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Strategic R&D risk choices of public and private firms

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ABSTRACT

This paper investigates the R&D (research and development) risk choices of private and public firms in a product differentiated mixed duopoly market. Using the canonical models of R&D risk choice in a mixed market, it compares market performances between Cournot and Bertrand. The main findings are (i) public firm always engages in higher R&D risks than private firm under Cournot, (ii) public firm mostly chooses higher R&D risks, but may choose lower R&D risks than private firm if the degree of product substitution is sufficiently large under Bertrand, (iii) both public and private firms are more willing to take R&D risks under Bertrand than under Cournot, and (iv) from the perspective of social welfare, private firm always assumes too low R&D risks under Cournot. However, it takes excessive risks if the degree of product substitution is large enough under Bertrand.

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1. Introduction

In most (developing as well as developed) countries, state-owned public firms and private firms coexist and compete against each other in industries such as telecommunication, postal service, transportation, automobile, steel, television, banking, housing, health care, insurance, education and so on (Matsumura & Tomaru, 2015). In these mixed industries, the private and public firms not only compete in quantity (or price), but also compete in product (or process) R&D (research and development). For example, in many countries the public and private firms have fierce R&D competition in the health care, medicine, biotechnology and energy industries.[1] Although in recent years the literature on mixed oligopoly is increasing, R&D competition in mixed industries has not attracted enough attention (Ishibashi & Kaneko, 2008).[2] This clearly contrasts with the key role of public firms in facilitating innovation and the development of national innovation systems.[3] The optimal choice of R&D expenditure has been extensively analysed in the R&D literature (d'Aspremont & Jacquemin, 1988; Boone, 2001; Whalley, 2011; Chen, Nie, & Wang, 2015; Menezes & Pereira, 2017; and so on). However, from the point view of firm managers, the

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decision may be not just how much to spend on R&D but also how to spend it.[4] In some cases, the main decision may be to choose among R&D strategies with different degrees of risk (Anderson & Cabral, 2007). Evidence from the telecommunication, integrated circuit and semiconductor industries in China (or the energy, airlines, rail and steel industries in many EU (European Union) countries), the public firms are willing to focus on basic research and the most private firms prefer to invest in application research but not basic research. As is known, unlike application research, basic research has high risk of failure (Nie & Yang, 2015; Xing, 2018). That is, the public firms are more willing to choose high-risk R&D programmes in these industries. However, the opposite may be true in some other industries.[5] This study examines strategic R&D risk choice when the private and public firms compete in differentiated industries. The purpose of this paper is threefold. First, the author aims at comparing the R&D incentives of private and public firms. Second, the author's objective is to investigate market performances between Cournot and Bertrand competition. The last goal is to analyse whether private optimality meets the requirements of social optimum.

Consider a two-stage game in the context of a mixed duopoly model. In the first stage, the private and public firms compete in product R&D. The aim of R&D is to increase market demand. In this stage, given a series of R&D programmes with different risk levels but an identical expected outcome, the firms choose the type of their R&D programmes (i.e., determine the R&D risk level). In the second stage, the private and public firms choose quantities (or prices) and compete in product market. It is assumed that the firms produce differentiated goods and the public (private) firm determines its choice variables so as to maximise the social welfare (the profits). This study mainly finds that: (i) in equilibrium the R&D risk level of the public firm is always higher than that of the private firm under Cournot competition. However, the degree of product substitution might reverse the R&D risk choices between public and private firms under Bertrand competition; (ii) the equilibrium R&D risk level of both public and private firms under Cournot competition is lower than under Bertrand competition; (iii) for the private firm, the private optimum of R&D risk is always lower than the social optimum under Cournot competition, but it is higher than the social optimum if the degree of product substitution is sufficiently large under Bertrand competition.

The remainder of this paper is organised as follows. Section 2 gives a brief literature review and Section 3 describes the basic model. Section 4 solves and then compares (Section 5) the public and private firms' R&D risk choices and the private and social optimum of R&D risk under Cournot (Bertrand) competition. Section 6 makes a comparison of R&D risk level under different types of market competition. The final section presents conclusions.

2. Literature review

In recent years, strategic R&D competition between private and public firms in mixed oligopoly has become an increasingly active field of interest (Zikos, 2007). Some scholars compare R&D spending of public and private firms, but their views are not uniform. Nett (1994) thinks that the private firm is more innovative than the public firm, while Poyago-Theotoky (1998) and Nie and Yang (2015) find that the public firm invests more into R&D than the private firm. Privatisation is a popular academic and realistic policy in mixed oligopolies. Several contributors have attempted to examine the relationship between privatisation and the R&D investment. Cato (2011) holds that privatisation reduces (increases) the cost-reducing investment if the market demand is sufficiently large (small); Heywood and Ye (2009) think that the optimal extent of privatisation is reduced due to the mixed duopolies conduct more R&D; Zhang (2015) shows that the R&D investment of the public (private) firm decreases (increases) with the degree of privatisation. Moreover, Buehler and Wey (2010, 2014) examine the effect of the public R&D on the private R&D in a mixed duopoly and derive sufficient conditions for the public investment to crow out the private investment. It is worth noting that the aforementioned studies mainly consider R&D expenditures in mixed oligopoly. For firm managers, another important decision concerns the risk associated with the R&D programmes.[6] Determination of the optimal R&D risk has received considerable attention in the context of private duopoly.[7] However, the issue of R&D risk has not received much attention in the mixed configuration. In this study, the author focuses on strategic risk choice of R&D in a mixed duopoly market. The novelty of this paper is as follows. First, the author compares the optimal R&D risk choice of public and private firms under both Bertrand and Cournot competition. The author shows that under Cournot competition the public firm always engages in higher R&D risks than the private firm, whereas under Bertrand competition the degree of product substitution might reverse the R&D risk choices between public and private firms. Second, the author examines the effect of competition modes on the optimal R&D risk choice and finds that under Bertrand competition both public and private firms are more willing to take R&D risks than under Cournot competition.[8] Third, the author compares the private optimum and the social optimum, and gives that relative to the social optimum, under Cournot competition the private firm always assumes too low R&D risks, whereas under Bertrand competition it takes excessive risks if the degree of product substitution is large enough.

3. The basic model

Consider an industry in which a private firm competes with a state-owned public firm.[9] The private firm is assumed to be interested in maximising profit. In contrast, the public firm is assumed to maximise social welfare.[10] The author denotes with subscript 1 the private firm and denotes with subscript 0 the public firm. Following Singh and Vives (1984), the author describes the representative consumer utility function as follows:

$$U(q_0, q_1) = a_0 q_0 + a_1 q_1 - \frac{1}{2}\left(q_0^2 + 2d q_0 q_1 + q_1^2\right) \tag{1}$$

In equation (1), q_i denotes the quantity of firm i's production ($i = 0, 1$), a_i ($a_i > 0$) is a positive parameter and d represents the degree of product substitutability. Note that, lower value of d corresponds to lower (higher) degree of product substitution (differentiation). The author assumes that $0<d<1$, which excludes the cases that the firms' products are perfect substitutes or completely unrelated.[11]

Given the utility function of representative consumer in equation (1), the resulting inverse demand function for product i is linear and given by:[12]

$$p_i = a_i - q_i - dq_j, \quad i, j = 0, 1, i \neq j \tag{2}$$

In equation (2), p_i is the price of product i. Thus, the corresponding direct demand function for product i can be written in the form:

$$q_i = \frac{1}{1-d^2}\left[a_i - p_i - d(a_j - p_j)\right], i, j = 0, 1, i \neq j \tag{3}$$

To increase the demand, the public and private firms conduct product R&D, which is able to shift their demand function upward due to improved product quality. The effect of R&D on demand function is outlined by:[13]

$$a_i = a + x_i, i = 0, 1 \tag{4}$$

In equation (4), x_i is the R&D outcome of firm i and a ($a>0$) is the price intercept of inverse demand function before R&D.

The profit functions of two firms are listed as follows:

$$\pi_i = (p_i - c_i)q_i - I(\mu_i, \sigma_i), i = 0, 1 \tag{5}$$

In equation (5), c_i is the marginal cost of firm i and $I(\mu_i, \sigma_i)$ is the cost expense for R&D investment by firm i.[14] $I(\mu_i, \sigma_i)$ is assumed to satisfy $\frac{\partial I(\mu_i,\sigma_i)}{\partial \mu_i} \geq 0$, $\frac{\partial I(\mu_i,\sigma_i)}{\partial \sigma_i} \geq 0$, $\frac{\partial I(\mu_i,\sigma_i)}{\partial \sigma_i}\big|_{\sigma_i=0} = 0$ and $\frac{\partial^2 I(\mu_i,\sigma_i)}{\partial {\sigma_i}^2} > 0$ $(\sigma_i \neq 0)$[15] (see Xing, 2014, 2017).[16] In addition, the R&D outcome (i.e., x_i) is assumed to be uncertain (but the firms know its probability distribution) when the public and private firms engage in product R&D. The probability distribution of x_i is $x_i \sim [\mu_i, \sigma_i]$,[17] in which μ_i ($\mu_i \geq 0$) is the expected value and σ_i ($\sigma_i \geq 0$) is the variance (i.e., $E(x_i) = \mu_i$ and $V(x_i) = \sigma_i$)[18]. Note that, the author uses the variance of R&D outcome to measure the risk of the R&D programme and assumes that both public and private firms are risk-neutral (Zhang et al., 2013; Xing, 2014).[19]

Social welfare is the sum of the consumer surplus and the firms' profit, which can be written as follows:

$$W = CS + \pi_0 + \pi_1 \tag{6}$$

In equation (6), CS is the consumer surplus.

The author considers a non-cooperative, two-stage game. In the first stage, each firm undertakes the demand-enhancing R&D and independently chooses the level of R&D risk.[20] In the second stage, the firms produce and simultaneously determine their quantity (price) under Cournot competition (Bertrand competition). The game will be solved by the backward induction. That is, the second stage problem is solved firstly and then the first stage will be considered.

4. The R&D risk choice under Cournot competition

This section considers the case that the product market involves Cournot competition. Using equations (2) and (4)–(6), the author obtains the profit function of firm 1 and the social welfare function:

$$\pi_1 = (a + x_1 - q_1 - dq_0 - c_1)q_1 - I(\mu_1, \sigma_1) \tag{7}$$

$$W = (a + x_0 - c_0)q_0 + (a + x_1 - c_1)q_1 - \frac{1}{2}\left(q_0^2 + 2dq_0q_1 + q_1^2\right) - I(\mu_0, \sigma_0) - I(\mu_1, \sigma_1) \tag{8}$$

In the second stage, the public and private firms non-cooperatively determine their product quantities. Given x_0 and x_1, the public firm chooses q_0 to maximise social welfare,[21] and the private firm chooses q_1 to maximise its profit. The first-order conditions are:

$$\frac{\partial \pi_1}{\partial q_1} = a + x_1 - c_1 - 2q_1 - dq_0 = 0 \tag{9}$$

$$\frac{\partial W}{\partial q_0} = a + x_0 - c_0 - q_0 - dq_1 = 0 \tag{10}$$

Solving equations (9) and (10) for q_0 and q_1 yields the Cournot–Nash equilibrium:[22]

$$q_0^C = \frac{2(a + x_0 - c_0) - d(a + x_1 - c_1)}{2 - d^2} \tag{11}$$

$$q_1^C = \frac{(a + x_1 - c_1) - d(a + x_0 - c_0)}{2 - d^2} \tag{12}$$

The resulting profit and social welfare are:

$$\pi_1^C = \frac{\left[a + x_1 - c_1 - d(a + x_0 - c_0)\right]^2}{(2 - d^2)^2} - I(\mu_1, \sigma_1) \tag{13}$$

$$\begin{aligned} W^C = \frac{1}{2(2-d^2)^2} \Big[& 2(2-d^2)\left\{\left[2(a + x_0 - c_0) - d(a + x_1 - c_1)\right](a + x_0 - c_0) \right. \\ & \left. + \left[(a + x_1 - c_1) - d(a + x_0 - c_0)\right](a + x_1 - c_1)\right\} - \left[2(a + x_0 - c_0) - d(a + x_1 - c_1)\right]^2 \\ & - 2d\left[2(a + x_0 - c_0) - d(a + x_1 - c_1)\right]\left[(a + x_1 - c_1) - d(a + x_0 - c_0)\right] \\ & - \left[(a + x_1 - c_1) - d(a + x_0 - c_0)\right]^2 \Big] - I(\mu_0, \sigma_0) - I(\mu_1, \sigma_1) \end{aligned} \tag{14}$$

In the first stage, the public and private firms choose the type of their R&D programmes from a series of R&D programmes with different risk levels but a same expected outcome (i.e., μ_i is a constant and $\mu_0 = \mu_1$).[23] This is equivalent to deciding on the R&D risk level (variance) (i.e., σ_i). Using equations (13) and (14), the author gets the expected profit of firm 1 and the expected social welfare:[24]

$$E\left(\pi_1^C\right) = \frac{\left[a+\mu_1-c_1-d(a+\mu_0-c_0)\right]^2+\sigma_1+d^2\sigma_0}{\left(2-d^2\right)^2}-I(\mu_1,\sigma_1) \tag{15}$$

$$\begin{aligned} E\left(W^C\right) = \frac{1}{2\left(2-d^2\right)^2}\Big[& 2\left(2-d^2\right)\left\{\left[2(a+\mu_0-c_0)-d(a+\mu_1-c_1)\right](a+\mu_0-c_0)\right. \\ & \left.+\left[(a+\mu_1-c_1)-d(a+\mu_0-c_0)\right](a+\mu_1-c_1)\right\}-\left[2(a+\mu_0-c_0)-d(a+\mu_1-c_1)\right]^2 \\ & -2d\left[2(a+\mu_0-c_0)-d(a+\mu_1-c_1)\right]\left[(a+\mu_1-c_1)-d(a+\mu_0-c_0)\right]-\left[(a+\mu_1-c_1)\right. \\ & \left.-d(a+\mu_0-c_0)\right]^2+\left(4-d^2\right)\sigma_0+\left(3-d^2\right)\sigma_1\Big]-I(\mu_0,\sigma_0)-I(\mu_1,\sigma_1) \end{aligned} \tag{16}$$

According to equations (2), (5), (11) and (12), the gross profit of firm 0 is $\Pi_0^C = \pi_0^C + I(\mu_0,\sigma_0) = 0$. Therefore, the expected gross profit of firm 0 is equal to 0 (i.e., $E(\Pi_0^C) = 0$). Combining equations (15) and (16), the author obtains the expected gross profit of firm 1 (the expected consumer surplus) $E(\Pi_1^C) = E(\pi_1^C) + I(\mu_1,\sigma_1)$ ($E(CS^C) = E(W^C) - E(\pi_0^C) - E(\pi_1^C)$), and can thus prove the following results.

Lemma 1. When the firms are involved in quantity competition, (i) the private firm's expected gross profit increases with its R&D risk (or the public firm's R&D risk); (ii) the public firm's expected gross profit does not depend on its R&D risk (or the private firm's R&D risk);[25] and (iii) the expected consumer surplus increases with the public firm's R&D risk (or the private firm's R&D risk).

Proof. See Appendix.

A numerical example is used to interpret Lemma 1. Consider two alternative programmes, a low-risk programme (programme A) that yields an R&D outcome 0.1 with probability 1, and a high-risk programme (programme B) that yields an R&D outcome 0.5 (0) with probability 0.2 (0.8). The expected R&D outcomes of these two programmes are equal. The intuition behind Lemma 1 is as follows. First, the author takes the result (i) of the lemma. Under Cournot competition, the inverse demand function for successful R&D shifts up relative to the inverse demand function before R&D. Moreover, this shift is greater when the R&D programme is a high-risk type than a low-risk type. When the private firm chooses the high-risk programme (given the public firm's choice) and succeeds, it can obtain more demand and set a higher price than when it chooses the low-risk programme (see Table 1 and Table 2). The benefit of R&D success can compensate for the loss caused by R&D failure. Therefore, the private firm expects to get more gross profits if it opts for the high-risk programme. In addition, when the public firm opts for the high-risk programme (given the private firm's choice) and fails, the private firm

Table 1. When d is small, the price, demand, gross profit and consumer surplus (or their expected values) are shown if firm i ($i = 0, 1$) chooses the high-risk (or low-risk) programme in Cournot competition case (given $a = 2$, $c_0 = 0.5$, $c_1 = 0.45$ and $d = 0.3$).[41]

		p_0^C	p_1^C	q_0^C	q_1^C	Π_0^C	Π_1^C	CS^C
Firm 0 chooses programme A	Firm 1 chooses programme B and succeeds	0.5000	1.2720	1.3534	0.8220	0.0000	0.6757	1.5874
	Firm 1 chooses programme B and fails	0.5000	1.0102	1.4319	0.5602	0.0000	0.3138	1.4228
	The expected value when firm 1 chooses programme B	0.5000	1.0626	1.4162	0.6126	0.0000	0.3862	1.4557
	The expected value when firm 1 chooses programme A	0.5000	1.0626	1.4162	0.6126	0.0000	0.3752	1.4507
Firm 0 chooses programme B	Firm 1 chooses programme B and succeeds	0.5000	1.2720	1.3534	0.8220	0.0000	0.6767	1.6079
	Firm 1 chooses programme B and fails	0.5000	1.0102	1.4319	0.5602	0.0000	0.3148	1.4432
	The expected value when firm 1 chooses programme B	0.5000	1.0626	1.4162	0.6126	0.0000	0.3872	1.4762
	The expected value when firm 1 chooses programme A	0.5000	1.0626	1.4162	0.6126	0.0000	0.3762	1.4712
Firm 1 chooses programme A	Firm 0 chooses programme B and succeeds	0.5000	0.9997	1.8351	0.5497	0.0000	0.3022	2.1375
	Firm 0 chooses programme B and fails	0.5000	1.0783	1.3115	0.6283	0.0000	0.3947	1.3046
	The expected value when firm 0 chooses programme B	0.5000	1.0626	1.4162	0.6126	0.0000	0.3762	1.4712
	The expected value when firm 0 chooses programme A	0.5000	1.0626	1.4162	0.6126	0.0000	0.3752	1.4507
Firm 1 chooses programme B	Firm 0 chooses programme B and succeeds	0.5000	0.9997	1.8351	0.5497	0.0000	0.3132	2.1425
	Firm 0 chooses programme B and fails	0.5000	1.0783	1.3115	0.6283	0.0000	0.4057	1.3096
	The expected value when firm 0 chooses programme B	0.5000	1.0626	1.4162	0.6126	0.0000	0.3872	1.4762
	The expected value when firm 0 chooses programme A	0.5000	1.0626	1.4162	0.6126	0.0000	0.3862	1.4557

Source: Author.

can have more demand and make a higher price than when it chooses the low-risk programme (see Table 1 and Table 2). The probability of failure in R&D when the public firm chooses the high-risk programme is higher than when it chooses the low-risk programme. Consequently, the private firm expects to get more gross profits when the public firm chooses the high-risk programme. Next, turn to the result (ii) of Lemma 1. Due to the public firm provides the level of output at which product price equals the marginal cost, the public firm's expected gross profit always equals zero and is not influenced by its R&D risk choice (or the private firm's choice). The third part of Lemma 1 can be interpreted as follows. When the public firm (the private firm) chooses the high-risk programme (given the private firm's (the public firm's) choice) and succeeds, the aggregate demand is more than when it chooses the low-risk programme (see Table 1 and Table 2). The increase in aggregate demand leads to an increase in consumer surplus. The increase of consumer surplus due to R&D success can compensate for the decrease due to R&D failure. As a result, the expected consumer surplus when the public firm (the private firm) chooses the high-risk programme is more than when it chooses the low-risk programme.

Table 2. When d is large, the price, demand, gross profit and consumer surplus (or their expected values) are shown if firm i ($i = 0, 1$) chooses the high-risk (or low-risk) programme in Cournot competition case (given $a = 2$, $c_0 = 0.5$, $c_1 = 0.45$ and $d = 0.72$).

		p_0^C	p_1^C	q_0^C	q_1^C	Π_0^C	Π_1^C	CS^C
Firm 0 chooses programme A	Firm 1 chooses programme B and succeeds	0.5000	1.0561	1.1636	0.6061	0.0000	0.3674	1.3685
	Firm 1 chooses programme B and fails	0.5000	0.7186	1.4066	0.2686	0.0000	0.0722	1.2974
	The expected value when firm 1 chooses programme B	0.5000	0.7861	1.3580	0.3361	0.0000	0.1312	1.3116
	The expected value when firm 1 chooses programme A	0.5000	0.7861	1.3580	0.3361	0.0000	0.1130	1.3072
Firm 0 chooses programme B	Firm 1 chooses programme B and succeeds	0.5000	1.0561	1.1636	0.6061	0.0000	0.3768	1.3907
	Firm 1 chooses programme B and fails	0.5000	0.7186	1.4066	0.2686	0.0000	0.0816	1.3197
	The expected value when firm 1 chooses programme B	0.5000	0.7861	1.3580	0.3361	0.0000	0.1406	1.3339
	The expected value when firm 1 chooses programme A	0.5000	0.7861	1.3580	0.3361	0.0000	0.1224	1.3295
Firm 1 chooses programme A	Firm 0 chooses programme B and succeeds	0.5000	0.5917	1.8979	0.1417	0.0000	0.0201	2.0048
	Firm 0 chooses programme B and fails	0.5000	0.8347	1.2230	0.3847	0.0000	0.1480	1.1606
	The expected value when firm 0 chooses programme B	0.5000	0.7861	1.3580	0.3361	0.0000	0.1224	1.3295
	The expected value when firm 0 chooses programme A	0.5000	0.7861	1.3580	0.3361	0.0000	0.1130	1.3072
Firm 1 chooses programme B	Firm 0 chooses programme B and succeeds	0.5000	0.5917	1.8979	0.1417	0.0000	0.0383	2.0092
	Firm 0 chooses programme B and fails	0.5000	0.8347	1.2230	0.3847	0.0000	0.1662	1.1650
	The expected value when firm 0 chooses programme B	0.5000	0.7861	1.3580	0.3361	0.0000	0.1406	1.3339
	The expected value when firm 0 chooses programme A	0.5000	0.7861	1.3580	0.3361	0.0000	0.1312	1.3116

Source: Author.

Because the firms are uncertain about the outcomes of R&D programmes in the first stage, the private (public) firm chooses the R&D risk level to maximise its expected profit (the expected social welfare). The first-order conditions give:

$$\frac{\partial E(\pi_1^C)}{\partial \sigma_1} = \frac{1}{(2-d^2)^2} - \frac{\partial I(\mu_1, \sigma_1)}{\partial \sigma_1} = 0 \tag{17}$$

$$\frac{\partial E(W^C)}{\partial \sigma_0} = \frac{4-d^2}{2(2-d^2)^2} - \frac{\partial I(\mu_0, \sigma_0)}{\partial \sigma_0} = 0 \tag{18}$$

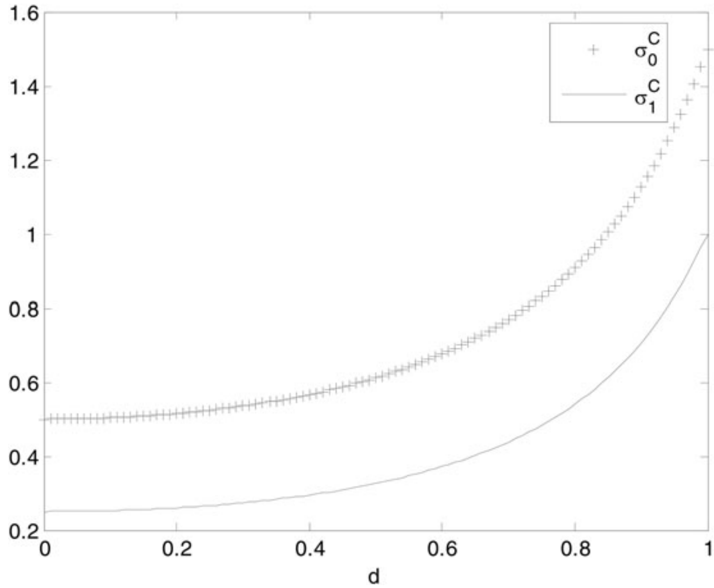

Figure 1. The equilibrium R&D risk levels under Cournot competition (when $I(\mu_i, \sigma_i) = 0.5\mu_i^2 + 0.5\sigma_i^2$, $i = 0, 1$). Source: Author.

Since μ_i is a constant and $\mu_0 = \mu_1$, the author sets:

$$h(\sigma_i) = \frac{\partial I(\mu_i, \sigma_i)}{\partial \sigma_i}, \quad i = 0, 1 \tag{19}$$

Using equations (17)–(19), the author has:

$$\frac{1}{(2-d^2)^2} - h\left(\sigma_1^C\right) = 0 \tag{20}$$

$$\frac{4-d^2}{2(2-d^2)^2} - h\left(\sigma_0^C\right) = 0 \tag{21}$$

where σ_i^C is the equilibrium R&D risk level of firm i in the mixed duopoly under Cournot competition.

Comparing the equilibrium R&D risk levels of private and public firms under Cournot competition, the author can derive the following result.

Proposition 1. For any given $d \in (0, 1)$, the public firm's equilibrium R&D risk level is higher than that of the private firm under Cournot competition (i.e., $\sigma_0^C > \sigma_1^C$).

Proof. See Appendix.

Proposition 1 implies that, in equilibrium the public firm is more willing to take risks than the private firm in a mixed duopoly with Cournot competition (see Figure 1).[26] This proposition can be interpreted as follows. In general, under R&D competition, when the private firm determines its R&D risk, it does not take into account the positive externalities of R&D on the public firm's expected profit or the expected

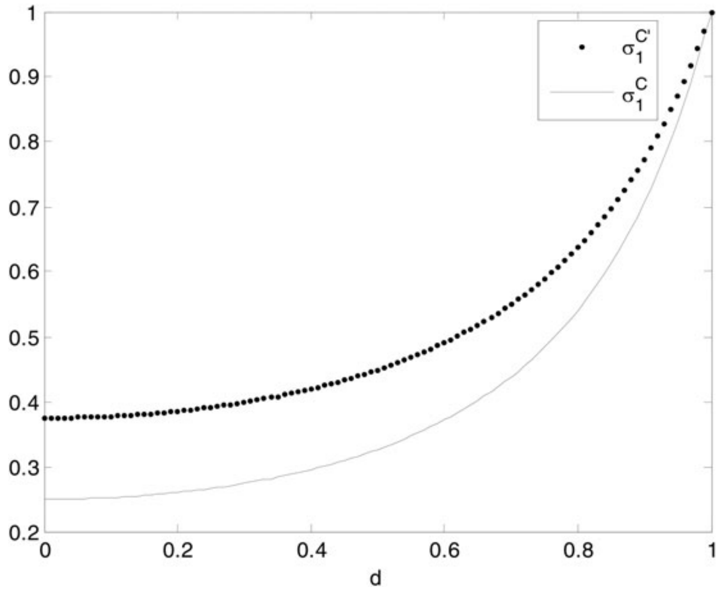

Figure 2. The private and social optimum of R&D risk for the private firm under Cournot competition (when $I(\mu_i, \sigma_i) = 0.5\mu_i^2 + 0.5\sigma_i^2$, $i = 0, 1$). Source: Author.

consumer surplus. However, the public firm cares about both the firms' expected profit and the expected consumer surplus and therefore maximises the sum of the two. Since the public firm's R&D risk has a positive effect on the sum of its expected gross profit and the expected consumer surplus under Cournot competition (Lemma 1),[27] the public firm is more willing to opt for a higher risk programme than the private firm.

Now the author examines the social efficiency of the private incentives on R&D. To do this, the author considers the choice of R&D risk for a social planner. His/her objective is to maximise the value of social welfare. Suppose that the social planner adjusts the R&D investment and the outputs are determined by competition. Because the outcomes of the R&D programme are uncertain, the social welfare is an expectation value. The expected social welfare is given in equation (16). Due to the public firm also maximises the expected social welfare when deciding on its R&D risk, it is in line with the social planner's objective. Thus, for the public firm its private optimum of R&D risk is equal to the social optimum. The author continues to solve the socially optimal R&D risk level for the private firm. Using equation (16), the author has:

$$\frac{\partial E(W^C)}{\partial \sigma_1} = \frac{3-d^2}{2(2-d^2)^2} - \frac{\partial I(\mu_1, \sigma_1)}{\partial \sigma_1} = 0 \tag{22}$$

According to equations (19) and (22), the author derives:

$$\frac{3-d^2}{2(2-d^2)^2} - h\left(\sigma_1^{C\prime}\right) = 0 \tag{23}$$

where $\sigma_1^{C\prime}$ is the socially optimal R&D risk level of the private firm under Cournot competition.

The comparisons between the private and social optimum of R&D risk for the private firm under Cournot competition will be summarised by Proposition 2.

Proposition 2. For any given $d \in (0, 1)$, the private firm's equilibrium R&D risk level is lower than its socially optimal R&D risk level under Cournot competition (i.e., $\sigma_1^C < \sigma_1^{C\prime}$).

Proof. See Appendix.

Proposition 2 shows that, from a social-welfare perspective, the equilibrium R&D risk level for the private firm is too low (i.e., the private firm is willing to assume less risks than it would be socially optimal) under Cournot competition (see Figure 2).[28] In order to increase social welfare, the social planner should encourage the private firms to undertake R&D programmes with high risk.[29] This proposition can be interpreted as follows. The private firm does not consider the positive externalities of R&D on its rival's expected profit or the expected consumer surplus when deciding on R&D risk, whereas the social planner cares about both the firms' expected profit and the expected consumer surplus and therefore maximises the sum of the two. The private firm's R&D risk has a positive effect on the sum of the public firm's expected gross profit and the expected consumer surplus under Cournot competition (Lemma 1).[30] This induces the private firm to take less risk than the social planner.

5. The R&D risk choice under Bertrand competition

In this section, the author analyses the case that the product market involves Bertrand competition. In the second stage, the public (private) firm determines price so as to maximise social welfare (its profit). Using equations (3)–(6), the author gets the profit function of firm 1 and the social welfare function:

$$\pi_1 = \frac{1}{1-d^2}(p_1 - c_1)\left[a + x_1 - p_1 - d(a + x_0 - p_0)\right] - I(\mu_1, \sigma_1) \tag{24}$$

$$\begin{aligned} W = \frac{1}{2(1-d^2)^2}\Big[& 2(1-d^2)\{(a + x_0 - c_0)\left[a + x_0 - p_0 - d(a + x_1 - p_1)\right] \\ & + (a + x_1 - c_1)\left[a + x_1 - p_1 - d(a + x_0 - p_0)\right]\} - \left[a + x_0 - p_0 - d(a + x_1 - p_1)\right]^2 \\ & - 2d\left[a + x_0 - p_0 - d(a + x_1 - p_1)\right]\left[a + x_1 - p_1 - d(a + x_0 - p_0)\right] \\ & - \left[a + x_1 - p_1 - d(a + x_0 - p_0)\right]^2\Big] - I(\mu_0, \sigma_0) - I(\mu_1, \sigma_1) \end{aligned} \tag{25}$$

The first-order conditions for private and public firms in the second stage are:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{1-d^2}\left[a + x_1 - d(a + x_0) + c_1 - 2p_1 + dp_0\right] = 0 \tag{26}$$

$$\frac{\partial W}{\partial p_0} = \frac{1}{1-d^2}(c_0 - dc_1 - p_0 + dp_1) = 0 \tag{27}$$

Solving equations (26) and (27) yields the Bertrand–Nash equilibrium:[31]

$$p_0^B = \frac{-d^2(a+x_0) + d(a+x_1) + 2c_0 - dc_1}{2-d^2} \tag{28}$$

$$p_1^B = \frac{-d(a+x_0-c_0) + (a+x_1) + (1-d^2)c_1}{2-d^2} \tag{29}$$

Thus, the resulting profit and social welfare are:

$$\pi_1^B = \frac{\left[(a+x_1-c_1) - d(a+x_0-c_0)\right]^2}{(1-d^2)(2-d^2)^2} - I(\mu_1, \sigma_1) \tag{30}$$

$$W^B = \frac{1}{2(1-d^2)^2(2-d^2)^2}\Big[2(1-d^2)(2-d^2)\{(2-d^2)[a+x_0-c_0-d(a+x_1-c_1)]$$

$$(a+x_0-c_0) + [a+x_1-c_1-d(a+x_0-c_0)](a+x_1-c_1)\} - (2-d^2)[a+x_0-c_0-d$$

$$(a+x_1-c_1)]\Big\{(2-d^2)[a+x_0-c_0-d(a+x_1-c_1)] + 2d[a+x_1-c_1-d(a+x_0-c_0)]\Big\}$$

$$-[a+x_1-c_1-d(a+x_0-c_0)]^2\Big] - I(\mu_0, \sigma_0) - I(\mu_1, \sigma_1) \tag{31}$$

Now let us turn to the R&D decision of public and private firms in the first stage. Using equations (30) and (31), the author obtains the expected profit of firm 1 and the expected social welfare:

$$E(\pi_1^B) = \frac{\left[(a+\mu_1-c_1) - d(a+\mu_0-c_0)\right]^2 + \sigma_1 + d^2\sigma_0}{(1-d^2)(2-d^2)^2} - I(\mu_1, \sigma_1) \tag{32}$$

$$E(W^B) = \frac{1}{2(1-d^2)^2(2-d^2)^2}\Big[2(1-d^2)(2-d^2)\Big\{(2-d^2)[a+\mu_0-c_0-d(a+\mu_1-c_1)]$$

$$(a+\mu_0-c_0) + [a+\mu_1-c_1-d(a+\mu_0-c_0)](a+\mu_1-c_1)\} - (2-d^2)$$

$$[a+\mu_0-c_0-d(a+\mu_1-c_1)]\Big\{(2-d^2)[a+\mu_0-c_0-d(a+\mu_1-c_1)]$$

$$+2d[a+\mu_1-c_1-d(a+\mu_0-c_0)]\} - [a+\mu_1-c_1-d(a+\mu_0-c_0)]^2$$

$$+(4-9d^2+7d^4-2d^6)\sigma_0 + (3-6d^2+4d^4-d^6)\sigma_1\Big] - I(\mu_0, \sigma_0) - I(\mu_1, \sigma_1) \tag{33}$$

According to equations (3), (5), (28) and (29), the author derives the profit of firm 0 $\pi_0^B = \frac{d[a+x_1-c_1-d(a+x_0-c_0)][a+x_0-c_0-d(a+x_1-c_1)]}{(1-d^2)(2-d^2)} - I(\mu_0, \sigma_0)$. Therefore, the firm 0's expected gross profit is given by $E(\Pi_0^B) = \frac{d[a+\mu_1-c_1-d(a+\mu_0-c_0)][a+\mu_0-c_0-d(a+\mu_1-c_1)]-d^2\sigma_0-d^2\sigma_1}{(1-d^2)(2-d^2)}$. Combining equations (32) and (33), the author has the expected gross profit of firm 1 $E(\Pi_1^B) = E(\pi_1^B) + I(\mu_1, \sigma_1)$ and the expected consumer surplus $E(CS^B) = E(W^B) - E(\pi_0^B) - E(\pi_1^B)$, and then can prove the following results.

Lemma 2. When the firms are involved in price competition, (i) the private firm's expected gross profit increases with its R&D risk (or the public firm's R&D risk); (ii) the public firm's expected gross profit decreases with its R&D risk (or the private firm's R&D risk); and (iii) the expected consumer surplus increases with the public firm's R&D risk (or the private firm's R&D risk).

Proof. See Appendix.

The author offers an explanation of Lemma 2 with the help of the numerical examples in Table 3 and Table 4. In Table 3 (or Table 4), programme A (programme B) is a low-risk programme (a high-risk programme), which is the same as in Table 1 and Table 2. First, the author takes the first part of the lemma. Under Bertrand competition, the direct demand function of private firm for successful R&D shifts up relative to that before R&D, and this shift is greater when the private firm's R&D programme is a high-risk type than a low-risk type. When the private firm chooses the high-risk programme (given the public firm's choice) and succeeds, it can set higher prices (see equation (29)) and will obtain more demand than when choosing the low-risk programme (see Table 3 and Table 4). Because the benefit of R&D success can compensate for the loss caused by failure, the private firm expects to get more gross profits when opting for the high-risk programme. In addition, the probability of failure in R&D if the public firm chooses the high-risk programme is higher than if it chooses the low-risk programme. When the public firm opts for the high-risk programme (given the private firm's choice) and fails, the private firm can set higher prices and will have more demand than when the public firm opts for the low-risk programme (see Table 3 and Table 4). Therefore, the private firm expects to obtain more gross profits when the public firm chooses the high-risk programme. Next, turn to the part (ii) of Lemma 2. When the public firm chooses the high-risk programme (given the private firm's choice) and is successful, it sets lower prices (see equation (28)) and will obtain more demand than when choosing the low-risk programme. Because the impact of R&D on the price of public firm is stronger than on its demand, the public firm expects to get less gross profit when opting for the high-risk programme. When the private firm opts for the high-risk programme (given the public firm's choice) and succeeds, the public firm will set higher prices (see equation (28)) and obtain less demand than when opting for the low-risk programme. Due to the effect of R&D on the price of public firm is weaker than on its demand, the public firm expects to get less gross profit when the private firm choosing the high-risk programme. These results are different from those in Cournot competition. The result (iii) of Lemma 2 is interpreted as follows. Under Bertrand competition, if the public firm

Table 3. When d is small, the price, demand, gross profit and consumer surplus (or their expected values) are shown if firm i ($i = 0, 1$) chooses the high-risk (or low-risk) programme in Bertrand competition case (given $a = 2$, $c_0 = 0.5$, $c_1 = 0.45$ and $d = 0.3$).

		p_0^B	p_1^B	q_0^B	q_1^B	Π_0^B	Π_1^B	CS^B
Firm 0 chooses programme A	Firm 1 chooses programme B and succeeds	0.7466	1.2720	1.0824	0.9033	0.2669	0.7425	1.2871
	Firm 1 chooses programme B and fails	0.6681	1.0102	1.2473	0.6156	0.2096	0.3449	1.1977
	The expected value when firm 1 chooses programme B	0.6838	1.0626	1.2143	0.6731	0.2211	0.4244	1.2155
	The expected value when firm 1 chooses programme A	0.6838	1.0626	1.2143	0.6731	0.2231	0.4123	1.2090
Firm 0 chooses programme B	Firm 1 chooses programme B and succeeds	0.7466	1.2720	1.0824	0.9033	0.2648	0.7436	1.3096
	Firm 1 chooses programme B and fails	0.6681	1.0102	1.2473	0.6156	0.2075	0.3460	1.2201
	The expected value when firm 1 chooses programme B	0.6838	1.0626	1.2143	0.6731	0.2190	0.4255	1.2380
	The expected value when firm 1 chooses programme A	0.6838	1.0626	1.2143	0.6731	0.2211	0.4134	1.2315
Firm 1 chooses programme A	Firm 0 chooses programme B and succeeds	0.6649	0.9997	1.6538	0.6041	0.2728	0.3321	1.8498
	Firm 0 chooses programme B and fails	0.6885	1.0783	1.1044	0.6904	0.2082	0.4338	1.0769
	The expected value when firm 0 chooses programme B	0.6838	1.0626	1.2143	0.6731	0.2211	0.4134	1.2315
	The expected value when firm 0 chooses programme A	0.6838	1.0626	1.2143	0.6731	0.2231	0.4123	1.2090
Firm 1 chooses programme B	Firm 0 chooses programme B and succeeds	0.6649	0.9997	1.6538	0.6041	0.2707	0.3442	1.8563
	Firm 0 chooses programme B and fails	0.6885	1.0783	1.1044	0.6904	0.2061	0.4458	1.0834
	The expected value when firm 0 chooses programme B	0.6838	1.0626	1.2143	0.6731	0.2190	0.4255	1.2380
	The expected value when firm 0 chooses programme A	0.6838	1.0626	1.2143	0.6731	0.2211	0.4244	1.2155

Source: Author.

(the private firm) chooses the high-risk programme (given the private firm's (the public firm's) choice) and succeeds, the aggregate demand is more than if it chooses the low-risk programme (see Table 3 and Table 4). Similar to the reason for part (iii) of Lemma 1, the expected consumer surplus when the public firm (the private firm) chooses the high-risk programme is more than when it chooses the low-risk programme.

The first-order conditions for maximisation problem are given by:

$$\frac{\partial E(\pi_1^B)}{\partial \sigma_1} = \frac{1}{(1-d^2)(2-d^2)^2} - \frac{\partial I(\mu_1, \sigma_1)}{\partial \sigma_1} = 0 \tag{34}$$

Table 4. When d is large, the price, demand, gross profit and consumer surplus (or their expected values) are shown if firm i ($i = 0, 1$) chooses the high-risk (or low-risk) programme in Bertrand competition case (given $a = 2$, $c_0 = 0.5$, $c_1 = 0.45$ and $d = 0.72$).

		p_0^B	p_1^B	q_0^B	q_1^B	Π_0^B	Π_1^B	CS^B
Firm 0 chooses programme A	Firm 1 chooses programme B and succeeds	0.9364	1.0561	0.2575	1.2585	0.1124	0.7628	1.0584
	Firm 1 chooses programme B and fails	0.6934	0.7186	1.0050	0.5578	0.1944	0.1498	1.0642
	The expected value when firm 1 chooses programme B	0.7420	0.7861	0.8555	0.6979	0.1780	0.2724	1.0630
	The expected value when firm 1 chooses programme A	0.7420	0.7861	0.8555	0.6979	0.2070	0.2346	1.0394
Firm 0 chooses programme B	Firm 1 chooses programme B and succeeds	0.9364	1.0561	0.2575	1.2585	0.0833	0.7824	1.1046
	Firm 1 chooses programme B and fails	0.6934	0.7186	1.0050	0.5578	0.1653	0.1695	1.1104
	The expected value when firm 1 chooses programme B	0.7420	0.7861	0.8555	0.6979	0.1489	0.2920	1.1093
	The expected value when firm 1 chooses programme A	0.7420	0.7861	0.8555	0.6979	0.1780	0.2542	1.0856
Firm 1 chooses programme A	Firm 0 chooses programme B and succeeds	0.6021	0.5917	1.6860	0.2943	0.1721	0.0417	1.8220
	Firm 0 chooses programme B and fails	0.7770	0.8347	0.6478	0.7988	0.1795	0.2542	0.9015
	The expected value when firm 0 chooses programme B	0.7420	0.7861	0.8555	0.6979	0.1780	0.2542	1.0856
	The expected value when firm 0 chooses programme A	0.7420	0.7861	0.8555	0.6979	0.2070	0.2346	1.0394
Firm 1 chooses programme B	Firm 0 chooses programme B and succeeds	0.6021	0.5917	1.6860	0.2943	0.1430	0.0796	1.8456
	Firm 0 chooses programme B and fails	0.7770	0.8347	0.6478	0.7988	0.1504	0.3452	0.9252
	The expected value when firm 0 chooses programme B	0.7420	0.7861	0.8555	0.6979	0.1489	0.2920	1.1093
	The expected value when firm 0 chooses programme A	0.7420	0.7861	0.8555	0.6979	0.1780	0.2724	1.0630

Source: Author.

$$\frac{\partial E(W^B)}{\partial \sigma_0} = \frac{4-9d^2+7d^4-2d^6}{2(1-d^2)^2(2-d^2)^2} - \frac{\partial I(\mu_0, \sigma_0)}{\partial \sigma_0} = 0 \tag{35}$$

Substituting equation (19) into equations (34) and (35) gives:

$$\frac{1}{(1-d^2)(2-d^2)^2} - h\left(\sigma_1^B\right) = 0 \tag{36}$$

$$\frac{4-9d^2+7d^4-2d^6}{2(1-d^2)^2(2-d^2)^2} - h\left(\sigma_0^B\right) = 0 \tag{37}$$

where σ_i^B is the equilibrium R&D risk level of firm i in the mixed duopoly under Bertrand competition.

Let us compare the equilibrium R&D risk levels of private and public firms under Bertrand competition and then the author can obtain the following proposition.

Proposition 3. There exists a threshold ($d^{\#} \in (0,1)$) of d such that (i) when d is smaller than $d^{\#}$ (i.e., $0<d<d^{\#}$), the public firm's equilibrium R&D risk level is higher than that of the private firm under Bertrand competition ($\sigma_0^B>\sigma_1^B$); and (ii) when d is larger than $d^{\#}$ (i.e., $d^{\#}<d<1$), the public firm's equilibrium R&D risk level is lower than that of the private firm under Bertrand competition (i.e., $\sigma_0^B<\sigma_1^B$).

Proof. See Appendix.

Proposition 3 states that, in a mixed duopoly with Bertrand competition, the public firm is more (less) willing to take risks in R&D than the private firm in equilibrium if the degree of product substitution is sufficiently small (large) (see Figure 3).[32] It is an interesting finding that the degree of product substitution might reverse the R&D risk choices between public and private firms under Bertrand competition. This is different from the result in Cournot competition. The intuition behind this proposition is as follows. The public firm's R&D risk has a negative effect on its expected gross profit under Bertrand competition (Lemma 2). This effect is weak if the degree of product substitution is small enough. In this case, the effect of the public firm's R&D risk on the sum of its expected gross profit and the expected consumer surplus is positive.[33] Since the private (public) firm maximises only its expected profit (the sum of firms' expected profit and the expected consumer surplus) when deciding on R&D risk, the public firm prefers to choose a riskier programme than the private firm. However, if the degree of product substitution is large enough, the public firm's R&D risk has a strong negative effect on its expected gross profit and this effect exceeds its positive effect on the expected consumer surplus. In this case, the effect of the public firm's R&D risk on the sum of its expected gross profit and the expected consumer surplus is negative.[34] Hence, the public firm prefers to choose a safer programme than the private firm.

Now turn to the situation where the social planner controls the R&D investment. The author considers the social efficiency of the private incentives on R&D. Suppose that the pricing is determined by competition. The expected social welfare is given in equation (33). For the public firm, its private optimum of R&D risk is equal to the social optimum. According to equation (33), the author obtains:

$$\frac{\partial E(W^C)}{\partial \sigma_1} = \frac{3-6d^2+4d^4-d^6}{2(1-d^2)^2(2-d^2)^2} - \frac{\partial I(\mu_1,\sigma_1)}{\partial \sigma_1} = 0 \tag{38}$$

Using equations (19) and (38), the author has:

$$\frac{3-6d^2+4d^4-d^6}{2(1-d^2)^2(2-d^2)^2} - h\left(\sigma_1^{B\prime}\right) = 0 \tag{39}$$

where $\sigma_1^{B\prime}$ is the socially optimal R&D risk level of private firm under Bertrand competition.

Comparing the private and social optimum of private firm under Bertrand competition, the author can prove the following results.

Proposition 4. There exists a threshold ($d^{\#} \in (0, 1)$) of d such that (i) when d is smaller than $d^{\#}$ (i.e., $0<d<d^{\#}$), for the private firm the equilibrium R&D risk level is lower than the social optimum under Bertrand competition (i.e., $\sigma_1^B<\sigma_1^{B\prime}$); and (ii) when d is larger than $d^{\#}$, (i.e., $d^{\#}<d<1$), for the private firm the equilibrium R&D risk level is higher than the social optimum under Bertrand competition (i.e., $\sigma_1^B>\sigma_1^{B\prime}$).

Proof. See Appendix.

The above proposition implies that, from a social-welfare perspective, the equilibrium R&D risk level for the private firm is too low (i.e., the private firm is willing to assume less risks than it would be socially optimal) if the degree of product substitution is sufficiently small under Bertrand competition (see Figure 4). However, it is too high (i.e., the private firm is willing to assume more risks than it would be socially optimal) if the degree of product substitution is sufficiently large under Bertrand competition (see Figure 4). To increase social welfare, the social planner should subsidy private firms who engage in high-risk R&D programmes if the degree of product substitution is small enough. However, he/she should tax private firms who undertake high-risk R&D programmes if the degree of product substitution is large enough.[35] Proposition 4 can be interpreted as follows. According to Lemma 2, the private firm's R&D risk has a negative effect on the public firm's expected gross profit and has a positive effect on the expected consumer surplus under Bertrand competition. If the degree of product substitution is small (large), the former effect is weak (strong) and the latter effect is strong (weak). In this case, the effect of the private firm's R&D risk on the sum of the public firm's expected gross profit and the expected consumer surplus is positive (negative).[36] The social planner cares about both the firms' expected profit and the expected consumer surplus and the private firm only considers its expected profit when they determine R&D risk. Thus, the social planner prefers the private firm to choose a riskier (safer) programme if the degree of product substitution is small (large).

6. Comparison

This section presents a comparison of the equilibrium R&D risk levels under Cournot and Bertrand competition. Using equations (20), (21), (36) and (37), the author can prove the following results.

Proposition 5. For any given $d \in (0, 1)$, in equilibrium the R&D risk levels for both public and private firms are higher if the product market involves Bertrand competition than if it involves Cournot competition (i.e., $\sigma_1^C<\sigma_1^B$ and $\sigma_0^C<\sigma_0^B$).

Proof. See Appendix.

This proposition tells us that, the modes of market competition may affect the R&D risk choice in a mixed duopoly, and further Bertrand competition makes the public and private firms more willing to take risks than Cournot competition.[37] Proposition 5 can be explained as follows. According to Lemmas 1 and 2, the private

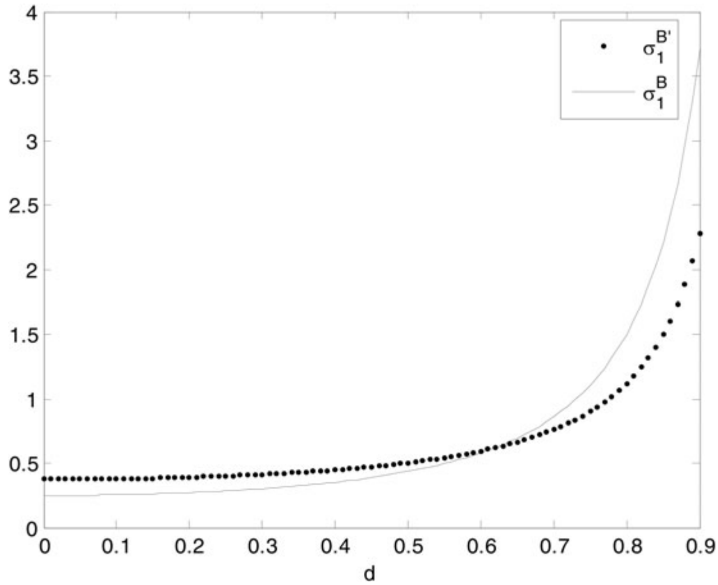

Figure 4. The private and social optimum of R&D risk for the private firm under Bertrand competition (when $I(\mu_i, \sigma_i) = 0.5\mu_i^2 + 0.5\sigma_i^2$, $i = 0, 1$). Source: Author.

firm's R&D risk has a positive effect on its expected gross profit under both Cournot and Bertrand competition. Since in Bertrand competition case this effect is stronger than in Cournot competition case,[38] Bertrand competition leads the private firm to choose a riskier R&D programme. In addition, Lemmas 1 and 2 also show that, the public firm's R&D risk has a positive effect on the private firm's expected gross profit and the expected consumer surplus under Cournot or Bertrand competition, and has no effect (a negative effect) on its expected gross profit under Cournot (Bertrand) competition. Obviously, $\frac{\partial E(\Pi_0^B)}{\partial \sigma_0} < \frac{\partial E(\Pi_0^C)}{\partial \sigma_0}$ (see Lemmas 1 and 2). Since, compared with Cournot competition case, the effect of the public firm's R&D risk on both the private firm's expected gross profit and the expected consumer surplus is greater in Bertrand competition case,[39] this in turn leads to a stronger effect on the sum of the firms' expected gross profit and the expected consumer surplus under Bertrand competition.[40] Therefore, the public firm takes more R&D risk under Bertrand competition.

7. Conclusions

The author constructs a two-stage duopoly model with differentiated products, in which the private and public firms compete in R&D firstly and then in quantity (or price). The author compares the R&D incentives of two firms under Cournot (or Bertrand) competition, and finds that in equilibrium the public firm always engages in higher R&D risks than the private firm under Cournot competition. However, the degree of product substitution might reverse the R&D risk choices between public and private firms under Bertrand competition. In addition, the author analyses the impact of competition modes on the R&D risk choices and concludes that both private and public firms under Cournot competition are willing to assume less R&D risks than under Bertrand competition. Finally, the author examines the social efficiency of private incentives on R&D, and finds that for the private firm its private optimum of R&D risk is always lower than the social optimum under Cournot competition. However, it exceeds the social optimum if the degree of product substitution is sufficiently large under Bertrand competition.

This study restricts attention to strategic R&D risk choice of public and private firms. It is also interesting to investigate how privatisation affects the R&D risk choice in a mixed duopoly. In addition, the firms are assumed to have multiple R&D programmes with different degree of risk but an identical expected outcome. A natural extending is that the firms have multiple R&D programmes with different degree of risk and different expected outcomes and determine both risk and expected outcomes in R&D stage. Finally, successful firms are not able to appropriate all of the gains from the outcomes of their R&D activities if technological spillovers occur in an industry. Thus, technological spillovers may weaken the firm's incentive to invest in R&D. It is important to study the impact of spillovers on the optimal R&D risk choice. However, such issues must remain for future research.

The main managerial implication is that in mixed duopoly market the private (public) firm's optimal decision is to choose safer (riskier) R&D programmes than its competitor under Cournot competition. Notice, however, that under Bertrand competition the firms should take the opposite tack if their products have a very high degree of substitution. This study also highlights that when the private (public) firm determines its R&D programmes, it should consider the modes of market competition and choose riskier R&D programmes under Bertrand competition than under Cournot competition. The practical implication for the social planner is that, when he/she carries out the R&D policy, the impacts of modes of market competition and degree of product substitution cannot be neglected. He/she would prefer the private firm to take more risk under Cournot competition. However, under Bertrand competition he/she would prefer the private firm to take less risk if the degree of product substitution is sufficiently large.

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Notes

1. For instance, in the Norwegian oil industry Statoil (a state-owned firm) competes against Norske Shell (a private firm) in large technological programmes. Statoil is a major investor in dual cycle energy production system based on fuel cells. Similarly, Norske

Shell is very active in R&D aimed at the development of energy systems based on SOFC (solid oxide fuel cells) (Godø, Nerdrum, Rapmund, & Nygaard, 2003).

2. Only a few scholars study the R&D activity in mixed markets (see Poyago-Theotoky, 1998; Ishibashi & Matsumura, 2006; Tomaru, 2007; Heywood & Ye, 2009; Cato, 2011; Tsai, Wang, & Chiou, 2016).
3. Gil-Moltó, Poyago-Theotoky, and Zikos (2011) and Kesavayuth and Zikos (2013) point out that the mixed market literature often ignores that public firms are key players in R&D-intensive industries such as health care, energy and bio-agriculture and Godø et al. (2003) provide case studies in the energy sector in European and OECD countries.
4. For example, there are usually different paths to achieve a given level of microprocessor speed in the microprocessor industry (Anderson & Cabral, 2007).
5. For example, in Chinese mobile phone industry, Huawei (a private firm) is more willing to engage in a number of high-risk programmes (e.g., smart phone chip and autonomous mobile operating system) than Lenovo (a public firm); in German pilotless automobile industry, Bosch, a private firm, prefers to take some riskier R&D programmes (e.g., electric power steering system and electronic stabilisation system) than Volkswagen (a public firm).
6. See some good examples in the studies of Cabral (2003) and Tishler (2008).
7. One line focuses on the choice of R&D risk in patent races, in which the invention time of technology is the variable of interest (see Klette & de Meza, 1986; Dasgupta & Maskin, 1987; Cabral, 1994; and so on), and the other line assumes that the R&D risk is denoted by the variance in R&D outcomes and investigates the strategic choice of R&D risk (see Cabral, 2003; Gerlach, Rønde, & Stahl, 2005; Tishler, 2008; Zhang, Mei, & Zhong, 2013; Xing, 2014; and so on). It is important to note, though, that these papers only consider private firms in the standard duopoly and overlook competition between public and private firms in the mixed configuration.
8. Comparison of Bertrand and Cournot outcomes is of fundamental importance in the industrial organisation literature. Qiu (1997), Symeonidis (2003), Chen and Nie (2014) and Lee and Choi (2016) examine the effect of product market competition (Bertrand and Cournot competition) on R&D investment. However, none of these studies analyse the impact of product market competition on the R&D risk choices.
9. The models in this paper inherit some features from previous mixed duopoly (or oligopoly) models (e.g., linear demand functions, utility function, and so on). However, there are at least three aspects that this study contributes to the existing mixed oligopoly literature: (i) it introduces the R&D risk variable into the mixed duopoly model and examines the strategic R&D risk choices; (ii) it considers the R&D risk choices not only in Cournot mixed duopoly but also in Bertrand mixed duopoly; (iii) it introduces a new R&D cost function into the mixed duopoly model.
10. Following the mixed duopoly literature (Matsumura & Matsushima, 2004; Cato, 2011; Kitahara & Matsumura, 2013), we assume that the public firm pursues maximisation of welfare.
11. The reason we make this assumption is that, we can observe product differentiation in most industries except for very few industries like power, steel and chemical products and so on.
12. The detailed derivation process of the inverse demand function is described in Tondji (2015).
13. Note that the R&D activity is assumed to be perfectly protected against imitation. That is, this study does not consider the R&D spillovers. This simplifying assumption has the purpose to show the only effect of product substitution on main propositions (see Proposition 3 and Proposition 4). However, the R&D spillovers are important factors affecting the firm's R&D risk choice. The author will study the impact of R&D spillovers on the optimal R&D risk choice in mixed oligopoly market in future.
14. The existence of risk may cause a loss to a firm. Harrington and Niehaus (2003) give the definition of risk cost and define it as the reduction of corporate value as a result of the

existence of risk. Combining the characteristics of technological innovation risk, Zhu (2011) gives the definition of the cost of technological innovation risk. She defines it as, in the process of technological innovation, due to the uncertainty of the risk, an enterprise's early investment to avoid possible deviations from expected goals and possible loss to choose to carry out this innovation. Similar to Harrington and Niehaus (2003) and Zhu (2011), the author thinks that the existence of R&D risk can bring the R&D risk cost and gives the R&D cost as a function of the R&D risk and the expected R&D outcome. Moreover, Liu (2007) gives methods for estimating the loss in technology innovation risk. However, there is little information in the published literature on the cost structure of R&D programmes in the case of considering R&D risk.

15. This can guarantee that the second-order conditions for R&D are met and the optimal solutions of R&D risk are interior (Xing, 2014).
16. It is worth noting that, in works of Xing (2014, 2017), we assume μ_i and σ_i are additively separable in $I(\mu_i, \sigma_i)$. However, this paper does not have such a requirement. Obviously, the R&D cost function is more general in this paper. In addition, these two works only examine the R&D risk choices in a private duopoly. Xing (2014) investigates the optimal R&D risk choices in a private duopoly market exhibiting network externalities, and Xing (2017) examines the R&D risk choices in a private duopoly market with technology spillovers. These works do not consider the public firms and compare the R&D incentive between public and private firms. Thus, they are significantly different from the application circumstances of this paper in a mixed duopoly.
17. Note that this study uses truncated distribution and assumes $x_i \geq 0$ $(i = 0, 1)$. Thus, the negative values of R&D what causes that the demand function shifts to the left are not allowed.
18. The covariance of x_0 and x_1 is assumed to equal zero in this study.
19. Note that the risk defined in this paper does not coincide with that in utility theory.
20. Although this study only considers demand-enhancing R&D, the main propositions still hold for cost-reducing R&D.
21. See the study of Han, Heywood and Ye (2017).
22. The second-order conditions are met (i.e., $\partial^2\pi_1/\partial q_1^2 = -2<0$ and $\partial^2 W/\partial q_0^2 = -1<0$) and the equilibrium in the second stage is locally stable. Moreover, to guarantee the positive demand for the firms, the author assumes that $c_0<a$, $c_1<a$ and $|c_0 - c_1|$ is small enough.
23. This is a similar assumption as in Tishler (2008), Zhang et al. (2013) and Xing (2014; 2017). However, a natural question is how this specification can be generalised into the other cases such as 'oligopolistic' firms have multiple R&D programmes with different degree of risk and 'different' expected outcomes. The author will extend this analysis in a further direction.
24. Zemlickienė, Bublienė and Jakubavičius (2018) give an advanced tool for verifying decisions on technology development at early stages of commercialisation.
25. In order to give economic explanations for main propositions more easily, the author considers the expected gross profit of public firm and further analyses the impact of R&D risk on it.
26. Some studies make a comparison of the R&D investment levels between public and private firms. Nett (1994) holds that the private firm has higher incentive to innovate than the public firm, whereas Poyago-Theotoky (1998) and Nie and Yang (2015) show that the private firm invests less than the public firm. However, they do not make a comparison of R&D risk.
27. According to Lemma 1, $\partial E(\Pi_0^C)/\partial\sigma_0 = 0$ and $\partial E(CS^C)/\partial\sigma_0>0$. Thus, $\partial E(\Pi_0^C)/\partial\sigma_0 + \partial E(CS^C)/\partial\sigma_0>0$. In addition, the author can derive $\partial E(\Pi_0^C)/\partial\sigma_0 + \partial\, E(\Pi_1^C)/\partial\sigma_0 + \partial E(CS^C)/\partial\sigma_0>\partial E(\Pi_1^C)/\partial\sigma_1>0$.
28. Zoledowska (2016) discusses the issues related to the existing or required support given by the State to firms to provide them conditions to innovate.

29. For example, the social planner subsidies the private firms who engage in the high-risk R&D programmes. Zikos (2010), Gil-Moltó et al. (2011), Kesavayuth and Zikos (2013) and Lee and Tomaru (2017) investigate R&D subsidies in mixed markets.
30. According to Lemma 1, $\partial E(\Pi_0^C)/\partial\sigma_1 = 0$ and $\partial E(CS^C)/\partial\sigma_1 > 0$. Thus, $\partial E(\Pi_0^C)/\partial\sigma_1 + \partial E(CS^C)/\partial\sigma_1 > 0$. In addition, the author can further obtain $\partial E(\Pi_0^C)/\partial\sigma_1 + \partial E(\Pi_1^C)/\partial\sigma_1 + \partial E(CS^C)/\partial\sigma_1 > \partial E(\Pi_1^C)/\partial\sigma_1 > 0$.
31. The second-order conditions are satisfied (i.e., $\partial^2\pi_1/\partial p_1^2 = -2/(1-d^2) < 0$ and $\partial^2 W/\partial p_0^2 = -1/(1-d^2) < 0$) and the equilibrium in the second stage is locally stable.
32. Combining Propositions 1 and 3, the author knows that whether the public firm is more willing to take risks than the private firm, depending on market competition and product characteristics. This can be used to understand why the public firms are more likely to invest in high-risk R&D programmes than the private firms in some industries, while the opposite may appear in other industries.
33. According to Lemma 2, $\partial E(\Pi_0^B)/\partial\sigma_0 < 0$ and $\partial E(CS^B)/\partial\sigma_0 > 0$. Then, the author can obtain $\partial E(\Pi_0^B)/\partial\sigma_0 + \partial E(CS^B)/\partial\sigma_0 > 0$ $(\partial E(\Pi_0^B)/\partial\sigma_0 + \partial E(\Pi_1^B)/\partial\sigma_0 + \partial E(CS^B)/\partial\sigma_0 > \partial E(\Pi_1^B)/\partial\sigma_1)$ if d is small.
34. The author can obtain $\partial E(\Pi_0^B)/\partial\sigma_0 + \partial E(CS^B)/\partial\sigma_0 < 0$ $(\partial E(\Pi_0^B)/\partial\sigma_0 + \partial E(\Pi_1^B)/\partial\sigma_0 + \partial E(CS^B)/\partial\sigma_0 < \partial E(\Pi_1^B)/\partial\sigma_1)$ if d is large.
35. Some scholars investigate the imposition of R&D tax on firms (Yang, Liu, & Yang, 2010).
36. That is, $\partial E(\Pi_0^B)/\partial\sigma_1 + \partial E(CS^B)/\partial\sigma_1 > (<) 0$ if d is small (large). In addition, the author can derive $\partial E(\Pi_0^B)/\partial\sigma_1 + \partial E(\Pi_1^B)/\partial\sigma_1 + \partial E(CS^B)/\partial\sigma_1 > (<) \partial E(\Pi_1^B)/\partial\sigma_1$ if d is small (large).
37. Qiu (1997) and Symeonidis (2003) compare the R&D investment under Bertrand and Cournot competition in the standard duopoly and show that Cournot competition induces more R&D effort than Bertrand competition.
38. That is, $\partial E(\Pi_1^B)/\partial\sigma_1 > \partial E(\Pi_1^C)/\partial\sigma_1$.
39. That is, $\partial E(\Pi_1^B)/\partial\sigma_0 > \partial E(\Pi_1^C)/\partial\sigma_0$ and $\partial E(CS^B)/\partial\sigma_0 > \partial E(CS^C)/\partial\sigma_0$.
40. That is, $\partial E(\Pi_0^B)/\partial\sigma_0 + \partial E(\Pi_1^B)/\partial\sigma_0 + \partial E(CS^B)/\partial\sigma_0 > \partial E(\Pi_0^C)/\partial\sigma_0 + \partial E(\Pi_1^C)/\partial\sigma_0 + \partial E(CS^C)/\partial\sigma_0$.
41. To help readers understand how the author gets the results of simulation, a description is given in combination with the calculation of q_0^C in the first and second parts of Table 1. However, the calculation of the rest in Table 1 (or the calculation of Tables 2, 3 and 4) can be similarly given. According to (11), the author can set $q_0^C(x_0, x_1) = [2(a + x_0 - c_0) - d(a + x_1 - c_1)]/(2 - d^2)$. Given firm 0 choosing programme A, (i) if firm 1 chooses programme B and succeeds, the firm 0's demand is $q_0^C(0.1, 0.5) \approx 1.3534$; (ii) if firm 1 chooses programme B and fails, the firm 0's demand is $q_0^C(0.1, 0) \approx 1.4319$; (iii) if firm 1 chooses programme B, the firm 0's expected demand is $E(q_0^C) = 0.2 \times q_0^C(0.1, 0.5) + 0.8 \times q_0^C(0.1, 0) \approx 1.4162$; and (iv) if firm 1 chooses programme A, the firm 0's expected demand is $E(q_0^C) = q_0^C(0.1, 0.1) \approx 1.4162$. In addition, given firm 0 choosing programme B, (i) if firm 1 chooses programme B and succeeds, the firm 0's demand is $0.2 \times q_0^C(0.5, 0.5) + 0.8 \times q_0^C(0, 0.5) \approx 1.3534$; (ii) if firm 1 chooses programme B and fails, the firm 0's demand is $0.2 \times q_0^C(0.5, 0) + 0.8 \times q_0^C(0, 0) \approx 1.4319$; (iii) if firm 1 chooses programme B, the firm 0's expected demand is $E(q_0^C) = 0.2 \times 0.2 \times q_0^C(0.5, 0.5) + 0.2 \times 0.8 \times q_0^C(0.5, 0) + 0.8 \times 0.2 \times q_0^C(0, 0.5) + 0.8 \times 0.8 \times q_0^C(0, 0) \approx 1.4162$; and (iv) if firm 1 chooses programme A, the firm 0's expected demand is $E(q_0^C) = 0.2 \times q_0^C(0.5, 0.1) + 0.8 \times q_0^C(0, 0.1) \approx 1.4162$.

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Appendix

A. Proof of Lemma 1: Because $\frac{\partial E(\Pi_1^C)}{\partial \sigma_1} = \frac{1}{(2-d^2)^2} > 0$, $\frac{\partial E(\Pi_1^C)}{\partial \sigma_0} = \frac{d^2}{(2-d^2)^2} > 0$, $\frac{\partial E(\Pi_0^C)}{\partial \sigma_0} = 0$, $\frac{\partial E(\Pi_0^C)}{\partial \sigma_1} = 0$, $\frac{\partial E(CS^C)}{\partial \sigma_0} = \frac{4-3d^2}{2(2-d^2)^2} > 0$ and $\frac{\partial E(CS^C)}{\partial \sigma_1} = \frac{1-d^2}{2(2-d^2)^2} > 0$.

B. Proof of Proposition 1: Using equations (20) and (21), $h(\sigma_0^C) - h(\sigma_1^C) = \frac{1}{2(2-d^2)}$. There is $\bar{\sigma}$ $(\bar{\sigma} > 0)$ making $(\sigma_0^C - \sigma_1^C)h'(\bar{\sigma}) = \frac{1}{2(2-d^2)}$. Since $h'(\bar{\sigma}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}|_{\sigma_i = \bar{\sigma}} > 0$ and $2-d^2 > 0$, $\sigma_0^C - \sigma_1^C > 0$.

C. Proof of Proposition 2: Using equations (20) and (23), $h(\sigma_1^{C'}) - h(\sigma_1^C) = \frac{1-d^2}{2(2-d^2)^2}$. There is $\underline{\sigma}$ $(\underline{\sigma} > 0)$ making $(\sigma_1^{C'} - \sigma_1^C)h'(\underline{\sigma}) = \frac{1-d^2}{2(2-d^2)^2}$. Since $h'(\underline{\sigma}) = \frac{\partial^2 I(\mu_1, \sigma_1)}{\partial \sigma_1^2}|_{\sigma_1 = \underline{\sigma}} > 0$ and $1-d^2 > 0$, $\sigma_1^{C'} - \sigma_1^C > 0$.

D. Proof of Lemma 2: Because $\frac{\partial E(\Pi_1^B)}{\partial \sigma_1} = \frac{1}{(1-d^2)(2-d^2)^2} > 0$, $\frac{\partial E(\Pi_1^B)}{\partial \sigma_0} = \frac{d^2}{(1-d^2)(2-d^2)^2} > 0$, $\frac{\partial E(\Pi_0^B)}{\partial \sigma_0} = -\frac{d^2}{(1-d^2)(2-d^2)} < 0$, $\frac{\partial E(\Pi_0^B)}{\partial \sigma_1} = -\frac{d^2}{(1-d^2)(2-d^2)} < 0$, $\frac{\partial E(CS^B)}{\partial \sigma_0} = \frac{4-7d^2+3d^4}{2(1-d^2)^2(2-d^2)^2} > 0$ and $\frac{\partial E(CS^B)}{\partial \sigma_1} = \frac{1-2d^4+d^6}{2(1-d^2)^2(2-d^2)^2} > 0$.

E. Proof of Proposition 3: Using equations (36) and (37), $h(\sigma_0^B) - h(\sigma_1^B) = \frac{2-7d^2+7d^4-2d^6}{2(1-d^2)^2(2-d^2)^2}$. There is $\hat{\sigma}$ $(\hat{\sigma} > 0)$ making $(\sigma_0^B - \sigma_1^B)h'(\hat{\sigma}) = \frac{2-7d^2+7d^4-2d^6}{2(1-d^2)^2(2-d^2)^2}$. There is $d^* \in (0,1)$ $(d^* \approx 0.7071)$ making $2-7d^2+7d^4-2d^6 > 0$ if $0 < d < d^*$, and making $2-7d^2+7d^4-2d^6 < 0$ if $d^* < d < 1$. Moreover, $h'(\hat{\sigma}) = \frac{\partial^2 I(\mu_i, \sigma_i)}{\partial \sigma_i^2}|_{\sigma_i = \hat{\sigma}} > 0$, $1-d^2 > 0$ and $2-d^2 > 0$. Thus, $\sigma_0^B - \sigma_1^B > 0$ when $0 < d < d^*$, and $\sigma_0^B - \sigma_1^B < 0$ when $d^* < d < 1$.

F. Proof of Proposition 4: Using equations (36) and (39), $h(\sigma_1^{B'}) - h(\sigma_1^B) = \frac{1-4d^2+4d^4-d^6}{2(1-d^2)^2(2-d^2)^2}$. There is $\underset{\sim}{\sigma}$ $(\underset{\sim}{\sigma} > 0)$ making $(\sigma_1^{B'} - \sigma_1^B)h'(\underset{\sim}{\sigma}) = \frac{1-4d^2+4d^4-d^6}{2(1-d^2)^2(2-d^2)^2}$. There is $d^{\#} \in (0,1)$ $(d^{\#} \approx 0.6180)$ making $1-4d^2+4d^4-d^6 > 0$ if $0 < d < d^{\#}$, and making $1-4d^2+4d^4-d^6 < 0$ if $d^{\#} < d < 1$. Moreover, $h'(\underset{\sim}{\sigma}) = \frac{\partial^2 I(\mu_1, \sigma_1)}{\partial \sigma_1^2}|_{\sigma_1 = \underset{\sim}{\sigma}} > 0$, $1-d^2 > 0$ and $2-d^2 > 0$. Thus, $\sigma_1^{B'} - \sigma_1^B > 0$ when $0 < d < d^{\#}$, and $\sigma_1^{B'} - \sigma_1^B < 0$ when $d^{\#} < d < 1$.

G. Proof of Proposition 5: Using equations (20) and (36), $h(\sigma_1^C) - h(\sigma_1^B) = -\frac{d^2}{(1-d^2)(2-d^2)^2}$. There is $\tilde{\sigma}$ $(\tilde{\sigma} > 0)$ making $(\sigma_1^C - \sigma_1^B)h'(\tilde{\sigma}) = -\frac{d^2}{(1-d^2)(2-d^2)^2} < 0$. Moreover, $h'(\tilde{\sigma}) = \frac{\partial^2 I(\mu_1, \sigma_1)}{\partial \sigma_1^2}|_{\sigma_1 = \tilde{\sigma}} > 0$. Thus, $\sigma_1^C - \sigma_1^B < 0$.

Using equations (21) and (37), $h(\sigma_0^C) - h(\sigma_0^B) = -\frac{d^4}{2(1-d^2)(2-d^2)^2}$. There is $[ssmile]\sigma$ $([ssmile]\sigma > 0)$ making $(\sigma_0^C - \sigma_0^B)h'([ssmile]\sigma) = -\frac{d^4}{2(1-d^2)(2-d^2)^2} < 0$. Moreover, $h'([ssmile]\sigma) = \frac{\partial^2 I(\mu_0, \sigma_0)}{\partial \sigma_0^2}|_{\sigma_0 = [ssmile]\sigma} > 0$. Thus, $\sigma_0^C - \sigma_0^B < 0$.