in different types of mixed-duopoly markets.

There are some previous theoretical and empirical papers on efficiency comparison of unit taxation and ad valorem taxation in mixed duopoly markets, or a closely related subject. It has been argued by De Fraja (2009) that when private and public firms interact, differences in objective function between them will lead to differences in observed dimensions of performance, often in unexpected and counterintuitive ways. Interaction between private and public entities is hugely important and it will be clearly a fundamental feature of developing economies for the foreseeable future.[1]

The objective maximisation problem of the private firm is different from the public firm and partially privatised firm. The private firm maximises profits, while the privatised firm takes both profits and social welfare into consideration as has been examined by Matsumura (1998). Matsumura shows that neither full privatisation (the government does not hold any shares) nor full nationalisation (the government holds all of the shares) is optimal under moderate conditions. As argued above, we can justify the viewpoint that the objective maximisation problem of the public firm is to promote the social welfare, while that of the partial privatisation government is to secure both the social welfare and its attainment of tax revenues.

The paper that comes closest to the comparison of efficiency of ad valorem and unit taxes in imperfectly competitive market is well written by Anderson et al. (2001).[2] The authors provide a simple proof that ad valorem taxes are welfare-superior to unit taxes in the short run when production costs are identical across firms.[3] The proof covers differentiated products and a wide range of market product. Cost asymmetries strengthen the case for ad valorem taxation under Cournot competition, but unit taxation may be welfare-superior under Bertrand competition with product differentiation. Ad valorem taxation is superior with free entry under Cournot competition, but not necessarily under price competition when consumers value variety.

Blackorby and Murty (2007) shows that if a monopoly sector is imbedded in a general equilibrium framework and profits are taxed at one hundred percent, then unit (specific) taxation and ad valorem taxation are welfare-wise equivalent. Under certain regularity conditions, for any fixed vector of profit shares, Blackorby and Murty (2013) shows that the utility possibility sets of economies with unit and ad valorem taxes are not generally identical. But it does not imply that one completely dominates the other. Rather, the two utility possibility frontiers cross each other. Additionally, by employing a standard partial equilibrium welfare analysis, we show that the Marshallian social surpluses resulting from the two tax structures are identical when the government can implement unrestricted transfers.

Aiura and Ogawa (2013) examine the choice of tax method between ad valorem tax and unit tax within the framework of spatial tax competition with cross-border shopping, and they show that the ad valorem tax method is a good strategy to compete for mobile consumers and consider that this choice leads to inferior outcome. The welfare dominance of ad valorem taxes over unit taxes in a single-market Cournot oligopoly is well-known. Hennessy and Lapan (2011) extends the analysis to multi-market oligopoly. Kotsogiannis and Serfes (2011) show that privatisation affects both optimal subsidy rate and resulting welfare.

For more complicated situations, with respect to multiproduct transactions, Hamilton (1999) shows that the ad valorem taxation is better than unit taxation under standard conditions. Anderson et al. (2001) analyse that the working of the two taxations is in oligopoly with differentiated products and price competition. However, cost asymmetries will make unit taxation a better instrument when consumers have preference for variety, have perfectly inelastic demand functions, and oligopolistic firms engage in price competition.

One important contribution in our paper is to allow for partial privatisation. Indeed, the common feature of most existing work on mixed oligopoly is to assume that the public firms care only about welfare. However in many countries, public firms are partially privatised, which implies they also care about profit. A natural way to formalise this is to assume that the public firm maximises a (convex) combination of welfare and profit, and to represent the extent of privatisation as the weight the public firms put on profits.

The rest of this paper is organised as follows. We formulate a reasonable model in section 2. Then we discuss effects of two tax policies on output and tax revenue in terms of three different-type markets in sections 3 and 4. And we also make efficiency comparisons in terms of welfare implications for different markets in section 5. Section 6 concludes the paper.

2. Basic model

Our mixed duopoly model contains a private firm and a public firm competing in quantities (Cournot model), which means that each firm's relevant strategic consideration pays attention only to quantity instead of price. The two firms produce homogeneous goods. The private firm is called firm 1 and supplies the product at quantity q_1, and the public firm is called firm 2 and supplies the product at quantity q_2. They both produce in conditions of no fixed costs, and constant marginal and average costs, which are normalised to 0 for the private firm and to $c > 0$ for the public firm. The additional cost is a constant which reflects the idea that the public firm is less efficient (possible causes will be discussed later).

We assume a linear inverse demand function $P(Q) = 1 - Q$, where $Q = q_1 + q_2$ and denotes the aggregate output and P is the price. On the demand side of the market, the representative consumer's utility is then a quadratic function given by:

$$U(Q) = q_1 + q_2 - \frac{1}{2}(q_1 + q_2)^2$$

Under a taxation system, there is a unit taxation rate of t $(t>0)$ per unit of output, or an ad valorem taxation rate of τ $(0<\tau<1)$ fraction of gross revenue. Suppose both the unit taxation and the ad valorem taxation are imposed on the producers. So, the firms' profits are given by the following formula under the imposition of unit taxation:

$$\pi_1^u = (P-t)q_1, \;\; \pi_2^u = (P-c-t)q_2$$

and are given by the following formula under ad valorem taxation:

$$\pi_1^a = ((1-\tau)P)q_1, \;\; \pi_2^a = ((1-\tau)P-c)q_2$$

The objective of the private firm is to maximise its profits. And without privatisation, the public firm, in contrast, hopes to maximise social welfare (W), which is defined as the sum of consumer surplus (CS), producer surplus (PS) and tax revenue (R).

The public firm's output is the solution of the following problem under a unit taxation scheme:

$$\underset{q_2}{Max}(W_u) = \underset{q_2}{Max}(CS + PS_u + R_u)$$

and is the solution of the following problem under an ad valorem taxation scheme:

$$\underset{q_2}{Max}(W_a) = \underset{q_2}{Max}(CS + PS_a + R_a)$$

where R_u is unit tax revenue and R_a is ad valorem tax revenue.

Furthermore, if there is privatisation, a natural way to formalise this is to assume that the public firm maximises a convex combination of welfare and profit. So now the privatised firm's output is the solution of the following problem under unit taxation policy:

$$\underset{q_2}{Max}(U_u) = \underset{q_2}{Max}\left(\alpha W_u + (1-\alpha)\pi_2^u\right)$$

and is the solution of the following problem under ad valorem taxation policy:

$$\underset{q_2}{Max}(U_a) = \underset{q_2}{Max}\left(\alpha W_a + (1-\alpha)\pi_2^a\right)$$

where $0 \leq \alpha \leq 1$ is the weight of the payoff of the government for the firm's objective. All possible values of α are as follows:

1. If $\alpha = 0$, it corresponds to full privatisation (profit maximisation);
2. If $\alpha = 1$, it corresponds to full nationalisation (welfare maximisation) which is the same with the public firm 2;
3. If $0 < \alpha < 1$, the firm is partially privatised.

The parameter α captures the degree of privatisation. Partial privatisation is defined as $0 < \alpha < 1$ (pointed out by Matsumura (1998)). Moreover, we assume that the government can indirectly control α through its share-holding. If the shares owned by the government increase, then α increases.

3. Effects of unit and ad valorem taxation on output and tax revenue without privatisation

In this section we compare unit and ad valorem taxation in terms of output and tax revenue under the model we have built in Section 2. Since we consider the long-run equilibrium, the analysis allows for the firms' choices to depend on the tax policy. And we consider the situation with no privatisation in this section. We first derive expressions for the consumer surplus (CS), producer surplus (PS), and tax revenue (R). They can be written as:

1. $CS = \int_0^{q_1+q_2} P(x)dx - P(q_1+q_2) = U(Q) - P(q_1+q_2)$ where $U(Q) = q_1 + q_2 - \frac{1}{2}(q_1+q_2)^2$, $P = 1 - q_1 - q_2$, so $CS = q_1 + q_2 - \frac{1}{2}(q_1+q_2)^2 - P(q_1+q_2)$
2. $PS_u = \pi_1^u + \pi_2^u + FC = \pi_1^u + \pi_2^u$, $PS_a = \pi_1^a + \pi_2^a + FC = \pi_1^a + \pi_2^a$
3. $R_u = t(q_1+q_2)$, $R_a = \tau P(q_1+q_2)$
4. $W = CS + PS + R$

3.1. Effects of taxations on output

In this section, we present clear proof to show that both of the two tax policies do not alter the equilibrium total output, however, they will affect the firms' equilibrium outputs.

3.1.1. Unit taxation analysis

In this section we examine how unit taxation affects the equilibrium total output.

Proposition 1. *In a Cournot mixed-duopoly market with homogeneous goods, if there does not exist privatisation, the imposition of unit taxation does not affect equilibrium total output and market price, but an increase in the unit taxation will always reduce the output of the private firm and increase the output of the public firm, which means that with an increase in taxation the less efficient (public) firm gains market share over the more efficient (private) firm.*

Proof. Because the two firms both produce positive quantity outputs in Cournot Model, firm 1 chooses q_1 and firm 2 chooses q_2, so we can find the two firms' profits under unit tax policy are:

$$\pi_1^u = (P-t)q_1, \;\; \pi_2^u = (P-c-t)q_2.$$

With the imposition of unit taxation, the private firm's output is the solution of the following problem:

$$\underset{q_1}{Max}(\pi_1^u) = \underset{q_1}{Max}((P-t)q_1)$$

The public firm's output is the solution of the following problem:

$$\underset{q_2}{Max}(W_u) = \underset{q_2}{Max}(CS + PS_u + R_u) = \underset{q_2}{Max}\left(q_1 + q_2 - \frac{1}{2}(q_1 + q_2)^2 - cq_2\right)$$

In order to make sense of these two equations, it shall be noted that the optimum will require the output levels to solve the following first-order conditions:

$$\frac{\partial \pi_1^u}{\partial q_1} = 1 - 2q_1 - q_2 - t = 0 \tag{1}$$

$$\frac{\partial W_u}{\partial q_2} = 1 - (q_1 + q_2) - c = 0 \tag{2}$$

The equation (2) means that $Q = q_1 + q_2 = 1 - c$ is a constant (we have assumed that the additional cost c is constant). So the imposition of the unit taxation does not affect the reaction function. It is straightforward that the total output Q remains unchanged no matter whether the imposition of the unit taxation is increasing or decreasing.

Thus, by combining the equations (1) and (2), we can get the following unique Nash Equilibrium output levels:

$$q_1^{u*} = c - t \tag{3}$$

$$q_2^{u*} = 1 - 2c + t \tag{4}$$

So an increase in unit taxation will always decrease the equilibrium output of the more efficient private firm 1 (q_1^{u*}). And in equilibrium, the less efficient firm 2 produces more with the more imposition of unit tax. A parallel proof establishes the claim for the ad valorem case.

3.1.2. Ad valorem taxation analysis

The ad valorem taxation analysis is similar to the unit taxation analysis. Utilising the same process of calculation as in the previous case, we have the following proposition.

Proposition 2. *In a Cournot mixed duopoly with homogeneous goods and if there is not privatisation, the imposition of ad valorem taxation also does not change total output and market price, and the equilibrium outputs of the private firm and the public firm also remain unchanged.*

Proof. Firm 1 chooses q_1 and firm 2 chooses q_2, so we can derive the following profit functions for two firms:

$$\pi_1^a = (1-\tau)Pq_1, \pi_2^a = ((1-\tau)P-c)q_2$$

With the imposition of ad valorem taxation, the private firm's output is the solution of the following problem:

$$\underset{q_1}{Max}\left(\pi_1^a\right) = \underset{q_1}{Max}\left((1-\tau)Pq_1\right)$$

The public firm's output is the solution of the following problem:

$$\underset{q_2}{Max}(W_a) = \underset{q_2}{Max}(CS + PS_a + R_a) = \underset{q_2}{Max}\left(q_1 + q_2 - \frac{1}{2}(q_1 + q_2)^2 - cq_2\right)$$

The proof is similar to that in Subsection 3.1.1, and under an ad valorem tax, the first order conditions for profit maximisation are:

$$\frac{\partial \pi_1^a}{\partial q_1} = (1-\tau)(1-2q_1-q_2) = 0 \tag{5}$$

$$\frac{\partial W_a}{\partial q_2} = 1 - (q_1 + q_2) - c = 0 \tag{6}$$

Equation (6) means that $Q = q_1 + q_2 = 1 - c$ is a constant. So the imposition of the ad valorem taxation also does not alter the reaction function. It is straightforward that the total output Q is unchanged no matter whether the imposition of the ad valorem taxation is increasing or decreasing.

Solving the above first-order conditions of equations (5) and (6), we obtain the following unique Nash Equilibrium quantities:

$$q_1^{a*} = c \tag{7}$$

$$q_2^{a*} = 1 - 2c \tag{8}$$

So the imposition of the ad valorem taxation will not affect the equilibrium outputs of the private firm 1 (q_1^{a*}) and the public firm 2 (q_2^{a*}).

Above all, the proposition 2 is proved. Both proposition 1 and 2 have summarised the effects of two taxation policies on output with no privatisation market.

3.2. Effects of taxations on tax revenue

The next proposition provides sufficient conditions to show that both taxation policies have positive effects on tax revenue.

Proposition 3. *In the mixed duopoly market with homogeneous goods, and if there does not exist privatisation, an increase in taxation (either unit or ad valorem) can always increase the tax revenue.*

Proof. We can also derive the expression of unit tax revenue from equations (7) and (8):

$$R_u = t\left(q_1^{u*} + q_2^{u*}\right) = t(1-c) \tag{9}$$

where $t > 0$.
Differentiating $R_u$with respect to t and making use of equation (9), we find that:

$$\frac{\partial R_u}{\partial t} = 1 - c > 0.$$

This result shows that R_u is always increasing in t, which implies that the imposition of unit taxation will always increase tax revenue.

Similarly, we can also derive the expression of ad valorem tax revenue:

$$R_a = \tau P\left(q_1^{a*} + q_2^{a*}\right) = \tau c(1-c) \tag{10}$$

From the expression of this tax revenue, we can calculate that

$$\frac{\partial R_a}{\partial \tau} = c(1-c) > 0.$$

So R_a is also increasing in τ, it means that the imposition of the ad valorem taxation also will always increase the tax revenue. So, an increase in either unit taxation or ad valorem taxation always increases tax revenue.

Thus, Proposition 3 is proved.

It is interesting to observe that in a corresponding quantity setting Cournot oligopoly model instead of mixed-duopoly model, an increase in ad valorem taxation will always decrease the total output and increase the market share of a more efficient firm. In contrast, in our mixed-duopoly model, total output will not be altered but the market share of the less efficient public firm rises if there is no privatisation. Moreover, the tax increase (either unit or ad valorem tax) always raises total tax revenue in our mixed-duopoly market. In contrast, in a Cournot oligopoly model it may fall if the initial taxation level is sufficiently high.

In the next section, we will compare same thing but consider privatisation in the market.

4. Effects of unit and ad valorem taxation on output and tax revenue with privatisation

In this section, we consider a much more complicated and practical problem—we allow privatisation to be involved in the market. According to all possible values of α, we will discuss two situations: full privatisation when $\alpha = 0$, and partial privatisation when $0< \alpha <1$. And either market type will lower the equilibrium total output under the Cournot Model. In addition, efficiency for tax revenue, which is more complicated, is also examined. The following proposition shows our results.

Proposition 4. *In the Cournot mixed-duopoly market with homogeneous goods,*

i. *Full privatisation ($\alpha = 0$) leads to a decrease in equilibrium total output with the imposition of either unit taxation or ad valorem taxation. Tax revenue will increase if the tax is fixed by an ad valorem tax rate, but it is indeterminate if the tax is fixed by a unit tax rate.*
ii. *Partial privatisation ($0 < \alpha < 1$) also leads to a decreasing equilibrium total output whether the tax is fixed by unit tax rate or by ad valorem tax rate. Tax revenue will increase if the tax is fixed by an ad valorem tax rate, but it is indeterminate if the tax is fixed by a unit tax rate.*

Proof. Based on the assumption, we know that the privatised firm's output is the solution of the following problem under the imposition of unit taxation:

$$\underset{q_2}{Max}(U_u) = \underset{q_2}{Max}\left(\alpha W_u + (1-\alpha)\pi_2^u\right)$$

with ad valorem taxation policy, the objective function is given by:

$$\underset{q_2}{Max}(U_a) = \underset{q_2}{Max}\left(\alpha W_a + (1-\alpha)\pi_2^a\right)$$

From the assumption we know that $0 \leq \alpha \leq 1$ is the weight of the payoff of the government for the firm's objection.

1. If $\alpha = 0$, it corresponds to full privatisation (profit maximisation).
2. If $\alpha = 1$, it corresponds to full nationalisation (welfare maximisation) which is the same total as with the public firm 2.
3. If $0 < \alpha < 1$, it means that the firm is partial privatised.

Since we have discussed situations of $\alpha = 1$ (full nationalisation) in previous sections, so we can just discuss the two situations of $\alpha = 0$ and $0 < \alpha < 1$ in this section.

4.1. Full privatisation ($\alpha = 0$)

Now the market changes to a privatised-duopoly market instead of mixed-duopoly. But in order to compare without loss of generality, we still have to consider this market.

4.1.1. Unit taxation analysis

The private firm's output is still the solution of the following problem:

$$\underset{q_1}{Max}\left(\pi_1^u\right) = \underset{q_1}{Max}\left((P - t)q_1\right)$$

The fully privatised firm's output is the solution of the following problem under the imposition of unit taxation:

$$\underset{q_2}{Max}\left(\pi_2^u\right) = \underset{q_2}{Max}\left((P - c - t)q_2\right)$$

Then we can find the Cournot Equilibrium from the first order conditions. And we assume that all firms produce a positive quantity outputs in the Cournot Model, so solving for an interior solution and denoting the equilibrium output levels by q_1^{u*} and q_2^{u*}, then we have

$$q_1^{u*} = \frac{1 + c - t}{3} \tag{11}$$

$$q_2^{u*} = \frac{1 - 2c - t}{3} \tag{12}$$

While the corresponding equilibrium total output and market price are:

$$Q^{u*} = q_1^{u*} + q_2^{u*} = \frac{2 - c - 2t}{3} \tag{13}$$

$$P^{u*} = 1 - q_1^{u*} - q_2^{u*} = \frac{1 + c + 2t}{3} \tag{14}$$

Straightforward computation yields that an increase in the unit tax rate will always decrease q_1^{u*}, q_2^{u*} and Q^{u*} from the expressions of q_1^{u*}, q_2^{u*} and Q^{u*}.

Based on these equilibrium outputs, we can also derive the expression of unit tax revenue from equations (11) and (12):

$$R_u = t\left(q_1^{u*} + q_2^{u*}\right) = t\frac{2 - c - 2t}{3} \tag{15}$$

where $t > 0$ and c is constant.

From equations (11) and (12) we can derive that $\frac{\partial R_u}{\partial t} = -\frac{4t+c-2}{3} > -\frac{2t}{3}$ (for $2 - c > 2t$), which can be positive, zero, or negative. So R_u's relationship with t is indeterminate.

4.1.2. Ad valorem taxation analysis

The private firm's output is still the solution of the following problem:

$$\underset{q_1}{Max}\left(\pi_1^a\right) = \underset{q_1}{Max}\left((1-\tau)Pq_1\right)$$

The fully privatised firm's output is the solution of the following problem under the imposition of ad valorem taxation:

$$\underset{q_2}{Max}\left(\pi_2^a\right) = \underset{q_2}{Max}\left(((1-\tau)P - c)q_2\right)$$

Then we can find the Cournot Equilibrium from the first order conditions. And we assume that all firms produce a positive quantity output in Cournot Model, so solving for an interior solution and denoting the equilibrium output levels by q_1^{a*} and q_2^{a*}, then we can obtain

$$q_1^{a*} = \frac{1-\tau+c}{3(1-\tau)} \tag{16}$$

$$q_2^{a*} = \frac{1-\tau-2c}{3(1-\tau)} \tag{17}$$

Then we can calculate the equilibrium total output and market price:

$$Q^{a*} = q_1^{a*} + q_2^{a*} = \frac{2-2\tau-c}{3(1-\tau)} \tag{18}$$

$$P^{a*} = 1 - q_1^{a*} - q_2^{a*} = \frac{c+1-\tau}{3(1-\tau)} \tag{19}$$

According to the previous assumption, we can conclude that

$$\frac{\partial q_1^{a*}}{\partial \tau} = \frac{c}{3(\tau-1)^2} > 0$$

We can similarly derive that

$$\frac{\partial q_2^{a*}}{\partial \tau} = -\frac{2c}{3(\tau-1)^2} < 0$$

$$\frac{\partial Q^{a*}}{\partial \tau} = -\frac{c}{3(\tau-1)^2} < 0$$

So we conclude that an imposition of the increase in the ad valorem taxation will always increase q_1^{a*}, but decrease q_2^{a*} and Q^{a*}.

We can also derive the expression of ad valorem tax revenue from equations (18) and (19):

$$R_a = \tau P^{a*}\left(q_1^{a*} + q_2^{a*}\right) = \frac{1}{9}\frac{\tau(2-2\tau-c)(1-\tau+c)}{(\tau-1)^2} \tag{20}$$

The equation (20) implies that

$$\frac{\partial R_a}{\partial \tau} = \frac{-2\tau^3 + 6\tau^2 - \tau c^2 - \tau c - 6\tau - c^2 + c + 2}{9(1-\tau)^3} = \frac{(1-\tau)\left[c + 2(1-\tau)^2\right] - (\tau+1)c^2}{9(1-\tau)^3}$$
$$> \frac{2c\left[c + 2(1-\tau)^2\right] - \tau c^2 - c^2}{9(1-\tau)^3} = \frac{4c(1-\tau)^2 + c^2(1-\tau)}{9(1-\tau)^3} > 0$$

so R_a is increasing in τ, it means that the imposition of ad valorem taxation will always increase the tax revenue.

4.2. Partial privatisation (0< α < 1)

4.2.1. Unit taxation analysis

The private firm's output is still the solution of the following problem:

$$\underset{q_1}{Max}\left(\pi_1^u\right) = \underset{q_1}{Max}\left((P-t)q_1\right)$$

The privatised firm's output is the solution of the following problem under the imposition of unit taxation:

$$\underset{q_2}{Max}(U_u) = \underset{q_2}{Max}\left(\alpha W_u + (1-\alpha)\pi_2^u\right)$$

Now we can derive the expression of U_u:

$$U_u = \alpha W_u + (1-\alpha)\pi_2^u = \alpha\left(q_1 + q_2 - \frac{1}{2}(q_1+q_2)^2 - cq_2\right) + (1-\alpha)(P-c-t)q_2$$

The first-order conditions for profit maximisation are:

$$\frac{\partial \pi_1^u}{\partial q_1} = 1 - 2q_1 - q_2 - t = 0$$

$$\frac{\partial U_u}{\partial q_2} = 1 - q_1 + (\alpha-2)q_2 - c + (\alpha-1)t = 0$$

so the Nash equilibrium output levels are given by:

$$q_1^{u*} = \frac{1-\alpha-t+c}{3-2\alpha} \tag{21}$$

$$q_2^{u*} = \frac{2\alpha t-2c-t+1}{3-2\alpha} \tag{22}$$

$$Q^{u*} = q_1^{u*} + q_2^{u*} = \frac{2\alpha t-\alpha-2t-c+2}{3-2\alpha} \tag{23}$$

Since $0< \alpha < 1$, so $(3—2\alpha)>0$ and $\frac{\partial q_1^{u*}}{\partial t} = \frac{-1}{3-2\alpha} < 0$ which means that the equilibrium output of the private firm is decreasing in t.

According the range of α, we know that $\frac{\partial q_2^{u*}}{\partial t} = \frac{2\alpha-1}{3-2\alpha}$ can be negative, positive or zero. It means that the sign of $\frac{\partial q_2^{u*}}{\partial t}$ is not determinate.

We can derive $\frac{\partial Q^{u*}}{\partial t} = \frac{2\alpha-2}{3-2\alpha} < 0$, which means that the equilibrium total output is decreasing in t.

We can also derive the expression of unit tax revenue from equations (21) and (22):

$$R_u = t\left(q_1^{u*} + q_2^{u*}\right) = t\frac{2\alpha t-\alpha-2t-c+2}{3-2\alpha} \tag{24}$$

where $t > 0$ and c is constant.

Straightforward calculation implies that (according to equations (21) and (22), $2 - c - \alpha > 2t - 2\alpha t$):

$$\frac{\partial R_u}{\partial t} = \frac{4\alpha t-4t-c-\alpha+2}{3-2\alpha} > \frac{(2\alpha-2)t}{3-2\alpha}$$

and $(2\alpha-2)t \subset (2t, 0)$. So the sign of $\frac{\partial R_u}{\partial t}$ is not determinate, which means that the effect of unit taxation on tax revenue is indeterminate (increasing, unchanged or decreasing are both possible).

4.2.2. Ad valorem taxation analysis

The private firm's output is still the solution of the following problem:

$$\underset{q_1}{Max}\left(\pi_1^a\right) = \underset{q_1}{Max}\left((1-\tau)Pq_1\right)$$

The partially privatised firm's objective maximisation function is:

$$\underset{q_2}{Max}(U_a) = \underset{q_2}{Max}\left(\alpha W_a + (1-\alpha)\pi_2^a\right)$$

Now we can derive the expression of U_a:

$$\begin{aligned} U_a = {} & \alpha W_a + (1-\alpha)\pi_2^a \\ & = \alpha\left(q_1 + q_2 - \frac{1}{2}(q_1+q_2)^2 - cq_2\right) + (1-\alpha)((1-\tau)P - c)q_2 \end{aligned}$$

The first-order conditions for profit maximisation are:

$$\frac{\partial \pi_1^a}{\partial q_1} = (1-\tau)(1-2q_1-q_2) = 0$$

$$\frac{\partial U_a}{\partial q_2} = \tau(\alpha + q_1 + 2q_2-\alpha q_1-2\alpha q_2-1) + \alpha q_2 - q_1 - 2q_2 + 1 = 0$$

Solving these Nash equilibrium conditions, the output levels are:

$$q_1^{a*} = \frac{\alpha\tau-\alpha + c-\tau + 1}{\alpha + 3(1-\alpha)(1-\tau)} \tag{25}$$

$$q_2^{a*} = \frac{\alpha\tau-2c-\tau + 1}{\alpha + 3(1-\alpha)(1-\tau)} \tag{26}$$

$$Q^{a*} = q_1^{a*} + q_2^{a*} = \frac{2\alpha\tau-\alpha-c-2\tau + 2}{\alpha + 3(1-\alpha)(1-\tau)} \tag{27}$$

From equations (25) to (27) we can show that

$$\frac{\partial q_1^{a*}}{\partial \tau} = \frac{(1-\alpha)(3c-\alpha)}{(3\alpha\tau-2\alpha-3\tau + 3)^2} > 0$$

$$\frac{\partial q_2^{a*}}{\partial \tau} = \frac{-2(1-\alpha)(3c-\alpha)}{(3\alpha\tau-2\alpha-3\tau + 3)^2} < 0$$

$$\frac{\partial Q^{a*}}{\partial \tau} = \frac{-(1-\alpha)(3c-\alpha)}{(3\alpha\tau-2\alpha-3\tau + 3)^2} < 0$$

so q_1^{a*} is increasing in τ, but q_2^{a*} and Q^{a*} are both decreasing in τ (the inequality $3c - \alpha > 0$ ensures that two taxation aren't negative).

According to equations (25) to (27), we can conclude that $1-Q^{a*} = q_1^{a*}$ and $1 - 2q_1^{a*} = q_2^{a*}$, so we can also derive the expression of ad valorem tax revenue:

$$R_a = \tau Q^{a*}(1-Q^{a*}) = \frac{\tau(2\alpha\tau-\alpha-c-2\tau + 2)(\alpha\tau-\alpha + c-\tau + 1)}{(3\alpha\tau-2\alpha-3\tau + 3)^2} = \tau\left(1-q_1^{a*}\right)q_1^{a*}$$

Thus, we can obtain

$$\frac{\partial R_a}{\partial \tau} = \left(1-q_1^{a*}\right)q_1^{a*} + \tau\left(\frac{\partial q_1^{a*}}{\partial \tau}-2q_1^{a*}\frac{\partial q_1^{a*}}{\partial \tau}\right) = \left(1-q_1^{a*}\right)q_1^{a*} + \tau q_2^{a*}\frac{\partial q_1^{a*}}{\partial \tau} > 0,$$

so tax revenue is increasing in the ad valorem tax.

Table 1. Tax efficiency comparison on equilibrium output and tax revenue.

	Full nationalisation in Cournot mixed-duopoly ($\alpha = 1$)		Full privatisation in Cournot privatised-duopoly ($\alpha = 0$)		Partial privatisation in Cournot mixed-duopoly ($0 < \alpha < 1$)	
	Unit taxation	Ad valorem taxation	Unit taxation	Ad valorem taxation	Unit taxation	Ad valorem taxation
Total output	Unchanged	Unchanged	Decreasing	Decreasing	Decreasing	Decreasing
Equilibrium output of a private firm	Decreasing	Unchanged	Decreasing	Increasing	Indeterminate	Increasing
Equilibrium output of a public firm	Increasing	Unchanged	Decreasing	Decreasing	Decreasing	Decreasing
Tax revenue	Increasing	Increasing	Indeterminate	Increasing	Indeterminate	Increasing

4.3. Comparisons of indirect taxation and market type

In this section, if the type of taxation is fixed and the assumption holds, then we show in the following Table 1 a comparison of the effects of taxes in terms of equilibrium output, tax revenue, and total surplus in different models by combining results given from previous sections. This table shows the most obvious explanations and results.

5. Efficiency comparison of unit and ad valorem taxation

This section considers the relative efficiency of unit taxation and ad valorem taxation in terms of welfare implications in different imperfectly competitive markets. The most important observation here is to suppose these two taxations can produce the same total output in equilibrium, which is also used by Anderson et al. (2001). Then we can show that ad valorem taxation is always welfare superior to unit taxation under full privatisation market ($\alpha = 0$), full nationalisation market ($\alpha = 1$), and the partial privatisation market ($0 < \alpha < 1$). The following propositions extend the efficiency result in Proposition 1 to 4 for Cournot in the long run. The following proofs also depict these results.

Proposition 5. *For full nationalisation ($\alpha = 1$), it always can produce equal equilibrium total output in the two different taxation schemes. Assuming that these two different taxation returns are equal under Cournot competition with homogeneous goods in a mixed duopoly market, then we can get proof that ad valorem taxation is welfare superior to unit taxation.*

Proof. From the assumption that these two different taxation returns are equal, we can derive that $t = \tau c$.

Now, under two taxation schemes, we have $W_a - W_u = c^2\tau > 0$, which implies that ad valorem taxation is welfare superior.

Proposition 6. *Considering Cournot competition with homogeneous goods in a mixed duopoly market, and assuming that these two different taxation schemes are selected such that they can produce equal equilibrium total output, so we can get* *that* $t = \frac{c\tau}{2(1-\tau)}$. *Under this condition, the ad valorem taxation is welfare superior to the unit taxation for full privatisation* ($\alpha = 0$).

Proof. When the condition $t = \frac{c\tau}{2(1-\tau)}$ is met, we can get equal equilibrium total output for this market type under the two different tax schemes. Firstly, we compare tax revenue.

For full privatisation ($\alpha = 0$), given $t = \frac{c\tau}{2(1-\tau)}$ we get that:

$$Q^{u*} = Q^{a*} = \frac{2}{3} - \frac{c}{3(1-\tau)}$$

The tax revenue difference is: $R_a - R_u = \frac{\tau(2-2\tau-c)^2}{18(1-\tau)^2}$ which is strictly positive.

Next, we calculate the social welfare differences under two kinds of tax. According to the previous results, we can get that $W_a - W_u = \frac{\tau c^2}{2(1-\tau)} > 0$. So we can conclude that the ad valorem is also welfare superior in this situation.

Proposition 7. *Considering Cournot competition with homogeneous goods in a mixed duopoly market, and assuming that these two different taxation schemes are selected such that they can produce equal equilibrium total output, so we can get that*

$$t = \frac{\tau}{2}\frac{3c-\alpha}{3\alpha\tau - 2\alpha - 3\tau + 3}.$$

Under this condition, the ad valorem taxation is welfare superior to the unit taxation for the partial privatisation ($0< \alpha < 1$). *(The implied condition here is* $\alpha < 3c$*, otherwise we will have a negative unit taxation)*

Proof. For partial privatisation ($0< \alpha < 1$), we derive that:

$$Q^{u*} = Q^{a*} = \frac{2\alpha\tau + \alpha c_1 - \alpha - 2\tau - c_1 - c_2 + 2}{3\alpha\tau - 2\alpha - 3\tau + 3}$$

The tax revenue difference is:

$$R_a - R_u = \frac{\tau(2\alpha\tau - \alpha - 2\tau - c + 2)^2}{2(3\alpha\tau - 2\alpha - 3\tau + 3)^2} > 0,$$

which indicate that the ad valorem taxation return is still higher .

At this time, the difference of social welfare is

$$W_a - W_u = \frac{\tau c(3c-\alpha)}{2(3\alpha\tau - 2\alpha - 3\tau + 3)}.$$

The denominator is always positive, while $(3c-\alpha)$ generates a positive sign in the numerator, so $W_a - W_u > 0$, which indicates that the ad valorem is also welfare superior in this situation.

From the proofs of Propositions 5 to 7, we can conclude that ad valorem taxation is always welfare superior to unit taxation under full nationalisation, full privatisation, and the partial privatisation markets.

6. Concluding remarks

In this paper we have investigated a quantity-setting duopoly involving a private firm and a public firm with homogeneous goods and with different values of the weight of the payoff of the government for the firm's objective. The analysis of the effects of unit taxation and ad valorem taxation in a mixed oligopoly (full nationalisation or partially privatised) yields some significantly different results in comparison to those conclusions obtained in a corresponding Cournot oligopoly.

Firstly, in full nationalisation in a Cournot mixed-duopoly ($\alpha = 1$): total output is unaffected by the imposition of or change in either taxation. However, with an increase in unit taxation, the less efficient (public) firm gains market share over the more efficient (private) firm. But in the case of ad valorem, the market share of private firm and public firm will remain unchanged. In addition, tax revenue always rises with a tax increase in either taxation. Either full privatisation ($\alpha = 0$) or partial privatisation ($0< \alpha < 1$) in a Cournot privatised-duopoly will reduce the equilibrium total output when either tax is levied.

In addition, by introducing taxes in a mixed duopoly model we show that privatisation lowers total output and equilibrium output of less efficient firms for either a fixed level of unit tax or a fixed ad valorem tax level. And the effect of both types of privatisation on tax revenue are indeterminate with a levy of unit taxation. But privatisation can increase the tax revenue with the imposition of ad valorem taxation.

Moreover, we use a straightforward approach that provides a clean proof of the unit and ad valorem efficiency claim. And we extend this result to the market involving privatisation. We examine that ad valorem taxation is always welfare superior to unit taxation for a full privatisation market ($\alpha = 0$), a full nationalisation market ($\alpha = 1$) and the partial privatisation market ($0< \alpha < 1$) in our paper.

One important contribution of our paper is to allow for partial privatisation. Our research has certain practical value. The central and western regions in China are relatively concentrated areas of crude oil, natural gas, and other resources, and yet are underdeveloped. A tax practice of resource tax in the form ad valorem duty combined with partial privatisation will, in our view, not only protect resources and prevent excessive development, but also can increase local fiscal revenue, enhance the level of social welfare and ensure the regional coordination and sustainable development.

On the side, the framework in this paper can be readily adapted to incorporate other effects of economic interest, such as differences across firms with the objective of producing differentiated goods. Anderson et al. (1992) have extended the differentiated products model to consider multiproduct firms, and tax issues may be addressed within that extended context. Our model can be extended to international trade to study other economic problems such as optimal tariffs, and the partial equilibrium setting will ideally be extended to the case of general equilibrium in order to study the taxation of goods in conjunction with the taxation of income and capital. Furthermore, we do not allow the Bertrand-Equilibria in which the results may have better comparison. Thus, extending our model to these directions remains for future research.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes

1. For recent researches on mixed oligopoly, see, for example, Nie (2014) and Tao et al. (2013).
2. Mujumdar and Pal (1998) show that ad valorem and specific taxes in a mixed oligopoly yield some significantly different results in comparison to those obtained in a corresponding Cournot oligopoly. Other papers comparing ad valorem and unit taxes include Aiura and Ogawa (2013), Hennessy and Lapan (2011), and Kotsogiannis and Serfes (2011).
3. By studying a differentiated goods oligopoly with asymmetric costs, Wang and Zhao (2009) show that unit taxation can be welfare superior to ad valorem taxation if the goods are sufficiently differentiated, the cost variance is sufficiently large, and the ad valorem tax rate is sufficiently high. Hsu and Wang (2011) note that, in a homogenous good oligopoly, if firms have different costs then the least efficient firms are more likely to be inactive under ad valorem taxation than under unit taxation, making it possible for unit taxation to be more efficient than ad valorem taxation.

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References

Aiura, H., & Ogawa, H. (2013). Unit tax versus ad valorem tax: A tax competition model with cross-border shopping. *Journal of Public Economics*, *105*, 30–38.

Amiram, D., Bauer, A., & Frank, M. M. (2013). Corporate tax avoidance and managerial incentives generated by shareholder dividend tax policy. Working Paper. Retrieved from https://pdfs.semanticscholar.org/e047/434bc5ac54659acdf0f6a0b9a6fd11d99005.pdf

Anderson, S., & de Palma, A. (1992). Multiple product firms: a nested logit approach. *Journal of Industrial Economics*, *40*(3), 261–276.

Anderson, S., de Palma, A., & Kreider, B. (2001). The efficiency of indirect taxes under imperfect competition. *Journal of Public Economics*, *81*(2), 231–251.

Blackorby, C., & Murty, S. (2007). Unit versus ad valorem taxes: Monopoly in general equilibrium. *Journal of Public Economics*, *91*(3–4), 817–822.

Blackorby, C., & Murty, S. (2013). Unit versus ad valorem taxes: The private ownership of monopoly in general equilibrium. *Journal of Public Economic Theory*, *15*(4), 547–579.

De Fraja, G. (2009). Mixed oligopoly: Old and new. In The Pros and Cons of Competition in the Public Sector (pp. 1–8). Swedish Competition Authority. Retrieved from http://www.konkurrensverket.se/globalassets/forskning/lankar-forslag-pa-uppsatsamne/the-pros-and-cons-of-competition-in-by-the-public-sector.pdf

De Fraja, G., & Delbono, F. (1989). Alternative strategies of a public enterprise in oligopoly. *Oxford Economic Papers*, *41*(1), 302–311.
Gordon, R. (2001). Taxes and privatization. CEPR DP2977.
Hamilton, S. (1999). Tax incidence under oligopoly: A comparison of policy approaches. *Journal of Public Economics*, *71*(2), 233–245.
Hennessy, D., & Lapan, H. E. (2011). Unit versus ad valorem taxes in multiproduct cournot oligopoly. *Journal of Public Economic Theory*, *13*(1), 125–138.
Hsu, J., & Wang, X. H. (2011). A note on the efficiency of indirect taxes in an asymmetric cournot oligopoly. *Review of Economics & Finance*, *1*, 53–57.
Koethenbuerger, M., & Stimmelmayr, M. (2014). Corporate deductibility provisions and managerial incentives. *Journal of Public Economics*, *111*(2), 120–130.
Kotsogiannis, C., & Serfes, K. (2011). The comparison of ad valorem and specific taxation under uncertainty. *Journal of Public Economic Theory*, *16*(1), 1–18.
Matsumura, T. (1998). Partial privatization in mixed duopoly. *Journal of Public Economics*, *70*, 475–476.
Mujumdar, S., & Pal, D. (1998). Effects of indirect taxation in a mixed oligopoly. *Economics Letters*, *58*(2), 199–204.
Nie, P.-Y. (2014). Effects of capacity constraints on mixed duopoly. *Journal of Economics*, *112*(3), 283–294.
Tao, A., Zhu, Y., & Zou, X. (2013). Welfare comparison of leader-follower models in a mixed duopoly. *Journal of Applied Mathematics*, *2013*, 320712.
Wang, X. H., & Zhao, J. (2009). On the efficiency of indirect taxes in differentiated oligopolies with asymmetric costs. *Journal of Economics*, *96*(3), 223–239.