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Modelling European sovereign default probabilities with copulas

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ABSTRACT

The goal of this article is to investigate a dependence among sovereign countries' risk of default. The analysis was based on data for 42 European countries during the period 1994–2013. Three models were used to calculate default probabilities: Li's based on transition matrix and prudent unconditional and conditional on previous defaults estimation technique for low default portfolios. The relationship was analysed through the use of different types of copulas. The analysis has shown no regularity in a selection of the optimal copula. The results differ based on the model and rating grade combination used.

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1. Introduction

The risk of falling into solvency difficulties should be treated in a context of attendant circumstances and the environment in which countries operate. A strong connection among sovereign countries through their involvement in the banking sector and granting intra-EU loans was one of the causes of the European financial crisis of 2008. That is why the main issue is not the financial disability of a single country, but more its impact on transmitting the crisis among other nations.

Much effort has been made to explore a relation between sovereign countries' defaults. This was an unsolved issue due to the requirement for dealing with high dimensional data. The usage of a standard binomial dependency measures such as Pearson, Kendall's Tau or Spearman correlation coefficients gave no answer of simultaneous co-movements of different types of risks. An introduction of a copula theory made this problem manageable. Little wonder the biggest rating companies like Standards & Poor's, Fitch, or Moody's have adopted this methodology in the field of risk of default.

A current discussion over sovereign default or solvency difficulties with the usage of copulas concentrates on modelling collateralised debt obligations (C.D.O.s) and is popular in the literature about credit default swaps (C.D.S.). Longstaff et al. (2011)

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have found that most of sovereign credit risks are influenced by global factors. Additionally, they have noticed that a cross dependency of credit risks is much stronger than a correlation between equity index returns among countries. Chen et al. (2011), by using a copula approach on C.D.S., have confirmed an asymmetric dependency between Latin American countries after the Argentinian crisis. An exchange rate dependency among Latin American countries supported by vine copula has been tested by Loaiza and Melo (2012). Similar investigations for the European market have made (Lucas et al., 2012), with the usage of a dynamic GH skewed multivariate copula, with time-varying volatility and correlations based on a period between January 2008 and June 2011. On the other hand, Boubaker and Jaghoubbi (2012) have investigated a correlation among 17 European stock markets between 2007 and 2011, with the usage of different types of copulas in connection with GARCH (1,1) model for marginal distributions. They have noticed the existence of a strong dependence between Greek and Italian, Portuguese, Belgian and Slovenian financial markets. A similar analysis was performed by Zhang (2014), based on 10-years' worth of government bond yields for 10 European countries. Some researchers were trying to analyse a systematic risk based on a CoVaR methodology. Reboredo and Ugolini (2015) have concluded that after the global crisis, the systematic risk for non-crisis countries has increased, unlike the crisis nations.

A copula approach as a solvency measure was adopted for the first time by Li to characterise a credit risk. He compared the results with the CreditMetrics approach and showed that CreditMetrics uses normal copula for their risk calculations. Because of its simplicity his technique was developed and applied, not only in finance to price inter alia C.D.O.s and C.D.O.s and in portfolio analysis, but also in the insurance analysis, meteorology and medicine.

Many researchers have applied the copula concept to financial instruments' credit ratings (e.g. Zitzmann, 2005; Berrada et.al., 2006). Skoglund and Chen have model joint probability of default but did not investigate the country at risk of default. This article fills this gap. Before 2008 European country insolvency sounded more than improbable. The global financial crisis changed the economic conditions. Some countries like Iceland and Greece went bankrupt, others are on the edge. Due to the financial linkage and dependency among different European nations, it is justified to model the joint probability of default among countries. For this purpose, we applied t-, Gumbel- and Clayton copula, which are able to model extreme co-movements and dependencies in tails, on available country ratings between 1994 and 2013 for all European countries, published by Standards & Poor's (S&P), one of the biggest rating companies in the world.

Moreover, we will perform an additional analysis for developed and emerging countries separately, which is not well researched. This will give a meaningful contribution to the current discussion. The results achieved will be compared with default probabilities calculated in line with an unconditional and conditional on previous defaults prudent estimation principle.

The rest of this article is organised as follows: at first current research over copula-based sovereign probability of default will be presented. Sections 3 and 4 introduce

an empirical background and the data is used for a further analysis. In Section 5 the outcome of the analysis will be presented. Section 6 will conclude with a summary and a discussion on the results achieved.

2. Copula

A copula approach is based on an idea of combining separate marginal distributions into a one multivariate sphere called a copula. Formally a foundation was given by Sklar in his theorem from 1959.

Taking $U = (u_1, u_2, \dots, u_n)$ as a random vector with distribution function F and marginal distribution functions (F_1, F_2, \dots, F_n) , then $F(u_1, \dots, u_n) = C(F_1(u_1), \dots, F_n(u_n))$ for all $(u_1, \dots, u_n) \in R^n$. Function C is an n dimensional copula function on $[0, 1]^n$ with uniform marginals. Sklar has proved that the above presented relation is alternate, that means considering distribution function F with marginal distribution functions (F_1, F_2, \dots, F_n) , that for each u_n on $[0, 1]^n$ exists a Copula C , where $C(u_1, \dots, u_n) = F(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n))$ with F_i^{-1} as a quasi-inverse function. The copula approach assumes also that C is unique, if marginal distribution functions (F_1, F_2, \dots, F_n) are continuous, otherwise C is explicitly defined on $Ran(F_1) \times Ran(F_2) \times \dots \times Ran(F_n)$, where $Ran(F_n)$ is a range of a function F_n . More details referring to copulas and copula families with their properties are presented in Nelsen (2006), Trivedi and Zimmer (2007), Cherubini et al. (2011), and Weiß (2010), etc.

The copula theory gained popularity for its flexibility and ability to model different kind of dependencies among variables, it was not limited to normally distributed. The possibilities of copula are almost unlimited. Below are presented the most commonly used types, from the big family of copulas.

2.1. Multivariate Gaussian copula

Considering random variable $U = (u_1, u_2, \dots, u_n)$ and Σ as a symmetric, positive define correlation matrix, then exists an n dimensional Multivariate Gaussian copula C^{Ga} such as:

$$C^{Ga}(u_1, u_2, \dots, u_n, \Sigma) = \Phi_{\Sigma} \left(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \dots, \Phi^{-1}(u_n) \right),$$

where Φ_{Σ} is a standardised multivariate normal distribution and analogously Φ^{-n} as an inverse multivariate normal distribution. Gaussian copula is unable to manage both lower and upper tail dependency. Hence, it is not suitable for modelling extreme co-movements.

2.2. T-copula

T-copula similar to Gaussian one can be derived from the n dimensional t-distribution of the underlay variables. Assuming a random variable $U = (u_1, u_2, \dots, u_n)$ and Σ as a symmetric, positive define correlation matrix, the t-copula C_n^T is given by:

$$C_n^T(u, v, \Sigma) = \int_{-\infty}^{t_v^{-1}(u_1)} \dots \int_{-\infty}^{t_v^{-1}(u_n)} \frac{\Gamma(v + \frac{n}{2})}{\Gamma(\frac{v}{2}) \sqrt{(\pi v)^n |\Sigma|}} \left(1 + \frac{X' \Sigma^{-1} X'}{v} \right)^{-\frac{(v+1)}{2}} dx,$$

where t_v^{-1} is an inverse univariate Student's t distribution with v degrees of freedom. This copula is able to capture joint extreme events, both in upper and lower tail, when $v \rightarrow \infty$.

2.3. Clayton copula

Clayton copula, similar to Gumbel copula, is a part of an Archimedean Family, which are determined by a generator $\varphi(u)$. In 1974 Kimberling proved that to generate Archimedean copula of any dimension, the generator had to be strictly monotone. McNail and Neslehova (2009) claimed that for a given dimension higher than three such condition is not sufficient and can lead to limited dependence characteristics, thus it is necessary and sufficient for an Archimedean copula generator to be complete monotone.

Let $\varphi(u)$ be a generator for Clayton copula such as $\varphi(u) = u^{-\alpha} - 1$ under the condition $\alpha > 0$, as only then generator is completely monotone, then Clayton's copula is formally described as: $C(u_1, u_2, \dots, u_n) = [\sum_{i=1}^n u_i^{-\alpha} - n + 1]^{-\frac{1}{\alpha}}$ with $\alpha > 0$. This copula is suitable for the observed strong low tail dependency with the corresponding correlation parameter $\tau = \frac{2\alpha}{1-\alpha}$. When $\alpha \rightarrow 0$ independency is assumed.

2.4. Gumbel copula

To create a Gumbel copula the generator is completely monotone when $\alpha > 1$ and has a form $\varphi(u) = (-\ln(u))^\alpha$. Therefore, the n dimensional Gumbel copula is defined in a way that:

$$C(u_1, u_2, \dots, u_n) = \exp \left\{ - \left[\sum_{i=1}^n (-\ln u_i)^\alpha \right]^{\frac{1}{\alpha}} \right\} \text{ with } \alpha > 1.$$

It is a favourable copula for those who are investigating an insurance risk measures because it can easily model extremes. In the case of $\alpha \rightarrow \infty$, it can capture an upper tail dependency given by $\lambda_U = 2 - 2^{\frac{1}{\alpha}}$, with the corresponding correlation measure $\tau = 1 - \frac{1}{\alpha}$.

3. Data

3.1. Ratings

The investigated sample contains ratings published for 21 emerging and 21 developed countries in the period 1994–2013. The only exemption is Belarus, which was rated in 2007 for the first time. The complete list of included nations is summarised in Table 1. Countries were rated at different points of time, e.g., in the mid-90s for the disintegrated Yugoslav Republic, and a few years earlier in the U.S.S.R. This influences the number of sample sizes within the years. Additionally, S&P does not publish

Table 1. Countries included in the analysis.

Country			
Developed	Emerging		
Austria	Italy	Armenia	Macedonia
Belgium	Luxembourg	Azerbaijan	Moldova
Cyprus	Malta	Belarus	Poland
Denmark	Netherlands	Bulgaria	Romania
Estonia	Norway	Croatia	Russia
Finland	Portugal	The Czech Republic	Serbia
France	Spain	Georgia	Slovakia
Germany	Sweden	Hungary	Slovenia
Greece	Switzerland	Kazakhstan	Turkey
Iceland	The United Kingdom	Latvia	Ukraine
Ireland		Lithuania	

Source: Authors' research based on information from IMF, emerging-europe.com, www.msci.com.

Table 2. Detailed Standard & Poor's rating abbreviations.

Rating	Abbreviations used in this paper	Definition
A.A.A.	A.A.A.	Extremely strong capacity to meet financial commitments. Highest Rating.
A.A.	A.A.	Very strong capacity to meet financial commitments.
A	A	Strong capacity to meet financial commitments, but somewhat susceptible to adverse economic conditions and changes in circumstances.
B.B.B.	B.B.B.	Adequate capacity to meet financial commitments, but more subject to adverse economic conditions.
B.B.B.-	B.B.B.	Considered lowest investment grade by market participants.
B.B.+	B.B.	Considered highest speculative grade by market participants.
B.B.	B.B.	Less vulnerable in the near-term but faces major ongoing uncertainties to adverse business, financial and economic conditions.
B	B	More vulnerable to adverse business, financial and economic conditions but currently has the capacity to meet financial commitments.
C.C.C.	C	Currently vulnerable and dependent on favourable business, financial and economic conditions to meet financial commitments.
C.C.	C	Currently highly vulnerable.
C	C	Currently highly vulnerable obligations and other defined circumstances.
D	C	Payment default on financial commitments.

Source: <http://www.standardandpoors.com/ratings/definitions-and-faqs/en/us>.

ratings for Armenia and Moldova and Moody's for Serbia and Macedonia. This will cause minor differences between the samples as well.

The main input factors used for the analysis are available ratings published by S&P. The ratings contain information, i.e., about country's credit worthiness, i.e., on probability of default on debt contracts. They range from A.A.A. to D, where A.A.A. is the highest rating with the highest probability of meeting all of financial commitments and the least susceptible to the changing economic environment conditions, while D means default on debt. The detailed S&P rating abbreviations are presented below in Table 2. Their usefulness and explanatory power were analysed by Canuto et al. (2012).

The original range of rating grades is quite extensive. Between the highest (A.A.A.) and the lowest rating (D) there are 23 other different grades. That is why in the analysis I have reduced the number of grades to 7 (see Table 2), this will make the analysis easier and will have no impact on the final outcome.

Six years after the outbreak of the global financial crisis there was no consistent definition of default and insolvency of a country. In this article we will use some

assumptions similar to those applied by Ciarlone et al. (2005): insolvency appears when a sovereign suffers a debt crisis, announces bankruptcy, or obtains strategic rescheduling, restructuring, or financial support from a financial institution such as the I.M.F. or the Paris Club. For that reason, in the further analysis, in addition to rating D, also credit rating C was considered as a determinant of a sovereign distress. Within the selected period only one clear case of bankruptcy has been declared by the rating agencies (Moldova 2002 – when Moldova violated the I.M.F.'s and World Bank's loan conditions). The other two were recorded as highly vulnerable (Greece 2012, Moldova 2001). After considering countries with a C and D rating, 21 cases of insolvency have been observed.

Within the investigated period 46% of all nations have been downgraded at least once in the yearly perspective, from which 63% are emerging and 37% are developed countries. It may seem that developing countries are unstable and more likely to change the rating. However, among the countries, which lowered the credit rating of more than once, developed countries (57%) dominate the emerging nations (42%). After 2008 many European countries suffered from financial difficulties, e.g., Greece, Italy, Iceland, and Spain. It was reflected in changes in ratings of 17% of developing countries against 53% of the developed countries. It is visible that developed nations were hit harder by the global financial crisis than the emerging economies.

3.2. Default probabilities calculated from transition matrix

It is commonly known that any changes made by the rating agencies are a signal for either positive or negative development for the valued entity. The rating grade itself gives, however, no sufficient information about threat of insolvency appearance. To make it more realistic, it is necessary to implement a dynamic factor into the model. It is done through a Markov Chain Process which presents a mathematical procedure of alteration from one grade into another, is presented in a transition matrix, and is described by the following formula:

$$Pr\{X_{t+1} = j \mid X_0 = i_0, \dots, X_t = i\} = Pr\{X_{t+1} = j \mid X_t = i\}$$

The methodology of creation of the migration matrix is described by Li (2000), Yanakieva and Antonov (2004), and Zitzmann (2005), etc. A transition matrix, based on historical ratings, reflects a movement between different rating classes in the window specified in advance time. In this case, a probability of a transition from a specific rating grades at the beginning of each year directly to the state of default. In Europe no country rated above A has defaulted within one year. The global crisis has caused some defaults among nations with B.B.B. or B.B. ratings. Irrespective of the global economic conditions or the time sovereigns rated with C are mostly threatened with default, even if default probabilities are very low, less than 6%. It must be strongly underlined that in Europe most of nations are highly developed and last 20 years was a time of prosperity and dynamic development for the whole region, also for the emerging nations. Some serious problems faced markets in Eastern and South Eastern Europe between 1997 and 2002, but it appears that these countries

despite their not very high ratings were more stable, reliable and resistant to global crisis compared to some developed countries like Greece, Spain, or Cyprus.

3.3. Default probabilities for low default portfolios

The financial crisis exposed that even high rated sovereigns are not resistant to default, e.g., financially stable and high rated Iceland suddenly announced bankruptcy in 2008. For that reason, it is important to find a way to include the probabilities of default also for high rated countries. Small number of defaults recorded for nations with rating greater than B confirmed that need. Engelmann and Rauhmeier (2011) have described two scenarios for the most prudent estimation of default probabilities for approaching this issue. The first one assumes no defaults among the sovereigns, while the second one, which is an extension of the first one, allows to include the number of defaults observed in each rating grade. Both models are based on a simple inequality $Pr_i < Pr_j$; where $i, j \in \{AAA, AA, A, BBB, BB, B, C\}$ and $i < j$, which means that probability of default for higher rated country should be lower than for those lower rated. To adjust default probabilities according to the first model (assuming no defaults in the investigated sample) the following formula will be

used $Pr \leq 1 - (1-\gamma)^{\frac{1}{\sum_{i=j}^c n_i}}$, where $j \in \{AAA, AA, A, BBB, BB, B, C\}$, with the frequencies n_i and the confidence level γ . It is visible that the probability of default (P.D.) calculated in this way depends strongly on the frequencies and γ . The higher the value of the confidence level γ , the higher the received default probabilities. The impact of the frequencies on P.D. calculation is contrary, the higher the frequencies the lower probability.

The above presented approach assumes occurring no defaults within the investigated period. Such assumption seems to be insufficient. For that reason, the most prudent estimation was extended by realised number defaults in each rating level. The probabilities of default are binomially distributed and are calculated based on the following for-

mula: $\sum_{k=0}^d \binom{\sum_{i=j}^D n_i}{k} Pr_j^k (1-Pr_j)^{\sum_{i=j}^D n_i - k}$, where $j \in \{AAA, AA, A, BBB, BB, B, C\}$,

with the frequencies n_i and the total number of observed defaults in each investigated rating range D. The probabilities of default calculated based on unconditional prudent estimation are systematically lower than the one based on transition matrix. The opposite results were achieved with conditional on previous defaults prudent estimation, where the defaults overperform.

4. Results

In order to determine copula parameters and verify the best fit, the empirical distribution was assumed, so that the results depend as much as possible from the underlying data and were biased from the parameter selection in the least possible way. Several types of copulas were fitted: t, Gumbel and Clayton. The selection of the best copula will be done based on a value of Akaike's information criterion (A.I.C.), which is useful to compare models with each other and its adequacy (Burzykowski, et al.,

2005). The model which generates A.I.C. with the lowest values is considered as the best one. Both measures are defined as a difference between number of estimated parameters (k) or a sample size (n) and maximised likelihood value of fitted copula (l): formally $AIC = 2k - 2\ln(L)$.

4.1. Transition matrix based copula

Fitting a copula and subsequent simulations were carried out using C.O.P.U.L.A. procedure featured by S.A.S. The primary sample contained seven variables (A.A.A., A.A., A, B.B.B., B.B., B, C), which has indicated at seven-dimensional copula. However, after calculating probabilities of default based on transition matrix for all 42 nations, the number of variables was reduced to four (B.B.B., B.B., B, C) because for other rating grades (A.A.A., A.A., A) no observations occurred. The same outcome was received for developed and emerging countries. It was necessary to reduce a number of analysed variables to three (B.B.B., B.B., C) and two (B, C) respectively. Fitting the proper copula to data is the first step in modelling via a copula approach. Three different copulas were fitted to data: t , Clayton and Gumbel. The outcome is summarised in Tables 3–5.

The results have shown that for 42 European countries the t copula is the most suitable for modelling dependence for given rating grades. It has generated the lowest AIC value among other copulas, with the acceptable level of standard error. The estimated parameters for Clayton copula are tending to null, which points at no dependency. A similar result was achieved for developed countries, for which t copula seems to be the appropriate as well (A.I.C. was the smallest). However, small differences between A.I.C. for other copulas (Clayton and Gumbel) suggest taking this choice with caution. The selection of the t copula as the right model will be verified in the further analysis. In case of emerging nations all copula functions were fitted

Table 3. Copula fitting results for 42 European countries.

Copula	Parameter	Std.error	p-Val	AIC	SBC
t	1,62	0,01	<.0001	-159,85	-153,24
Clayton	1,05E-08	n/c	n/c	2,00	2,94
Gumbel	1,33	0,09	<.0001	-23,91	-22,97

Source: Author's calculations.

Table 4. Copula fitting results for developed countries.

	Parameter	Std.error	p-Val	AIC	SBC
t	2,05	0,001	<.0001	-129,74	-125,96
Clayton	27,22	3,92	<.0001	-127,61	-126,66
Gumbel	3,89	0,45	<.0001	-128,34	-127,39

Source: Author's calculations.

Table 5. Copula fitting results for emerging countries.

	Parameter	Std.error	p-Val	AIC	SBC
t	100	n/c	n/c	-0,30	1,59
Clayton	1,05E-08	n/c	n/c	2,00	2,94
Gumbel	1,39	0,28	<.0001	0,16	1,10

Source: Author's calculations.

successfully. A.I.C. criterion has pointed at the t copula as the optimal one, which is associated with a significant p-value and smaller standard error equals 0.001. The t copula is characterised by the similar dependency in lower and upper tail, which means that both low and high rating grades tend to move together.

However, due to the fact that Akaike's information criterion is not *sensu stricto* a sufficient measure for goodness of fit for copulas, they are only a kind of a proxy or an impression the user might have by evaluating a fit of a chosen copula. The final check for adequate reflection of the underlying structure will be confirmed through simulating from the fit copula and comparison of the received correlation.

In order to confirm the proper selection of the copula, it is necessary to simulate the data with the given properties and to compare them with the input data. The obtained correlation matrices have confirmed t copula as the most suitable for explaining the interdependences among the probabilities of default for all European nations as well for European developed and emerging markets. The difference between Spearman correlation coefficient generated from the simulated data and the original one was the smallest.

4.2. Low default portfolio based copula

The same method as described above was applied to default probabilities calculated according the rules for low defaults portfolios. In the first step copulas for combination of all rating grades (A.A.A.-C) for all 42 countries were fit. We used the same rating combination as was applied for default probabilities based on transition matrix: for 42 nations (B.B.B., B.B., B, C), for developed nations (B.B.B., B.B., C) and for emerging nations (B, C). The results are summarised below in Tables 6 and 7.

Depending on rating grades combination and default probabilities estimation technique, similar to results described in Section 5.1, there is no one specific copula which can model the dependencies. It varies from t, through Clayton to the Gumbel

Table 6. Copula fitting results for 42 European countries calculated based on the default probabilities estimated for low default portfolios.

	Unconditional		Conditional	
	(A.A.A.-C)	(B.B.B.-C)	(A.A.A.-C)	(B.B.B.-C)
t	n/c	-20,32	n/c	-45,79
Clayton	-30,49	-12,59	2,00	2,00
Gumbel	-19,21	-22,42	-74,28	-21,74

Source: Author's calculations.

Table 7. Copula fitting results for 21 developed and emerging European countries calculated based on the default probabilities estimated for low default portfolios with the combination of ratings used in Section 5.1.

	Developed		Emerging	
	unconditional (B.B.B. B.B. C)	conditional (B.B.B. B.B. C)	unconditional (B-C)	Conditional (B-C)
t	-48,83	-63,27	-0,43	2,98
Clayton	-16,51	-19,91	1,18	0,59
Gumbel	-14,08	-25,41	-0,38	1,34

Source: Author's calculations.

copula. An interesting result was achieved by developed countries for ratings B.B.B., B.B., C, where t copula reached the best fit. Because the parameter α in both cases tends to infinity, no dependency in upper and lower limit can be assumed. Similar as above the generated correlation matrices have confirmed fitting results as well as lack of normal copula among the optimal ones.

5. Conclusion

In this analysis we have calculated default probabilities in three ways: according to the method proposed by Li based on a transition matrix and proposed by Engelmann and Rauhmeier for low defaults portfolios the most prudent unconditional and conditional on previous defaults estimation method. In the next step some Archimedean copulas (Clayton, Gumbel) as well as a t-copula were fit to the empirical data. What is seen from above it is not possible to point at one define type of copula, which explains the dependences in the optimal way. The shape of rating pairs is not so unique due to the limited number of observations, low volatility in rating migrations and limited number of defaults recorded in the past 20 years in Europe.

Almost all researchers who have worked with copulas admit that copulas are a very useful and powerful tool. Despite the difficult times, which copulas had after the global crisis, when many have accused models based on copulas as the real cause of the crisis, copulas still gained in popularity, especially by modelling high dimensional data with complex intra-correlation dependences.

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