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# The Multiphase Queuing System of the Rijeka Airport

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## ABSTRACT

The paper gives an overview of the real system as a multiphase single server queuing problem, which is a rare case in papers dealing with the application of the queueing theory. The methodological and scientific contribution of this paper is primarily in setting up the model of the real problem applying the multiphase queueing theory. The research of service system at Rijeka Airport may allow the airport to be more competitive by increasing service quality. The existing performance measures have been evaluated in order to improve Rijeka Airport queueing system, as a record number of passengers is to be expected in the next few years. Performance indicators have pointed out how the system handles congestion. The research is also focused on defining potential bottlenecks and comparing the results with IATA guidelines in terms of maximum waiting times.

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## 1 Introduction

Significant growth of world's airline industry has induced the need for accurate planning and management even in the airports with low passenger flow. According to the International Air Transportation Association (IATA) forecast, the amount of passengers could double to 8.2 billion in 2037 [6]. The participation of low-cost carriers has also been increased by 61% over the last ten years [5]. As the annual demand is rising, airport operations managers have to ensure efficient management strategies in order to shorten overall queuing process time and achieve higher level of service.

Although smaller airports have limited facilities and human resources, they are still trying to continuously increase service quality and passengers' satisfaction. The major service quality indicator is passengers' waiting time. Therefore, managers should avoid long waiting times which can lead to delays and cause negative effect on passengers' total perception of service quality.

At every airport, departing passengers have to pass through several service points, such as check-in and bag-

gage drop, security and passport control, and finally leave the system through the boarding gate. All these phases are characterized by service and waiting times. From the passengers' perspective, check-in and security controls are key points which may be considered as the bottleneck of every airport terminal system. The number of working servers in each phase should guarantee minimum processing times as well as minimum queue length with direct impact on related costs [7].

The main purpose of this paper was to evaluate Rijeka Airport's single-server multiphase queuing system with Poisson arrivals and exponential service times in order to examine performance measures. The arrivals are served on a FIFO basis with infinite queue length. The first step of research was to conduct the analysis of servicing the passengers at the airport as well as to give the insight into the current state of service in each phase at Rijeka Airport. In particular, the examination whether single phases of the service meet customer arrivals, whether there is congestion at certain phases of the service, or there is a lack of capacity utilization at certain phases. Therefore, in order to test the service system of the Rijeka airport, it was necessary

to set it up and present it as a multiphase service system in which users from one phase go to the next phase of service until they leave the airport. It was important to observe each phase individually, as this was the only way to identify problems, possible congestion or unused capacity in certain phases of service and finally to propose a solution to the problem and to improve the functioning of the observed system. Section 2 gives a brief overview of methodology used to study the multiphase queuing system. In Section 3 the results of analysis are presented, while Section 4 reports some concluding remarks and future work.

## 2 Theoretical background

Queues are made when the amounts of arrivals exceeds the rate of service delivery [2]. Many queuing models have been developed, giving an optimal solution to minimize waiting times and the number of customers in the queue. That can be achieved by speeding up the service rate in each server or by adding additional server [1]. The most common method used to optimize performance and avoid congestions is a queuing theory which can predict system behaviour due to changes. Furthermore, queuing models can be useful for both manufacturing and service areas.

When analysing queuing systems, the assumption that service rates are the same for all the servers in the system can be true exclusively in a case of electronically or mechanically controlled service process. On the other hand, there are plenty of queuing systems with human controlled servers, where each server may have different service time distributions [3]. Therefore, there is a need for modelling such multiphase queuing system where customer must pass through several different phases in a particular order to complete the service process, as shown in Figure 1. The queuing system in Figure 1 can also be with multiple servers in each phase.

In order to calculate performance measures, basic parameters are determined. By dividing the average arrival rate  $\lambda$  with average service rate  $\mu$ , traffic intensity  $\rho$  is obtained using the following equation:

$$\rho_i = \frac{\lambda_i}{\mu_i} < 1; i = 1, 2, \dots, k \tag{1}$$

Accordingly, traffic intensity must not be greater or equal to 1, thus the average arrival rate has to be smaller

than the average service rate. In that case, the system is classified as stable. If the above-mentioned criterion has not been met, the number of servers should be increased until the system stability is achieved. Subsequently, when the traffic intensity rate is known, it is possible to determine performance indicators of multiphase single-server queuing system.

The average number of customers in the system (waiting and being served), notated by  $L_s$  is obtained by solving the equation as follows:

$$L_s = \sum_{i=1}^k \frac{\rho_i}{1 - \rho_i}; i = 1, 2, \dots, k \tag{2}$$

whereas the average number of customers waiting in the queue  $L_Q$  can be calculated by the equation below:

$$L_Q = \sum_{i=1}^k \frac{\rho_i^2}{1 - \rho_i}; i = 1, 2, \dots, k \tag{3}$$

Equation 4 is used to determine the average time a customer spends in the system  $W_s$  (waiting time plus service time). The smaller value of  $W_s$  gives the more efficient existing queuing system [4].

$$W_s = \sum_{i=1}^k \frac{1}{1 - \rho_i} \cdot \frac{1}{\mu_i}; i = 1, 2, \dots, k \tag{4}$$

The average time a customer spends waiting in the queue  $W_Q$  is expressed by following equation:

$$W_Q = \sum_{i=1}^k \frac{\rho_i}{\mu_i \cdot (1 - \rho_i)}; i = 1, 2, \dots, k \tag{5}$$

In addition, the notation  $p_n$  represents the probability of  $n$  customers in each phase, given as:

$$p_{ni} = (1 - \rho_i) \cdot \rho_i^n; i = 1, 2, \dots, k \tag{6}$$

By multiplying the values  $p_{ni}$  of each phase, the probability of  $n$  customers in the system is obtained. The number of unoccupied waiting spots needed in each phase, can be calculated by cumulative sum of probabilities  $p_n$  or by solving the following equation:

$$N_i \geq \frac{\ln(1 - \beta)}{\ln \rho_i} - 1; i = 1, 2, \dots, k \tag{7}$$

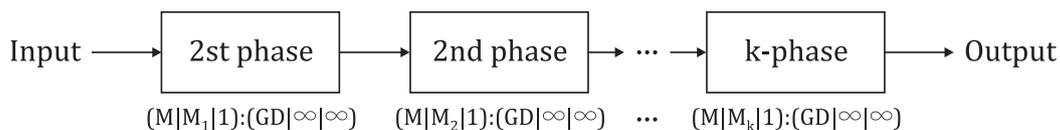


Figure 1 Queuing system with k service phases and infinite queue length

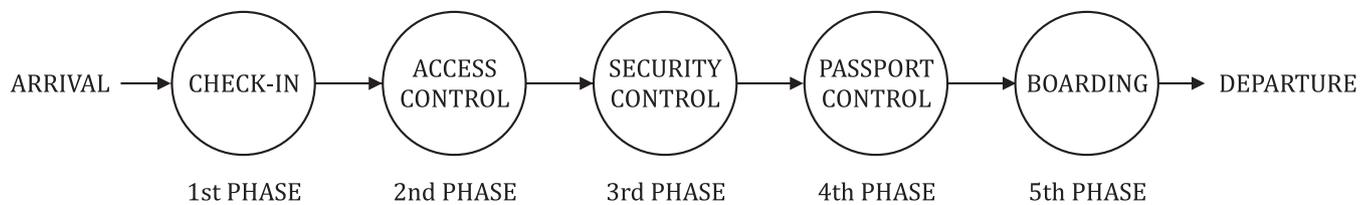


Figure 2 Queuing system at Rijeka Airport

Source: Authors

In such a way the length of any queue can be predicted. The main goal is to define optimal number of servers while reducing waiting time and wait-related costs as much as possible [8].

### 3 Results and findings

At Rijeka airport, within the terminal area, passengers have to pass through five different phases, as presented in Figure 2.

Although there are several servers in each phase, only one server handle arriving passengers, as data are collected in off-season when the number of passengers is considerably smaller. The research was carried out on a sample of 112 passengers. The arrival and service rates, given in Table 1, vary depending on the phase thus obtaining different traffic intensity.

The average arrival rate of 22.39 passengers per hour is the lowest of all, since web-checked passengers are not obliged to report to check-in counter. Those passengers can proceed directly to the second phase i.e. access control point. That is the reason why the average arrival rate in the second phase is higher than the arrival rate in the first phase. The average arrival rate remains the same in the phases two, three and four. These service points are physically near to each other, so passengers have no other place to go until they complete phase number four. On the other hand, the average arrival rate is the highest in the last phase due to the fact that passengers don't want to miss their flight so they are in a rush to go on board.

At service phases two and five can be served the greatest number of passengers, with the amount of 196.72 and 251.92 passengers per hour respectively. It is understand-

able that the average service rate is the lowest in the first phase, due to differing service time, since many different scenarios could occur, such as: large groups with a lot of luggage wanting to sit together, passengers with reduced mobility having special requests, infants having baby equipment, unaccompanied minors, pets travelling in the cabin or in the aircraft cargo hold, etc. According to Table 1, servers in the third and fourth phase are the busiest, with traffic intensity of 53.48% and 51.71% respectively, while the last phase provides the lowest congestion of 15.27%. Although the average arrival rate in this phase is the highest, the average service rate is much higher, thus reducing congestion. There is no need for adding additional server in any phase, since the traffic intensity in each phase is lower than 1, so the system is stable. By adding additional server, the overall efficiency is increasing as well as related costs. Performance indicators of Rijeka Airport's queuing system are presented in the Table 2. These calculations are conducted to explain the correlation between performance indicators.

Table 2 shows that the third phase has the most unfavourable impact on the entire system, since the average number of 0.61 passengers in the queue is the highest of all. The average waiting time in the queue of 0.98 minutes is quite longer in comparison with other phases. The last phase provides the shortest queue of 0.03 passengers and the lowest waiting time of 0.04 minutes. The total average time a passenger spends in the system of 5.76 minutes is acceptable. Comparing data in Table 2 with the International Air Transportation Association (IATA) Guidelines shown in Table 3, waiting times at Rijeka Airport are quite short and acceptable, since the waiting time at each phase is within the IATA limits.

Table 1 Overview of basic parameters

Phase number	Phase description	$\lambda$ (pass./hour)	$\mu$ (pass./hour)	$\rho$
1	Check - in	22.39	61.09	0.3665
2	Access control	37.50	196.72	0.1906
3	Security control	37.50	70.12	0.5348
4	Passport control	37.50	72.52	0.5171
5	Boarding	38.46	251.92	0.1527

Source: Authors

**Table 2** Performance indicators of Rijeka Airport’s queuing system

Phase number	Phase description	$L_s$	$L_q$	$W_s$ (min)	$W_q$ (min)
1	Check – in	0.58	0.21	1.55	0.57
2	Access control	0.23	0.04	0.38	0.07
3	Security control	1.15	0.61	1.84	0.98
4	Passport control	1.07	0.55	1.71	0.89
5	Boarding	0.18	0.03	0.28	0.04
	Total	3.21	1.44	5.76	2.55

Source: Authors

**Table 3** Maximum waiting time – IATA Guidelines (in minutes)

Phase description	Short to acceptable	Acceptable to long
Check – in	0 – 12	12 – 30
Security control	0 – 3	3 – 7
Passport control	0 – 5	5 – 10

Source: Adapted by authors according to [1]

The probabilities of the certain number of passengers in each phase are given in Table 4. In each phase the probabilities that the server will be idle are the highest. As shown in Table 4 both, the highest and lowest values are obtained in the fifth phase. The server in the fifth phase will be idle with the probability of 84.73%, while there is a 0.001% chance that five passengers are in the last phase. As the number of passengers in each phase is rising, probabilities are getting lower. The probability of  $n$  passengers in the system is obtained by multiplying the values  $p_{ni}$  of each phase. Accordingly, the probability of no passengers in the entire system is 9.76%.

Table 5 shows the number of unoccupied waiting spots needed in each phase. Calculations are made with a 95% certainty that a passenger could join the queue. Four unoccupied waiting spots have to be provided in the third and fourth phase, which is almost twice as big as the results in other phases.

Another way of defining  $N_i$  is by cumulative sum of probabilities  $p_n$ . For example, cumulative sum of probabilities in the third phase is 0.9768 which means it is possible to provide four waiting spots with even higher certainty than  $\beta$ .

**Table 4** Probability of  $n$  ( $n=0,\dots,5$ ) passengers in the  $i$ -th phase ( $i=1,\dots,5$ )

$p$	Check – in	Access control	Security control	Passport control	Boarding
$p_{0i}$	0.6335	0.8094	0.4652	0.4829	0.8473
$p_{1i}$	0.2322	0.1543	0.2488	0.2497	0.1294
$p_{2i}$	0.0851	0.0294	0.1331	0.1294	0.0198
$p_{3i}$	0.0312	0.0056	0.0712	0.0668	0.0030
$p_{4i}$	0.0114	0.0011	0.0381	0.0345	0.0005
$p_{5i}$	0.0042	0.0002	0.0204	0.0179	0.0001

Source: Authors

**Table 5** The number of unoccupied waiting spots needed in each phase

Phase number	Phase description	$N_i$ ( $\beta = 0.95$ )
1	Check – in	$\leq 2$
2	Access control	$\leq 1$
3	Security control	$\leq 4$
4	Passport control	$\leq 4$
5	Boarding	$\leq 1$

Source: Authors

## 4 Conclusion

In this paper the effectiveness of Rijeka Airport's multiphase queuing system is evaluated. The main problem of the research was to analyse the present queueing system at Rijeka Airport, i.e. to determine whether the service system works stable, without congestion, taking into account the existing capacity and the average number of passengers per day during winter. Furthermore, if congestion exists, determine where it occurs, at what stage. Analysing performance indicators it can be concluded that all the servers have shown very good results in dealing with congestion. It has been proven that the passenger waiting times for service in each phase is provided within the recommended limits by IATA guidelines.

The results of the research also show that the arrivals intensity is highest in the fifth and lowest in the first phase. The largest number of passengers per hour can be served in the fifth and the least in the first phase. Based on these data, it was obtained that the third channel was the most used, the most congested with 53.48%, while the fifth channel was used the least with 15.27%. In terms of performance indicators, the number of passengers in the queue and the time spent in the queue is also highest in the third phase and least in the fifth phase. Hence the system handles congestion very well, no congestion occurs at any stage, since even the most congested channels (security and passport control) are only used around 50% and could pose a potential bottlenecks in peak period.

Since this study was directed on a relatively small sample of the passengers in the winter period and indicated the occurrence of overcapacity, future research could be focused on the same service system during the sum-

mer peak season because of significant difference in the number of passengers in winter and summer time due to seasonality. In that case, a research should be carried out as a multiphase queuing system with multiple servers in each phase.

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