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INFLUENCE OF SUBGRADE REACTION COEFFICIENT MODELLING ON SIMPLE 3D FRAME SUBJECTED TO SYMMETRIC HORIZONTAL LOAD

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Abstract: The aim of this paper is to show the influence of the subgrade reaction coefficient modelling on the simple 3D frame subjected to horizontal symmetrical static load. Since the work represents a continuation of the research on this topic, it was decided that the construction, in this case too, was built on the square foundations on granular soil. Different values of subgrade reaction coefficient obtained for the square foundation are the consequence of using different expressions defined by individual authors. Thus, one diagram shows each author's relation between the width of the foundations and the value of the subgrade reaction coefficient. The response of the characteristic structural framework additionally loaded with horizontal static force is then described further in the paper.

Key words: horizontal loaded 3D frame, subgrade reaction coefficient, numerical modelling

UTJECAJ MODELIRANJA KOEFICIJENTA REAKCIJE TLA NA JEDNOSTAVNI 3D OKVIR IZLOŽEN HORIZONTALNOM SIMETRIČNOM OPTEREĆENJU

Sažetak: Cilj ovog rada je prikazati utjecaj modeliranja koeficijenta reakcije podloge na jednostavan 3D okvir izložen horizontalnom simetričnom statičkom djelovanju. Budući da rad predstavlja nastavak istraživanja na ovu temu, radi jednostavnosti usvojeno je da je konstrukcija i u ovom slučaju također izvedena na kvadratnim temeljima na granuliranom tlu. Različite vrijednosti koeficijenta reakcije tla dobivene za kvadratne temelje posljedica su korištenja različitih izraza definiranih od strane pojedinih autora. Tako su na jednom dijagramu prikazane za svakog autora vrijednosti koje nam daju odnos između dimenzija kvadratnih temelja i koeficijenta reakcije tla. Odgovor karakterističnog okvira, dodatno opterećenog horizontalnim djelovanjem, na te utjecaje prikazan je u radu.

Ključne riječi: horizontalno opterećen 3D okvir, koeficijent reakcije tla, numeričko modeliranje

1. Introduction

For any structure one has to consider the way of the system foundation, and consequently its connection to the soil. Mostly, in the common building, the shallow foundations' system is used. In that case, Winkler's springs model is implemented for modelling the connection between the soil and the foundation [1], [2], [3]. From the numerical point of view, it is necessary to find out the correct value of the Winkler's spring stiffness through the so-called subgrade reaction coefficient [4], [5], [6], [7]. In its essence, it corresponds to the relation between stress under the foundation and its deflection [8], [9], [10], [11], [12], [13], [14], [15], [16]. These phenomena were investigated by a lot of authors and each solution is a little bit different. So, the implementation of the average value was interesting for the investigation. As the result of our scientific project, which is still dealing with above phenomena, this paper is written.

As already mentioned in [17] we should primarily be aware of the trust we put into the accuracy and plausibility of contemporary static and dynamic computational calculations. According to that, we have to avoid to be like those who blindly trust to each decimal if it was computer-generated, or each textbook formula or expression, even if it proved to be a printing error. If we are talking about the name of the subgrade reaction coefficient, someone would think that coefficient is nondimensional value, which would not be the wright way of thinking. From the investigation of the various literature, different name for the same thing was used. The good explanation could be found in more detail in the paper [18] where the authors were discussing about the term that should be used. As it is mentioned this value has been called by many different names in various publications such as modulus of subgrade reaction, subgrade reaction, subgrade modulus, coefficient of subgrade reaction, Winkler foundation, Winkler subgrade, K value, etc.. Furthermore, for the purpose of this paper this value will be called subgrade reaction coefficient k_s.

Another intention of the authors was to demonstrate what consequences the chosen intensity of the subgrade reaction coefficient $k_s$has on the structure. Because of that, the software SE_Calc was developed and tested examples are shown in the paper [10]. The software calculates the subgrade reaction coefficient k_s for different expression (1) to (5) given by named authors. The first implications of the results were analysed on the simple 2D frames [10], [11], [12]. It showed some dissipations of the values in the moment diagrams and the displacements [19], [20], especially on the extreme values. This phenomenon has a significant influence on the process of structural elements dimensioning and structural detailing.

The comparison of the results was the most interesting aim of that problem. Because of that, the model with the average subgrade reaction coefficient k_s was chosen to be the basic one [11], [12]. It was found out that, from the point of departure of the results on simple 2D frames using the average subgrade reaction coefficient k_s, it seems to be a correct approach. A negative aspect of using the average value was that it didn't represent the exact value of the subgrade reaction coefficient k_s, but the statistical one.

2. Expressions for the subgrade reaction coefficient

As already mentioned, the software SE_Calc was developed for the purpose of determining the subgrade reaction coefficients k_s by different authors' expressions [10] shown by the formulas (1) to (5). It was tested on various examples from the literature [10], [21] and lately upgraded by the possibility of the average value calculation [22]. For the purpose of structural elements dimensioning and better structural behaviour understanding, it was interesting to show each coefficient value and the average one on the same figure (Figure 1), or separately under each Tab (Figure 1). The software also has the possibility to show all values in the same table.

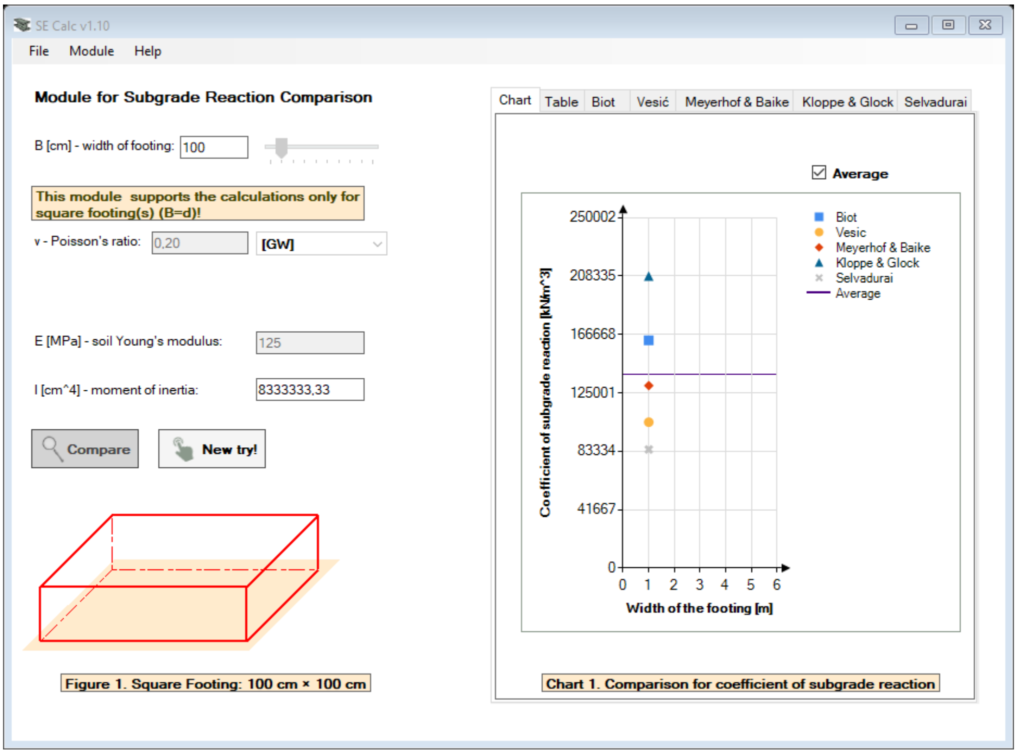

Figure 1. Subgrade reaction coefficient value comparison

The authors and their expressions are listed below in order used by the software SE_Calc. The first one is Biot [10]

$$k_s = \frac{0.95\cdot E_s}{B\cdot\left(1-\vartheta^2\right)}\cdot\left[\frac{E_s\cdot B^4}{\left(1-\vartheta^2\right)\cdot E_b\cdot I}\right]^{0.108} \tag{1}$$

where, like in other expressions the k_s represent the calculated value of the subgrade reaction coefficient, E_s the elastic modulus of soil, E_b the modulus of elasticity of the footing, B the dimension of the square footing, I the moment of inertia of the footing and ϑ the Poisson ratio.

The second one is Vesić, the most common author used in our area [10].

$$k_s = \frac{0.65\cdot E_s}{B\cdot\left(1-\vartheta^2\right)}\cdot\sqrt[12]{\frac{E_s\cdot B^4}{E_b\cdot I}} \tag{2}$$

The third one are Meyerhof & Baike [10]

$$k_s = \frac{E_s}{B\cdot\left(1-\vartheta^2\right)} \tag{3}$$

The forth one are Kloppe & Glock [10]

$$k_s = \frac{2 \cdot E_s}{B \cdot (1+\vartheta)} \tag{4}$$

The fifth one is Selvadurai [10]

$$k_s = \frac{0.65 \cdot E_s}{B \cdot (1-\vartheta^2)} \tag{5}$$

From the above expressions, it is obvious that the results are dissipate. In the case for the granulated soil and dimensions of the foundation from the examples the difference between the smallest and the biggest value is almost 2.5 times.

3. Examples

As this paper presents the part of the scientific project which investigate the structural behaviour regards to the subgrade reaction coefficient k_s change, the same examples from the paper [22] had to be used. The only difference was that the structure is now additionally loaded with the horizontal static force of 40kN, which represents about 10% of the construction self-weight.

So, again horizontally loaded symmetrical and non-symmetrical simple 3D frame (Figure 2) were analysed and the results were compared with the structures loaded only with gravity load. Clearly, that the seismic analyses would be interesting, but it belongs in area of further investigation and therefore is not mentioned here.

The 3D frame in both directions has the range l = 6.00m, and total height of h = 4.00m. Column dimensions are 30x30cm, while the dimensions of the beams are 30x50cm. Beams are connected with concrete plate 22cm thick. Everything is made from concrete C25/30. The dimensions of foundations are 1.00m x 1.00m and the thickness is 0.60m. The 22cm thick concrete plate is loaded with uniformly distributed load of 10kN/m2, horizontal concentrated static load of 40kN and its own self-weight.

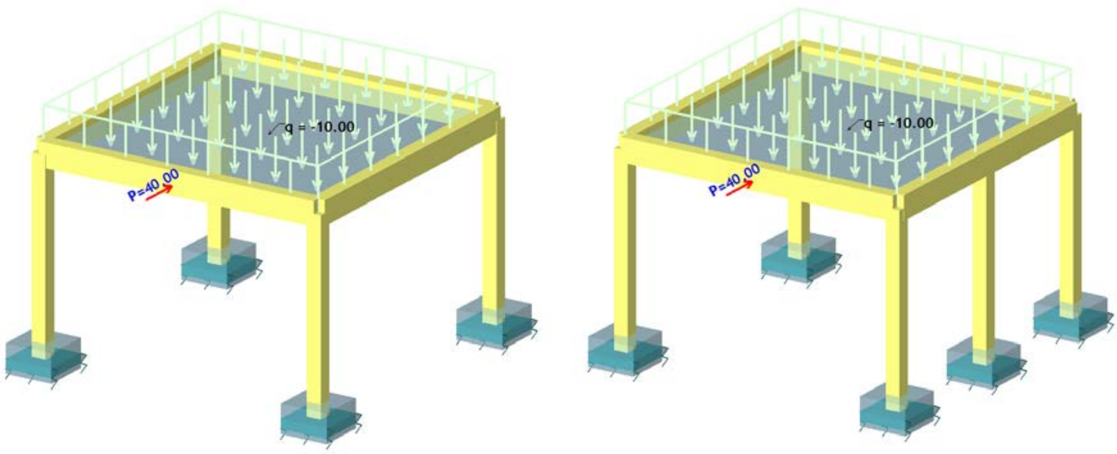

Figure 2. Symmetric (left) and non-symmetric (right) 3D frame

3.1 Symmetric 3D frame

Since all data for the numerical modelling of the simple 3D frames are known, model with the average value of the subgrade reaction coefficient k_s is chosen as the basic one. For the purpose of the structural analyses, software Tower – 3D Model Builder is used [23]. It uses 1D linear finite elements for beam modelling and the 2D finite elements for surface modelling. The chosen finite element mesh density for the surface structures is 0.20x0.20m. As input value, necessary for modelling the connection between soil and foundation by rigidity of Winkler spring, for the software [23] the value of the subgrade reaction coefficient k_s was obtained by SE_Calc [24].

Comparison of the characteristic values between the frame without horizontal load [22] and with horizontal load are shown in Table 1. For easier understanding of the obtained results, the Figure 3 shows the characteristic moment values of the right frame for the average value of the subgrade reaction coefficient k_s for the frame without horizontal load (Figure 3, left) and with horizontal load (Figure 3, right).

Table 1. Subgrade reaction coefficient and moment values comparison on symmetric system

Author	**Subgrade reaction coefficient [kN/m^3]**	**Frame without horizontal load**			**Frame with horizontal load**		
		M [kN/m]	**M [kN/m]**	**M [kN/m]**	**M [kN/m]**	**M [kN/m]**	**M [kN/m]**
		Column - Beam	*Column - Foundation*	*Middle of the beam*	*Column - Beam*	*Column - Foundation*	*Middle of the beam*
Vesic	104107.95	29.99	5.28	97.74	56.59	18.67	98.27
Biot	162490.76	30.75	7.06	97.31	55.42	22.39	97.77
Meyerhof & Baike	130208.33	30.36	6.15	97.53	55.96	20.55	98.02
Kloppe & Glock	208333.33	31.19	8.12	97.06	54.94	24.37	97.49
Selvadurai	84635.42	29.65	4.54	97.93	57.25	16.93	98.50
Average	137955.16	30.46	6.38	97.47	55.81	21.04	97.96

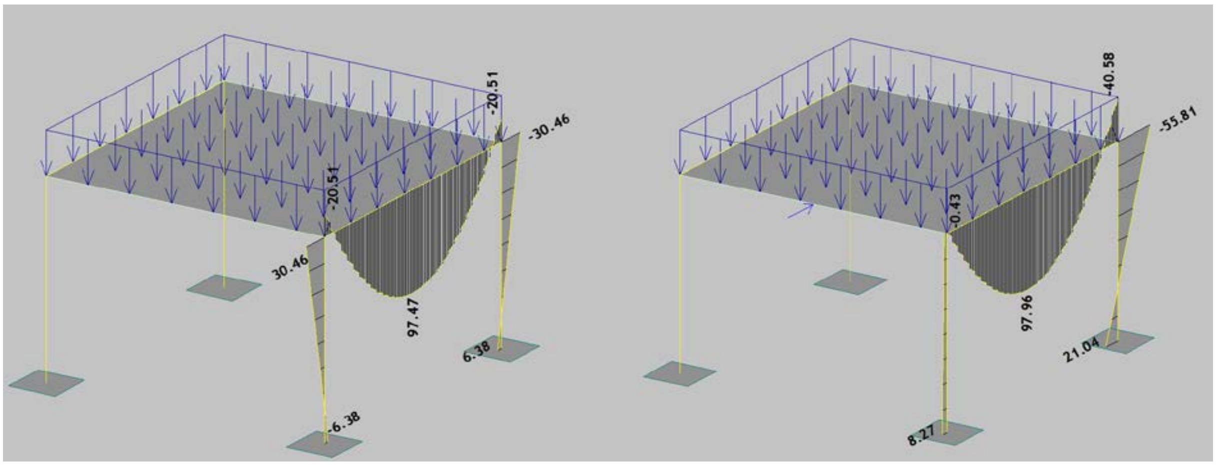

Figure 3. Moment diagram on symmetric frame without (left) and with (right) horizontal load

In the Table 2, the effects of subgrade reaction coefficient k_s change on the maximum and minimum normal stress under the foundation are shown for the case of the frame without horizontal load [22] and with horizontal load. As it is shown, the difference of the maximum and minimum stresses under the foundation is significant (about 35% higher of the maximum stress, and 65% lower the minimum stress).

Table 2. Stress under the foundation comparison on symmetric system

Author	**Frame without horizontal load**		**Frame with horizontal load**	
	Stress [kN/m^2]	**Stress [kN/m^2]**	**Stress [kN/m^2]**	**Stress [kN/m^2]**
	σ_{max}	σ_{min}	σ_{max}	σ_{min}
Vesic	245.77	125.78	331.03	58.23
Biot	265.87	105.42	361.53	26.18
Meyerhof & Baike	255.59	115.85	346.29	42.18
Kloppe & Glock	277.82	93.27	378.45	8.44
Selvadurai	237.36	134.27	317.27	72.74
Average	258.22	113.17	350.26	38.00

3.2 Non-symmetric 3D frame

For better understanding of the simple 3D structure behaviour the non-symmetric model is made. One column was added in the middle of the right side frame (Figure 2, right picture). The observed portal frame was not the same as it was in the paper [22]. That was the reason why the results written in Table 3 for a frame without horizontal load are not the same like the ones from the Table 4 in paper [22]. To avoid the reader doubt in this case the observed portal is the one shown in Figure 4, and in this case also for the average value of the subgrade reaction coefficient k_s.

As it was mentioned in the previous example, the only difference between worked examples and the one published in the paper [22] is the horizontal force in amount of 40kN (Figure 4, right side picture). Comparison of the results between the frames without and with horizontal load is shownat characteristic point in Table 3 and Table 4.

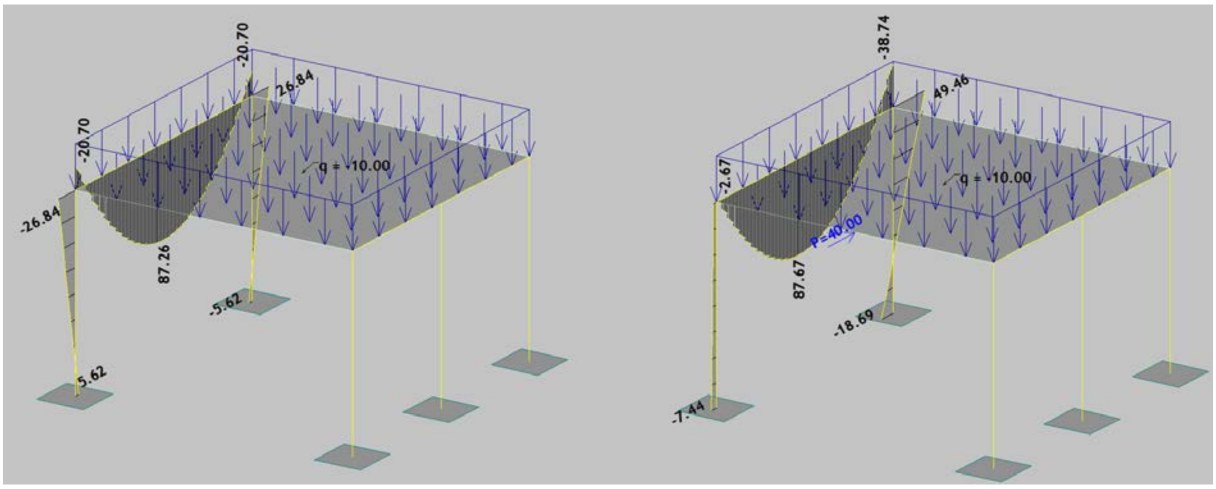

Figure 4. Moment diagram on non-symmetric frame without (left) and with (right) horizontal load

Table 3. Subgrade reaction coefficient and moment values comparison on non-symmetric system

Author	Subgrade reaction coefficient [kN/m³]	Frame without horizontal load			Frame with horizontal load		
		M [kN/m]	M [kN/m]	M [kN/m]	M [kN/m]	M [kN/m]	M [kN/m]
		Column - Beam	*Column - Foundation*	*Middle of the beam*	*Column - Beam*	*Column - Foundation*	*Middle of the beam*
Vesic	104107.95	26.51	4.67	87.75	50.26	16.61	88.21
Biot	162490.76	27.05	6.21	86.98	49.07	19.88	87.37
Meyerhof & Baike	130208.33	26.77	5.42	87.36	49.61	18.26	87.78
Kloppe & Glock	208333.33	27.37	7.13	86.57	48.57	21.62	86.94
Selvadurai	84635.42	26.29	4.02	88.13	50.92	15.07	88.61
Average	137955.16	26.84	5.62	87.26	49.46	18.69	87.67

Table 4. Stress under the foundation comparison on symmetric system

Author	Frame without horizontal load		Frame with horizontal load	
	Stress [kN/m²]	Stress [kN/m²]	Stress [kN/m²]	Stress [kN/m²]
	σ_{max}	σ_{min}	σ_{max}	σ_{min}
Vesic	258.52	93.35	327.23	42.33
Biot	280.08	82.28	357.00	21.60
Meyerhof & Baike	269.09	87.83	342.10	31.76
Kloppe & Glock	292.75	76.06	373.61	8.39
Selvadurai	249.39	98.28	313.88	52.21
Average	271.91	86.39	345.98	29.08

If we are talking about the normal stress under the foundation, the behaviour of the stress distribution is almost the same (about 35% higher maximum stress under the foundation, and 65% lower minimum stress under the foundation) as in the case of the symmetric frame. The answer on the question why is it happening could be found in the additional horizontal loading which causes additional moment of 160kNm. From another point of view, the moment distribution on beam and column shows almost the same behaviour in the moment results dissipation from the values obtained by the example with the average value of subgrade reaction coefficient k_s. Naturally, the horizontal load slightlyincreases that dissipation. In the interest for further research it will be preferable to investigate this effect within other software [25].

4. Conclusion

This paper presents the part of the scientific project which investigate the structural behaviour regards to the subgrade reaction coefficient k_s change. For that reason in this paper the response of the symmetrical and non-symmetrical 3D frame, additionally loaded with a horizontal static load, were analysed. The intensity of that load was about 10% of the structure self-weight. For the simplicity, the structure had the same geometry and soil characteristic as the one loaded only with gravity load.

It should be stated that Winkler spring model is used for modelling the connection between the concrete foundation and the soil. The capacity of the spring was defined by subgrade reaction coefficient k_s. Its values were obtained by developed software SE_Calc which calculate different values by expressions given by different authors. The average value was the basic value for the comparison of the results. Like in the case of the 2D frame and the 3D frame only with gravity load the calculation results showed that the higher value of the soil reaction coefficient gives less peak moments which is more expressed on the non-symmetrical system.

Comparison of the normal stress distribution under the foundation of the gravity loaded and additionally horizontal loaded 3D frame are made. The behaviour is almost the same for the symmetric and non-symmetric 3D frame. The maximum normal stress under the foundation was about 35% higher, and minimum normal stress was about 65% lower. The answer on the question why it was happening could easily be found in the additional horizontal loading, which causes additional moment.

Like in the case of the 2D frame and simple 3D frame, also in this case with a horizontal load it was shown that from the point of departure of the results the approach of using the average subgrade reaction coefficient seems to be correct. Generally, from the stress under the foundation point of view, the higher subgrade reaction coefficient gives the higher stress and lower vertical displacement (settlement) of the foundation for these kinds of structures.

For further investigation, 2D multi-span and 3D multi-storey constructions should be analysed under symmetric, asymmetric and seismic horizontal loading.

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