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Prioritization of low-carbon suppliers based on Pythagorean fuzzy group decision making with self-confidence level

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ABSTRACT
Business decisions often require economic analysis involving uncertainties. This study brings forward the multi-attribute group decision making (MAGDM) framework based upon Pythagorean fuzzy (PF) sets with self-confidence of decision makers. By incorporating the ideas of the order-inducing variables of the induced ordered weighted averaging (IOWA) operator, we propose two PF confidence aggregation methods, namely PF confidence induced ordered weighted averaging (PFCIOWA) operator and PF confidence induced hybrid weighted averaging (PFCIHWA) operator. The focal property of the devised operators is their ability to take into consideration both the evaluation data and its corresponding confidence levels. Moreover, a MAGDM method based on the developed operators is adopted. Finally, the practicality of the method is tested by using low carbon supplier selection problems. The new approach is compared against the existing ones in order to check its applicability and validity. As an empirical case, the low carbon supplier selection problem is solved.

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Pythagorean fuzzy set; self-confidence level; IOWA operator; MAGDM; low carbon supplier selection

1. Introduction

MAGDM is a widely used tool in human activity, whose main purpose is to make a choice among a finite set of available alternatives using the preference information submitted by multiple experts (decision makers). However, the process of MAGDM tends to be ambiguous and inaccurate as it involves complexity of human cognitive thinking, which makes it difficult for decision makers or experts to give precise evaluations or preference information in the evaluation process. To handle these problems, intuitionistic fuzzy set (IFS) proposed by Atanassov (1986) can be considered as an appealing tool to deal with the fuzziness and inaccuracy in the data. Given the ability to handle different types of uncertain information, theoretical development and empirical applications of IFS have seen increasing spread across the domains. To

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date, it has been widely applied in economics, management and many other fields. Especially, Sirbiladze and Badagadze (2017) developed several probabilistic aggregation operators for IFS and studied their application in supplier selection problems. Büyüközkan and Göçer (2017) investigated the axiomatic design method for supplier selection problems under the IFS environment. Recently, Yu and Liao (2016) conducted a detailed scientometric review on the development of IFS from various perspectives. Based on the study by Yu and Liao (2016), Liu and Liao (2017) carried out an in-depth bibliometric survey of the decision-making process involving intuitionistic fuzzy and other types of fuzzy information theories covering the years 1970 ∼ 2015.

Considering the trends outlined in the literature review above, the usefulness and powerfulness of the IFS can be clearly described. However, the real-life applications involve situations where the sum of the degree of membership (μ) and the degree of non-membership (ν) of an IF element given by experts or decision makers is higher than unity, yet square of the sum is still lower than or equal to unity. In these cases, IFS is not suitable for use to describe experts' or decision makers' evaluation or preference information. In order to overcome this shortcoming, the Pythagorean fuzzy set (PFS) proposed by Yager (2014) can be considered as a useful extension of IFS. More specifically, the restriction on the sum of the two parameters (degrees) is changed from $0 \leq \mu + \nu \leq 1$ into $\mu^2 + \nu^2 \leq 1$. Based on the relaxed constraint, the PFS is more flexible than IFS because the PFS is able to depict imprecise and ambiguous information that the latter cannot. Consequently, the PFS theory has been perceived as an appropriate tool to handle MAGDM problems with uncertainty (Garg, 2017). Thus, a number of MAGDM approaches and techniques under PFS environment are developed, for example, Zhang and Xu (2014) extended the traditional TOPSIS technique to handle the PF MAGDM problems. Zhang (2016a) developed a closeness indicator for ranking the Pythagorean fuzzy numbers (PFNs). Yet another ranking method was devised by Peng and Yang (2015) to facilitate PF MAGDM problems. Zhang (2016b) proposed a new ranking method and a novel similarity measure for PFNs. Later, Peng and Yang (2016) defined several Choquet integral PF operators. Zeng, Chen, and Li (2016) proposed a hybrid PF TOPSIS method based on distance measure and aggregation operator. Chen (2018) introduced novel VIKOR (Vlse Kriterijumska Optimizacija IKompromisno Resenje)-based methods for PF multiple criteria decision problems. Zeng (2017) presented a PF MAGDM method based on probabilistic information and the ordered weighted averaging (OWA) (Yager, 1988) approach. Peng and Dai (2017) proposed the stochastic PF decision making framework built upon prospect theory and regret theory. Wei (2017) developed some PF interaction aggregation operators and studied their application.

Obviously, there have been a number of extensions proposed for PF aggregation methods and decision making frameworks. However, it can be noted that all the above mentioned studies do not consider the information regarding the self-confidence associated with the ratings submitted by the decision makers (or experts). In other words, it is implicitly assumed that the decision makers (or experts) feature uniform levels of familiarity of the attributes describing the assessed objects. But in real-life situations, decision makers (or experts) often have different academic and professional backgrounds, which will result in their differences and inconsistencies for the evaluated object (Liu, Dong, Chiclana, Cabrerizo, & Herrera-Viedma, 2017; Xia, Xu, & Chen, 2011; Yu, 2014). Thus it is important to consider that the preference values are provided with the different self-confidence levels of decision makers. In order to overcome this drawback and enhance the scientific validity of evaluation, decision makers (or experts) may provide the preference information on the objects under evaluation along with the corresponding confidence levels defining the degrees to which they are confident with the ratings they provided given the circumstances surrounding the evaluation. On the other hand, addressing the vagueness and imprecision by using the powerful PFS in decision making process concerning the low carbon supplier evaluation problem is an interesting topic. Considering the reasons discussed above, this paper proposes two new PF aggregation methods, viz. PFCIOWA operator and PFCIHWA operator, to aggregate information in the evaluation of a set of alternatives when information is expressed in PFNs and supplemented with confidence levels. Moreover, a MAGDM method based on the proposed operators is developed. Finally, the practicality of the method is tested by using low-carbon supplier selection problems.

The paper further proceeds in the following manner. Preliminaries for the IF, PFS theory and IOWA operator are discussed in Section 2. The PFCIOWA and PFCIHWA operators are developed in Section 3. Section 4 proposes a MAGDM method involving the developed operators. Next, Section 5 embarks on an illustrative numerical example for realising the practicality of the proposed methodology. Finally, Section 6 puts the concluding remarks alongside directions for further research.

2. Preliminaries

Definition 1. Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse, an IFS I in X is defined in the following way:

$$I = \left\{ \langle x, (\mu_I(x), \nu_I(x)) \rangle | x \in X \right\}. \tag{1}$$

where functions $\mu_I(x)$ and $\nu_I(x)$ are termed the membership degree and non-membership degree, respectively, defining to which extent element x belongs to the set I. The degrees of membership and non-membership are restricted so as to maintain $0 \le \mu_I(x) + \nu_I(x) \le 1$, and $\pi_I(x) = 1 - \mu_I(x) - \nu_I(x)$ is termed the degree of hesitancy of x to the set I.

Yager (2014) offered the PFS as a generalization of the IFS which basically extends the domain of the parameters defining a certain IFS. More specifically, the sum of squares of the parameters is restricted rather than just the sum of the parameters. Formally, the PFS is defined as follows:

Definition 2. Assuming $X = \{x_1, x_2, \dots, x_n\}$ is a fixed set, a certain PFS P is defined in the following manner:

$$P = \left\{ \langle x, (\mu_P(x), \nu_P(x)) \rangle | x \in X \right\}. \tag{2}$$

where functions $\mu_P(x)$ and $\nu_P(x)$ represent the degrees of membership and non-membership, respectively, for element x to set P. The degrees of membership and non-membership are restricted so that condition $0 \le (\mu_P(x))^2 + (\nu_P(x))^2 \le 1$ is maintained. The degree of indeterminacy for x to set P is calculated residually as $\pi_P(x) = \sqrt{1-(\mu_P(x))^2-(\nu_P(x))^2}$.

A shorthand notation has been proposed by Zhang and Xu (2014): a Pythagorean fuzzy number (PFN) is a collection of two parameters $\alpha = (\mu_\alpha, \nu_\alpha)$ such that $\mu_\alpha \in [0,1]$, $\nu_\alpha \in [0,1]$ and $(\mu_\alpha)^2 + (\nu_\alpha)^2 \leq 1$. Furthermore, some basic operation laws and comparison method of PFNs also were introduced.

Definition 3. The operations for arbitrarily given PFNs $\alpha = (\mu_\alpha, \nu_\alpha), \alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ are defined as following:

1. $\alpha_1 \oplus \alpha_2 = (\sqrt{\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2 \cdot \mu_{\alpha_2}^2}, \nu_{\alpha_1} \cdot \nu_{\alpha_2})$;
2. $\alpha_1 \otimes \alpha_2 = (\mu_{\alpha_1} \cdot \mu_{\alpha_2}, \sqrt{\nu_{\alpha_1}^2 + \nu_{\alpha_2}^2 - \nu_{\alpha_1}^2 \cdot \nu_{\alpha_2}^2})$;
3. $\lambda \cdot \alpha = (\sqrt{1-(1-\mu_\alpha^2)^\lambda}, (\nu_\alpha)^\lambda)$, $\lambda>0$;
4. $\alpha^\lambda = ((\mu_\alpha)^\lambda, \sqrt{1-(1-\nu_\alpha^2)^\lambda})$, $\lambda>0$.

Multi-criteria decision making involves ranking of alternatives. In case information is expressed in the PFNs, certain ranking rules of such numbers are needed. The following definition presents an instance of these rules:

Definition 4. Following Zhang and Xu (2014), a certain PFN $\alpha = (\mu_\alpha, \nu_\alpha)$ can be associated with score and accuracy functions which are obtained as $s(\alpha) = (\mu_\alpha)^2 - (\nu_\alpha)^2$ and $h(\alpha) = (\mu_\alpha)^2 + (\nu_\alpha)^2$ respectively. Therefore, any two PFNs $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ can be compared: if $s(\alpha_1)>s(\alpha_2)$, then $\alpha_1>\alpha_2$; if $s(\alpha_1) = s(\alpha_2)$, then $\begin{cases} h(\alpha_1)<h(\alpha_2) \Rightarrow \alpha_1<\alpha_2 \\ h(\alpha_1)>h(\alpha_2) \Rightarrow \alpha_1>\alpha_2 \end{cases}$.

The operational rules above have been applied in a number of frameworks which allowed for construction of different PF aggregation operators. Among these, Yager (2014) introduced the Pythagorean fuzzy weighted averaging (PFWA) operator which has been a celebrated tool for handling PF information. The definition below discusses the operator:

Definition 5. For a certain collection of PFNs $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j}), j = 1, 2, \ldots, n,$ an n-dimensional PFWA operator is defined as a mapping $PFWA{:}\Omega^n \to \Omega$ (Ω is the set of all PFNs) which considers weighting vector W with its elements restricted by $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$:

$$PFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{j=1}^{n} w_j \alpha_j = \left(\sqrt{1 - \prod_{j=1}^{n} \left(1-\mu_{\alpha_j}^2\right)^{w_j}}, \prod_{j=1}^{n} \nu_{\alpha_j}^{w_j} \right). \tag{3}$$

Yager and Filev (1999) introduced the IOWA operator which aggregates information by re-ordering the arguments according to order-inducing variables. Contrasted to the OWA operator, the IOWA operator does not consider the values of the arguments during the aggregation process. Instead, the order-inducing variables provide a rule for re-ordering of and attaching corresponding weights to the arguments. Due to its suitability for decision analysis, applications of the IOWA operator can be found across different areas of performance management and frameworks involving multiple decision making approaches (Aggarwal, 2015; Merigó & Gil-Lafuente, 2013; Xian, Zhang, & Xue, 2016; Zeng, Merigó, Palacios-Marqués, Jin, & Gu, 2017; Zhou, Tao, Chen, & Liu, 2014). The IOWA operator can be defined in the following manner:

Definition 6. An n-dimensional IOWA operator maps a collection of real numbers to a single real number IOWA:$R^n \times R^n \rightarrow R$ (R is the set of real numbers) with respect to an associated weighting vector W with its elements bounded according to $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{4}$$

where b_j actually represents a certain value a_i from the IOWA pair $\langle u_i, a_i \rangle$ featuring the j-th largest u_i, note that the order inducing variable u_i and the argument variable a_i are the values presented as the initial decision information.

Following the ideal of the IOWA operator, Xu, Yu, Zeng, and Liu (2017) introduced the Pythagorean fuzzy induced ordered weighted averaging (PFIOWA) operator.

Definition 7. For a certain collection of PFNs $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j})(j = 1, 2, \ldots, n)$, a PFIOWA operator is defined by a weighting vector $W = (w_1, \ldots, w_n)$ and an order-inducing vector $U = (u_1, \ldots, u_n)$, such that:

$$PFIOWA(\langle u_1, \alpha_1 \rangle, \langle u_2, \alpha_2 \rangle, \ldots, \langle u_n, \alpha_n \rangle) = \sum_{j=1}^{n} w_j \beta_j \tag{5}$$

where $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$, $(\beta_1, \ldots, \beta_n)$ is recorded $(\alpha_1, \ldots, \alpha_n)$ as per decreasing order of $(u_1, \ldots, u_n)$.

3. Pythagorean fuzzy information aggregation method with confidence level

The order-induced variables used for the IOWA operator can be constructed on either cardinal or ordinal scale, and are often used to represent any property related to each alternative-criterion pair, such as confidence levels, importance, or consistency of an alternative (Yager & Filev, 1999). Following this kind of reasoning allows us to directly construct the Pythagorean fuzzy confidence level aggregation method taking the PFIOWA operator as the underlying aggregation principle. However, the role of the order-induced variables in IOWA and PFIOWA operators is limited to establishing the order of arguments to be aggregated, which often results in the loss of information related to inherent changes between the order-inducing variables. More specifically, the differences in the magnitude of the order-inducing variables do not affect the aggregation results. So we shall propose a new confidence aggregation operator with PF information, named the PFCIOWA operator, which can not only consider the confidence level represented with order-induced variables in the actual aggregation step, but also aggregate the PF information. Its definition is given below.

Definition 8. Let there be a collection of PFNs $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j}), j = 1, 2, \ldots, n$. Further on, let l_j be the confidence levels associated with each PFN α_j such that $0 \leq l_j \leq 1$. An n-dimensional PFCIOWA operator is then defined as a mapping PFCIOWA:$R^n \times \Omega^n \rightarrow \Omega$ (R and Ω are the sets of real numbers and PFNs, respectively) that applies weighting vector W with its elements satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$PFCIOWA(\langle l_1, \alpha_1 \rangle, \langle l_2, \alpha_2 \rangle, \ldots, \langle l_n, \alpha_n \rangle) = \sum_{j=1}^{n} w_j l_{\sigma(j)} \alpha_{\sigma(j)}. \tag{6}$$

where $(\sigma(1), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ carried out with respect to the confidence levels such that $l_{\sigma(j-1)} \geq l_{\sigma(j)}$ for any j, and $\alpha_{\sigma(j)}$ is a PFN corresponding to the confidence level $l_{\sigma(j)}$, $j = 1, 2, \ldots, n$.

Next we take up an example to illustrate the aggregation process based on the PFCIOWA operator.

Example 1. Let the PFNs and confidence levels to be aggregated in the form $\langle l_i, \alpha_i \rangle$ be$\langle 0.9, (0.3, 0.8) \rangle$, $\langle 0.5, (0.6, 0.6) \rangle$, $\langle 0.6, (0.4, 0.7) \rangle$, $\langle 0.8, (0.7, 0.2) \rangle$ and the weighting vector is $W = (0.3, 0.2, 0.1, 0.4)$. We first record the confidence levels and have:

$$l_{\sigma(1)} = l_1 = 0.9, \; l_{\sigma(2)} = l_4 = 0.8, \; l_{\sigma(3)} = l_3 = 0.6, \; l_{\sigma(4)} = l_2 = 0.5.$$

Thus,

$$\begin{aligned} \alpha_{\sigma(1)} &= \alpha_1, \\ \alpha_{\sigma(2)} &= \alpha_4, \\ \alpha_{\sigma(3)} &= \alpha_3, \\ \alpha_{\sigma(4)} &= \alpha_2, \end{aligned}$$

So, by Eq. (6), we get the final aggregation result:

$$\begin{gathered} PFCIOWA\big(\langle 0.9, (0.3, 0.8) \rangle, \langle 0.5, (0.6, 0.6) \rangle, \langle 0.6, (0.4, 0.7) \rangle, \langle 0.8, (0.7, 0.2) \rangle\big) \\ = 0.3 \times 0.9 \otimes (0.3, 0.8) \oplus 0.2 \times 0.8 \otimes (0.7, 0.2) \oplus \\ \oplus 0.1 \times 0.6 \otimes (0.4, 0.7) \oplus 0.4 \times 0.5 \otimes (0.6, 0.6) \\ = (0.456, 0.643). \end{gathered}$$

Differently from the IOWA and the PFIOWA operators where the order-inducing variables simply decide the ordering of the arguments, the PFCIOWA operator uses the information provided by the confidence indicators when constructing the weights. Also, the information expressed in terms of PFNs can by aggregated which makes the PFCIOWA operator suitable for handling uncertain information.

Considering the operational rules defined for the PFNs, one can prove that the PFCIOWA operator features the desirable properties for aggregation operators. These include monotonicity, idempotency, boundedness and independence on permutation. The theorems below prove the aforementioned properties for the PFCIOWA operator.

Theorem 1. (Monotonicity). Let there be the two collections of PFNs denoted by α_j and $\beta_j, j = 1, 2, \ldots, n$. Further on, let there be $\alpha_j \leq \beta_j$ for all j, then

$$PFCIOWA(\langle l_1, \alpha_1 \rangle, \ldots, \langle l_n, \alpha_n \rangle) \leq PFCIOWA(\langle l_1, \beta_1 \rangle, \ldots, \langle l_n, \beta_n \rangle). \tag{7}$$

Theorem 2. (Idempotency). If all PFNs $\alpha_j, j = 1, 2, \ldots, n$, are equal, i.e. $\alpha_j = \alpha$ for all j, then

$$PFCIOWA(\langle l_1, \alpha_1 \rangle, \ldots, \langle l_n, \alpha_n \rangle) = \alpha. \tag{8}$$

Theorem 3. (Boundedness). The PFCIOWA operator returns the aggregate value which lies in between the values provided by max and min operators, that is

$$\min(\alpha_1, \ldots, \alpha_n) \leq PFCIOWA(\langle l_1, \alpha_1 \rangle, \ldots, \langle l_n, \alpha_n \rangle) \leq \max(\alpha_1, \ldots, \alpha_n). \tag{9}$$

Theorem 4. (Independence on permutation). Say a collection of PFNs $(\langle l_1, \beta_1 \rangle, \ldots, \langle l_n, \beta_n \rangle)$ is any permutation of another collection of PFNs $(\langle l_1, \alpha_1 \rangle, \ldots, \langle l_n, \alpha_n \rangle)$, $j = 1, 2, \ldots, n$, then

$$PFCIOWA(\langle l_1, \alpha_1 \rangle, \ldots, \langle l_n, \alpha_n \rangle) = PFCIOWA(\langle l_1, \beta_1 \rangle, \ldots, \langle l_n, \beta_n \rangle). \tag{10}$$

Note that the proofs of these theorems are straightforward and we, thus, omit them here for the sake of brevity.

Obviously, the PFCIOWA operator only weights the induced order arguments, but ignores the importance of the arguments themselves. Next we introduce the PFCIHWA operator, in which the weights of the induced order arguments and themselves are all taken into account.

Definition 9. Say there is a collection of PFNs $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j}), j = 1, 2, \ldots, n$. Then, the n-dimensional PFCIHWA operator is defined as a mapping $PFCIHWA: R^n \times \Omega^n \rightarrow \Omega$ (R and Ω are the sets of real numbers and PFNs, respectively) that has an associated weight vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$PFCIHWA(\langle l_1, \alpha_1 \rangle, \langle l_2, \alpha_2 \rangle, \ldots, \langle l_n, \alpha_n \rangle) = \sum_{j=1}^{n} w_j l_{\sigma(j)} \beta_{\sigma(j)}, \tag{11}$$

where $\beta_{\sigma(j)} = n\omega_j\alpha_{\sigma(j)}$, $\alpha_{\sigma(j)}$ is the PFN corresponding to the confidence level $l_{\sigma(j)}$, $j = 1, 2, \ldots, n$, ω_j is the corresponding weight of argument PFN $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j}), j = 1, 2, \ldots, n$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$,$n$ is balancing coefficient, ensuring balance between the arguments aggregated and the result of aggregation.

The example below is provided to give an intuition on the aggregation of a set of PFNs according to the rules of the proposed operator.

Example 2. Three experts (whose weights vector is $\omega = (0.4, 0.5, 0.1)^T$) give their PFNs evaluations and corresponding confidence levels in the form $\langle 0.7, (0.5, 0.8)\rangle$, $\langle 0.9, (0.7, 0.7)\rangle$ and $\langle 0.5, (0.6, 0.3)\rangle$, respectively. We first record the confidence levels and have:

$$l_{\sigma(1)} = 0.9, l_{\sigma(2)} = 0.7, l_{\sigma(3)} = 0.5,$$

In this example, the balancing coefficient is $n = 3$ and based on the operational laws of PFNs, we have

$$\beta_{\sigma(1)} = n\omega_1\alpha_{\sigma(1)} = \left(\sqrt{1-(1-0.7^2)^{3\times0.5}}, 0.7^{3\times0.5}\right) = (0.797, 0.586),$$
$$\beta_{\sigma(2)} = n\omega_2\alpha_{\sigma(2)} = \left(\sqrt{1-(1-0.5^2)^{3\times0.4}}, 0.8^{3\times0.4}\right) = (0.540, 0.765),$$
$$\beta_{\sigma(3)} = n\omega_3\alpha_{\sigma(3)} = \left(\sqrt{1-(1-0.6^2)^{3\times0.1}}, 0.3^{3\times0.1}\right) = (0.354, 0.697),$$

Suppose the underlying vector of weight coefficients for the PFCIHWA operator is given by $W = (0.2, 0.5, 0.3)^T$. Therefore, by applying the Eq. (11), we get

$$PFCIHWA(\langle 0.7, (0.5, 0.8)\rangle, \langle 0.9, (0.7, 0.7)\rangle, \langle 0.5, (0.6, 0.3)\rangle) = (0.525, 0.783).$$

Given the principles outlined in Definition 9 and Example 2, it can be concluded that that the PFCIHWA operator comprises the three main stages of data processing: first, the initial data are weighted by means of the pre-specified weights reflecting the importance of the arguments, the data are then re-ordered according to order-inducing variables and, finally, the weights are applied upon the re-ordered arguments to reflect the importance of the positions of the arguments as well as the associated levels of confidence. Once the data are processed, they can be aggregated into a single value (a PFN). Therefore, the PFCIHWA operator simultaneously captures the difference in the importance of arguments and their positions defined by the order-inducing variables. Later, we are to examine the relationship between the PFCIHWA and PFCIOWA operators.

Theorem 5. The PFCIOWA operator is a particular case of the PFCIHWA operator.

Proof. Let the weights of the initial data be uniform, i.e. $\omega = (1/n, 1/n, \ldots, 1/n)^T$, then

$$\begin{aligned} PFCIHWA(\langle l_1, \alpha_1 \rangle, \dots, \langle l_n, \alpha_n \rangle) &= \sum_{j=1}^{n} w_j l_{\sigma(j)} \beta_{\sigma(j)} \\ &= \sum_{j=1}^{n} w_j l_{\sigma(j)} (n\omega_j \alpha_{\sigma(j)}) \\ &= \sum_{j=1}^{n} w_j l_{\sigma(j)} \alpha_{\sigma(j)} \\ &= PFCIOWA(\langle l_1, \alpha_1 \rangle, \dots, \langle l_n, \alpha_n \rangle) \end{aligned}.$$

which proves Theorem 5.

Note that if there exists no differences in confidence levels, i.e. $l_1 = l_2 = \dots = l_n = l$, then the PFCIHWA operator boils down to the PF combined weighted average (PFCWA) operator, which can be defined in the following way:

$$PFCWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \sum_{j=1}^{n} w_j \beta_j, \tag{12}$$

where $\beta_j = n\omega_j \alpha_j$, ω_j stands for the corresponding weight of argument PFN $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j}), j = 1, 2, \dots, n$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$,$n$ is balancing coefficient, which ensures the balance between arguments and the result of aggregation. Assuming equal importance of the arguments, we set $\omega = (1/n, 1/n, \dots, 1/n)^T$, then the PFCWA operator boils down to the PFWA operator. Thus, we showed that the PFCIOWA, PFCWA and PFWA operators can all be derived as special cases of the PFCIHWA operator. Moreover, a number of instances of aggregation operators can be devised by modifying the PFCIHWA operator in the spirit of the recent studies on aggregation operators (Liu & Peng, 2017; Merigó, Guillén, & Sarabia, 2015; Wang, Wang, & Li, 2016; Zeng, 2016; Zeng et al., 2016).

4. Pythagorean fuzzy MAGDM model based on the PFCIHWA operator

In this section, we will focus on studying the application of the PFICHWA operator in a MAGDM problems. Consider a MAGDM problem which involves m alternatives denoted as $A = \{A_1, A_2, \dots, A_m\}$ and compared against n attributes denoted as $C = \{C_1, C_2, \dots, C_n\}$ with weights arranged into vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ satisfying $\omega_k \geq 0$ and $\sum_{k=1}^{n} \omega_k = 1$. Furthermore, let there be multiple decision makers (experts) denoted by set $E = \{e_1, e_2, \dots, e_t\}$ and associated with the corresponding weights arranged into weight vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_t)^T$, $\lambda_k \geq 0$, $\sum_{k=1}^{t} \lambda_k = 1$. The process can be outlined in terms of the following steps:

Step 1. The ratings and the information on the confidence are provided by each expert in regard to alternative A_i against attribute C_j. In this way, the individual PF decision matrix is established $(\langle l_{ij}^{(k)}, \alpha_{ij}^{(k)} \rangle)_{m \times n}$ for each $k = 1, 2, \dots, t$, where $\alpha_{ij}^{(k)}$ denotes the rating expressed in PFNs: $\alpha_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)})$, and $l_{ij}^{(k)}$ is the corresponding confidence level for $\alpha_{ij}^{(k)}$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

Step 2. The individual decision matrices $(\langle l_{ij}^{(k)}, \alpha_{ij}^{(k)} \rangle)_{m \times n}$, $k = 1, 2, \dots, t$, are then summarised into the aggregate PF decision matrix $(\alpha_{ij})_{m \times n}$, where

$$\alpha_{ij} = PFCIHWA\left(\langle l_{ij}^{(1)}, \alpha_{ij}^{(1)}\rangle, \langle l_{ij}^{(2)}, \alpha_{ij}^{(2)}\rangle, \ldots, \langle l_{ij}^{(t)}, \alpha_{ij}^{(t)}\rangle\right). \tag{13}$$

Step 3. The PF utility scores are calculated for each alternative by applying the PFWA operator on the PF aggregate ratings α_{ij}, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$. The utility score for the i-th alternative α_i $(i = 1, 2, \ldots, m)$ is obtained as:

$$\alpha_i = PFWA(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}). \tag{14}$$

Step 4. The alternatives are ranked and the best one is identified by applying the score and accuracy functions given in Definition 4 on each α_i, $i = 1, 2, \ldots, m$. The higher values imply more preferable alternatives.

5. An application for the choice of low-carbon supplier

Environmental considerations have become more important across different regions and sectors due to the global climate change (Anastasiadis, Konstantinopoulos, Kondylis, Vokas, & Salame, 2018; Zhao et al., 2017). In recent years, much attention has been paid to analysis of the low carbon supply chain amid the pursuit for curbing the global warming and environmental protection awareness (Govindan & Sivakumar, 2016; Krishnendu, Ravi, Surendra, & Lakshman, 2012; Rao, Xiao, Xie, Goh, & Zheng, 2017). The creation of sustainable supply chain and subsequent reduction of the environmental pressures highly rely on identifying and selecting suitable low-carbon suppliers. Generally speaking, the supplier selection process can be represented by a MADM problem because discordant and numerous attributes should be considered and evaluated in the decision process (Qin, Liu, & Pedrycz, 2017; Song, Xu, & Liu, 2017; Wan, Xu, & Dong, 2017). So far, low carbon supplier evaluation and selection has been implemented by means of the MADM methods, yet it was often assumed that the attribute information is certain and precise (Asadabadi, 2017; Davood, Seyed, & Ashkan, 2016; Jain, Panchal, & Kumar, 2014). However, the rapid development of the economy and complex commercial environment makes it difficult for decision makers to give precise evaluations or preference information due to the ambiguity of human thinking involved. Recently, the induced intuitionistic ordered weighted averaging (IIOWA) operator was applied by Tong and Wang (2016) when solving low carbon supplier selection problem involving intuitionistic preference information. As mentioned in the Introduction, the relaxed restrictions on the (non-)membership degrees of the PFSs allow for an extended domain thus making PFS superior to intuitionistic fuzzy set (IFS) in describing imprecise and ambiguous information. Therefore, it is interesting and necessary to investigate the low carbon supplier selection problem in the PF situation.

To illustrate the possibilities for solving the low carbon supplier problem by means of reasoning based on the PFSs, we consider the following MADM problem. The proposed framework relies on the principles outlined in Section 4. Let us assume there are a group of experts who are to provide ratings for different suppliers in regard to the objectives of the low carbon supply chain. Specifically, let there be the four potential suppliers denoted by A_i, $i = 1, 2, \ldots, 4$, and the five

Table 1. PF decision matrix for expert 1.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.5, (0.6, 0.6)\rangle$	$\langle 0.7, (0.8, 0.6)\rangle$	$\langle 0.8, (0.4, 0.7)\rangle$	$\langle 0.5, (0.8, 0.1)\rangle$	$\langle 0.3, (0.9, 0.2)\rangle$
A_2	$\langle 0.4, (0.5, 0.7)\rangle$	$\langle 0.7, (0.3, 0.8)\rangle$	$\langle 0.5, (0.7, 0.7)\rangle$	$\langle 0.6, (0.4, 0.8)\rangle$	$\langle 0.2, (0.5, 0.4)\rangle$
A_3	$\langle 0.2, (0.5, 0.4)\rangle$	$\langle 0.6, (0.6, 0.4)\rangle$	$\langle 0.8, (0.7, 0.6)\rangle$	$\langle 0.5, (0.5, 0.7)\rangle$	$\langle 0.4, (0.9, 0.2)\rangle$
A_4	$\langle 0.7, (0.8, 0.3)\rangle$	$\langle 0.7, (0.3, 0.4)\rangle$	$\langle 0.5, (0.4, 0.5)\rangle$	$\langle 0.3, (0.4, 0.6)\rangle$	$\langle 0.5, (0.6, 0.6)\rangle$

Table 2. PF decision matrix for expert 2.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.6, (0.3, 0.8)\rangle$	$\langle 0.8, (0.4, 0.5)\rangle$	$\langle 0.7, (0.2, 0.6)\rangle$	$\langle 0.5, (0.7, 0.2)\rangle$	$\langle 0.4, (0.8, 0.1)\rangle$
A_2	$\langle 0.7, (0.6, 0.2)\rangle$	$\langle 0.8, (0.4, 0.4)\rangle$	$\langle 0.4, (0.3, 0.6)\rangle$	$\langle 0.6, (0.7, 0.5)\rangle$	$\langle 0.3, (0.6, 0.2)\rangle$
A_3	$\langle 0.3, (0.7, 0.4)\rangle$	$\langle 0.8, (0.9, 0.2)\rangle$	$\langle 0.9, (0.9, 0.1)\rangle$	$\langle 0.7, (0.3, 0.8)\rangle$	$\langle 0.6, (0.7, 0.4)\rangle$
A_4	$\langle 0.8, (0.5, 0.4)\rangle$	$\langle 0.6, (0.8, 0.1)\rangle$	$\langle 0.8, (0.7, 0.1)\rangle$	$\langle 0.6, (0.2, 0.7)\rangle$	$\langle 0.7, (0.3, 0.5)\rangle$

Table 3. PF decision matrix for expert 3.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.3, (0.5, 0.6)\rangle$	$\langle 0.8, (0.4, 0.7)\rangle$	$\langle 0.4, (0.7, 0.4)\rangle$	$\langle 0.4, (0.9, 0.3)\rangle$	$\langle 0.6, (0.6, 0.5)\rangle$
A_2	$\langle 0.7, (0.7, 0.3)\rangle$	$\langle 0.3, (0.7, 0.1)\rangle$	$\langle 0.8, (0.8, 0.1)\rangle$	$\langle 0.7, (0.6, 0.2)\rangle$	$\langle 0.4, (0.4, 0.8)\rangle$
A_3	$\langle 0.6, (0.3, 0.8)\rangle$	$\langle 0.7, (0.8, 0.2)\rangle$	$\langle 0.6, (0.9, 0.1)\rangle$	$\langle 0.6, (0.4, 0.6)\rangle$	$\langle 0.7, (0.8, 0.1)\rangle$
A_4	$\langle 0.3, (0.9, 0.3)\rangle$	$\langle 0.6, (0.5, 0.5)\rangle$	$\langle 0.7, (0.3, 0.4)\rangle$	$\langle 0.4, (0.8, 0.3)\rangle$	$\langle 0.3, (0.4, 0.5)\rangle$

Table 4. The aggregate PF decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.533,0.637)	(0.568,0.598)	(0.672,0.209)	(0.803,0.172)	(0.787,0.189)
A_2	(0.638,0.315)	(0.429,0.481)	(0.737,0.289)	(0.568,0.449)	(0.537,0.350)
A_3	(0.562,0.527)	(0.832,0.196)	(0.837,0.238)	(0.393,0.697)	(0.797,0.206)
A_4	(0.779,0.310)	(0.672,0.209)	(0.496,0.256)	(0.687,0.402)	(0.511,0.534)

criteria: low carbon technology (C_1), cost (C_2), risk factor (C_3), capacity (C_4) and Economic efficiency (C_5). Due to uncertainties associated with the phenomenon under analysis, PFNs are used to express the ratings provided by the group of experts ($\lambda_1 = 0.35,\ \lambda_2 = 0.25,\ \lambda_3 = 0.4$). The resulting individual decision matrices are given in Tables 1–3.

The decision making process proceeds by aggregating the individual decision matrices into the collective one by exploiting the PFCIHWA operator. Let the underlying weight vector be $W = (0.243, 0.514, 0.243)^T$, as suggested by the normal distribution method (Xu, 2005). Table 4 presents the results.

In this problem, the five attributes have different importance as represented by the weight vector $\omega = (0.22, 0.18, 0.25, 0.15, 0.20)^T$. Based on this information, we can aggregate the collective information from Table 4 by employing the PFWA operator. The resulting PF utility for each alternativeA_i $(i = 1, 2, 3, 4)$ is given below:

$$\alpha_1 = (0.6884, 0.3072), \alpha_2 = (0.6144, 0.3583),$$
$$\alpha_3 = (0.7492, 0.3125), \alpha_4 = (0.6462, 0.3191).$$

Finally, the PF utility scores are defuzzified by applying the score function for each $\alpha_i, i = 1, 2, 3, 4$:

Table 5. The aggregate PF decision matrix under complete confidence.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.466,0.676)	(0.574,0.559)	(0.680,0.204)	(0.813,0.175)	(0.810,0.171)
A_2	(0.624,0.287)	(0.515,0.305)	(0.635,0.361)	(0.633,0.417)	(0.542,0.331)
A_3	(0.596,0.469)	(0.844,0.221)	(0.880,0.141)	(0.397,0.703)	(0.808,0.214)
A_4	(0.767,0.327)	(0.680,0.204)	(0.580,0.202)	(0.552,0.517)	(0.437,0.507)

$$s(\alpha_1) = 0.3976, s(\alpha_2) = 0.2490, s(\alpha_3) = 0.4637, s(\alpha_4) = 0.3157.$$

Since

$$s(\alpha_3) \succ s(\alpha_1) \succ s(\alpha_4) \succ s(\alpha_2),$$

we have

$$A_3 \succ A_1 \succ A_4 \succ A_2.$$

The latter ordering implies A_3 is the best candidate supplier.

We now reiterate the exercise by assuming that all the decision makers are equally confident about the ratings they provide during the decision making process, i.e. we set $l_{ij} = 1$. In this case, the PFCWA operator is used to integrate the views of three experts. The new collective decision matrix is presented in Table 5.

Given the new aggregate decision matrix, we re-calculate the utility of each alternative. The utility values $\alpha_i, i = 1, 2, 3, 4$ are obtained by exploiting the PFWA operator:

$$\alpha_1 = (0.6939, 0.3003), \alpha_2 = (0.5963, 0.3344),$$
$$\alpha_3 = (0.7782, 0.2755), \alpha_4 = (0.6303, 0.3114).$$

The score function is applied on PF utility scores $\alpha_i, i = 1, 2, 3, 4$:

$$s(\alpha_1) = 0.3912, s(\alpha_2) = 0.2438, s(\alpha_3) = 0.5297, s(\alpha_4) = 0.3004.$$

Given

$$s(\alpha_3) \succ s(\alpha_1) \succ s(\alpha_4) \succ s(\alpha_2),$$

we establish the following order of preference:

$$A_3 \succ A_1 \succ A_4 \succ A_2.$$

Again, alternative A_3 turns out to be the most preferable one. In this case, the results coincide with those rendered by the PFCIHWA operator. However, the scores attached to each alternative are different. The differences in the utility scores can be explained by the fact that the PFCIHWA operator considers the differences in confidence levels during the aggregation of ratings provided by the experts, whereas the PFCWA operator aggregates the ratings under assumption of 100% confidence in the assessment.

6. Conclusion

This study proposed the Pythagorean fuzzy MAGDM method which allows considering decision makers' confidence levels associated with the subjective ratings. By implementing the improvements in regard to information aggregation rules based on order-inducing variables in IWOA operator, two PF confidence aggregation operators, namely the PFCIOWA and PFCIHWA operators are developed. The desirable properties and different families of each operator have also been studied. It is observed that some existing PF aggregation operators, including the PFWA and the PFCIOWA operators are all special cases of the PFCIHWA operator proposed in this study. Moreover, a MAGDM method based on the PFCIHWA operator is presented to handle PF problems.

Finally, an illustrative example concerning low carbon supplier selection has been provided to demonstrate possibilities for application of the proposed method in supply chain management. The comparative analysis was conducted by assuming complete confidence and comparing the results to those under incomplete confidence. It is shown that the PFS is an effective tool to describe vague and uncertain information in MAGDM problems and inclusion of the information on the confidence alters the results. Altogether, the devised operators are relevant to complicated instances of MAGDM due to their ability to aggregate preference information expressed in PFNs with corresponding confidence levels. Inclusion of confidence levels indicated by experts themselves improves the robustness of the analysis. In future research, extensions of the proposed operators for different types of information and application of the presented operator in different areas will be analysed.

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References

Aggarwal, M. (2015). A new family of induced OWA operators. *International Journal of Intelligent Systems*, *30*(2), 170–205. doi:10.1002/int.21693

Anastasiadis, A., Konstantinopoulos, S., Kondylis, G., Vokas, G. A., & Salame, M. J. (2018). Carbon tax, system marginal price and environmental policies on smart microgrid operation. *Management of Environmental Quality: An International Journal*, *29*(1), 76–88. doi:10.1108/MEQ-11-2016-0086

Asadabadi, M. R. (2017). A customer based supplier selection process that combines quality function deployment, the analytic network process and a Markov chain. *European Journal of Operational Research*, *263*(3), 1049–1062. doi:10.1016/j.ejor.2017.06.006

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. doi:10.1016/S0165-0114(86)80034-3

Büyüközkan, G., & Göçer, F. (2017). Application of a new combined intuitionistic fuzzy MCDM approach based on axiomatic design methodology for the supplier selection problem. *Applied Soft Computing*, *52*, 1222–1238. doi:10.1016/j.asoc.2016.08.051

Chen, T. Y. (2018). Remoteness index-based Pythagorean fuzzy VIKOR methods with a generalized distance measure for multiple criteria decision analysis. *Information Fusion*, *41*, 129–150. doi:10.1016/j.inffus.2017.09.003

Davood, M., Seyed, H. G., & Ashkan, H. (2016). A game theoretic analysis in capacity-constrained supplier-selection and cooperation by considering the total supply chain inventory costs. *International Journal of Production Economics*, *181*, 87–97.

Garg, H. (2017). Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t- norm and t-conorm for multicriteria decision-making process. *International Journal of Intelligent Systems*, *32*(6), 597–630. doi:10.1002/int.21860

Govindan, K., & Sivakumar, R. (2016). Green supplier selection and order allocation in a low-carbon paper industry: integrated multi-criteria heterogeneous decision-making and multi-objective linear programming approaches. *Annals of Operations Research*, *238*(1–2), 243–276. doi:10.1007/s10479-015-2004-4

Jain, V., Panchal, G. B., & Kumar, S. (2014). Universal supplier selection via multi-dimensional auction mechanisms for two-way competition in oligopoly market of supply chain. *Omega*, *47*(3), 127–137. doi:10.1016/j.omega.2013.10.005

Krishnendu, S., Ravi, S., Surendra, S. Y., & Lakshman, S. T. (2012). Supplier selection using fuzzy AHP and fuzzy multi-objective linear programming for developing low carbon supply chain. *Expert Systems with Applications*, *39*(3), 8182–8192.

Liu, W. Q., Dong, Y. C., Chiclana, F., Cabrerizo, F., & Herrera-Viedma, E. (2017). Group decision-making based on heterogeneous preference relations with self-confidence. *Fuzzy Optimization and Decision Making*, *16*(4), 429–447. doi:10.1007/s10700-016-9254-8

Liu, W. S., & Liao, H. C. (2017). A bibliometric analysis of fuzzy decision research during 1970–2015. *International Journal of Fuzzy Systems*, *19*(1), 1–14. doi:10.1007/s40815-016-0272-z

Liu, C. P., & Peng, B. (2017). A method for group decision making based on interval-valued intuitionistic fuzzy geometric distance measures. *Informatica*, *28*(3), 453–470. doi:10.15388/Informatica.2017.138

Merigó, J. M., & Gil-Lafuente, A. M. (2013). Induced 2-tuple linguistic generalized aggregation operators and their application in decision-making. *Information Sciences*, *236*, 1–16. doi:10.1016/j.ins.2013.02.039

Merigó, J. M., Guillén, M., & Sarabia, J. M. (2015). The ordered weighted average in the variance and the covariance. *International Journal of Intelligent Systems*, *30*(9), 985–1005. doi:10.1002/int.21716

Peng, X. D., & Dai, J. G. (2017). Approaches to pythagorean fuzzy stochastic multi-criteria decision making based on prospect theory and regret theory with new distance measure and score function. *International Journal of Intelligent Systems*, *32*(11), 1187–1214. doi:10.1002/int.21896

Peng, X. D., & Yang, Y. (2015). Some results for pythagorean fuzzy sets. *International Journal of Intelligent Systems*, *30*(11), 1133–1160. doi:10.1002/int.21738

Peng, X. D., & Yang, Y. (2016). Pythagorean fuzzy Choquet integral based MABAC method for multiple attribute group decision making. *International Journal of Intelligent Systems*, *31*(10), 989–1020. doi:10.1002/int.21814

Qin, J. D., Liu, X. W., & Pedrycz, W. (2017). An extended TODIM multi-criteria group decision making method for green supplier selection in interval type-2 fuzzy environment. *European Journal of Operational Research*, *258*(2), 626–638. doi:10.1016/j.ejor.2016.09.059

Rao, C. J., Xiao, X. P., Xie, M., Goh, M., & Zheng, J. J. (2017). Low carbon supplier selection under multi-source and multi-attribute procurement. *Journal of Intelligent & Fuzzy Systems*, *32*(6), 4009–4022. doi:10.3233/JIFS-151813

Sirbiladze, G., & Badagadze, O. (2017). Intuitionistic fuzzy probabilistic aggregation operators based on the choquet integral: application in multicriteria decision-making. *International Journal of Information Technology & Decision Making*, *16*(1), 245–279. doi:10.1142/S0219622016500449

Song, W. Y., Xu, Z. T., & Liu, H. C. (2017). Developing sustainable supplier selection criteria for solar air-conditioner manufacturer: An integrated approach. *Renewable and Sustainable Energy Reviews*, *79*, 1461–1471. doi:10.1016/j.rser.2017.05.081

Tong, X., & Wang, Z. J. (2016). A group decision framework with intuitionistic preference relations and its application to low carbon supplier selection. *International Journal of Environmental Research and Public Health*, *13*, 923. doi:10.3390/ijerph13090923

Wan, S. P., Xu, G. L., & Dong, J. Y. (2017). Supplier selection using ANP and ELECTRE II in interval 2-tuple linguistic environment. *Information Sciences*, *385-386*, 19–38. doi:10.1016/j.ins.2016.12.032

Wang, Z. J., Wang, Y. H., & Li, K. W. (2016). An acceptable consistency-based framework for group decision making with intuitionistic preference relations. *Group Decision and Negotiation*, *25*(1), 181–202. doi:10.1007/s10726-015-9438-6

Wei, G. W. (2017). Pythagorean fuzzy interaction aggregation operators and their application to multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems*, *33*(4), 2119–2132. doi:10.3233/JIFS-162030

Xia, M. M., Xu, Z. S., & Chen, N. (2011). Induced aggregation under confidence levels. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *19* (02), 201–227. doi:10.1142/S0218488511006976

Xian, S., Zhang, J., & Xue, W. (2016). Fuzzy linguistic induced generalized OWA operator and its application in fuzzy linguistic decision making. *International Journal of Intelligent Systems*, 2016, *31*(8), 749–762. doi:10.1002/int.21801

Xu, Z. S. (2005). An overview of methods for determining OWA weights. *International Journal of Intelligent Systems*, *20*(8), 843–865. doi:10.1002/int.20097

Xu, Q. F., Yu, K. F., Zeng, Z. S., & Liu, J. (2017). Pythagorean fuzzy induced generalized OWA operator and its application to multi-attribute group decision making. *International Journal of Innovative Computing Information and Control*, *13*(5), 1527–1536.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man, and Cybernetics*, *18*(1), 183–190. doi: 10.1109/21.87068

Yager, R. R. (2014). Pythagorean membership grades in multi-criteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*(4), 958–965. doi:10.1109/TFUZZ.2013.2278989

Yager, R. R., & Filev, D. P. (1999). Induced ordered weighted averaging operators. *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, *29* (2), 141–150. doi:10.1109/3477.752789

Yu, D. J. (2014). Intuitionistic fuzzy information aggregation under confidence levels. *Applied Soft Computing*, *19*(6), 147–160. doi:10.1016/j.asoc.2014.02.001

Yu, D. J., & Liao, H. C. (2016). Visualization and quantitative research on intuitionistic fuzzy studies. *Journal of Intelligent & Fuzzy Systems*, *30*(6), 3653–3663. doi:10.3233/IFS-162111

Zeng, S. Z. (2016). An extension of OWAD operator and its application to uncertain multiple-attribute group decision-making. *Cybernetics and Systems*, *47*(5), 363–375.

Zeng, S. Z. (2017). Pythagorean fuzzy multiattribute group decision making with probabilistic information and OWA approach. *International Journal of Intelligent Systems*, *32*(11), 1136–1150. doi:10.1002/int.21886

Zeng, S. Z., Chen, J. P., & Li, X. S. (2016). A hybrid method for Pythagorean fuzzy multiple-criteria decision making. *International Journal of Information Technology & Decision Making*, *15*(2), 403–422. doi:10.1142/S0219622016500012

Zeng, S. Z., Merigó, J. M., Palacios-Marqués, D., Jin, H., & Gu, F. (2017). Intuitionistic fuzzy induced ordered weighted averaging distance operator and its application to decision making. *Journal of Intelligent & Fuzzy Systems*, *32*(1), 11–22. doi:10.3233/JIFS-141219

Zhang, X. L. (2016). A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making. *International Journal of Intelligent Systems*, *31*(6), 593–611. doi:10.1002/int.21796

Zhang, X. L. (2016). Multicriteria Pythagorean fuzzy decision analysis: a hierarchical QUALIFLEX approach with the closeness index-based ranking methods. *Information Sciences*, *330*, 104–124. doi:10.1016/j.ins.2015.10.012

Zhang, X. L., & Xu, Z. S. (2014). Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, *29*(12), 1061–1078. doi:10.1002/int.21676

Zhao, Q. R., Chen, Q. H., Xiao, Y. T., Tian, G. Q., Chu, X. L., & Liu, Q. M. (2017). Saving forests through development? Fuelwood consumption and the energy-ladder hypothesis in rural Southern China. *Transformations in Business & Economics*, *16* (3), 199–219.

Zhou, L. G., Tao, Z. F., Chen, H. Y., & Liu, J. P. (2014). Intuitionistic fuzzy ordered weighted cosine similarity measure. *Group Decision and Negotiation*, *23*(4), 879–900. doi:10.1007/s10726-013-9359-1