Routledge
Taylor & Francis Group

Economic Research-Ekonomska Istraživanja

ISSN: 1331-677X (Print) 1848-9664 (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

Environmental R&D subsidy, spillovers and privatization in a mixed duopoly

Mingqing Xing, Tingting Tan & Xia Wang

To cite this article: Mingqing Xing, Tingting Tan & Xia Wang (2019) Environmental R&D subsidy, spillovers and privatization in a mixed duopoly, Economic Research-Ekonomska Istraživanja, 32:1, 2989-3015, DOI: 10.1080/1331677X.2019.1658530

To link to this article: https://doi.org/10.1080/1331677X.2019.1658530

Published online: 09 Sep 2019.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2019, VOL. 32, NO. 1, 2989–3015
https://doi.org/10.1080/1331677X.2019.1658530

Environmental R&D subsidy, spillovers and privatization in a mixed duopoly

Mingqing Xing[a], Tingting Tan[b] and Xia Wang[c]

[a]School of Economics and Management, and Neural Decision Science Laboratory, Weifang University, Weifang, China; [b]Business School, Huzhou University, Huzhou, China; [c]School of Chemical and Environmental Engineering, Weifang University, Weifang, China

ABSTRACT

The study considers an environmental R&D subsidy in a mixed duopoly with spillovers. Public and private firms compete in environmental R&D investments and the government sets the subsidy to environmental R&D. This study examines three cases: (a) the public firm cares for the environment and maximises the welfare; (b) it does not care for the environment and maximises the sum of consumer and producer surpluses net of subsidy; and (c) it is privatised and maximises its own profit. The main findings are as follows: 1) the optimal subsidies are positive in all three cases; 2) the optimal subsidy always increases with spillovers in case (a), but it may decrease with spillovers in cases (b) and (c); 3) the optimal subsidy is higher in case (a) than in case (b) if the seriousness of environmental damage generated by pollution is small, however the opposite case may be the case if it is large; 4) the use of subsidy results in higher total R&D and lower environmental damage in all three cases; 5) privatisation lowers optimal subsidy (or environmental damage) no matter whether the public firm cares for the environment or not; and 6) privatisation reduces welfare if the public firm cares for the environment, but may raise welfare if it does not care for the environment.

ARTICLE HISTORY
Received 14 September 2018
Accepted 5 July 2019

KEYWORDS
Environmental R&D subsidy; R&D competition; mixed duopoly; spillover; privatization; Cournot competition

JEL CLASSIFICATION
O3; L1; H23

1. Introduction

With the global climate warming[1] and environmental pollution increasing, people are paying more attention to reducing pollution and improving environmental quality. Thus, most countries encourage the development and promotion of clean technologies. Since the market cannot internalise the environmental damages, the intervention of the government (regulator) is necessary (Ben Youssef & Dinar, 2011). In order to reduce pollution, the environmental R&D subsidy is a commonly used environmental regulatory instrument by the government. In many countries, the highly polluting industries usually include both public and private firms.[2] For example, in the steel industry of China, there are both public firms (e.g., Baosteel Group Co.) and private

CONTACT Mingqing Xing mqxing1979@163.com

firms (e.g., Shagang Group Co.). The Chinese government has subsidised the steel firms (whether they are public or private firms) that are in favour of eliminating backward production capacity in environmental protection equipment and technological transformation.

Environmental R&D activities have significant externalities (i.e., the spillover effect) in some industries.[3] When the spillovers in environmental R&D occur, a firm benefits not only from its own R&D effort but also from its rivals' efforts.[4] This may affect the environmental R&D subsidy policy of the government. Thus, the first objective of this study is to examine the effect of spillovers on the optimal subsidy to environmental R&D. Empirical evidence indicates that the public firms' concern for the environment differs widely across countries (Pal & Saha, 2015). For instance, Chang et al. (2015) analyse data from Chinese firms in some highly polluting industries and find that public firms tend to invest more in pollution reducing activities than private firms, while Hettige, Huq, Pargal, and Wheeler (1996) and Ohori (2006) point out that several public firms showed much higher pollution intensities than private firms in some other developing countries.[5] A public firm's attitude towards the environment obviously impacts its environmental R&D behaviour. When the government formulates the environmental R&D subsidy policy, it should take into account the public firms' environmental attitude. Thus, our second objective is to analyse how the environmental attitude of a public firm affects the R&D subsidy policy. Today, privatisation (or partial privatisation) of public firms has been a feature of government policy in many developing as well as developed countries (Xu, Cho, & Lee, 2016). Does the decision whether to privatise a public firm affect the environmental R&D subsidy policy, environmental R&D, environmental damage and welfare? The third objective of this study is thus to investigate the effect of privatisation on them in mixed markets.

Environmental R&D subsidy is an important regulatory instrument in encouraging the development of cleaner technologies. It is being given more attention by governments in many polluting industries including those characterised by mixed oligopolies.[6] However, this issue has not received much attention in the literature on mixed oligopoly, which is our focus of the present paper. The main purpose of this paper is to introduce the environmental R&D subsidy into a mixed duopoly and examine the optimal subsidy under different environmental concerns of a public firm.[7] This study first considers the following two scenarios: (a) the public firm cares for the environment; and (b) it does not. Next, it considers the case where the public firm is privatised (case c) to analyse the impact of privatisation on subsidy.

The main findings are as follows. Firstly, we find that the optimal subsidy is positive and always increases with spillovers in case (a). However, it may decrease with spillovers in case (b). This is different from the result that the spillovers always have a positive effect on the cost-reducing R&D subsidy in a mixed duopoly (Gil-Moltó, Poyago-Theotoky, & Zikos, 2006, 2011). Secondly, we suggest that the government should provide higher subsidy in case (a) than in case (b) if the seriousness of environmental damage is small, but the opposite may be the case if it is large. Thirdly, we show that the use of subsidy raises the private firm's R&D and total R&D[8] and improves environmental quality in both (a) and (b) but it reduces (does not affect)

the public firm's R&D in case (a) (case (b)).[9] Fourthly, privatisation leads to lower optimal subsidy regardless of the public firm's environmental attitude. This result is consistent with most literature on cost-reducing R&D subsidy in a mixed oligopoly (Gil-Moltó et al., 2006, 2011; Gil-Moltó, Poyago-Theotoky, Rodrigues-Neto, & Zikos, 2018). Finally, we find that under a policy of providing optimal subsidy to environmental R&D, after privatisation the total R&D and welfare reduce in case (a) and they may rise in case (b). However, privatisation always leads to lower environmental damage in both case (a) and case (b). Gil-Moltó et al. (2006, 2011) find that when the optimal subsidy to cost-reducing R&D is provided, the total R&D and welfare always decrease after privatisation. Obviously, our results in case (b) are different from theirs. Moreover, the effect of privatisation on the total R&D, environmental damage and welfare could be different when other environmental policies are used. For instance, if an environmental tax is employed, privatisation may damage the environment (Pal & Saha, 2015; Tsai, Wang, & Chiou, 2016; Xu et al., 2016; Haruna & Goel, 2019) and it always leads to reductions in the total R&D (Haruna & Goel, 2019) and welfare (Tsai et al., 2016).

The remainder of this paper is organised as follows. Section 2 gives a brief literature review and section 3 describes the basic model. Section 4 (section 5) solves the optimal R&D subsidy before (after) privatisation of the public firm. Section 6 makes a comparison of optimal R&D subsidy, environmental damage, welfare and others in sections 4 and 5. The final section presents conclusions.

2. Literature review

More and more countries recognise the importance of innovation in economic growth and promotion of international competitiveness. The use of subsidies to R&D is an important policy for the government to encourage firms to increase their R&D investments (Yang & Nie, 2015; Sekuła, 2017).[10] In recent years, the R&D subsidy in a mixed oligopoly, where private and public firms compete in R&D, has become an increasingly active field of interest.

Some scholars are concerned about the cost-reducing R&D subsidy policy in mixed oligopolies. Gil-Moltó et al. (2006, 2011) investigate the use of R&D subsidy in a mixed duopoly with spillovers and show that the optimal subsidy is positive and increases with spillovers, but it is higher compared with a private duopoly. When there is more than one private firm in a mixed oligopoly, Gil-Moltó et al. (2018) find that the optimal subsidy is also higher than in a private oligopoly. Lee et al. (2017) analyse R&D and output subsidies in a mixed duopoly and find that a R&D subsidy is welfare-inferior to an output subsidy, but Lee and Muminov (2017) give an agreement-based incentive R&D subsidy to internalise spillovers and obtain the opposite conclusion, and Kesavayuth and Zikos (2013) show that a R&D subsidy is socially superior (inferior) to an output subsidy if the spillovers are high (low). Moreover, Zikos (2007) and Lee and Tomaru (2017) investigate the policy mix of R&D and output subsidies and show that a tax on R&D with a subsidy on output can achieve the first-best allocation. These studies focus on the cost-reducing R&D subsidy in a mixed oligopoly. However, all of them do not consider the subsidy to

environmental R&D and the effect of the public firm's environmental concerns to it.

As people attach importance to the environment and enhance the awareness of environmental protection, more mixed oligopoly studies incorporate environment issues into their analysis. One aspect relates to environmental R&D in mixed markets. Some studies reveal the influence of corporate social responsibility (CSR), privatisation and technological efficiency on public or private firms' environmental R&D. Lambertini and Tampieri (2010) find that the presence of a CSR firm may induce the other firms to invest in green technology in a Cournot oligopoly with pollution. Tsai et al. (2016) show that privatisation cannot induce public and private firms to carry out more R&D concurrently and it may even curb both firms' R&D. Haruna and Goel (2019) think that the public firm has positive emissions-reducing R&D effort, but whether emissions reduction of the private firm is positive or zero depends on the efficiency of R&D technology. However, they do not examine the impact of subsidies on environmental R&D.

The second aspect is the literature on environmental policy (or regulation) in a mixed oligopoly.[11] Some scholars study how privatising a public firm affects the environmental tax policy and its effects on the environmental quality and welfare under such a policy. Bárcena-Ruiz and Garzón (2006) show that privatisation results in higher environmental tax and lower environmental damage. Pal and Saha (2015) think that the optimal privatisation damages the environment most if the public firm is unconcerned about the environment, but privatisation improves the environment if it cares for it. Tsai et al. (2016) find that privatisation may lead to a poorer environment if the imposition of environmental tax inadequately internalises pollution externality, and it reduces overall social welfare. Some other scholars investigate the effect of privatisation on the optimal environmental regulation (Naito & Ogawa, 2009) and on the welfare effects of emission tax and emission quota (Kato, 2010) and find that they critically depend on the degree of privatisation. These studies mainly focus on regulatory instruments such as environmental tax. However, the environmental R&D subsidy as an important regulatory instrument in mixed markets is neglected, which is the focus of the present paper.

3. The model

Consider an industry with two firms (firm 0 and firm 1). Firm 0 is entirely public (or privately) owned and firm 1 is entirely privately owned. They produce a homogeneous good. The inverse demand function is linear and given as follows:

$$P(Q) = a - Q \tag{1}$$

In (1), Q denotes the total output ($Q = q_0 + q_1$); q_i ($i = 0, 1$) is the firm i's output; and $a \geq Q$. We assume that both firms have the following production cost functions: $C(q_i) = cq_i + q_i^2$ ($i = 0, 1$).[12]

Production processes in both firms generate environmental pollution. For simplicity, we assume that producing one unit of output causes one unit of pollution. However, firms can reduce emissions by undertaking environmental R&D.[13] Both

firms use end-of-pipe technology in pollution abatement.[14] The emission generated by firm i after environmental R&D is as follows:[15] $e_i = q_i - x_i - \beta x_j$, where x_i is the environmental R&D effort for firm i and β ($\beta \in [0,1]$) is the degree of spillovers. Since there exist knowledge spillovers in environmental R&D, a firm can receive benefits not only from its own environmental R&D effort but also its competitor's effort. The environmental R&D expenditure for firm i is $\Gamma(x_i) = \frac{1}{2}x_i^2$. The total environmental damage is given by:[16]

$$D(E) = \frac{d}{2}\left[(q_0 - x_0 - \beta x_1) + (q_1 - x_1 - \beta x_0)\right]^2 = \frac{d}{2}\left[Q - (1+\beta)X\right]^2 \tag{2}$$

In (2), E denotes the total emission ($E = e_0 + e_1$); X denotes the total R&D effort ($X = x_0 + x_1$); and d ($d>0$) measures the seriousness of environmental damage generated by pollution,[17] which is assumed to be not very large.[18] Using (2), we get the following marginal environmental damage $D'(E) = dE$.

The government subsidises the environmental R&D level of each firm.[19] Each firm receives a subsidy s per unit of environmental R&D output, $S_i = sx_i$. Thus, the profit function for firm i is:[20]

$$\begin{aligned} \pi_i &= P(Q)q_i - C(q_i) - \Gamma(x_i) + S_i \\ &= P(Q)q_i - (cq_i + q_i^2) - \frac{1}{2}x_i^2 + sx_i, \quad i = 0,1 \end{aligned} \tag{3}$$

The social welfare is defined as the sum of consumer surplus (CS) and producer surplus ($PS = \pi_0 + \pi_1$) net of environmental R&D subsidies ($S_0 + S_1$) and total environmental damage ($D(E)$):

$$\begin{aligned} SW &= CS + PS - (S_0 + S_1) - D(E) = \int_0^Q P(z)dz - C(q_0) - C(q_1) - \Gamma(x_0) - \Gamma(x_1) - D(E) \\ &= \frac{1}{2}(q_0 + q_1)^2 + (a - c - 2q_0 - q_1)q_0 + (a - c - q_0 - 2q_1)q_1 - \frac{1}{2}(x_0^2 + x_1^2) \\ &\quad - \frac{d}{2}\left[(q_0 - x_0 - \beta x_1) + (q_1 - x_1 - \beta x_0)\right]^2 \end{aligned} \tag{4}$$

Notice that the environmental R&D subsidy cancels out in the social welfare function when we aggregate (see (4)). This implies that the subsidy to environmental R&D has no direct effect on the social welfare. However, it may have indirect effect on the social welfare via two firms' output or environmental R&D.

The objective of the public firm is assumed to maximise the social welfare in most mixed oligopoly literature on the environment. However, several scholars point out that the public firm may not share the same objectives as the government.[21] Empirical evidence shows that the public firms' concern for the environment differs widely across countries (Hettige et al., 1996; Ohori, 2006; Chang et al., 2015). We assume that the public firm's objective function is as follows:[22]

$$\Omega = CS + PS - (S_0 + S_1) - kD(E) = \int_0^Q P(z)dz - C(q_0) - C(q_1) - \Gamma(x_0) - \Gamma(x_1) - kD(E), k \in \{0, 1\} \tag{5}$$

In (5), if $k = 1$, the public firm cares about the environmental damages, and its objective is to maximise the social welfare. This case corresponds to the standard case in which the objective of the public firm is the same as the government. However, if $k = 0$, the public firm is not concerned about environmental damages. In this case, its objective is to maximise the sum of consumer and producer surpluses net of environmental subsidy. This study will consider both scenarios to investigate how the optimal environmental R&D subsidy may vary between two cases.[23]

In order to examine how privatisation of the public firm affects the optimal subsidy to environmental R&D, we also consider the private duopoly case, in which firm 0 is also entirely privately owned. In this case, both firm 0 and firm 1 pursue maximum profits.

The game has three stages. In the first stage, the government determines the environmental R&D subsidy rate (s) to maximise social welfare. In the second stage, each firm decides on the environmental R&D level (x_i) simultaneously and independently. In the third stage, firms choose the output (q_i) simultaneously and independently.

4. Mixed duopoly

In this section, firm 0 is entirely public owned, whereas firm 1 is entirely privately owned. According to the public firm's attitude towards the environment, we consider the following two scenarios: one in which the public firm cares about environmental damages (i.e., $k = 1$) (case (a)) and the other where it does not (i.e., $k = 0$) (case (b)). The public and private firms pursue different objectives in this section. The objective of the public firm is to maximise social welfare (the sum of consumer and producer surpluses net of environmental subsidy) if it cares (does not care) for the environment as shown in (5). However, the objective of the private firm is to maximise its profits as shown in (3) when $i = 1$. As usual, we solve the game by the backward induction.

4.1. Production

First, we examine the final stage. In stage 3, the public (private) firm determines production level to maximise social welfare (5) (profit (3)). Differentiating (5) (profit (3)) with respect to q_0 (q_1), we have the following first-order conditions:

$$\begin{aligned} \frac{\partial \Omega}{\partial q_0} &= P(Q) - C'(q_0) - kD'(Q - (1+\beta)X) \\ &= m + kd(1+\beta)(x_1 + x_0) - (3 + kd)q_0 - (1 + kd)q_1 = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= P(Q) + P'(Q)q_1 - C'(q_1) \\ &= m - q_0 - 4q_1 = 0 \end{aligned} \tag{7}$$

In (6) and (7), $m = a-c$.[24] Solving above conditions gives:

$$q_0^t(x_0, x_1) = \frac{(3-kd)m + 4kd(1+\beta)(x_1+x_0)}{11+3kd} \tag{8}$$

$$q_1^t(x_0, x_1) = \frac{(2+kd)m - kd(1+\beta)(x_1+x_0)}{11+3kd} \tag{9}$$

Note that superscript $t = a$ ($t = b$) if $k = 1$ ($k = 0$). Using (8) and (9), we carry out comparative statics and obtain $\frac{\partial q_0^a}{\partial x_0} = \frac{\partial q_0^a}{\partial x_1} = \frac{\partial q_0^a}{\partial X} = \frac{4d(1+\beta)}{11+3d} > 0$, $\frac{\partial q_1^a}{\partial x_1} = \frac{\partial q_1^a}{\partial x_0} = \frac{\partial q_1^a}{\partial X} = -\frac{d(1+\beta)}{11+3d} < 0$, $\frac{\partial q_0^b}{\partial x_0} = \frac{\partial q_0^b}{\partial x_1} = \frac{\partial q_0^b}{\partial X} = 0$ and $\frac{\partial q_1^b}{\partial x_1} = \frac{\partial q_1^b}{\partial x_0} = \frac{\partial q_1^b}{\partial X} = 0$. These results show that in the mixed duopoly regime, the public (private) firm's output increases (decreases) with its own (its rival's or the total) environmental R&D effort if $k = 1$, whereas the public (private) firm's output does not depend on them if $k = 0$. The reason is as follows. For $k = 1$, an increase in X (x_0 or x_1) reduces the effective marginal cost of the public firm (i.e., a decrease in $C'(q_0) + D'(E)$), so that the public firm has a stronger incentive to produce, while the private firm decreases its output due to the strategic substitution (β can strengthen this effect).[25] In addition, for $k = 0$, an increase in X (x_0 or x_1) does not affect the effective marginal cost of both firms, so that the firms' environmental R&D investments play no role in changing the production allocation.

4.2. R&D

We proceed to consider the R&D stage. By substituting (8) and (9) into the objective functions of two firms (3) and (5), we derive the following first-order conditions for the firms' optimal R&D decisions:

$$\begin{aligned}\frac{\partial \Omega(x_0, x_1)}{\partial x_0} &= (1+\beta)kD'(Q^t - (1+\beta)X) - \left[P'(Q^t)q_1^t + kD'(Q^t-(1+\beta)X)\right]\frac{\partial q_1^t}{\partial X} - \Gamma'(x_0) \\ &= \frac{kd(1+\beta)}{(11+3kd)^2}\left[(53+19kd)m - (121+43kd)(1+\beta)(x_0+x_1)\right] - x_0 = 0\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\partial \pi_1(x_0, x_1)}{\partial x_1} &= s + P'(Q^t)q_1^t\frac{\partial q_0^t}{\partial X} - \Gamma'(x_1) \\ &= -\frac{4kd(1+\beta)}{(11+3kd)^2}\left[(2+kd)m - kd(1+\beta)(x_1+x_0)\right] + s - x_1 = 0\end{aligned} \tag{11}$$

Solving (10) and (11) yields:

$$x_0^t(s) = \frac{kd(1+\beta)\left\{\left[(53+19kd)(11+3kd) + 8(11+4kd)kd(1+\beta)^2\right]m - (121+43kd)(11+3kd)(1+\beta)s\right\}}{(11+3kd)\left[(11+3kd)^2 + kd(121+39kd)(1+\beta)^2\right]} \tag{12}$$

$$x_1^t(s) = \frac{\left[(11+3kd)^2 + kd(121+43kd)(1+\beta)^2\right](11+3kd)s - 4kd(1+\beta)\left[(2+kd)(11+3kd) + 2(11+4kd)kd(1+\beta)^2\right]m}{(11+3kd)\left[(11+3kd)^2 + kd(121+39kd)(1+\beta)^2\right]} \tag{13}$$

Substituting (12) and (13) into (8) and (9), we obtain:

$$q_0^t(s) = \frac{\left[(3-kd)(11+3kd) + (33+7kd)kd(1+\beta)^2\right]m + 4kd(11+3kd)(1+\beta)s}{(11+3kd)^2 + kd(121+39kd)(1+\beta)^2} \tag{14}$$

$$q_1^t(s) = \frac{\left[(2+kd)(11+3kd) + 2(11+4kd)kd(1+\beta)^2\right]m - kd(11+3kd)(1+\beta)s}{(11+3kd)^2 + kd(121+39kd)(1+\beta)^2} \tag{15}$$

Note that to guarantee the firms' R&D efforts, outputs and emissions are not negative, s must satisfy $\underline{s} \le s \le \bar{s}$ in this section, in which $\underline{s} = \frac{4kd(1+\beta)[(2+kd)(11+3kd)+2kd(1+\beta)^2(11+4kd)]m}{(11+3kd)[(11+3kd)^2+kd(1+\beta)^2(121+43kd)]}$ and

$$\bar{s} = \frac{\left(\begin{array}{c}[(11+3kd)+4kd(1+\beta)][(2+kd)(11+3kd)+2(11+4kd)kd(1+\beta)^2] - kd\beta(1+\\ \beta)[(53+19kd)(11+3kd)+8(11+4kd)kd(1+\beta)^2]\end{array}\right)m}{(11+3kd)\{[kd(1+\beta)+(11+3kd)](11+3kd)+kd(121+43kd)(1-\beta)(1+\beta)^2\}}.$$

Then, the total environmental damage is calculated as:

$$D^t(s) = \frac{d}{2}\left(\frac{5\left[(11+3kd)+2kd(1+\beta)^2\right]m - 11(1+\beta)(11+3kd)s}{(11+3kd)^2 + kd(121+39kd)(1+\beta)^2}\right)^2 \tag{16}$$

Lemma 1. *In the mixed duopoly, (i) when the public firm cares about the environment, the private (public) firm's environmental R&D increases (decreases) with the subsidy,*[26] *and the total environmental R&D increases with the subsidy; and (ii) when the public firm does not care about the environment, the private firm's (or the total) environmental R&D increases with the subsidy, and the public firm's environmental R&D does not depend on the subsidy.*

Proof. See Appendix.

The government determines the subsidy rate at the first stage, and this subsidy rate affects future environmental R&D investment of the private (or public) firm. The private firm's R&D effort increases with larger subsidy rate in both case (a) and case (b), whereas the public firm's R&D effort decreases with larger subsidy rate in case (a) and is not affected by the subsidy in case (b). This effect plays a salient role between the firms and the government.

The reason for Lemma 1 is as follows. In the first line of (10) ((11)), the term $(1+\beta)kD'(Q^t-(1+\beta)X) - [P'(Q^t)q_1^t + kD'(Q^t-(1+\beta)X)]\frac{\partial q_1^t}{\partial X}$ $(s + P'(Q^t)q_1^t\frac{\partial q_0^t}{\partial X})$ is the marginal benefit of the public (private) firm from environmental R&D, whereas

$\Gamma'(x_0)$ ($\Gamma'(x_1)$) is the marginal cost. Since q_i^t is independent from s (see (8) and (9)), an increase in s definitely enhances the marginal benefit to the private firm, so that it conducts more environmental R&D (for both $k = 1$ and $k = 0$). The same result does not apply to the public firm. Since the subsidy is also a cost in the objective function of the public firm (see (5)), this cancels out the positive effect of the subsidy. As a result, the subsidy has only an indirect effect on the public firm's environmental R&D behaviour through the private firm's environmental R&D.

In the linear demand model,[27] for $k = 1$ the increase in the private firm's environmental R&D contributes to a decrease in the public firm's marginal benefit. Accordingly, the public firm reduces its environmental R&D due to strategic substitution. However, for $k = 0$, the increase in the private firm's environmental R&D does not affect the public firm's marginal benefit, so that the public firm does not conduct environmental R&D. In addition, for $k = 1$, the increase in the private firm's environmental R&D level outweighs the decrease in the public firm's level for all values of $\beta \in [0, 1]$. Thus, the total environmental R&D level always increases with the subsidy.[28] For $k = 0$, the total environmental R&D level obviously increases with the subsidy.[29]

Lemma 2. *In the mixed duopoly, (i) when the public firm cares about the environment, the public (private) firm's output increases (decreases) with the subsidy, and the total output increases with the subsidy; and (ii) when the public firm does not care about the environment, the public (private) firm's output and the total output do not depend on the subsidy.*

Proof. See Appendix.

The intuition for part (i) of Lemma 2 is the following. The subsidy has no direct effect on social welfare (see (4)), so that it does not directly affect the public firm's output if it cares for the environment. The subsidy affects the public firm's output via the effect it exerts on the total environmental R&D output (see Footnote 23). Since the total environmental R&D increases with the subsidy (see Lemma 1), the public firm's output increases as the subsidy increases if it cares for the environment.[30] The increase in the subsidy leads to an increase in the total environmental R&D and the public firm's output, which in turn leads to a decrease in the private firm's output due to strategic substitution (see Footnote 25). Because the positive effect of the subsidy on the public firm's output dominates the negative effect on the private firm's output, the total output increases with the subsidy.

The reason for part (ii) of lemma is that the subsidy affects the firms' output via the effect it exerts on the environmental R&D. When the public firm is unconcerned about the environment, the environmental R&D has no effect on the private (or public) firm's output (see (8) and (9)). Thus, the public (or private) firm's output and the total output do not depend on the subsidy. Note that this result is different from that in the case when the public firm cares about the environment. This implies that whether the environmental R&D subsidy affects the public (or private) firm's output or not depends on the public firm's environmental attitude in a mixed duopoly.

Lemma 3. *In the mixed duopoly, regardless of whether the public firm cares about the environment or not, the total environmental damage decreases with the subsidy.*

Proof. See Appendix.

The reason for this is as follows. If the public firm cares for the environment, two effects are determining this result: (i) the subsidy will increase the total output (see Lemma 2), which leads to an increase in the total environmental damage; and (ii) the subsidy will increase the total environmental R&D (see Lemma 1), which leads to a decrease in the total environmental damage. The former effect is compensated by the latter effect. Thus, the subsidy has a positive effect on the environmental improvement. In addition, if the public firm does not care for the environment, the subsidy has no effect on the total output (see Lemma 2). The subsidy affects the total environmental damage only via the environmental R&D output. The subsidy will increase the total environmental R&D (see Lemma 1), which leads to a decrease in the total environmental damage.[31]

Lemma 4. *In the mixed duopoly, (i) when the public firm cares for the environment, the public firm's profit increases (increases first and then decreases) with the subsidy if the degree of spillover is small (large) enough, and the private firm's profit (or the total profit) increases with the subsidy; and (ii) when the public firm does not care for the environment, the public firm's profit does not depend on the subsidy, and the private firm's profit (or the total profit) increases with the subsidy.*

Proof. See Appendix.

The intuition for this lemma is as follows. First, we consider the part (i) of the lemma. If the public firm cares for the environment, the public firm's output (or the total output) increases and the private firm's output decreases with the subsidy (see Lemma 2). It follows that the price decreases and the public (private) firm's production cost increases (decreases) with the subsidy. This leads to a reduction in the public (or private) firm's gross profit with an increase of subsidy. Using Lemma 1, the public (private) firm's environmental R&D decreases (increases) with the subsidy. It follows that the public (private) firm's R&D cost also decreases (increases) with the subsidy. In addition, it also follows that the subsidy received by the public firm increases (increases first and then decreases) with the subsidy if the degree of spillover is small (large) enough, and the subsidy received by the private firm always increases with the subsidy.

A firm's profit equals the sum of its gross profit and subsidy it received net of its R&D cost. For the public firm, if the degree of spillover is small enough, the increase of subsidy it received and the decrease of R&D cost can compensate for the decrease of gross profit. However, if both the degree of spillover and the subsidy rate are large, the decrease of R&D cost can't compensate for the decrease of gross profit and subsidy it received. Thus, the public firm's profit increases (increases and then decreases) with the subsidy if the degree of spillover is small (large) enough. For the private firm, the increase of subsidy it received can compensate for the decrease of gross profit and the increases of R&D cost. Thus, the private firm's profit increases with the subsidy. Because the effect of subsidy on the total subsidy received by firms dominates its effect on the total gross profit and the total R&D cost, the total profit increases with the subsidy.

Now, we turn to part (ii) of Lemma 4. If the public firm does not care for the environment, the subsidy does not affect the firms' output (see Lemma 2). It follows that the subsidy does not affect the price and the firms' production cost. Thus, the subsidy also does not affect the firms' gross profit. Since the public firm does not invest in environmental R&D if it is unconcerned about the environment, the subsidy does not affect the subsidy it received and its R&D cost. Thus, the public firm's profit does not depend on the subsidy. However, for the private firm, both the subsidy it received and its R&D cost increase with the subsidy. Because the increase of subsidy it received can compensate for the increase of its R&D cost, the private firm's profit increases with the subsidy. Obviously, the total profit also increases with the subsidy.

Lemma 5. *In the mixed duopoly, regardless of whether the public firm cares about the environment or not, the social welfare increases (decreases) with the subsidy if the subsidy rate is small (large).*

Proof. See Appendix.

This lemma implies that the relationship between the social welfare and the subsidy rate is inverse U shape. The reason for the lemma is as follows. If the public firm cares for the environment, with the increase of subsidy, consumer and producer surpluses (or the total subsidy) increase and environmental damage decreases (see Lemma 3).[32] If the subsidy rate is small (large), the positive effect of subsidy on social welfare (higher consumer surplus, higher producer surplus and lower environmental damage) dominates (is dominated by) its negative effect on social welfare (higher total subsidy). Thus, the social welfare increases (decreases) with the subsidy if the subsidy rate is small (large).

In addition, if the public firm does not care for the environment, with the increase of subsidy, consumer surplus does not change, producer surplus (or the total subsidy) increases and environmental damage decreases. If the subsidy rate is small (large), the positive effect of subsidy on social welfare (higher producer surplus and lower environmental damage) dominates (is dominated by) its negative effect on social welfare (higher total subsidy). Thus, the social welfare increases (decreases) with the subsidy if the subsidy rate is small (large).

4.3. R&D subsidy

In stage 1, the government sets the environmental R&D subsidy rate (s) to maximise social welfare. The first-order condition is:

$$\begin{aligned}\frac{dSW}{ds} &= \left[P(Q^t)-C'(q_0^t)-D'(E^t)\right]\frac{\partial q_0^t}{\partial X}\frac{\partial X^t}{\partial s}+\left[P(Q^t)-C'(q_1^t)-D'(E^t)\right]\frac{\partial q_1^t}{\partial X}\frac{\partial X^t}{\partial s}\\ &\quad+\left[-\Gamma'(x_0^t)+(1+\beta)D'(E^t)\right]\frac{\partial x_0^t}{\partial s}+\left[-\Gamma'(x_1^t)+(1+\beta)D'(E^t)\right]\frac{\partial x_1^t}{\partial s} \qquad (17)\\ &= \frac{(1+\beta)\tau m-(11+3kd)\varpi s}{(11+3kd)\left[(11+3kd)^2+kd(121+39kd)(1+\beta)^2\right]^2}=0\end{aligned}$$

Note that expressions for ϖ and τ in (17) are too long and they are given in the Appendix.

For $k = 1$, combined with the first-order condition of firm 0 in the second stage, equation (17) shows that the government sets s so as to equalise both firms' environmental R&D costs (i.e., $\Gamma'(x_0^a) = \Gamma'(x_1^a)$).[33] Using (11) and (17), we have $s = [-P'(Q^a)q_1^a - D'(E^a)]\frac{\partial q_1^a}{\partial X} - P'(Q^a)q_1^a\frac{\partial q_0^a}{\partial X} + (1+\beta)D'(E^a)$. In addition, for $k = 0$, the government sets s so as to meet $\Gamma'(x_1^b) = (1+\beta)D'(E^b)$.[34] Combining with (11), we have $s = (1+\beta)D'(E^b)$. Note that, E^a and E^b are shown in Footnote 31. Obviously, the optimal subsidies may be different in cases (a) and (b) as the firms' environmental R&D investments play an important (play no) role in changing the production allocation if $k = 1$ ($k = 0$). Specifically, the subsidy can (cannot) change the production allocation through the firms' environmental R&D if $k = 1$ ($k = 0$). The change of production allocation will affect consumer and producer surpluses, so that the subsidy indirectly affects consumer and producer surpluses if $k = 1$, whereas it does not affect them if $k = 0$. This may lead to the difference of optimal subsidies in cases (a) and (b).

Solving (17) gives the optimal environmental R&D subsidy:

$$s^t = \frac{(1+\beta)\tau m}{(11+3kd)\varpi} \tag{18}$$

Using (18), we can prove the following results.

Proposition 1. *In the mixed duopoly, (i) $s^a>0$ and $s^b>0$; (ii) s^a always increases with β; and (iii) s^b increases with β if d is small, whereas it may decrease with β if d is large.*

Proof. See Appendix.

Gil-Moltó et al. (2006, 2011) examine the use of cost-reducing R&D subsidy in a mixed market and also show that the optimal subsidy is positive. However, we find that the optimal environmental R&D subsidy may decrease with spillovers when the public firm does not care for the environment.[35] This is different from the finding of Gil-Moltó et al. (2006, 2011), who think that the optimal subsidy always increases with spillovers. Accordingly, when the government examines the effect of spillovers on the R&D subsidy policy, it should distinguish the types of subsidy (a cost-reducing R&D subsidy or an environmental R&D subsidy).[36] In addition, Proposition 1 also implies that, when the government investigates how the spillovers affect the environmental R&D policy, it should consider the environmental concerns of firms and the degree of environmental damage caused by production.

The economic explanation of this proposition is as follows. The result of part (i) stems from the role that the environmental R&D subsidy plays in correcting two types of market failures: (i) the under-investment in environmental R&D by private firms;[37] and (ii) the excess environmental damage caused by production (as defined by Kato (2013)). From Lemmas 1 and 3, regardless of the public firm's environmental attitude, the environmental R&D subsidy really increases the total environmental

R&D and decreases the total (or the marginal) environmental damage. Thus, the government should provide the subsidy to environmental R&D in both cases (a) and (b).

Next, we consider part (ii). When the public firm cares for the environment, the spillovers always weaken the positive effect of the subsidy on the total environmental R&D (denoted by the effect I of β),[38] and strengthen (weaken) the negative effect of the subsidy on the marginal damage if d is small or d is large and β is small (if both d and β are large)[39] (denoted by the effect II of β) (see Footnotes 28 and 31). Obviously, when both d and β are large, the government should raise subsidy with larger spillovers. In addition, when d is small or d is large and β is small, the effect I of β exceeds the effect II of β. In this case, the government should also raise subsidy as the spillovers increase. In summary, the optimal subsidy of the government increases with the spillovers in case (a).

Finally, we explain part (iii). Unlike case (a), when the public firm does not care for the environment, the spillovers do not change the effect of the subsidy on the total environmental R&D (see Footnote 29),[40] but it can change the negative effect of the subsidy on the total (or the marginal) environmental damage (see Footnote 31). To maximise the social welfare, the government sets the subsidy rate to equal $(1+\beta)D'(E^b)$. With an increase of β, $1+\beta$ increases, while $D'(E^b)$ decreases. The effect of β on the marginal environmental damage is small if d is small. This leads to an increase of $(1+\beta)D'(E^b)$. Thus, the optimal subsidy increases with the spillovers if d is small. However, if d is large, the effect of β on the marginal environmental damage is large and may exceed it on $1+\beta$. In this case, $(1+\beta)D'(E^b)$ may decrease as β increases. Thus, the optimal subsidy may decrease with the spillovers if d is large.

We further compare the optimal subsidy of the government in cases (a) and (b), and obtain the following result.

Proposition 2. *In the mixed duopoly, s^a is larger than s^b if d is small, whereas s^a may be smaller than s^b if d is large.*

Proof. See Appendix.

This is an interesting result. On the surface, the government only needs to provide a lower environmental R&D subsidy in case (a) than case (b) due to the public firm investing more into environmental R&D if it cares about the environment than if it does not. However, we show that the opposite appears if the seriousness of environmental damage is small. This deserves the attention of policymakers. They can not only determine the level of environmental R&D subsidy according to the environmental attitude of firms in a mixed market, but also need to take into account other factors such as the degree of environmental damage caused by production.

The intuition for Proposition 2 is as follows. The subsidy can decrease the total environmental damage under both case (a) and case (b) (see Lemma 3). Moreover, the subsidy can increase the total output if the public firm is concerned about the environment (see Lemma 2), and therefore can also increase consumer surplus. However, the subsidy does not impact consumer surplus if the public firm is unconcerned about the environment. When d is small, the impact of subsidy on total

environmental damage is also small. Thus, rendering investing in environmental R&D is more socially profitable in case (a) than in case (b). However, when d is large, the impact of subsidy on total environmental damage is also large. Since an increase in the total output also increases the total emission (given X) in case (a), the effect of subsidy on total environmental damage may be higher if the public firm does not care for the environment than if it does. In this case, the marginal benefit from subsidy in case (b) may exceed that in case (a).[41]

5. Private duopoly

In this section, the public firm is completely privatised (case c), i.e., both firm 0 and firm 1 are private firms. They seek to maximise their own profits. The objective function for firm i is as shown in (3) when $i = 0, 1$.

5.1. Production

In the third stage, the firms choose their outputs to maximise their revenue given in (3). The corresponding first-order conditions are derived as:

$$\frac{\partial \pi_i}{\partial q_i} = P(Q) + P'(Q)q_i - C'(q_i) = m - q_j - 4q_i = 0, i = 0, 1, i \neq j \tag{19}$$

Solving (19) yields equilibrium outputs:

$$q_0^c = q_1^c = \frac{1}{5}m \tag{20}$$

Using (20), we can easily prove $\frac{\partial q_i^c}{\partial x_i} = \frac{\partial q_i^c}{\partial x_j} = \frac{\partial q_i^c}{\partial X} = 0$ $(i = 0, 1,\ i \neq j)$. This indicates that in the private duopoly regime, each firm's output does not depend on its (the rival's or the total) environmental R&D effort. Rather, the firms' environmental R&D investments play no role in changing the production allocation. Note that these results are different from those in case (a), but they are similar to those in case (b).

5.2. R&D

In the second stage, each firm determines its environmental R&D level so as to maximise its profits. The first-order conditions are:

$$\frac{\partial \pi_i(x_0, x_1)}{\partial x_i} = s - \Gamma'(x_i) = s - x_i = 0, i = 0, 1 \tag{21}$$

The above equation system shows that the firms' environmental R&D behaviour completely depends on the subsidy. Firm i conducts its R&D to maximise the pure profits from investments (i.e., $sx_i - \Gamma(x_i)$). In other words, the firm i's optimal behaviour in R&D stage is characterised as:

$$s = \Gamma'(x_i), i = 0, 1 \tag{22}$$

which indicates that firms themselves adjust their investments to be equalised.

Solving (21) gives equilibrium R&D levels:

$$x_0^c(s) = x_1^c(s) = s \tag{23}$$

Note that to ensure the firms' emissions are not negative, s must satisfy $0 \leq s \leq \frac{m}{5(1+\beta)}$ in this section.

Now, substituting the above expressions for q_i^c and x_i^c $(i = 0, 1)$ into (2), we obtain the total environmental damage as follows:

$$D^c(s) = \frac{2d}{25}[m - 5(1+\beta)s]^2 \tag{24}$$

Lemma 6. *In the private duopoly, each firm's environmental R&D and the total environmental R&D output increase with the subsidy.*

Proof. See Appendix.

In the private duopoly, the environmental R&D subsidy is a net inflow for both firms. This lemma can be interpreted as follows. In (21), the term s is the marginal benefit of firm i from environmental R&D and the term $\Gamma'(x_i)$ is the marginal cost. Since q_i^c is independent from s (see (20)), an increase in s enhances the marginal benefit to firm i. It follows that firm i conducts more environmental R&D.[42]

Note that, in the private duopoly (case (c)) the effect of subsidy on the firm 1's environmental R&D incentives is similar as in the mixed duopoly (cases (a) and (b)). However, the effect of subsidy on the firm 0's environmental R&D incentives is different from that in the mixed duopoly (case (a) and (b)). For example, the subsidy catalyses the environmental R&D of firm 0 in case (c), but curbs (does not affect) its environmental R&D in case (a) (case (b)). This implies that, privatisation may change the effect form of subsidy on the public firm's environmental R&D incentives.

Lemma 7. *In the private duopoly, each firm's output and the total output do not depend on the subsidy.*

Proof. See Appendix.

The reason is as follows. In the private duopoly, the environmental R&D subsidy affects each firm's output via the effect it exerts on the environmental R&D output. However, the environmental R&D has no effect on each firm's output (see (20)). It follows that each firm's output and the total output are not affected by the subsidy. Note that, unlike this result, the subsidy can affect each firm's output and the total output in case (a).

Lemma 8. *In the private duopoly, the total environmental damage decreases with the subsidy.*

Proof. See Appendix.

The intuition for this result is as follows. In the private duopoly, the environmental R&D subsidy has no effect on the total output (see Lemma 7). Accordingly, the subsidy affects the total environmental damage only through the environmental R&D output. The subsidy increases the total environmental R&D (see Lemma 6), which leads to a decrease in the total environmental damage.[43] Thus, the subsidy has a positive effect on the environmental improvement.[44]

5.3. R&D subsidy

In the first stage, the government maximises the social welfare function by choosing optimal subsidy. The first-order condition is as follows:

$$\begin{aligned}\frac{dSW}{ds} &= \left[-\Gamma'(x_0^c) + (1+\beta)D'(E^c)\right]\frac{\partial x_0^c}{\partial s} + \left[-\Gamma'(x_1^a) + (1+\beta)D'(E^c)\right]\frac{\partial x_1^c}{\partial s} \\ &= \frac{2}{5}\left[2(1+\beta)dm - 10(1+\beta)^2 ds - 5s\right] = 0\end{aligned} \tag{25}$$

Symmetry of investments is ensured by the firms' own optimal actions (see (22)). This leads to the obvious policy recommendation that the government sets the subsidy rate to meet:

$$\Gamma'(x_0) = \Gamma'(x_1) = (1+\beta)D'(E^c) \tag{26}$$

This comes from the limited role of environmental R&D in the sense that the investments never influence the production allocation. In (26), E^c is shown in Footnote 43.

Solving (25), we get the following optimal environmental R&D subsidy:

$$s^c = \frac{2(1+\beta)dm}{5\left[1+2(1+\beta)^2 d\right]} \tag{27}$$

Proposition 3. *In the private duopoly, (i) $s^c > 0$; and (ii) s^c increases with the spillovers if d is small, whereas it decreases with the spillovers if d is large.*

Proof. See Appendix.

The economic intuition is as follows. In the private duopoly, the subsidy increases the total environmental R&D and decreases the total environmental damage (see Lemmas 6 and 8). Thus, the subsidy can correct (or partly correct) the market failures (i) and (ii).[45] In addition, the spillovers do not change the effect of subsidy on total environmental R&D (see Footnote 42),[46] but can change the effect of subsidy on total (or marginal) environmental damage (see Footnote 43). In order to maximise social welfare, the government sets the subsidy rate to equal $(1+\beta)D'(E^c)$ (see (26)). With an increase of β, $1+\beta$ increases but $D'(E^c)$ decreases. Since the impact of β on the marginal damage is small if d is small, this leads to an increase of $(1+\beta)D'(E^c)$. Thus, the optimal subsidy increases with β if d is small. However, the impact of β on

the marginal damage is large if d is large and exceeds it on $1+\beta$. In this case, $(1+\beta)D'(E^c)$ decreases as β increases. Thus, the optimal subsidy decreases with β if d is large.

Combining Propositions 1 and 3, when the public firm does not care for the environment, the spillovers have a similar effect on the optimal subsidy in the cases of mixed duopoly and private duopoly. However, when the public firm cares for the environment, it may have a different effect on the optimal subsidy in the cases of mixed duopoly and private duopoly. This implies that the public firm's attitude towards the environment is a critical factor determining the effect of privatisation on the relationship between spillovers and environmental R&D subsidy policy.

6. Comparisons

In this section, we compare the optimal subsidy, total R&D, environmental damage, welfare and others in the mixed duopoly (cases (a) and (b)) and in the private duopoly (case (c)) and further examine the effect of privatising the public firm on them.

Proposition 4. *The optimal subsidy to environmental R&D in the mixed duopoly (whether the public firm cares about the environment or not) is larger than in the private duopoly (i.e., $s^a>s^c$ and $s^b>s^c$).*

In other words, privatisation of the public firm leads to a decrease in the optimal subsidy, regardless of the public firm's attitude to the environment. The policy implication of this proposition is obvious. The privatisation policy of the government can affect its environmental R&D subsidy policy. The government should lower the environmental R&D subsidy rate accordingly if it intends to implement the privatisation policy in a polluting industry.

The reason for Proposition 4 is as follows. It is easy to compare the optimal subsidy in cases (a) and (c). In the latter case, the task that the government should undertake is only to equalise the social marginal cost of environmental R&D and the marginal environmental damage, because symmetry of investments is ensured by the firms' own optimal actions. On the other hand, it follows from a comparison between the firm 1's first-order conditions in cases (a) and (c) (see (11) and (21)) that firm 1 has less incentive to conduct environmental R&D in case (a) than in case (c) because of the term $P'(Q^a)q_1^a\frac{\partial q_0^a}{\partial X}<0$. Thus, compared to case (c), the government must raise the subsidy rate to ensure $\Gamma'(x_0)=\Gamma'(x_1)$ in case (a).

Now, we consider the optimal subsidy in cases (b) and (c). The government sets the subsidy rate to meet $s=(1+\beta)D'(E)$ in both case (b) and case (c). Since the total output is more in case (b) than in case (c),[47] subsidising R&D in case (b) is more socially profitable than in case (c). Thus, the optimal subsidy of the government is larger in case (b) than in case (c).

Lemma 9. *When the optimal subsidy to environmental R&D is provided, there are: (i) $X^a>X^c$; (ii) $Q^a>Q^c$; and (iii) $PS^a<(>)PS^c$ if d is small (large).*

The reason for this lemma is as follows. We first consider part (i). Since the public firm which cares for the environment maximises the social welfare but not its profits, it invests more and produces more in case (a) than when it is privatised in case (c)

(i.e., $x_0^a|_{s=s^a} > x_0^c|_{s=s^c}$ and $q_0^a|_{s=s^a} > q_0^c|_{s=s^c}$). The private firm invests more in case (a) than in case (c) (i.e., $x_1^a|_{s=s^a} > x_1^c|_{s=s^c}$) because of higher subsidy rate in the mixed duopoly (see Proposition 4). It follows that privatisation of the public firm which cares for the environment leads to a decrease in the total environmental R&D.

Now, turn to part (ii) of the lemma. The private firm produces less in case (a) than in case (c) (i.e., $q_1^a|_{s=s^a} < q_1^c|_{s=s^c}$) because of weaker competition in the private duopoly. However, the effect of privatisation on the public firm's output dominates that on the private firm's output. Thus, privatisation of the public firm which cares for the environment reduces the total output.[48]

Finally, we interpret part (iii). The private firm produces more and achieves more profit in case (c) than in case (a) (i.e., $\pi_1^a|_{s=s^a} < \pi_1^c|_{s=s^c}$) because of weaker competition in case (c), whereas whether the public firm achieves more profit or not in case (a) than where it is privatised in case (c) depends on d.[49] If d is small, the environmental damage caused by production is slight and the difference of the optimal subsidy in cases (a) and (c) is small. Since the public firm is more concerned about its profit in case (c) than in case (a), it achieves more profit in case (c). It follows that privatisation of the public firm raises the total profit. However, if d is large, the environmental damage caused by production is serious and the optimal subsidy is much higher in case (a) than in case (c). Since the public firm can obtain much more subsidy in case (a) than in case (c), it achieves much more profit in case (a). Although the private firm achievev less profit in case (a), the higher profit achieved by the public firm can compensate this if d is large enough. Thus, the total profit is higher in case (a) than in case (c) if d is large.

Lemma 10. *When the optimal subsidy to environmental R&D is provided, there are: (i) $X^b < X^c$; (ii) $Q^b > Q^c$; and (iii) $PS^b < PS^c$.*

The intuition behind this lemma is as follows. We begin by taking into account part (i). If the public firm does not care for the environment, the subsidy does not affect its environmental R&D behaviour and thus it does not invest in case (b) (see (12)). However, the subsidy can stimulate R&D investment by the private firm in both case (b) and case (c) because it is a net inflow for a private firm. The public firm obviously invests less in case (b) than it is, privatised, in case (c) (i.e., $x_0^b|_{s=s^b} < x_0^c|_{s=s^c}$), but the private firm invests more in case (b) than in case (c) (i.e., $x_1^b|_{s=s^b} > x_1^c|_{s=s^c}$) because of higher subsidy rate in the mixed duopoly. Since more R&D by the public firm can compensate less R&D by the private firm in case (c), privatisation of the public firm raises the total environmental R&D. This is different from the result that privatisation leads to reductions in total R&D under the environmental tax policy (Haruna & Goel, 2019).[50] Combining part (i) of Lemmas 9 and 10, we find that the effect of privatisation on the total environmental R&D depends on the public firm's attitude towards the environment.

Next, consider part (ii) of the lemma. Similar reasons as in part (ii) of Lemma 9, privatisation of the public firm which does not care for the environment also decreases the total output.

Finally, we give an explanation of part (iii). The private firm achieves more profit in case (c) than in case (b) (i.e., $\pi_1^b|_{s=s^b} < \pi_1^c|_{s=s^c}$) because of weaker competition in

case (c). In addition, if the public firm does not care for the environment, it also achieves more profit in case (c) than in case (b). It follows that privatisation of the public firm raises the total profit. Note that this is different from the result in the case where the public firm cares for the environment.

Proposition 5. *When the optimal subsidy to environmental R&D is provided, there are: (i) $D^a>D^c$ and $D^b>D^c$; and (ii) $SW^a>SW^c$ and $SW^b>(<)SW^c$ if d is small (large).*

This proposition implies that under a policy of providing optimal subsidy to environmental R&D, privatisation of the public firm leads to a decline in environmental damage despite its environmental attitude. This is different from the result that privatisation may damage the environment under the environmental tax policy (Pal & Saha, 2015; Tsai et al., 2016; Xu et al., 2016; Haruna & Goel, 2019). However, whether privatisation reduces or raises social welfare may depend on the public firm's environmental attitude and the seriousness of environmental damage. Privatisation always reduces social welfare if the public firm cares for the environment, whereas it raises social welfare if the public firm is unconcerned for the environment and the seriousness of environmental damage is large. This is different from the result of Gil-Moltó et al. (2006, 2011), who show that under a policy of providing optimal subsidy to cost-reducing R&D, privatisation always induces a decline in social welfare. Proposition 5 suggests that, when the government investigates the effect of privatisation on environmental damage and social welfare in polluting industries, it should take into account the environmental attitude of public firms and the seriousness of environmental damage.

The reason for this proposition is as follows. We first consider part (i). Although the total environmental R&D is more in case (a) than in case (c), the total output is also higher in the former case (see Lemma 9). For given output, an increase in the total R&D can reduce the total emissions. However, the reduction of emissions (due to the increase of total R&D) cannot compensate for the rise of emissions (caused by the increase of total output) in case (a). Thus, the total emissions are higher in case (a) than in case (c) (i.e., $E^a|_{s=s^a}>E^c|_{s=s^c}$). It follows that the total environmental damage is higher in case (a) than in case (c). In addition, the total environmental R&D is less in case (b) than in case (c), whereas the total output is higher in the former case (see Lemma 10). Thus, the total emissions are higher in case (b) than in case (c). This leads to lower total environmental damage in the private duopoly.

Secondly, turn to part (ii) of proposition 5. Privatisation of the public firm, regardless of its attitude to the environment, will lead to lower total output (and thus lower consumer surplus) (see Lemmas 9 and 10) and lower total environmental damage. In addition, privatisation leads to higher (lower) producer surplus if the public firm cares for the environment and d is small (large) (see Lemma 9) and it leads to higher producer surplus if the public firm does not care for the environment (see Lemma 10). If the public firm cares for the environment and d is small, the negative effect (lower consumer surplus) dominates the positive effect (lower total environmental damage and higher producer surplus). If the public firm cares for the environment and d is large, the negative effect (lower consumer and producer surpluses) dominates

the positive effect (lower total environmental damage). It follows that privatisation of the public firm which is concerned about the environment will reduce the social welfare. However, if the public firm does not care for the environment, whether the negative effect (lower consumer surplus) dominates the positive effect (lower total environmental damage and higher producer surplus) or not depends on d. It transpires that the negative effect dominates (does not dominate) the positive effect when d is small (large). Thus, privatisation of the public firm which is unconcerned about the environment will reduce (raise) the social welfare if d is small (large).

7. Conclusions

The theory of mixed oligopoly with environmental policy issues has attracted much interest recently. This study examines the use of subsidy to environmental R&D in a mixed duopoly market. Firstly, we consider the following two scenarios with regard to the public firm's attitude towards environment – one in which the public firm cares about the environment (case (a)) and the other where it does not (case (b)). We show that the optimal subsidy is positive and always increases with spillovers in case (a), whereas it may decrease with spillovers in case (b). Moreover, whether it is larger or smaller in case (a) than in case (b) depends on the seriousness of the environmental damage. We also examine the effect of the subsidy on the environmental R&D incentives and find that the subsidy does not always promote all firms' R&D. For example, the public firm's R&D decreases with (does not depend on) the subsidy in case (a) (case (b)). In addition, we analyse how the subsidy affects the total R&D and environmental damage, and find that the use of subsidy leads to an increase in total R&D and a decrease in environmental damage in both case (a) and case (b). Secondly, we consider the case where the public firm is completely privatised (case (c)) and explore the effect of privatisation on the optimal subsidy, total R&D, environmental damage and welfare. We find that privatisation leads to a decrease of the optimal subsidy no matter how the public firm treats environmental damage. In addition, when the optimal subsidy is provided, privatisation lowers the environmental damage regardless of the public firm's environmental attitude. However, whether it reduces or raises the total R&D and welfare relies on the public firm's attitude to the environment.

This study uses a simplest framework to examine the optimal environmental R&D subsidy in a mixed duopoly. Several extensions of this analysis are possible: (i) we only consider the Cournot duopolistic competition model in this paper. An extension is that we also analyse the optimal subsidy under Bertrand competition and compare them in quantity and price competition;[51] (ii) we assume that the environmental concern of the public firm is exogenous. We can extend this study to examine the optimal subsidy if it is endogenous; (iii) with the increase of environmental awareness, more consumers prefer to buy environmentally friendly products. This stimulates firms to carry out more environmental R&D investments in polluting industries. Thus, an extension is that we study the environmental R&D subsidy policy by considering environmental awareness; (iv) this study can be extended to the case where public and private firms compete in the foreign market while cooperating in the

domestic market or where a domestic public (private) firm competes with a private (public) foreign firm.[52] However, these extensions require much effort and are tasks that remain for future research.

Notes

1. Some researches have reported that, from the beginning of 20th century to the present, the average temperature of the earth's surface has increased by about 1.1f (0.6 °C); in the past 40 years, the average temperature has risen by about 0.5f (0.2–0.3 °C); in 20th century, the degree of global warming was more than in any period of the past 400–600 years.
2. For example, public firms coexist with private firms in many highly polluting industries (such as mining, petrochemicals, textiles and clothing, biomedicine, metal and non-metal) in China (Chang et al., 2015).
3. Environmental R&D information disclosure, technology imitation, researchers flow and technology exchange of polluting firms will lead to environmental R&D spillovers.
4. See the related studies of Lambertini, Poyago-Theotoky, and Tampieri, (2017) and Furková and Chocholatá (2017).
5. State-owned pulp and paper firms have higher pollution intensities than private firms in Indonesia, Bangladesh and Thailand (see Hettige et al., 1996).
6. This is the case in China's steel, coal, textile, power and other industries.
7. The public firm's attitude towards the environment affects its environmental R&D behaviour and indirectly affects its rival's R&D behaviour in a mixed duopoly. Accordingly, the government should take into account the public firm's concern for environment when making subsidy polices for environmental R&D. Pal and Saha (2015) distinguish the public firm's environmental attitudes when investigating the relationship between privatisation and environmental damage. However, their study does not involve the environmental R&D subsidy policy.
8. When analysing the effect of cost-reducing R&D subsidy on the private firm's R&D and total R&D in a mixed oligopoly, Gil-Moltó et al. (2006; 2011; 2018) and Lee et al. (2017) obtain a similar result. However, their studies do not involve the environmental R&D subsidy and the public firm's environmental concerns.
9. This is different from the result that the cost-reducing R&D subsidy may increase the public firm's R&D in a mixed duopoly (Gil-Moltó et al., 2006, 2011; Lee, Muminov, & Tomaru, 2017).
10. The R&D subsidies are the second largest type of government aid to industry in OECD countries (Nezu, 1997).
11. Note that the environmental policy in markets where all firms are privately owned has been analyzed by many scholars (such as Barrett, 1994; Markusen, Morey, & Olewiler, 1995; Ulph, 1996; Hoel, 1997; Eerola, 2006; Chappin, Vermeulen, Meeus, & Hekkert, 2009; Bárcena-Ruiz & Garzón, 2013; and Moore, Porten, Plummer, Brandes, & Baird, 2014). However, they do not consider environmental policy in mixed markets where public and private firms coexist.
12. The inclusion of a quadratic term in firm's cost function is standard in the literature on mixed oligopoly (Zikos, 2007; Heywood & Ye, 2009; Gil-Moltó et al., 2011; Andree, 2013; Nie, 2014; Naya, 2015).
13. Note that the objective of a firm who invests in cost-reducing R&D is to lower its marginal cost of production (Zikos, 2007; Gil-Moltó et al., 2006, 2011). However, the objective of a firm which invests in environmental R&D is to reduce its emissions (Poyago-Theotoky, 2007; Ouchida & Goto, 2014). Unlike the cost-reducing R&D, the environmental R&D (e.g., the R&D for end-of-pipe technology) does not necessarily improve productive efficiency.

14. There are generally two emission-reducing technologies in the existing literature: end-of-pipe and clean technologies (Skea, 2000; Requate, 2005; Tsai, Tu, & Chiou, 2015; Tsai et al., 2016).
15. Poyago-theotoky (2007) and Ouchida and Goto (2014) also consider the spillover effect in environmental R&D activity.
16. This form of environmental damage function is also employed by Poyago-theotoky (2007), Naito and Ogawa (2009), Pal and Saha (2015), Ouchida (2016) and others.
17. Pal and Saha (2015) call it the increment in marginal environmental damage due to pollution.
18. We assume that $0<d<0.64$. This assumption ensures that the emissions for firms are not negative and we can obtain interior solutions in all three cases (cases (a), (b) and (c)), irrespective of the degree of spillovers.
19. Unlike the existing studies on cost-reducing R&D subsidy (Gil-Moltó et al., 2011), the main purpose of the government subsidising environmental R&D is to promote firms to carry out environmental R&D investments and improve environmental quality.
20. This study assumes that the regulator does not impose environmental taxes. This simplifying assumption has the purpose to show the only effect of R&D subsidy on the environmental R&D, output and environmental damage.
21. The objective divergence between the public firms and the government is analyzed by Ohori (2006), Wang and Wang (2009), Kato (2010) and Pal and Saha (2015).
22. Pal and Saha (2015) have adopted a similar method in the construction of the public firm's objective function. However, they consider the environmental tax policy but not the environmental R&D subsidy policy.
23. Note that the subsidy to environmental R&D cancels out in the public firm's objective function (see (5)). It follows that the subsidy has no direct effect on the public firm's environmental R&D or output behavior. However, it can indirectly affect the public firm's environmental R&D and output through its effect on the private firm's R&D behavior. Gil-Moltó et al. (2011) give a similar analysis when they explain the technical efficiency of cost-reducing R&D subsidy.
24. Note that this study assumes $a>c$.
25. Using (6) and (7), we derive the reaction functions of firms in output stage: $q_0(q_1) = [m + kd(1+\beta)(x_1+x_0) - (1+kd)q_1]/(3+kd)$ and $q_1(q_0) = (m-q_0)/4$. Obviously, an increase of q_0 (q_1) will lead to a decrease of q_1 (q_0).
26. Gil-Moltó et al. (2006, 2011) examine the impact of R&D subsidy on cost-reducing R&D in a mixed market and find that the public firm's R&D increases with the subsidy if the degree of spillovers is large. Obviously, their result is different from the result when we consider the subsidy to environmental R&D.
27. According to (10) and (11), we obtain the following reaction functions of two firms in R&D stage: $x_0(x_1) = [kd(53+19kd)(1+\beta)m - kd(121+43kd)(1+\beta)^2 x_1]/[kd(121+43kd)(1+\beta)^2 + (11+3kd)^2]$ and $x_1(x_0) = [(11+3kd)^2 s - 4kd(2+kd)(1+\beta)m + 4k^2d^2(1+\beta)^2 x_0]/[(11+3kd)^2 - 4k^2d^2(1+\beta)^2]$.
28. Because $\partial|\partial x_0^a(s)/\partial s|/\partial\beta = 2d(121+43d)(11+3d)^2(1+\beta)/[(11+3d)^2 + d(121+39d)(1+\beta)^2]^2 > 0$, $\partial|\partial x_1^a(s)/\partial s|/\partial\beta = 8d^2(11+3d)^2(1+\beta)/[(11+3d)^2 + d(121+39d)(1+\beta)^2]^2 > 0$ and $\partial|\partial[x_0^a(s)+x_1^a(s)]/\partial s|/\partial\beta = -2d(11+3d)^2(121+39d)(1+\beta)/[(11+3d)^2 + d(121+39d)(1+\beta)^2]^2 < 0$, the spillovers strengthen the negative (positive) effect of s on $x_0^a(s)$ ($x_1^a(s)$) and weaken the positive effect of s on $x_0^a(s)+x_1^a(s)$.
29. Because $\partial|\partial x_0^b(s)/\partial s|/\partial\beta = \partial|\partial x_1^b(s)/\partial s|/\partial\beta = \partial|\partial[x_0^b(s)+x_1^b(s)]/\partial s|/\partial\beta = 0$, the spillovers do not change the effect of s on $x_0^b(s)$, $x_1^b(s)$ and $x_0^b(s)+x_1^b(s)$.
30. According to (2), an increase in total environmental R&D output leads to a decrease in environmental damage caused by increased unit production. Moreover, the social welfare increases as the total environmental damage decreases. Thus, from the view point of social welfare, the public firm will increase its output if the total environmental R&D output increases.

31. $E^t(s) = q_0^t(s) + q_1^t(s) - (1+\beta)[x_0^t(s) + x_1^t(s)]$ $(t = a, b)$. Combining with (12), (13), (14) and (15), we can obtain $E^a(s) = \{5[(11+3d) + 2d(1+\beta)^2]m - 11(1+\beta)(11+3d)s\}/[(11+3d)^2 + d(121+39d)(1+\beta)^2]$ and $E^b(s) = [5m - 11(1+\beta)s]/11$. Using (16), we have $D^a(s) = d(E^a(s))^2/2$ and $D^b(s) = d(E^b(s))^2/2$. The marginal environmental damage is given by $dE^a(s)$ ($dE^b(s)$) in case (a) (case (b)). We get $\partial|\partial E^a(s)/\partial s|/\partial\beta > (<) 0$ if $0 < d < \left(22 \times \sqrt{127} - 209\right)/147 \approx 0.2648$ or $d > \left(22 \times \sqrt{127} - 209\right)/147$ and $\beta < \sqrt{(11+3d)^2/[d(121+39d)]} - 1$ (if $d > \left(22 \times \sqrt{127} - 209\right)/147$ and $\beta > \sqrt{(11+3d)^2/[d(121+39d)]} - 1$). It follows that the spillovers strengthen (weaken) the negative effect of s on $E^a(s)$, $dE^a(s)$ and $D^a(s)$ if d is small or d is large and β is small (if both d and β are large). In addition, we obtain $\partial|\partial E^b(s)/\partial s|/\partial\beta > 0$. Thus, the spillovers strengthen the negative effect of s on $E^b(s)$, $dE^b(s)$ and $D^b(s)$.
32. Because the total output (the total environmental R&D) increases with s (see Lemmas 1 and 2), consumer surplus (the total subsidy) also increases with s.
33. $\Gamma'(x_0^a) = \Gamma'(x_1^a)$ because of $dSW/ds = \{[-P'(Q^a)q_1^a - D'(E^a)]\frac{\partial q_1^a}{\partial X} - \Gamma'(x_1^a) + (1+\beta)D'(E^a)\}\frac{\partial x_1^a}{\partial s} = [\Gamma'(x_0^a) - \Gamma'(x_1^a)]\frac{\partial x_1^a}{\partial s} = 0$ and $\frac{\partial x_1^a}{\partial s} > 0$ in case (a).
34. $\Gamma'(x_1^b) = (1+\beta)D'(E^b)$ because of $dSW/ds = [-\Gamma'(x_1^b) + (1+\beta)D'(E^b)]\frac{\partial x_1^b}{\partial s} = 0$ and $\frac{\partial x_1^b}{\partial s} > 0$ in case (b).
35. For example, s^b decreases with β in interval $[0.5, 1]$ if $d \geq 0.5$.
36. The cost-reducing R&D subsidy can change the production allocation through the firms' R&D. This change will affect consumer and producer surpluses, so that the subsidy indirectly affects the social welfare (see Gil-Moltó et al., 2011). However, this subsidy cannot change the social welfare through the environmental damage as the environmental R&D subsidy does. This may lead to different results under the two types of subsidy.
37. Private firms do not take into account the decrease in environmental damage as a consequence of the investment on environmental R&D (as environmental damage does not belong to their objective function). This will rest in underinvestment in environmental R&D. Ulph (1999) defines the R&D undervaluation effect. However, he does not consider the environmental damage.
38. This means that, the larger the β, the weaker the effect of the R&D subsidy on correcting market failure (i).
39. This means that, if d is small or d is large and β is small (if both d and β are large), the larger the β, the stronger (weaker) the effect of the R&D subsidy on correcting market failure (ii).
40. This means that the effect of the subsidy on correcting the market failure (i) does not depend on β.
41. In addition, we compare the welfare (environmental damage) in cases (a) and (b) and find that when the optimal subsidy is provided, the welfare (environmental damage) is higher (lower) if the public firm cares for the environment than if it does not.
42. Because $\partial|\partial x_0^c(s)/\partial s|/\partial\beta = \partial|\partial x_1^c(s)/\partial s|/\partial\beta = \partial|\partial[x_0^c(s) + x_1^c(s)]/\partial s|/\partial\beta = 0$, the spillovers do not change the positive effect of s on $x_0^c(s)$, $x_1^c(s)$ and $x_0^c(s) + x_1^c(s)$.
43. Using (20) and (23), we obtain $E^c(s) = q_0^c + q_1^c - (1+\beta)[x_0^c(s) + x_1^c(s)] = 2[m - 5(1+\beta)s]/5$. According to (24), we get $D^c(s) = d(E^c(s))^2/2$. The marginal environmental damage is given by $dE^c(s)$. Because $\partial|\partial E^c(s)/\partial s|/\partial\beta = 2 > 0$, the spillovers strengthen the negative effect of s on $E^c(s)$. Thus, the spillovers also strengthen the negative effect of s on $dE^c(s)$ and $D^c(s)$.
44. Substituting (20) and (23) into (3) and (4) and then taking comparative statics on the profits and welfare, we find that each firm's profit and the total profit increase with the subsidy, and the relationship between the welfare and the subsidy is inverse U shape (given $0 < d < 0.64$, $0 \leq \beta \leq 1$ and $0 \leq s \leq m/[5(1+\beta)]$).
45. The market failure (i) is the under-investment in environmental R&D by firms and the market failure (ii) is the excess environmental damage caused by production.

46. This means that the effect of the subsidy on correcting the market failure (i) does not depend on β.
47. This is because $(q_0^b + q_1^b)|_{s=s^b} = 5m/11 > 2m/5 = (q_0^c + q_1^c)|_{s=s^c}$.
48. It follows that privatisation of the public firm which cares for the environment lowers consumer surplus (i.e., $CS^a|_{s=s^a} > CS^c|_{s=s^c}$).
49. If d is small (large), $\pi_0^a|_{s=s^a} < (>) \pi_0^c|_{s=s^c}$.
50. In addition, Gil-Moltó et al. (2011) consider the effect of privatisation on cost-reducing R&D and show that privatisation always reduces the total cost-reducing R&D. This is different from our result.
51. Chen and Nie (2014) study duopoly innovation under both Cournot and Bertrand competition.
52. See the related studies of Zhang, Zhong, and Mei (2016), Cieślik (2016) and Wang and Chiou (2016).

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This study is supported by the Social Science Planning Research Project of Shandong [grant number 18CCZJ18; 19CDNJ38; 19BJCJ27]; Natural Science Foundation of Shandong Province [grant number ZR2017BEE038]; Shandong Institute of Humanities and Social Sciences Research Projects [grant number J16WE27]; and MOE (Ministry of Education in China) Project of Humanities and Social Sciences [grant number 17YJC790137].

References

Andree, K. (2013). A note on merger in mixed duopoly: Bertrand versus Cournot. *Journal of Economics*, *108*(3), 291–298. doi:10.1007/s00712-012-0280-x

Bárcena-Ruiz, J. C., & Garzón, M. B. (2006). Mixed oligopoly and environmental policy. *Spanish Economic Review*, *8*(2), 139–160. doi:10.1007/s10108-006-9006-y

Bárcena-Ruiz, J. C., & Garzón, M. B. (2013). Multiproduct firms and environmental policy coordination. *Environmental and Resource Economics*, *59*(3), 1–25. doi:10.1007/s10640-013-9736-x

Barrett, S. (1994). Strategic environmental policy and international trade. *Journal of Public Economics*, *54*(3), 325–338. doi:10.1016/0047-2727(94)90039-6

Ben Youssef, S., & Dinar, Z. (2011). Regulation of a duopoly and environmental R&D. *Environmental Economics*, *2*(3), 47–51.

Chang, L., Li, W., & Lu, X. (2015). Government engagement, environmental policy, and environmental performance: Evidence from the most polluting Chinese listed firms. *Business Strategy and the Environment*, *24*(1), 1–19. doi:10.1002/bse.1802

Chappin, M. M. H., Vermeulen, W. J. V., Meeus, M. T. H., & Hekkert, M. P. (2009). Enhancing our understanding of the role of environmental policy in environmental innovation: Adoption explained by the accumulation of policy instruments and agent-based factors. *Environmental Science & Policy*, *12*(7), 934–947. doi:10.1016/j.envsci.2009.06.001

Chen, Y. H., & Nie, P. Y. (2014). Duopoly innovation under product externalities. *Economic Research-Ekonomska Istraživanja*, *27*(1), 232–243. doi:10.1080/1331677X.2014.952092

Cieślik, A. (2016). Exports versus FDI in Smith-Motta framework. *Equilibrium*, *11*(2), 189–218. doi:10.12775/EQUIL.2016.009

Eerola, E. (2006). International trade agreements, environmental policy, and relocation of production. *Resource and Energy Economics*, *28*(4), 333–350. doi:10.1016/j.reseneeco.2005.12.002

Furková, A., & Chocholatá, M. (2017). Interregional R&D spillovers and regional convergence: A spatial econometric evidence from the EU regions. *Equilibrium*, *12*(1), 9–24. doi:10.24136/eq.v12i1.1

Gil-Moltó, M. J., Poyago-Theotoky, J., Rodrigues-Neto, J. A., & Zikos, V. (2018). Mixed oligopoly, privatization and R&D subsidization. Retrieved from https://ssrn.com/abstract=3162021

Gil-Moltó, M. J., Poyago-Theotoky, J., & Zikos, V. (2006). Public policy towards R&D in a mixed duopoly with spillovers. Retrieved from https://pdfs.semanticscholar.org/1d10/ab93aa21935797e0669d1e177131b42e622c.pdf

Gil-Moltó, M. J., Poyago-Theotoky, J., & Zikos, V. (2011). R&D subsidies, spillovers and privatization in mixed markets. *Southern Economic Journal*, *78*(1), 233–255.

Haruna, S., & Goel, R. K. (2019). Optimal pollution control in a mixed oligopoly with research spillovers. *Australian Economic Papers*, *58*(1), 21–40. doi:10.1111/1467-8454.12138

Hettige, H., Huq, M., Pargal, S., & Wheeler, D. (1996). Determinants of pollution abatement in developing countries: Evidence from South and Southeast Asia. *World Development*, *24*(12), 1891–1904. doi:10.1016/S0305-750X(96)00076-9

Heywood, J. S., & Ye, G. L. (2009). Partial privatization in a mixed oligopoly with an R&D rivalry. *Bulletin of Economic Research*, *61*(2), 165–178.

Hoel, M. (1997). Environmental policy with endogenous plant locations. *The Scandinavian Journal of Economics*, *99*(2), 241–259. doi:10.1111/1467-9442.00061

Kato, K. (2010). Partial privatization and environmental policies. MPRA Paper 27630, University Library of Munich, Germany.

Kato, K. (2013). Optimal degree of privatization and the environmental problem. *Journal of Economics*, *110*(2), 165–180. doi:10.1007/s00712-012-0316-2

Kesavayuth, D., & Zikos, V. (2013). R&D versus output subsidies in mixed markets. *Economics Letters*, *118*(2), 293–296.

Lambertini, L., Poyago-Theotoky, J., & Tampieri, A. (2017). Cournot competition and "green" innovation: An inverted-U relationship. *Energy Economics*, *32*(2), 297–319. doi:10.1016/j.eneco.2017.09.022

Lambertini, L., & Tampieri, A. (2010). Corporate social responsibility in a mixed oligopoly. Quaderni DSE Working Paper No. 723. Retrieved from https://ssrn.com/abstract=1729130

Lee, S. H., & Muminov, T. K. (2017). R&D Output sharing in a mixed duopoly and incentive subsidy policy. Retrieved from https://mpra.ub.uni-muenchen.de/81732/1/MPRA_paper_81732.pdf

Lee, S. H., Muminov, T. K., & Tomaru, Y. (2017). Partial privatization and subsidization in a mixed duopoly: R&D versus output subsidies. *Hitotsubashi Journal of Economics*, *58*(2), 163–177.

Lee, S. H., & Tomaru, Y. (2017). Output and R&D subsidies in a mixed oligopoly. *Operations Research Letters*, *45*(3), 238–241.

Markusen, J. R., Morey, E. R., & Olewiler, N. (1995). Competition in regional environmental policies when plant locations are endogenous. *Journal of Public Economics*, *56*(1), 55–77. doi:10.1016/0047-2727(94)01419-O

Moore, M. L., Porten, S. V. D., Plummer, R., Brandes, O., & Baird, J. (2014). Water policy reform and innovation: A systematic review. *Environmental Science & Policy*, *38*, 263–271. doi:10.1016/j.envsci.2014.01.007

Naito, T., & Ogawa, H. (2009). Direct versus indirect environmental regulation in a partially privatized mixed duopoly. *Environmental Economics and Policy Studies*, *10*(2–4), 87–100.

Naya, J. M. (2015). Endogenous timing in a mixed duopoly model. *Journal of Economics*, *116*(2), 165–174. doi:10.1007/s00712-014-0416-2

Nezu, R. (1997). Trends and patterns of public support to industry in the OECD Area. *STI Review*, *21*, 12–24.

Nie, P. Y. (2014). Effects of capacity constraints on mixed duopoly. *Journal of Economics*, *112*(3), 283–294. doi:10.1007/s00712-013-0362-4

Ohori, S. (2006). Trade liberalization, consumption externalities and the environment: A mixed duopoly approach. *Economics Bulletin*, *17*, 1–9.
Ouchida, Y., & Goto, D. (2014). Do emission subsidies reduce emission? In the context of environmental R&D organization. *Economic Modelling*, *36*, 511–516.
Ouchida, Y. (2016). Environmental R&D organization in a differentiated Cournot duopoly. In T. Naito (Ed.), *Sustainable growth and development in a regional economy* (pp. 215–238). Tokyo, Japan: Springer.
Pal, R., & Saha, B. (2015). Pollution tax, partial privatization and environment. *Resource and Energy Economics*, *40*, 19–35.
Poyago-Theotoky, J. A. (2007). The organization of R&D and environmental policy. *Journal of Economic Behavior and Organization*, *62*(1), 63–75.
Requate, T. (2005). Dynamic incentives by environmental policy instruments-a survey. *Ecological Economics*, *54*(2–3), 175–195. doi:10.1016/j.ecolecon.2004.12.028
Sekuła, A. (2017). Does system of local government subsidisation fulfil revenue equalisation function? Evidence from Poland. *Oeconomia Copernicana*, *8*(4), 523–535. doi:10.24136/oc.v8i4.32
Skea, J. (2000). Environmental technology. In H. Folmer & H. L. Gabel (Eds.), *Principle of environmental and resource economics* (pp. 338–362). Cheltenham, UK: Edward Elgar.
Tsai, T. H., Tu, K. I., & Chiou, J. R. (2015). Tariffs and environmental taxes in the presence of environmental R&D. *Environmental and Resource Economics*, *60*(3), 413–431.
Tsai, T. H., Wang, C. C., & Chiou, J. R. (2016). Can privatization be a catalyst for environmental R&D and result in a cleaner environment? *Resource and Energy Economics*, *43*, 1–13.
Ulph, A. (1996). Environmental policy and international trade when governments and producers act strategically. *Journal of Environmental Economics and Management*, *30*(3), 265–281. doi:10.1006/jeem.1996.0018
Ulph, D. (1999). Competition, innovation and research joint ventures. Paper prepared for presentation at the Final Workshop of the TSER Network, "Innovation R&D and Productivity", Brussels; 26–27 March.
Wang, C. C., & Chiou, J. R. (2016). Taxation on a mixed oligopoly in the presence of foreign ownership. *Asia-Pacific Journal of Accounting & Economics*, *23*(3), 342–355. doi:10.1080/16081625.2016.1188448
Wang, L. F. S., & Wang, J. (2009). Environmental taxes in a differentiated mixed duopoly. *Economic Systems*, *33*(4), 389–396. doi:10.1016/j.ecosys.2009.08.002
Xu, L. L., Cho, S., & Lee, S. H. (2016). Emission tax and optimal privatization in Cournot-Bertrand comparison. *Economic Modelling*, *55*, 73–82.
Yang, Y. C., & Nie, P. Y. (2015). R&D subsidies under asymmetric Cournot competition. *Economic Research-Ekonomska Istraživanja*, *28*(1), 830–842.
Zhang, W., Zhong, W. J., & Mei, S. (2016). R&D subsidy policy of domestic firm considering foreign firm competition. *Journal of Southeast University (English Edition)*, *32*(4), 513–519.
Zikos, V. (2007). A reappraisal of the irrelevance result in mixed duopoly: A note on R&D competition. *Economics Bulletin*, *12*(8), 1–6.

Appendix

A. Proof of Lemma 1: Because $\frac{\partial x_0^a(s)}{\partial s} = -\frac{d(121+43d)(1+\beta)^2}{(11+3d)^2+d(121+39d)(1+\beta)^2} < 0$, $\frac{\partial x_1^a(s)}{\partial s} = \frac{(11+3d)^2+d(121+43d)(1+\beta)^2}{(11+3d)^2+d(121+39d)(1+\beta)^2} > 0$, $\frac{\partial[x_0^a(s)+x_1^a(s)]}{\partial s} = \frac{(11+3d)^2}{(11+3d)^2+d(121+39d)(1+\beta)^2} > 0$, $\frac{\partial x_0^b(s)}{\partial s} = 0$, $\frac{\partial x_1^b(s)}{\partial s} = 1 > 0$ and $\frac{\partial[x_0^b(s)+x_1^b(s)]}{\partial s} = 1 > 0$.

B. Proof of Lemma 2: Because $\frac{\partial q_0^a(s)}{\partial s} = \frac{4d(11+3d)(1+\beta)}{(11+3d)^2+d(121+39d)(1+\beta)^2} > 0$, $\frac{\partial q_1^a(s)}{\partial s} = -\frac{d(11+3d)(1+\beta)}{(11+3d)^2+d(121+39d)(1+\beta)^2} < 0$, $\frac{\partial[q_0^a(s)+q_1^a(s)]}{\partial s} = \frac{3d(11+3d)(1+\beta)}{(11+3d)^2+d(121+39d)(1+\beta)^2} > 0$ and $\frac{\partial q_0^b(s)}{\partial s} = \frac{\partial q_1^b(s)}{\partial s} = \frac{\partial[q_0^b(s)+q_1^b(s)]}{\partial s} = 0$.

C. Proof of Lemma 3: Because $\frac{\partial D^a(s)}{\partial s} = -\frac{11(1+\beta)(11+3d)d[5[(11+3d)+2d(1+\beta)^2]m-11(1+\beta)(11+3d)s]}{[(11+3d)^2+d(121+39d)(1+\beta)^2]^2} < 0$ and $\frac{\partial D^b(s)}{\partial s} = -\frac{d(1+\beta)[5m-11(1+\beta)s]}{11} < 0$.

D. Proof of Lemma 4: Substituting (12), (13), (14) and (15) into (3), we obtain $\pi_0^t(s)$ and $\pi_1^t(s)$ $(t = a, b)$. First, we prove the part (i) of this lemma. Obviously, $\frac{\partial \pi_0^a(s)}{\partial s}$, $\frac{\partial \pi_1^a(s)}{\partial s}$ and $\frac{\partial[\pi_0^a(s)+\pi_1^a(s)]}{\partial s}$ are linear functions of s. $\frac{\partial \pi_0^a(s)}{\partial s}|_{s=\underline{s}} > 0$ for all $\beta \in [0,1]$ and $\frac{\partial \pi_0^a(s)}{\partial s}|_{s=\bar{s}} > (<) 0$ if β is small (large) enough. Thus, if β is small enough, $\frac{\partial \pi_0^a(s)}{\partial s} > 0$ for all $s \in [\underline{s}, \bar{s}]$. However, if β is large enough, there exists a $\tilde{s} \in (\underline{s}, \bar{s})$ making that $\frac{\partial \pi_0^a(s)}{\partial s} > (<) 0$ for all $s \in (\underline{s}, \tilde{s})(s \in (\tilde{s}, \bar{s}))$. It follows that in interval $[\underline{s}, \bar{s}]$, $\pi_0^a(s)$ increases (increases first and then decreases) with s if β is small (large) enough. In addition, because $\frac{\partial \pi_1^a(s)}{\partial s}|_{s=\underline{s}} > 0$, $\frac{\partial \pi_1^a(s)}{\partial s}|_{s=\bar{s}} > 0$, $\frac{\partial[\pi_0^a(s)+\pi_1^a(s)]}{\partial s}|_{s=\underline{s}} > 0$ and $\frac{\partial[\pi_0^a(s)+\pi_1^a(s)]}{\partial s}|_{s=\bar{s}} > 0$, $\frac{\partial \pi_1^a(s)}{\partial s} > 0$ and $\frac{\partial[\pi_0^a(s)+\pi_1^a(s)]}{\partial s} > 0$ for all $s \in [\underline{s}, \bar{s}]$. It follows that $\pi_1^a(s)$ and $\pi_0^a(s) + \pi_1^a(s)$ increase with s in interval $[\underline{s}, \bar{s}]$. Secondly, we prove part (ii) of the lemma. This is because $\frac{\partial \pi_0^b(s)}{\partial s} = 0$ and $\frac{\partial \pi_1^b(s)}{\partial s} = \frac{\partial[\pi_0^a(s)+\pi_1^a(s)]}{\partial s} = s \geq 0$.

E. Proof of Lemma 5: Substituting (12), (13), (14) and (15) into (4), we obtain $SW^t(s)$ $(t = a, b)$. Obviously, $\frac{\partial SW^t(s)}{\partial s}$ is a linear function of s. Because $\frac{\partial SW^t(s)}{\partial s}|_{s=\underline{s}} > 0$ and $\frac{\partial SW^t(s)}{\partial s}|_{s=\bar{s}} < 0$, there exists a $\hat{s} \in (\underline{s}, \bar{s})$ making that $\frac{\partial SW^t(s)}{\partial s} > (<) 0$ for all $s \in (\underline{s}, \hat{s})(s \in (\hat{s}, \bar{s}))$. It follows that $SW^t(s)$ increases (decreases) with s if $s \in (\underline{s}, \hat{s})(s \in (\hat{s}, \bar{s}))$.

F. Proof of Lemma 6: Because $\frac{\partial x_0^c(s)}{\partial s} = \frac{\partial x_1^c(s)}{\partial s} = 1 > 0$ and $\frac{\partial[x_0^c(s)+x_1^c(s)]}{\partial s} = 2 > 0$.

G. Proof of Lemma 7: Because $\frac{\partial q_0^c}{\partial s} = \frac{\partial q_1^c}{\partial s} = \frac{\partial(q_0^c+q_1^c)}{\partial s} = 0$.

H. Proof of Lemma 8: Because $\frac{\partial D^c(s)}{\partial s} = -\frac{4d(1+\beta)[m-5(1+\beta)s]}{5} < 0$.

I. Proof of Proposition 1: (i) Because $m = a-c > 0$, $0 \leq \beta \leq 1$ and $0 < d < 0.64$, $s^a > 0$ and $s^b > 0$; (ii) Because $\frac{\partial s^a}{\partial \beta} > 0$ for all $\beta \in [0,1]$ and $d \in (0, 0.64)$; and (iii) Using (18), $s^b = \frac{5d(1+\beta)m}{11[1+d(1+\beta)^2]}$. Thus, $\frac{\partial s^b}{\partial \beta} = \frac{5dm[1-d(1+\beta)^2]}{11[1+d(1+\beta)^2]^2} > (<) 0$ if $d < (>) \frac{1}{(1+\beta)^2}$.

J. Proof of Proposition 2: $s^a > s^b$ if d is small because of $s^a - s^b > 0$ for $0 < d \leq 0.04$ and $0 \leq \beta \leq 1$. In addition, we can prove $s^a - s^b < 0$ for $0.2 \leq d < 0.64$ and $0 \leq \beta \leq 0.9$. Thus, s^a may be smaller that s^b if d is large.

K. Proof of Proposition 3: (i) Because $m = a-c > 0$, $d > 0$ and $0 \leq \beta \leq 1$, $s^c > 0$; (ii) Because $\frac{\partial s^c}{\partial \beta} = \frac{2[1-2(1+\beta)^2 d]dm}{5[1+2(1+\beta)^2 d]^2} > (<) 0$ if $d < (>) \frac{1}{2(1+\beta)^2}$.

L. Expressions of ϖ and τ: $\varpi = ([(11+3kd)^2(43k^2d^2+121d)+k^2d^2(121+43kd)^2(1+\beta)^2](1+\beta)^2+[(11+3kd)^2+kd(121+43kd)(1+\beta)^2]^2)$
and

$$\tau = \begin{pmatrix} (11+3kd)\{(19k^2d^2-2kd+55d)(11+3kd)^2+2kd[48k^3d^3+143k^2d^2+(165d-121)kd+ \\ 605d](1+\beta)^2\}+k^2d^2(121+43kd)[(53+19kd)(11+3kd)+8kd(11+4kd)(1+\beta)^2](1+\beta)^2 \\ +4kd[(11+3kd)^2+kd(121+43kd)(1+\beta)^2][(2+kd)(11+3kd)+2kd(11+4kd)(1+\beta)^2] \end{pmatrix}.$$