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APPLICATION OF THE CROSS ITERATIVE METHOD ON SIMPLE SYSTEMS

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Abstract: This paper gives a brief overview of the different possibilities of applying standard engineering calculation methods. The intention was to show how a developed balancing method can be used in different fields on the example of Cross iterative method. In the field of structural theory, in this paper it deals with the procedure of calculating statically indeterminate structures by balancing moments in nodes, and in the field of pressure system hydraulics it deals with the problems of analysis of flow in pipe networks.

Key words: calculation methods, balancing, Cross method, hydraulics of pressure systems, pipe networks

PRIMJENA CROSSOVE ITERATIVNE METODE NA JEDNOSTAVNIM SUSTAVIMA

Sažetak: U radu se daje kratak osvrt na različite mogućnosti primjene standardnih inženjerskih metoda proračuna. Na primjeru Crossove iterativne metode htjelo se prikazati kako se razvijena metoda uravnoteženja može koristiti u različitim područjima. U ovom radu ona se u području teorije konstrukcija dotiče postupka proračuna statički neodređenih konstrukcija kroz uravnoteženje momenata u čvorovima, a u području hidraulike sustava pod tlakom bavi se problemima analize tečenja u cijevnim mrežama.

Ključne riječi: metode proračuna, uravnoteženje, Crossova metoda, hidraulika sustava pod tlakom, cijevne mreže

1. Introduction

Approach to the analysis and calculation of any structure involves the use of appropriate methods. Solutions to the analysed problems are obtained by properly using these methods. The paper emphasizes the possibility of using the same calculation method in different engineering problems.

So, the discussion is about a method that can be applied to problems of balancing moments in nodes of statically indeterminate structures as well as problems of hydraulic analysis of pipe networks, and it is the Hardy Cross method [4], [5]. In 1929, the author of the method published papers demonstrating the application of this iterative method in the analysis of continuous frames by distributing fixity moments. However, it was later realized that this method could be used for analysis of hydraulic problems, and a study on the topic of analysis of flow in pipe networks was published in 1949.

2. The Cross iterative method

The term iterative method refers to groups of methods using analytical procedures that solve a given set of equations in a successive manner in order to solve statically indeterminate rod structures [1], [2], [6], [7]. The Cross iterative moment distribution method has been most widely applied in solving static as well as hydraulic problems [1], [3].

2.1 Example - rigid body statics

Application of this method to the characteristic statically indeterminate system in Figure 1 is given below. For given values: F = 200kN, span l = 4m, EI = const. distribution values (moment distributions) η are specified below.

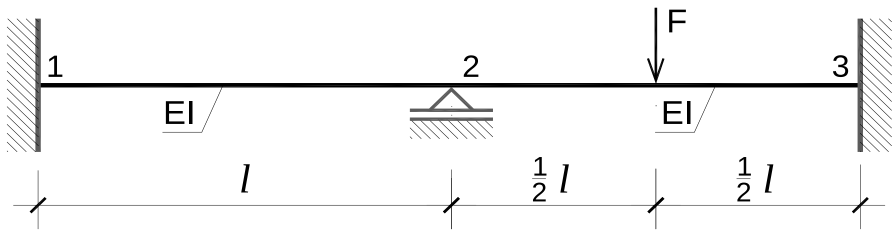

Figure 1. Statically indeterminate system

$$\eta_{21} = \frac{K_{21}}{\sum K} = \frac{4 \cdot \left(\frac{EI}{4}\right)}{4 \cdot \left(\frac{EI}{4}\right) + 4 \cdot \left(\frac{EI}{4}\right)} = 0.5, \qquad \eta_{23} = \frac{K_{23}}{\sum K} = \frac{4 \cdot \left(\frac{EI}{4}\right)}{4 \cdot \left(\frac{EI}{4}\right) + 4 \cdot \left(\frac{EI}{4}\right)} = 0.5. \tag{1}$$

It is important to note that the following condition must be satisfied:

$$\sum \eta_{čvor} = 1.0. \tag{2}$$

Then, since all the nodes are still fixed, the fixity moments are calculated for each element, which in this case have an absolute value of 100 kNm. For the positive sign of moments, it is adopted to be in conformity with full displacement method. The obtained values of full fixity forces (moments) are written in step 1.

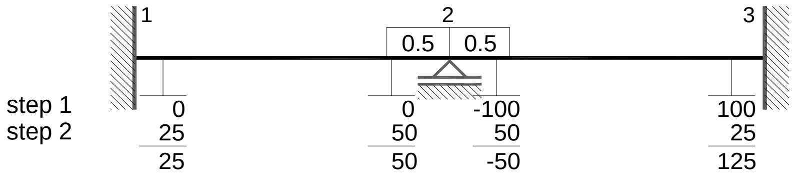

Figure 2. Iterative scheme

In step 2, the sum of moments in node 2 is calculated and balanced with the moment of the same intensity and opposite sign. So, the sum of moments is -100 and it is balanced with a moment of 100. It is distributed with respect to the distribution factor of node 2 to 50 left and 50 right. This moment of 50 kNm acting at node 2 is transferred to the other end of the rod to node 1 and 3 (50% of value). The final diagram is shown in Figure 3.

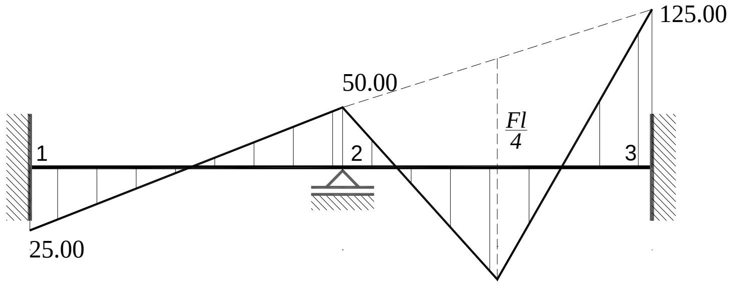

Figure 3. Moment diagram

2.2 Example - hydraulic analysis of pipe networks

In each loop the algebraic sum of pressure losses must equal zero. However, since line flows are assumed arbitrarily, taking into account only continuity, the algebraic sum of losses will be non-zero. The difference is due to an error in the assumed flow, or due to the difference (ΔQ) between the actual and the assumed flow. For this reason, the following must be satisfied:

$$\sum_{i=1}^{n} s_i \cdot \ell_i \cdot \left(Q_i + \Delta Q\right)^2 = 0. \tag{3}$$

Here, the index *i* indicates a line in the loop. Cross assumed that the correction for all lines of one loop was the same. Now:

$$\sum_{i=1}^{n} s_i \cdot \ell_i \cdot Q_i^2 + 2 \cdot \Delta Q \cdot \sum_{i=1}^{n} s_i \cdot \ell_i \cdot Q_i + \left(\Delta Q\right)^2 = 0. \tag{4}$$

The term $\left(\Delta Q\right)^2$ can be ignored as a small second order quantity, so it follows:

$$\Delta Q = -\frac{\sum_{i=1}^{n} s_i \cdot \ell_i \cdot Q_i^2}{2 \cdot \sum_{i=1}^{n} s_i \cdot \ell_i \cdot Q_i} = -\frac{\sum_{i} h_i \text{(considering the sign)}}{2 \cdot \sum_{i=1}^{n} s_i \cdot \ell_i \cdot Q_i \text{ (not taking into account the sign)}}. \tag{5}$$

The specific resistance coefficient of the pipe line is defined by the following expression:

$$s = 0.0826 \cdot \frac{\ell}{d^5}. \tag{6}$$

In the numerical example below, the task is to determine the amount of water flowing through individual pipes as well as the pressure values at the nodes. The pipe network and the set flows in [l/s] are shown in Figure 4, and the set data are given in Tables 1 and 2. The input data are: absolute roughness k = 0.06 mm; set pressure at node A, p_A = 70 m a.s.l.; kinematic viscosity $v = 1.14 \cdot 10^{-6}$ m^2/s.

Table 1. Lengths and diameters of pipe lines

PIPE	AB	BC	CD	DE	EF	AF	BE
LENGTH [m]	600	600	200	600	600	200	200
DIAMETER [mm]	250	150	100	150	150	200	100

Table 2. Data for nodes

NODE	A	B	C	D	E	F
m a.s.l.	30	25	20	20	22	25

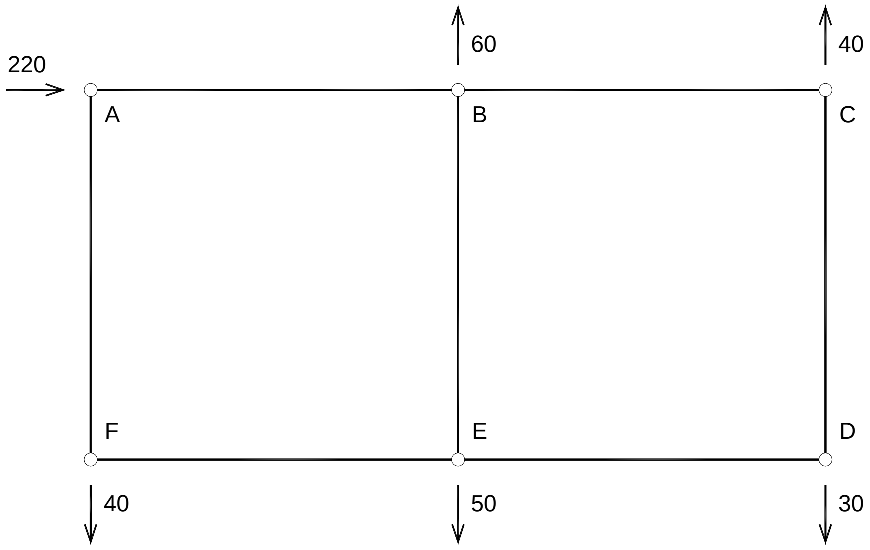

Figure 4. Pipe network

The calculation procedure is conducted in the following steps:

1. Define loops. The simplest way is to define loops so as to be adjacent, or to have at least one branch in common. For example, loop 1 is defined by nodes ABEFA and loop 2 is defined by nodes BCDEB (Figure 5).
2. Assign estimated flows in pipes. For each loop, it is sufficient to determine one flow value, while the remaining part of the flow is determined by the condition of preservation of continuity at nodes. For example, if one takes into account that the total inflow to the system is 220 l/s, and assuming that the flow Q_{AB} = 120 l/s, then Q_{AF} = 100 l/s.
3. The pressure loss coefficient is calculated for each pipe.

$$K = \frac{\lambda \cdot \ell}{2 \cdot g \cdot D \cdot A^2}. \tag{7}$$

$$h_L = K \cdot Q \cdot |Q|. \tag{8}$$

The coefficient λ is determined from the Moody diagram. Barr's equation can be used as an alternative.

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.70 \cdot D} + \frac{5.1286}{\mathrm{Re}^{0.89}}\right). \tag{9}$$

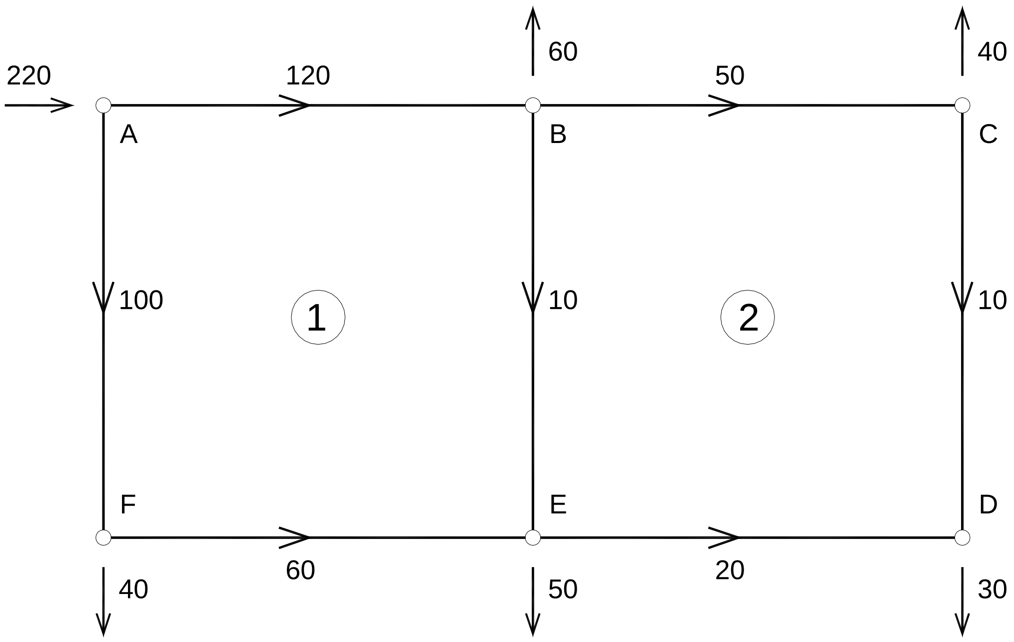

Figure 5. Distribution of flows by lines

If the Reynolds number has a relatively small value $(\leq 10^5)$, calculation can be continued by iterative methods, changing the value for λ so that the result converges to the solution.

After that, as shown below, the calculation is performed in tables (Tables 3 to 5). Line losses are calculated from the expression:

$$h_L = r \cdot Q^2, \tag{10}$$

where *r* represents the coefficient of resistance of the pipe line calculated as:

$$r = s \cdot \ell. \tag{11}$$

The symbol *s* represents the specific conductivity of the pipe line, while *l* is the length of the particular pipe line.

Table 3. Calculation for loop 1

LOOP	PIPE	k/D	Q	Re	λ	K	h_L	h_L/Q
			[m³/s]	(*10⁵)			[m]	[m/m³/s]
1	AB	0.00024	0.12	5.36	0.01585	805	11.59	96.56
	BE	0.00060	0.01	1.12	0.02066	34180	3.41	341.46
	EF	0.00040	-0.06	4.48	0.01744	11399	-41.00	683.26
	FA	0.00030	-0.10	5.58	0.01640	847	-8.47	84.72
						Σ =	-34.46	1205.99

For the first loop the value of ΔQ is:

$$\Delta Q = \frac{-\sum h}{2\cdot\sum\frac{h}{Q}} = \frac{-(-34.46)}{2\cdot 1205.99} = 0.01429 = 14.29\, l/s. \tag{12}$$

Table 4. Calculation for loop 2

LOOP	PIPE	k/D	Q	Re	λ	K	h_L	h_L/Q
			[m³/s]	(*10⁵)			[m]	[m/m³/s]
2	BC	0.00040	0.05	3.72	0.01761	11511	28.75	574.98
	CD	0.00060	0.01	1.12	0.02066	34180	3.41	341.46
	DE	0.00040	-0.02	1.49	0.01898	12402	-4.96	247.79
	EB	0.00060	-0.024	2.71	0.01927	31877	-18.70	771.61
						Σ =	8.51	1935.85

For the second loop the value of ΔQ is:

$$\Delta Q = \frac{-\sum h}{2\cdot\sum\frac{h}{Q}} = \frac{-(8.51)}{2\cdot 1935.85} = -2.20\, l/s. \tag{13}$$

The repeated calculation (first iteration) for the first loop is given in Table 5.

Table 5. Calculation for loop 1 - first iteration

LOOP	PIPE	k/D	Q	Re	λ	K	h_L	h_L/Q
			[m³/s]	(*10⁵)			[m]	[m/m³/s]
1	AB	0.00024	0.1343	6.00	0.01575	800	14.42	107.35
	BE	0.00060	0.0243	2.71	0.01927	31873	18.78	773.38
	EF	0.00040	-0.0451	3.24	0.01776	11608	-21.96	504.63
	FA	0.00030	-0.0857	4.79	0.01654	855	-6.28	73.21
						Σ =	4.97	1458.57

For the first loop the repeated calculation gives the value ΔQ of:

$$\Delta Q = \frac{-\sum h}{2 \cdot \sum \frac{h}{Q}} = \frac{-(4.97)}{2 \cdot 1458.57} = -1.70 l/s. \tag{14}$$

The calculation procedure is repeated for loop 2. Iterations are continued. The specified solution obtained for Σh < 0.01 m, is shown in tables 6 and 7. Depending on the problem being solved, a result with higher tolerance is also acceptable.

Table 6. Calculation results by pipe lines

PIPE	**Q**	**h_L**
	[m^3/s]	**[m]**
AB	0.1337	14.30
BE	0.0237	17.96
FE	0.0477	26.20
AF	0.0863	6.35
BC	0.0377	16.61
CD	0.0011	4.38
ED	0.0186	4.32

Table 7. Calculation results by nodes

NODE	**PRESSURE**
	[m]
A	40.00
B	30.70
C	19.09
D	14.71
E	15.74
F	38.65

3. Conclusion

The aim of any method is to solve given problems in a uniform manner. Here, the intention was to emphasize the possibility of using the principle of the appropriate method to solve problems in different research fields. These examples demonstrate the possibility of using the Cross iterative procedure for solving problems in the field of structural theory on statically indeterminate systems and for hydraulic analyses of flow in pipe networks.

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