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Economic description of tolerance in a society with asymmetric social cost functions

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ABSTRACT

The evolutionary game dynamics of social tolerance among heterogeneous economic agents have been discussed in an economic interaction model with asymmetric social cost functions, where the individual cost depends only on the share of intolerant people in the opposite group. We show that, very different from the symmetric function case studied previously , economic interactions between individuals in a society with asymmetric social cost functions can be exactly solved in phase plane, and rich behaviours can be revealed by using algebraic approach. Our contribution consists in offering the explicit formula of evolutionary trajectories in the phase plane for the first time. The property of equilibrium is shown to be closely related to the group populations. Based on the explicit formula in the phase plane, the equilibriums of the evolutionary dynamics can be easily identified, and the evolutionary trajectory can be exactly analysed. We also show that the explicit solutions obtained would be especially suited to effective control of the evolutionary dynamics of social tolerance. The necessary and sufficient conditions of the full tolerance equilibrium under asymmetric social cost function are also discussed, which provides guidance and reference to set policies and development strategy of social tolerance.

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1. Introduction

Tolerance, which is defined as a generic ability to accept diversity (Akerlof & Kranton, 2000; Florida, 2004), has attracted more and more attention over the last decade and is increasingly recognised as an important influence factor of economic growth (Berggren & Elinder, 2012; Shi & Peng, 2014). Using empirical analysis, it has been suggested that intolerant behaviour obstructs the free movement of talent (Florida, 2004) and favours corruption and political patronage (Tabellini, 2010). Moving from an intolerant to a tolerant society would always increase aggregate income (Corneo & Jeanne, 2009), technological performance (Berggren & Nilsson, 2013) and social development (Bjørnskov, 2004).

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Assuming that all agents are purely driven by the economic incentive, the discussion on tolerance at the individual level paves a path towards the control of tolerance, explains many social phenomena related to tolerance (Garofalo, Di Dio, & Correani, 2010; Muldoon, Borgida, & Cuffaro, 2012; Shi & Pan, 2017a) and reveals that economic reasoning can offer original and unique insights into the determinants of tolerance. Evolutionary game theory is introduced to describe the dynamics of tolerance (Cerqueti, Correani, & Garofalo, 2013; Garofalo et al., 2010). Consistent with the empirical results, it has been demonstrated in the evolutionary game that a fully tolerant society assures prosperity and that moving from intolerance to tolerance can increase income and social development because intolerant behaviour reduces trust and cooperation among economic agents, which consequently reduces the total welfare of the society.

Evolutionary game theory emphasises the dynamic process of adjustment and convergence of behavioural decision making, which is very suitable for describing the dynamics of tolerance in real economic life. It is well known that the economic agents are often assumed to be completely rational in traditional game theory. However, the complete rationality of the economic agents is difficult to achieve in real economic life (Juul, Kianercy, Bernhardsson, & Pigolotti, 2013). Advantageously, evolutionary game theory is based on 'limited rationality' and does not require complete rationality of the economic agents. Evolutionary game theory combines game analysis with dynamic evolution, and emphasises dynamic evolutionary equilibrium rather than traditional static equilibrium (Nakajima & Masuda, 2015).

Recently, an evolutionary game model of tolerance among two group of heterogeneous economic agents has been studied (Cerqueti et al., 2013; Shi, Pan, & Peng, 2017) by introducing a symmetric double-channel social cost function $c_i(x_1, x_2)$, where x_1 and x_2 are the share of tolerant agents in group 1 and group 2, respectively. We note that the social cost function may be asymmetric in some cases (Akerlof & Kranton, 2000; Garofalo et al., 2010), where the individual cost depends only on the share of intolerant people in the opposite group. In the circumstances, the asymmetric social cost function obeys the following properties: (1) $\frac{\partial c_1}{\partial x_2}<0$ and $\frac{\partial c_2}{\partial x_1}<0$; (2) $\frac{\partial^2 c_1}{\partial x_2^2} \geq 0$ and $\frac{\partial^2 c_2}{\partial x_1^2} \geq 0$; (3) $c_1 = 0$ if $x_2 = 1$ and $c_2 = 0$ if $x_1 = 1$. Property (1) states that the social cost decreases when the share of tolerant people in the opposite group increases. The second property put convexity condition (in a single variable sense). Property (3) states that the social cost becomes zero if there are no intolerant people in the opposite group. In general, the cost may depend on both variables (Cerqueti et al., 2013), in such a case stronger conditions are required for convexity. Obviously, the properties of such asymmetric social cost function are quite different from the properties of the symmetric case (Cerqueti et al., 2013), and it has been shown (Shi et al., 2017) that the asymmetric function may give a reasonable description of politically separated groups. So it is interesting to study the evolution of social tolerance in the asymmetric function case.

In the present work, the game of social tolerance among heterogeneous economic agents in an economic interaction model with asymmetric social cost functions is discussed, and our contribution consists in offering the evolutionary trajectories in the phase plane which are described by explicit formula for the first time. In fact, the obtained explicit formula is one of only a few exact solutions in the topic of tolerance. Due to the highly nonlinearity of the evolution equations, the explicit formula cannot be revealed using traditional dynamic analysis method or linearised equations of the problem. Using the explicit formula, we show that the property of equilibriums depends on not only the distribution of economic factors such as aggregate wealth and social costs, but also the group populations. Based on the explicit formula in the phase plane, the equilibriums of the evolutionary dynamics can be easily identified, and the evolutionary trajectory can be exactly analysed. We also show that the explicit solutions obtained would be especially suited to effective control of the evolutionary dynamics of social tolerance. Finally, from the policy perspective, government policies favour a society with full tolerance, which requires effective control of the evolutionary dynamics of social tolerance. In our case, slope of the linear relation between x_1 and x_2 is shown to be determined by population numbers of each group and the distribution of aggregate wealth, which can be implemented via cultural and economic integration (Cerqueti et al., 2013). The necessary and sufficient conditions of the full tolerance equilibrium under asymmetric social cost function are also discussed, which provides guidance and reference to set policies and development strategy of social tolerance.

Before passing to the process of constructing evolutionary dynamics, we would like to highlight the difference between the present model and the model with local social cost functions (Shi & Pan, 2017). In fact, the difference is twofold. First, in the model with local social cost functions, the individual cost depends only on the share of intolerant people in a person's own group. Thus the individual cost comes from within the group. The present model studies the other scenario that the individual cost depends only on the share of intolerant people in the opposite group. In this case, the individual cost comes from outside the group. Second, the model with local social cost functions shows an exponential relationship between x_1 and x_2 in the phase space, which makes it difficult to achieve effective control of the evolutionary dynamics of social tolerance due to the irregular trajectory. In the present model, the slope of the linear relation between x_1 and x_2 determined by population numbers of each group and the distribution of aggregate wealth can be implemented via cultural and economic integration (Cerqueti et al., 2013), which leads to effective control of the evolutionary dynamics of social tolerance.

2. Evolutionary game model of tolerance

To model the evolutionary dynamics of social tolerance, we follow the model of two groups discussed by Cerqueti et al. (2013), and the theory of replicators is used that the strategy (tolerant and intolerant behaviour) that gives a higher payoff tends to spread in the society. We consider that N economic agents is divided into two differentiated groups with the corresponding populations N_1 and N_2. The populations of each group are assumed to be large enough and changeless with time. Each agent can be tolerant or intolerant towards the agents of another group, and the share of tolerant and intolerant agents in group i is indicated by x_i and $\tilde{x}_i$, respectively, where $\tilde{x}_i$ denotes the share of intolerant agents in group i. Two agents interact after being randomly matched, producing aggregate wealth $R_{ij} = R_{ji}$, which depends on the capital contribution of both agents. The capital contribution of agents in group i is denoted by k_i, and the relative capital contribution that agents in group i interact with agents in group j is defined as $\delta_{ij} \equiv k_i/(k_i + k_j)$, and determines the shares of the aggregate wealth R_{ij}. That is, the agent in group i shares $\delta_{ij}R_{ij}$ when he interacts with the agent in group j. The social tolerance influences the net gain of each agent in the following cases: (1) If the two agents are of the same group, then each agent obtains $R_{ii}/2$; (2) If the two agents are of different group and both tolerance, them suffer economic costs, including a psychological cost and a social cost, with the exception of $\delta_{ij}R_{ij}$. The psychological cost is chosen to be $\alpha_i = R_{ii}/2$ (Akerlof & Kranton, 2000; Cerqueti et al., 2013) in terms of loss of identity. The social cost, which describes the social reaction of intolerant agents adverse to the agents of the opposite group, is chosen to be asymmetric here. In the present work, simple asymmetric social cost functions $c_1 = \beta(1-x_2)$ and $c_2 = \beta(1-x_1)$ are used in comparison to the symmetric double-channel function $c_i = \beta(1-x_1x_2)$ used by Cerqueti et al. (2013); (3) If the two agents are of different group, no wealth is produced if any of them is intolerant.

According to the theory of replicators which has a good feature to provide an intuitive and meaningful economic analysis, the evolutionary dynamics of social tolerance can be described by

$$\dot{x}_i = x_i\tilde{x}_i[E(R_{x_i})-E(R_{\tilde{x}_i})], \tag{1}$$

where $E(R_{x_i})$ and $E(R_{\tilde{x}_i})$ are the expected net gain of tolerant and intolerant individuals in group i, respectively. We should note that the theory of replicators requires a large population, and the game model (2) implicitly assumes that tolerant and intolerant behaviour spreads based on the payoff of two strategies. According to the normalisation condition, we have $x_i + \tilde{x}_i = 1$ and $x_i, \tilde{x}_i \in [0, 1]$, for each $i = 1, 2$.

Considering the randomly match, in which all agents have the same probability to be selected, and we denote $P_{x_ix_j}$ (or $P_{x_i\tilde{x}_j}$) the probability for a tolerant agent of group i matches a tolerant (or intolerant) agent of group j, then these expected net gains can be calculated as follows:

$$\begin{aligned}
E(R_{x_1}) &= P_{x_1x_1}R_{11}/2 + P_{x_1\tilde{x}_1}R_{11}/2 + \left[\delta_{12}R_{12}-R_{11}/2-\beta(1-x_2)\right]P_{x_1x_2},\\
E(R_{x_2}) &= P_{x_2x_2}R_{22}/2 + P_{x_2\tilde{x}_2}R_{22}/2 + \left[\delta_{21}R_{21}-R_{22}/2-\beta(1-x_1)\right]P_{x_2x_1},\\
E(R_{\tilde{x}_1}) &= P_{\tilde{x}_1x_1}R_{11}/2 + P_{\tilde{x}_1\tilde{x}_1}R_{11}/2,\\
E(R_{\tilde{x}_2}) &= P_{\tilde{x}_2x_2}R_{22}/2 + P_{\tilde{x}_2\tilde{x}_2}R_{22}/2,
\end{aligned} \tag{2}$$

where $\delta_{12} + \delta_{21} = 1$, and the randomly match probabilities are

$$\begin{aligned}
&P_{x_1x_1} = \frac{x_1N_1-1}{N-1}, \quad P_{x_1\tilde{x}_1} = \frac{\tilde{x}_1N_1}{N-1}, \quad P_{x_1x_2} = \frac{x_2N_2}{N-1}, \quad P_{x_1\tilde{x}_2} = \frac{\tilde{x}_2N_2}{N-1},\\
&P_{\tilde{x}_1x_1} = \frac{x_1N_1}{N-1}, \quad P_{\tilde{x}_1\tilde{x}_1} = \frac{\tilde{x}_1N_1-1}{N-1}, \quad P_{\tilde{x}_1x_2} = \frac{x_2N_2}{N-1}, \quad P_{\tilde{x}_1\tilde{x}_2} = \frac{\tilde{x}_2N_2}{N-1},\\
&P_{x_2x_1} = \frac{x_1N_1}{N-1}, \quad P_{x_2\tilde{x}_1} = \frac{\tilde{x}_1N_1}{N-1}, \quad P_{x_2x_2} = \frac{x_2N_2-1}{N-1}, \quad P_{x_2\tilde{x}_2} = \frac{\tilde{x}_2N_2}{N-1},\\
&P_{\tilde{x}_2x_1} = \frac{x_1N_1}{N-1}, \quad P_{\tilde{x}_2\tilde{x}_1} = \frac{\tilde{x}_1N_1}{N-1}, \quad P_{\tilde{x}_2x_2} = \frac{x_2N_2}{N-1}, \quad P_{\tilde{x}_2\tilde{x}_2} = \frac{\tilde{x}_2N_2-1}{N-1}.
\end{aligned}$$

Given the above expected net gains, the motion of tolerant population with respect to time will be then modelled by the following differential equations:

$$\begin{aligned}\dot{x}_1 &= \frac{x_1\tilde{x}_1x_2N_2}{N-1}\left[\delta_{12}R_{12}-R_{11}/2-\beta(1-x_2)\right],\\ \dot{x}_2 &= \frac{x_2\tilde{x}_2x_1N_1}{N-1}\left[\delta_{21}R_{21}-R_{22}/2-\beta(1-x_1)\right].\end{aligned} \tag{3}$$

These equations give a complete description of the evolutionary dynamics of social tolerance. The equilibriums of Equations (3) are:

$$P_1=(1,1),\quad P_2=(\xi_1,0),\quad P_3=(0,\xi_2),\quad P_4=(1-\Omega_2,1-\Omega_1),\quad P_5=(0,0),$$

where $\Omega_1=[\delta_{12}R_{12}-R_{11}/2]/\beta$, $\Omega_2=[\delta_{21}R_{21}-R_{22}/2]/\beta$, and $\xi_1,\xi_2\in(0,1]$ are arbitrary constants. There are two more equilibriums $P_6=(1,\xi_3)$ and $P_7=(\xi_4,1)$ with $\xi_3,\xi_4\in(0,1]$ if $\Omega_1=0$ and $\Omega_2=0$, respectively. These equilibriums have precise economic and social meanings. For example, P_1 is the full tolerance equilibrium, P_2 and P_3 depict situations that one group is wholly populated by intolerant agents while another group is arbitrary tolerant.

3. Solutions in the phase plane

In comparison to the evolutionary dynamics of social tolerance under double-channel functions $c_i=\beta(1-x_1x_2)$ discussed by Cerqueti et al. (2013), Equations (3) which describe the evolutionary dynamics of social tolerance under asymmetric social cost function is integrable. In what follows, we give a description of the derivation of the solution in phase space.

Proposition 1. The solution of evolutionary social tolerance under asymmetric social cost function in phase plane is

$$\exp\left(N_1x_1-N_2x_2\right)(1-x_1)^{N_1\Omega_2}=C(1-x_2)^{N_2\Omega_1}, \tag{4}$$

where C is an integration constant that depends on initial values.

Proof. In the phase space, Equations (3) become:

$$\frac{x_1-(1-\Omega_2)}{\hat{x}_1N_2}\mathrm{d}x_1=\frac{x_2-(1-\Omega_1)}{\hat{x}_2N_1}\mathrm{d}x_2, \tag{5}$$

Integration of Equation (5) and using the following formulas

$$\begin{aligned}\int\frac{x_1-(1-\Omega_2)}{x_1-1}\mathrm{d}x_1 &= x_1+\Omega_2\ln|x_1-1|,\\ \int\frac{x_2-(1-\Omega_1)}{x_2-1}\mathrm{d}x_2 &= x_2+\Omega_1\ln|x_2-1|,\end{aligned}$$

we can obtain

$$x_1 + \Omega_2 \ln|x_1 - 1| = \frac{N_2}{N_1}[x_2 + \Omega_1 \ln|x_2 - 1|] + C.$$

After some simplification we obtain Equation (4).

4. Applications of the explicit solutions in the phase plane

The applications of the explicit formula derived are twofold. First, the steady states of the evolutionary dynamics can be easily identified from the solution in phase plane, and the evolutionary trajectory in the phase plane can be exactly analysed. Second, the explicit solutions obtained would be especially suited to effective control of the evolutionary dynamics of social tolerance.

Here we take the steady state (1, 0.5) as an example to illustrate the first application. From the initial value $x_1 = 1$ one can easily determine the integration constant as $C = 0$. In this case the solution of evolutionary social tolerance under asymmetric social cost function in phase plane becomes $\exp(N_1x_1 - N_2x_2)(1-x_1)^{N_1\Omega_2} = 0$. Due to the non-zero characteristics of $\exp(N_1x_1 - N_2x_2)$, we can obtain that $x_1 \equiv 1$, which is stable and not changing with time. Furthermore, using the same method, two general steady states $P_6 = (1, \xi_3)$ and $P_7 = (\xi_4, 1)$ with $\xi_3, \xi_4 \in (0, 1]$ can be easily determined to be stable.

The evolutionary trajectories in the phase plane can also be obtained by using the explicit solution. For a more general form, Equation (4) in phase space is shown in Figure 1 with the parameters $N_2/N_1 = 1$ and $C = -0.2$. We can clearly identify the steady states P_1, P_2, P_3 and P_4 for the case $\Omega_2 = 0.5$, and we can see that the steady state P_4 is a saddle point here. Comparison between the evolutionary trajectories for $\Omega_2 = 0.5$ and $\Omega_2 = 0.8$ shows that the evolutionary dynamics is sensitively dependent on Ω_1 and Ω_2, which are determined by population numbers of each group and the distribution of aggregate wealth.

The existence of multiple steady states always leads to complex trajectories. In fact, it may be quite difficult to give an analytical description of the evolutionary dynamics. This difficulty exists extensively within many subjects, including engineering

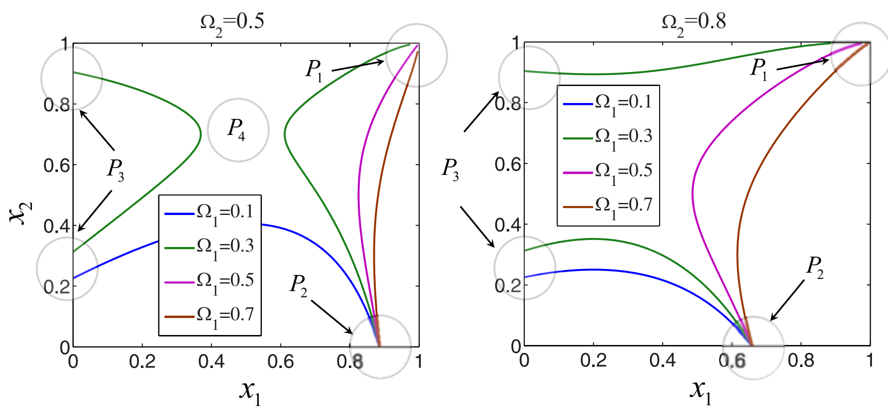

Figure 1. Different solutions of evolutionary social tolerance in phase plane.

science and physics (Xiong et al., 2012). Traditional way of dealing with such issue is the perturbation method which requires the uniform stability of the steady states. According to the Kolmogorov–Arnold–Moser theorem, the evolutionary trajectories are confined to closed tori that translate into regular closed orbits on Poincare sections for a regular oscillation due to the integrability of evolutionary dynamics in the present case (Xiong & Wu, 2018; You, 1999), and our results provides the possibility for the analytical description of the evolutionary dynamics of social tolerance.

Now we turn to show the second application that the explicit solutions obtained would be especially suited for effective control of the evolutionary dynamics of social tolerance. In fact, evolutionary trajectories in the phase space give the relationship between social tolerances of the two groups, which is quite important in tolerance control.

Usually, the relationship between social tolerances of the two groups is complicated and rising or falling trends can be discussed. For some parameters, the relationship between social tolerances of the two groups becomes linear, and in this case one can easily achieve tolerance control via economic integration (Shi & Peng, 2014). Here a typical example is shown in Figure 2 with the parameters $N_2/N_1 = 5$, $\Omega_2 = 0.5$, and $C = 0.002$, and we find a linear relation between x_1 and x_2 in the phase space when x_1 and x_2 are both small. Such a linear relation between x_1 and x_2 in the phase space can be analytically confirmed via the explicit formula (4). The slope of the linear relation between x_1 and x_2 is determined by Ω_1 and Ω_2, which are determined by population numbers of each group and the distribution of aggregate wealth.

Population numbers of each group are crucial for the evolutionary dynamics of social tolerance. However, these contents have not been well discussed (Akerlof & Kranton, 2000; Cerqueti et al., 2013; Shi & Pan, 2018b). In what follows, we will show that tolerance control also can be achieved by the adjustment of population numbers of each group. Here we take the parameter case $N_2 \ll N_1$ as an example.

For the case $N_2 \ll N_1$, the solution (4) becomes $x_1 + \Omega_2 \ln|x_1 - 1| = C + o(N_2/N_1)$ where the value of x_1 depend mainly upon the integration constant. In this case, the variation range of x_1 is quite narrow and can be well controlled. A similar situation occurs when $N_2 \gg N_1$ where the variations range of x_2 is quite narrow.

Evolutionary trajectories of x_1 and x_2 in the phase space are shown in Figure 3 with the parameters $N_2/N_1 = 0.1$, $\Omega_2 = 0.5$, and $C = -0.2$. We confirm that the

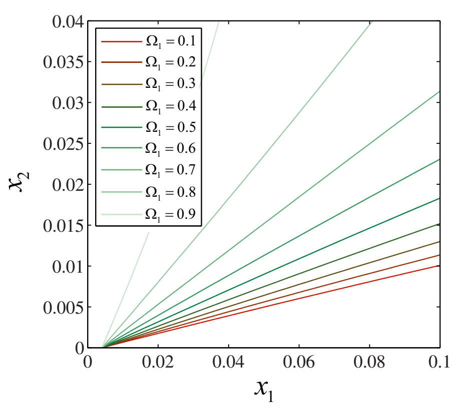

Figure 2. Linear relationship between x_2 and x_1 in the phase space.

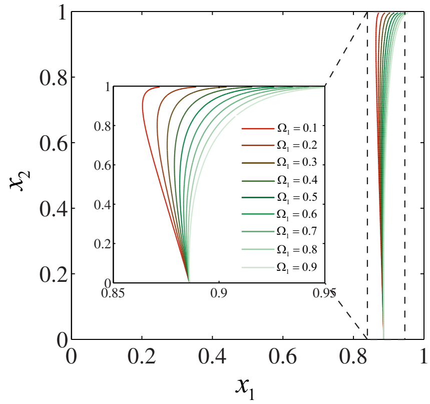

Figure 3. Evolutionary trajectories in the phase space

variation range of x_1 in these evolutionary trajectories are indeed quite narrow, although Ω_1 range from 0.1 to 0.9.

5. Necessary and sufficient conditions of full tolerance steady state

The full tolerance steady state $(1, 1)$ is of particular interest. In this section we focus our attention on discussing the necessary condition and sufficient condition of the full tolerance steady state $(1, 1)$.

Proposition 2. The necessary condition of achieving full tolerance is

$$R_{11} + R_{22} - 2R_{12} < 0$$

Proof. To verify the stability of steady state $(1, 1)$, we linearised the evolution Equation (3) by introducing infinitesimals δx_1 and δx_2 around the steady state:

$$\delta \dot{x}_1 = -\lambda_1 \delta x_1, \delta \dot{x}_2 = -\lambda_2 \delta x_2, \tag{7}$$

where

$$\lambda_1 = \frac{(\delta_{12}R_{12} - R_{11}/2)N_2}{N-1}, \lambda_2 = \frac{(\delta_{21}R_{21} - R_{22}/2)N_1}{N-1},$$

The solutions of Equations (7) are

$$\delta x_1 \propto \exp(-\lambda_1 t), \delta x_2 \propto \exp(-\lambda_2 t) \tag{8}$$

The necessary conditions of achieving full tolerance are $\lambda_1 > 0$ and $\lambda_2 > 0$. After some simplification we obtain Inequality (6).

The economics meaning is that tolerance is impossible if $R_{11} + R_{22} - 2R_{12} > 0$ where R_{12} is not sufficiently high to produce a tendency of mixed interaction in economic incentive. This necessary condition of achieving full tolerance is exactly the result obtained by Cerqueti et al. (2013). In fact, sufficient conditions of achieving full tolerance are in the same situation (Shi & Pan, 2018a). A sufficient condition of achieving full tolerance is also summarised as follows:

Proposition 3. A sufficient condition of achieving full tolerance at any starting point (x_1^0, x_2^0) is

$$\begin{aligned} &\beta < \delta_{12} R_{12} - R_{11}, \\ &\beta < R_{12} - \delta_{12} R_{12} - R_{22}, \end{aligned} \tag{9}$$

Proof. According to Equations 3, $\delta_{12}R_{12} - R_{11}/2 - \beta > 0$ ensures $\dot{x}_1 > 0$ for any x_1 while $\delta_{21}R_{21} - R_{22}/2 - \beta > 0$ ensures $\dot{x}_2 > 0$ for any x_2, so the inequalities $\delta_{12}R_{12} - R_{11}/2 - \beta > 0$ and $\delta_{21}R_{21} - R_{22}/2 - \beta > 0$ give a sufficient condition of achieving full tolerance at any starting point. After some simplification we can obtain Inequality (9).

6. Conclusions

The evolutionary dynamics of tolerance among heterogeneous economic agents is an interesting topic in both economics and sociology, and several theoretical approaches have been proposed recently. We discuss the dynamics of social tolerance among heterogeneous economic agents in an economic interaction model with asymmetric social cost functions, which is very different from the double-channel function case discussed by Cerqueti et al. (2013). In the economic interaction model with asymmetric social cost functions, the evolutionary trajectories can be exactly solved in the phase plane, and we obtain an explicit formula for the evolutionary trajectories in the phase plane. We show that the property of equilibriums depends on not only the distribution of economic factors such as aggregate wealth and social costs, but also the group populations. It is worth noting that the explicit formula cannot be revealed using traditional dynamic analysis method or linearised equations of the problem, due to the highly nonlinearity of the evolution equations. The applications of the solution in the phase plane are discussed, and we show that the equilibriums of the evolutionary dynamics can be easily identified and analysed from the solution in phase plane, which would be especially suited to effective control of the evolutionary dynamics of social tolerance. Especially, the slope of the linear relation between x_1 and x_2 is shown to be determined by population numbers of each group and the distribution of aggregate wealth, which can be implemented via cultural and economic integration (Cerqueti et al., 2013). The necessary and sufficient conditions of the full tolerance equilibrium under asymmetric social cost function are also discussed, which provides guidance and reference to set policies and development strategy of social tolerance that a lower social cost (determined by Equation (9)) gives a sufficient condition of achieving full tolerance at any starting point.

Disclosure statement

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