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SYSTEM OPTIMUM FUZZY TRAFFIC ASSIGNMENT PROBLEM

ABSTRACT

This paper focuses on converting the system optimum traffic assignment problem (SO-TAP) to system optimum fuzzy traffic assignment problem (SO-FTAP). The SO-TAP aims to minimize the total system travel time on road network between the specified origin and destination points. Link travel time is taken as a linear function of fuzzy link flow; thus each link travel time is constructed as a triangular fuzzy number. The objective function is expressed in terms of link flows and link travel times in a non-linear form while satisfying the flow conservation constraints. The parameters of the problem are path lengths, number of lanes, average speed of a vehicle, vehicle length, clearance, spacing, link capacity and free flow travel time. Considering a road network, the path lengths and number of lanes are taken as crisp numbers. The average speed of a vehicle and vehicle length are imprecise in nature, so these are taken as triangular fuzzy numbers. Since the remaining parameters, that are clearance, spacing, link capacity and free flow travel time are determined by the average speed of a vehicle and vehicle length, they will be triangular fuzzy numbers. Finally, the original SO-TAP is converted to a fuzzy quadratic programming (FQP) problem, and it is solved using an existing approach from literature. A numerical experiment is illustrated.

KEY WORDS

fuzzy traffic assignment problem; average speed; vehicle length; link capacity; travel time;

1. INTRODUCTION

The Traffic Assignment Problem (TAP) describes the distribution of vehicles in traffic through a network comprising a set of nodes and a set of directed links connecting these nodes. In traffic systems, the drivers do not want to deal with congestion while travelling from an origin point to a destination point. To overcome this congestion effect, many researchers have studied this problem. At this stage, the notion of equilibrium should be given to solve TAP. The equilibrium in the TAP is satisfied by performance/demand analysis which is a special case of supply/demand equilibrium in economics. Performance and demand are functions in terms of flow and its corresponding value of delay. In a performance function, delay increases with the flow while in a demand function delay decreases as the flow increases. To solve TAP, it is important to find a certain point at intersection of the performance and demand functions. While the flow is low on a link, the delay will be low, and accordingly, more vehicles will come to that link. On the contrary, when the flow is very high, there will be a long queue, the delay will be high, and no vehicle will tend to join that queue. A specified flow x^* creates a delay t^* using performance function and in demand function t^* corresponds to x^*. This (x^*,t^*) point will be an equilibrium point and it is a solution of TAP, which is also known as transportation network equilibrium problem. The intersection point of these functions is given in *Figure 1*. When a traffic system is in a disequilibrium state, there will be forces tending to direct the system toward an equilibrium state, i.e. when an interaction among individual vehicles in roadway increases, they become very close, come to a halt and traffic congestion will occur. Since drivers do not want to waste their time in congested traffic, they could tend to change their routes to find any alternative path to reach their destination point, and therefore a new equilibrium state will be observed.

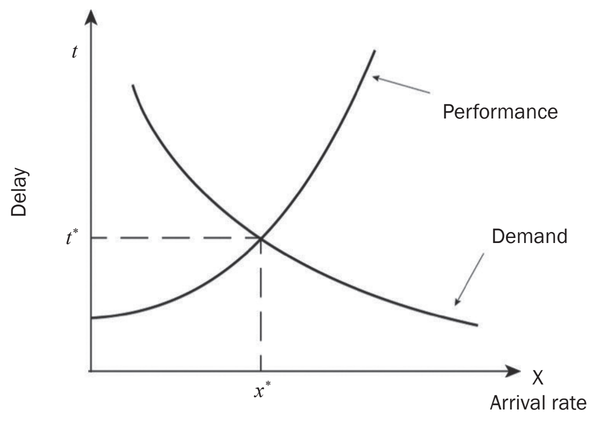

Figure 1 – Illustration of performance and demand functions and their intersection point [1]

In literature, TAP can be modelled as user equilibrium (UE) or system optimum (SO). The aim of these models is to minimize the travel time through the network while satisfying the flow conservation constraints. However, the objective function of both models has different form and feature. UE-TAP is based on the Wardrop's first principle, that no driver can improve their travel time by unilaterally changing routes [1]. Each individual driver non-cooperatively seeks to minimize their own travel time between origin-destination (O-D) points in the UE-TAP, i.e. each driver wants to select the fastest route to reach their destination point. Another assumption for UE-TAP is that the travel times of all the used paths between O-D pairs are equal or less than travel times of unused paths. In this problem, the drivers are assumed to have full information of the network conditions and traffic congestions, i.e. they are considered to know all paths between each O-D pair which seems not realistic. SO-TAP is based on the Wardrop's second principle that the drivers cooperate with one another in order to minimize the total system travel time. The main difference between UE and SO is that in the SO condition, while drivers of ambulances, fire trucks, etc., reduce their travel times, other vehicles can experience redundantly high travel times in emergency cases. It will be an optimum solution whether this case affects positively the total travel time of a system [2]. Since the congestion effects are ignored, both UE and SO models would give the same result which is known as all-or-nothing assignment.

The travel time between O-D pairs is calculated as summation of each link travel time forming a path. The link travel time can be determined using a linear/non-linear function in terms of flow, density, or volume-capacity ratio. In literature, Branston [3] gave some types of link travel time functions in his review article. Carey and Ge [4] used linear and quadratic forms of travel time functions which are defined in terms of vehicles and estimated mean flow rate. Kachroo and Sastry [5] used density-based travel time function for intelligent transportation systems.

Traffic assignment is modelled in two categories: static and dynamic. In static models, the flow is assumed not to vary with time. Within static models, it is assumed that the inflow (traffic demand) is constant through time. Thus, the inflow is always equal to the outflow and accordingly, the travel time increases while the inflow increases. The traffic flow includes a combination of drivers and vehicle behaviour. Facilities of traffic flow are given as interrupted and uninterrupted. In an interrupted flow, the vehicles are arranged by external interruptions such as signals or stop/yield signs. In uninterrupted flow, there is an interaction only between vehicle-vehicle and vehicle-roadway, such as freeways. As a characteristic of traffic flow, the microscopic traffic flow model analyses both a single driver on different features of the road and interaction between a driver and another driver on the road. The car-following models and cellular automaton models are examined in this model. Another characteristic of the traffic flow, macroscopic traffic flow model examines the traffic flow from a global perspective instead of considering individually each vehicle. The parameters of macroscopic traffic flow model are flow, density, and speed. Flow is the number of vehicles which traverse between a specified O-D pair in a unit time. Density is defined as the number of vehicles occupying a given length of the link. Speed of vehicle is the distance traversed per unit of time. Under uninterrupted flow conditions, the fundamental diagram of traffic flow theory is $flow = speed \times density$

The idea of traffic equilibrium was introduced by Frank Knight in 1924. In 1956, Beckmann et al. [6] formulated the first mathematical model of network equilibrium model and it is known in literature as Beckmann's transformation. This mathematical model is used in UE-TAP and aims to minimize the sum of integrals of link travel time functions while satisfying a set of flow conservations and non-negativity conditions. Leblanc et al. [2] presented an efficient method for solving non-linear programming of large-scale road network equilibrium assignment, and average travel time function is taken as used in the U.S. Federal Highway Administration traffic assignment models. Also, Leblanc [7] showed some large-scale mathematical programming models used in transportation systems. In the same year, Branston [3] reviewed the state-of-the-art of the measurement and formulation of link capacity functions for use in traffic assignment procedures. Daganzo [8] generalised the algorithm presented by Leblanc [2] to the networks with link capacities and illustrated the modification with a simple numerical example. Furthermore, Daganzo [9] generalised the results obtained in [8] by showing how to incorporate link capacities into any equilibrium traffic assignment algorithm. Daganzo [10] examined a general form of link travel time functions considered in the dynamic traffic assignment literature. Peeta and Mahmassani [11] formulated two dynamic time-dependent network traffic assignment models which are SO and UE. Vandaele et al. [12] used queuing theory to describe uninterrupted traffic flows.

In order to represent real life human perception and deal with vague, ambiguous or imprecise information, fuzzy theory can be used as a powerful tool. Many studies have been presented to apply fuzzy theory to traffic assignment. Binetti and De Mitri [13] presented a path choice model and used fuzzy numbers to represent travel costs. Ridwan [14] introduced a route choice based on fuzzy preference model which is the first application of fuzzy individual choice in traffic assignment. Liu and Kao [15] examined the network flow problem by using fuzzy arc lengths. Murat and Uludag [16] studied the route choice model of transportation network using fuzzy logic and logistic regression models and these models are compared with each other. Ramazani et al. [17] introduced a traffic assignment algorithm assuming driver perception of travel time that affects route choices, and fuzziness is used to define the drivers' perceived travel time. Also, the fuzzy equilibrium is suggested to predict the network flow, and fuzzy incremental traffic assignment is developed. Miralinaghi et al. [18] proposed a new TAP based on the fuzzy equilibrium condition. In this condition, the users' perceived travel times are assumed to follow the fuzzy values. For relaxing UE assumptions, they used fuzzy and probability theories.

This paper focuses on SO-TAP as a macroscopic static traffic model under uninterrupted flow conditions over a sampled network considering linear link travel time functions in terms of flow. The parameters of SO-TAP taken for constructing the link travel time functions are path length, number of lanes, average speed, vehicle length, clearance, spacing, link capacity and free flow travel time. In the SO-TAP, the objective function is obtained by the sum of multiplying each link travel time function and link traffic flow, and as a result it becomes non-linear, while constraints are linear and satisfy the flow conservation. Since the collected data contain some errors or have uncertainty arising from the lack of information, all parameters in the problem are considered as triangular fuzzy numbers defined by Zadeh [19] to overcome these imprecisions. Considering a road network, the average speed and vehicle length are taken as triangular fuzzy numbers. Due to the structure of the parameters, path lengths and number of lanes are not fuzzified. Consequently, the SO-FTAP is presented as an FQP problem with all parameters being fuzzy. Modifying the proposed method in [20] which is developed for fully fuzzy linear programming (FFLP) problems, a solution approach for SO-FTAP is presented. Also, a numerical experiment to illustrate this approach is provided.

This paper is organized as follows: brief information about fuzzy numbers and some definitions are given in Section 2. Section 3 presents the mathematical model of SO-TAP and SO-FTAP, with the solution process being presented, respectively, and in the last Section, a numerical experiment is illustrated.

2. PRELIMINARIES

Definition 1. Let X be a universal set which is subset of real numbers, $X \subseteq \Re$, and elements of X be denoted by $x: x \in X$. A fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(X)), x \in X\}$ where $\mu_{\tilde{A}}(X)$ is called membership function which is mapped as $\mu_{\tilde{A}}(X): X \to [0,1]$.

Definition 2. A fuzzy set $\tilde{A}$ is convex if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ where all $x_1, x_2 \in X$ and $\lambda \in [0,1]$.

Definition 3 [21]. α-level (α-cut) set A_α is determined by members whose membership is not less than α, such that $A_\alpha = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$ where $\alpha \in [0,1]$ and A_α is a crisp set.

Definition 4. Let $\tilde{A}$ be a fuzzy set in X. If there is at least one element $x \in X$ such that $\mu_{\tilde{A}}(X) = 1$, then $\tilde{A}$ is called a normal fuzzy set.

Definition 5 [22]. A fuzzy number $\tilde{A}$ is a convex normal fuzzy subset of X with the membership function $\mu_{\tilde{A}}(X)$ satisfying the following conditions:

i. There is at least one element $r \in X$ such that $\mu_{\tilde{A}}(r) = 1$ where r is called the mean value of $\tilde{A}$,
ii. $\mu_{\tilde{A}}(x)$ is non-decreasing on $(-\infty, r)$ and non-increasing on (r, ∞),
iii. $\mu_{\tilde{A}}(x)$ is piecewise continuous,
iv. $\int_{-\infty}^{\infty} \mu_{\tilde{A}}(x)dx < \infty$.

Definition 6. A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is called a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x < a_2 \\ \frac{x - a_3}{a_2 - a_3}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

where $a_1 \leq a_2 \leq a_3$ and $a_1, a_2, a_3 \in \Re$.

Definition 7. A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is non-negative if $a_1 \geq 0$.

Definition 8. Two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are equal iff $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$.

Definition 9 [21]. Let X, Y be universal sets, $X, Y \subseteq \Re$ and $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$, $\tilde{B} = \{(y, \mu_{\tilde{B}}(y)): y \in Y\}$ are two fuzzy sets. Then $\tilde{R} = \{((x,y), \mu_{\tilde{R}}(x,y)): (x,y) \in X \times Y\}$ is a fuzzy relation on $\tilde{A}$ and $\tilde{B}$ if $\forall (x,y) \in X \times Y, \mu_{\tilde{R}}(x,y) \leq \mu_{\tilde{A}}(x)$ and $\mu_{\tilde{R}}(x,y) \leq \mu_{\tilde{B}}(y)$ where the membership function of the fuzzy relation is defined as $\mu_{\tilde{R}}(x,y): X \times Y \to [0,1]$.

Definition 10 [21]. Let X, Y be universal sets, $\tilde{P}(Y)$ be the set of all fuzzy sets in Y (power set) and $\tilde{f}: X \to \tilde{P}(Y)$ is a mapping. $\tilde{f}$ is a fuzzy function iff $\mu_{\tilde{f}(x)}(y) = \mu_{\tilde{R}}(x,y)$, $\forall (x,y) \in X \times Y$.

Definition 11 [20]. A ranking function R is mapping from the set of triangular fuzzy numbers to the set of real numbers where a natural order exists. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number, then $\mathrm{R}(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}$.

Definition 12. Let $\tilde{A}$ and $\tilde{B}$ be arbitrary triangular fuzzy numbers. Ordering these triangular fuzzy numbers is given as follows: $\tilde{A} \overset{f}{<} \tilde{B}$ iff $\mathrm{R}(\tilde{A}) < \mathrm{R}(\tilde{B})$,

$\tilde{A} \overset{f}{>} \tilde{B}$ iff $\mathrm{R}(\tilde{A}) > \mathrm{R}(\tilde{B})$, $\tilde{A} \equiv \tilde{B}$ iff $\mathrm{R}(\tilde{A}) = \mathrm{R}(\tilde{B})$, where the symbols " $\overset{f}{<}$ " and " $\overset{f}{>}$ " represent the fuzzy order relations.

Definition 13. Some arithmetic operations on triangular fuzzy numbers are given as follows:

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two arbitrary triangular fuzzy numbers, then $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$, $-\tilde{A} = (-a_3, -a_2, -a_1)$, $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$. The multiplication of two triangular fuzzy numbers based on the extension principle is given by the following equation $\tilde{A}\tilde{B} = (a, b, c)$ where $b = a_2 b_2$, $a = \min\{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$ and $c = \max\{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$. The multiplication of a triangular fuzzy number by a scalar λ is given as $\lambda\tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3)$, for $\lambda>0$, $\lambda\tilde{A} = (\lambda a_3, \lambda a_2, \lambda a_1)$ for $\lambda<0$. The division of two triangular fuzzy numbers based on the extension principle is given as follows: if $0 \notin \tilde{B}$, $\frac{\tilde{A}}{\tilde{B}} = \tilde{A}\left(\frac{1}{\tilde{B}}\right) = (a, b, c)$, where $b = \frac{a_2}{b_2}$, $a = \min\left\{\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}\right\}$ and $c = \max\left\{\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}\right\}$.

Definition 14 [20]. Suppose that $\tilde{A}$ and $\tilde{B}$ are two triangular fuzzy numbers, then $\mathrm{R}(\tilde{A} + \tilde{B}) = \mathrm{R}(\tilde{A}) + \mathrm{R}(\tilde{B})$. This property can be extended for n triangular fuzzy numbers $\tilde{A}_1, \tilde{A}_2, \ldots \tilde{A}_n$ as $\mathrm{R}\left(\sum_{i=1}^{n} \tilde{A}_i\right) = \sum_{i=1}^{n} \mathrm{R}(\tilde{A}_i)$.

3. MATHEMATICAL MODEL

The following notations are defined:

Index Sets:

$N=(V,I)$ – the network where V is the set of nodes and I is the set of directed links connecting the nodes;

O – the set of origin points, $o \in O$;

D – the set of destination points, $d \in D$;

OD – the set of origin-destination pairs, $(o,d) \in OD$;

I – the set of directed links in the network N, $i \in I$;

K – the set of paths comprised of links, $k \in K$;

Parameters:

F^{od} – the inflow (traffic demand) between origin o and destination d;

$$\delta_{i,k}^{od} = \begin{cases} 1, \text{if link } i \text{ is part of path } k \text{ connecting } od \text{ pairs} \\ 0, \text{otherwise} \end{cases}$$

Decision variables:

x_i – the flow on link i;

t_i – the travel time on link i;

f_k^{od} – the flow on path k connecting origin o and destination d;

c_k^{od} – the travel time on path k connecting origin o and destination d.

3.1 Mathematical model of SO-TAP

The SO-TAP aims to minimize the total system travel time of vehicles on a specified network between O-D pairs and finds link flows x_i while satisfying the flow conservation constraints. The mathematical model of SO-TAP is given in *Model 1*.

$$\mathrm{Min} \sum_i x_i t_i(x_i) \quad (1)$$

subject to

$$\sum_k f_k^{od} = F^{od}; \quad \forall (o,d) \in OD \quad (1a)$$

$$x_i = \sum_k f_k^{od} \delta_{i,k}^{od}, \quad \forall (o,d) \in OD, \forall i \in I \quad (1b)$$

$$c_k^{od} = \sum_i t_i \delta_{i,k}^{od}, \quad \forall (o,d) \in OD, \forall k \in K \quad (1c)$$

$$f_k^{od} \geq 0, \quad \forall (o,d) \in OD, \forall k \in K \quad (1d)$$

Here, the constraints represent the flow conservation (*1a*), path-link incidence relationships (*1b* and *1c*), and non-negativity (*1d*), respectively. *Constraint 1a* states the equality between the inflow and outflow, i.e. F^{od} on a specified network. *Constraint 1b* defines the flow on link i by using the path flow. In *Constraint 1c* the path travel times are determined by using the link travel times. *Constraint 1d* guarantees a reliable solution space. It is important that this formulation assuming the travel time on a considered link is a function of the flow on that link, and this link travel time function is assumed to be positive and increasing. To determine the link travel time functions belonging to a given network, some parameters such as path length, vehicle speed, link capacity, etc. can be collected by observing a specified network at certain time intervals or by taking data from the transportation departments. However, these collected data can contain some errors or have uncertainty arising from the lack of information. To overcome these imprecise data, the parameters are considered fuzzy.

3.2 Mathematical model of SO-FTAP

Consider a road network consisting of links. In the case of low traffic density, vehicles have high speed and there is no interaction between vehicles on the links. Thus, the average high speed is taken as triangular fuzzy number such that $\tilde{u}_1 = (u_{11}, u_{12}, u_{13})$ km/h. Using the average high speed $\tilde{u}_1$ and length of link i L_i km, the travel time of link i $\tilde{T}_i^1$, also known as free flow travel time, is computed in terms of minutes as

$$\tilde{T}_i^1 = 60 \frac{\text{Length of link } i}{\text{Average speed in low traffic density}} = 60 \frac{L_i}{\tilde{u}_1}$$

In the case of high traffic density, links become congested and the interaction arises between vehicles, and this situation causes vehicles to slow down. In this case, the average low speed is taken as a triangular fuzzy number such that $\tilde{u}_2 = (u_{21}, u_{22}, u_{23})$ km/h. In this paper, the assumption is that the lower bound of $\tilde{u}_1$ is equal to the upper bound of $\tilde{u}_2$, i.e. u_{23}=u_{11}. Because of the congested traffic, vehicles need to maintain a safe following distance which is known as clearance. The definition of clearance accepted in our country is that the distance between two moving vehicles must be at least half the value of average speed of the following vehicle in terms of metres. From this definition, clearance $\tilde{c}$ is computed as $\tilde{c}=\frac{\tilde{u}_2}{2}$ metres. To find spacing $\tilde{s}$, we need the average length of vehicle $\tilde{l}$ in terms of metres, and so $\tilde{s}=\tilde{c}+\tilde{l}$ metres.

The number of vehicles per lane on link i is

$$\tilde{n}_i = 1000\frac{\text{Length of link } i}{\text{Spacing}} = 1000\frac{L_i}{\tilde{s}}.$$

If link i has more than one lane, the total number of vehicles on that link is $\tilde{N}_i = w_i\tilde{n}_i$ where w_i is the number of lanes on link i. In the congested traffic, the travel time on link i is computed in terms of minutes as

$$\tilde{T}_i^2 = 60\frac{\text{Length of link } i}{\text{Average speed in high traffic density}} = 60\frac{L_i}{\tilde{u}_2}$$

The maximum flow traversing from specified link i per minutes, which is also called capacity, is

$$\tilde{q}_i = \frac{\text{Total number of vehicles on link } i}{\text{Travel time on link } i \text{ in high traffic density}} = \frac{\tilde{N}_i}{\tilde{T}_i^2}$$

Thus, the fuzzy link travel time function on link i using linear equation is constructed as follows:

$$\tilde{t}_i = \tilde{t}_i(\tilde{x}_i) = \left(\frac{\tilde{T}_i^2 - \tilde{T}_i^1}{\tilde{q}_i}\right)\tilde{x}_i + \tilde{T}_i^1$$

where variables $\tilde{x}_i = (x_{i1}, x_{i2}, x_{i3})$ and $\tilde{t}_i = (t_{i1}, t_{i2}, t_{i3})$ are link flow and link travel time, respectively. These variables will be obtained as triangular fuzzy numbers. Due to the assumption of u_{23}=u_{11} and the equalities of $\tilde{T}_i^1$ and $\tilde{T}_i^2$ given above, the upper bound of $\tilde{T}_i^1$ will be equal to the lower bound of $\tilde{T}_i^2$, and so their difference will be non-negative. Thus, the fuzzy link travel time function is presented as

$$\tilde{t}_i(\tilde{x}_i) = \tilde{\alpha}_i\tilde{x}_i + \tilde{\beta}_i \tag{2}$$

Since parameters $\tilde{\alpha}_i, \tilde{\beta}_i$ and variables $\tilde{x}_i$ are non-negative, the fuzzy link travel time function can be rewritten as

$$\tilde{t}_i(\tilde{x}_i) = (\alpha_{i1}x_{i1} + \beta_{i1},\ \alpha_{i2}x_{i2} + \beta_{i2},\ \alpha_{i3}x_{i3} + \beta_{i3}) \tag{3}$$

These calculations are made for all links over the network and then each fuzzy link travel time function is obtained. The aim is to minimize the objective function of this model which is the summation of multiplication of each fuzzy link travel time function and fuzzy link flow. Therefore, the objective function of SO-FTAP is

$$\mathrm{Min}\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i) \tag{4}$$

and the constraints are

$$\sum_k \tilde{f}_k^{od} = \tilde{F}^{od},\ \ \forall(o,d)\in OD \tag{4a}$$

$$\tilde{x}_i = \sum_k \tilde{f}_k^{od}\delta_{i,k}^{od},\ \ \forall(o,d)\in OD,\ \ \forall i\in I \tag{4b}$$

$$\tilde{c}_k^{od} = \sum_i \tilde{t}_i\delta_{i,k}^{od},\ \ \forall(o,d)\in OD,\ \ \forall k\in K \tag{4c}$$

$$\tilde{f}_k^{od} \geq (0,0,0),\ \ \forall(o,d)\in OD,\ \ \forall k\in K \tag{4d}$$

where $(\tilde{x}_i, \tilde{t}_i, \tilde{f}_k^{od}, \tilde{c}_k^{od})$ are non-negative triangular fuzzy variables, $\tilde{F}^{od}$ is a non-negative triangular fuzzy parameter, and $\delta_{i,k}^{od}$ is a binary parameter. The solution of SO-FTAP (*4*) gives the results on how many vehicles exist on each link and how long the link travel time is in the fuzzy version.

4. SOLUTION METHOD

The method proposed in [20] which deals with the solution of the FFLP problem was adapted. In this paper, we modified the method in [20] to find a solution of SO-FTAP (*4*) which is a Fully Fuzzy Quadratic Programming (FFQP) problem.

Definition 15: The fuzzy feasible solution of *Problem 4* will be $\tilde{X}^* = [\tilde{x}_j^*]_{n\times 1}$ if it satisfies the following characteristics:

i. $\tilde{x}_j^*$ is a non-negative triangular fuzzy number,

ii. $\tilde{x}_j^*$ satisfies all constraints of *Problem 4*.

Problem *4* is non-feasible if it has no feasible solution, i.e. the feasible region is empty.

Definition 16: The fuzzy optimal solution of *Problem 4* will be $\tilde{X}^* = [\tilde{x}_j^*]_{n\times 1}$ if it satisfies the following characteristics:

i. $\tilde{x}_j^*$ is a non-negative triangular fuzzy number,

ii. $\tilde{x}_j^*$ satisfies all constraints of *Problem 4*,

iii. If there exists any non-negative $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ which satisfies all constraints of *Problem 4*, then $\mathrm{R}\left(\sum_i \tilde{x}_i^*\tilde{t}_i(\tilde{x}_i^*)\right) \leq \mathrm{R}\left(\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i)\right)$.

Definition 17: Let $\tilde{X}^* = [\tilde{x}_j^*]_{n\times 1}$ be a fuzzy optimal solution of *Problem 4*. If there exists $\tilde{Y}^* = [\tilde{y}_j^*]_{n\times 1}$ such that

i. $\tilde{y}_j^*$ is a non-negative triangular fuzzy number,

ii. $\tilde{y}_j^*$ satisfies all constraints of *Problem 4*,

iii. $\mathrm{R}\left(\sum_i \tilde{x}_i^*\tilde{t}_i(\tilde{x}_i^*)\right) = \mathrm{R}\left(\sum_i \tilde{y}_i^*\tilde{t}_i(\tilde{y}_i^*)\right)$,

then $\tilde{Y}^* = [\tilde{y}_j^*]_{n\times 1}$ is said to be an alternative fuzzy optimal solution of *Problem 4*.

Definition 18: $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n)$ is said to be an approximate fuzzy optimal solution of *Problem 4* if it satisfies the following conditions:

i. $\tilde{\mathbf{x}}$ is not a feasible fuzzy solution;

ii. for each constraint;

- the lower bound of the triangular fuzzy number in the right-hand side of the constraint is less than or equal to the upper bound of triangular fuzzy number in the left-hand side of the constraint;
- the upper bound of the triangular fuzzy number in the right-hand side of the constraint is greater than or equal to the lower bound of the triangular fuzzy number in the left-hand side of the constraint.

Here, condition (i) implies that the fuzzy number obtained in the left-hand side of at least one equality constraint is not equal to its right-hand side fuzzy number.

It can be observed that the FFLP problem given in [20] has different types (=, ≥ or ≤) of constraints and its objective function is in a linear form, whereas *Problem 4* has only equality constraints and its objective function is non-linear. In [20], after applying the ranking function to the objective function of the FFLP problem, a crisp objective function is obtained and then the optimal solution is found by solving a crisp LP problem. In this paper, a ranking function to non-linear objective function of *4* was applied and then a crisp non-linear objective function was obtained. This crisp QP problem is solved by using GAMS Software and the optimal solution is found. Based on these adaptations, the solution process of SO-FTAP is given below:

Step 1: Using *Problem 4*, construct the SO-FTAP.

Step 2: Because parameter $\tilde{F}^{od}$ and decision variables $\left(\tilde{x}_i,\tilde{t}_i,\tilde{f}_k^{od},\tilde{c}_k^{od}\right)$ are non-negative triangular fuzzy numbers, then SO-FTAP can be written as

$$\operatorname{Min}\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i) \tag{5}$$

subject to

$$\sum_k\left(f_{k1}^{od},f_{k2}^{od},f_{k3}^{od}\right)=\left(F_1^{od},F_2^{od},F_3^{od}\right);\ \ \forall(o,d)\in OD \tag{5a}$$

$$(x_{i1},x_{i2},x_{i3})=\sum_k\left(f_{k1}^{od},f_{k2}^{od},f_{k3}^{od}\right)\delta_{i,k}^{od},\ \ \forall(o,d)\in OD,\forall i\in I \tag{5b}$$

$$\left(c_{k1}^{od},c_{k2}^{od},c_{k3}^{od}\right)=\sum_i(t_{i1},t_{i2},t_{i3})\delta_{i,k}^{od},\forall(o,d)\in OD,\forall k\in K \tag{5c}$$

Step 3: Problem 5 can be converted into the following crisp QP *Problem 6* by using arithmetic operations:

$$\operatorname{Min}\mathrm{R}\left(\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i)\right) \tag{6}$$

subject to

$$\left.\begin{aligned}\sum_k f_{k1}^{od}&=F_1^{od}\\ \sum_k f_{k2}^{od}&=F_2^{od}\\ \sum_k f_{k3}^{od}&=F_3^{od}\end{aligned}\right\}\forall(o,d)\in OD \tag{6a}$$

$$\left.\begin{aligned}x_{i1}&=\sum_k f_{k1}^{od}\delta_{i,k}^{od}\\ x_{i2}&=\sum_k f_{k2}^{od}\delta_{i,k}^{od}\\ x_{i3}&=\sum_k f_{k3}^{od}\delta_{i,k}^{od}\end{aligned}\right\}\forall(o,d)\in OD,\forall i\in I \tag{6b}$$

$$\left.\begin{aligned}c_{k1}^{od}&=\sum_i t_{i1}\delta_{i,k}^{od}\\ c_{k2}^{od}&=\sum_i t_{i2}\delta_{i,k}^{od}\\ c_{k3}^{od}&=\sum_i t_{i3}\delta_{i,k}^{od}\end{aligned}\right\}\forall(o,d)\in OD,\forall k\in K \tag{6c}$$

$$\left.\begin{aligned}f_{k2}^{od}-f_{k1}^{od}&\ge 0\\ f_{k3}^{od}-f_{k2}^{od}&\ge 0\end{aligned}\right\}\forall(o,d)\in OD,\forall k\in K \tag{6d}$$

$$\left.\begin{aligned}x_{i2}-x_{i1}&\ge 0\\ x_{i3}-x_{i2}&\ge 0\end{aligned}\right\}\forall i\in I \tag{6e}$$

$$\left.\begin{aligned}c_{k2}^{od}-c_{k1}^{od}&\ge 0\\ c_{k3}^{od}-c_{k2}^{od}&\ge 0\end{aligned}\right\}\forall(o,d)\in OD,\forall k\in K \tag{6f}$$

where the ranking function of *Objective function 6* is constructed as

$$\begin{aligned}&\mathrm{R}\left(\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i)\right)=\mathrm{R}\left(\sum_i\left(\tilde{\alpha}_i,\tilde{x}_i^2+\tilde{\beta}_i\tilde{x}_i\right)\right)=\\ &\mathrm{R}\left(\sum_i\left(\alpha_{i1},x_{i1}^2+\beta_{i1}x_{i1},\alpha_{i2},x_{i2}^2+\beta_{i2}x_{i2},\alpha_{i3},x_{i3}^2+\beta_{i3}x_{i3}\right)\right)=\\ &\sum_i\frac{\alpha_{i1},x_{i1}^2+\beta_{i1}x_{i1}+2\left(\alpha_{i2},x_{i2}^2+\beta_{i2}x_{i2}\right)+\alpha_{i3},x_{i3}^2+\beta_{i3}x_{i3}}{4}\end{aligned} \tag{7}$$

Step 4: If there is an optimal solution x_{i1}, x_{i2} and x_{i3} of *Problem 6*, then find the fuzzy optimal solution by substituting the values of x_{i1}, x_{i2} and x_{i3} in $\tilde{x}_i=(x_{i1},x_{i2},x_{i3})$ and determine the fuzzy optimal value by substituting $\tilde{x}_i$ in the *objective function 4* and STOP. If *Problem 6* does not have any feasible solution, then continue.

Step 5: Rearrange each constraint of *Problem 4* by introducing new non-negative triangular fuzzy variables $\tilde{S}_r$ to the left side, and $\tilde{S}_r^{'}$ to the right side, respectively:

$$\operatorname{Min}\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i) \tag{8}$$

subject to

$$\sum_k \tilde{f}_k^{od}+\tilde{S}_r=\tilde{F}^{od}+\tilde{S}_r^{'}\ \ \forall(o,d)\in OD,\ \ r=1,\dots,\eta \tag{8a}$$

$$\begin{aligned}&\tilde{x}_i+\tilde{S}_r=\sum_k \tilde{f}_{k1}^{od}\delta_{i,k}^{od}+\tilde{S}_r^{'}\\ &\forall(o,d)\in OD,\ \ \forall i\in I,\ \ r=\eta+1,\dots,\mu\end{aligned} \tag{8b}$$

$$\begin{aligned}&\tilde{c}_k^{od}+\tilde{S}_r=\sum_i \tilde{t}_i\delta_{i,k}^{od}+\tilde{S}_r^{'}\\ &\forall(o,d)\in OD,\ \ \forall k\in K,\ \ r=\mu+1,\dots,\theta\end{aligned} \tag{8c}$$

Note that *Problem 8* has θ constraints for one pair of (o,d).

Step 6: Because parameter $\tilde{F}^{od}$ and decision variables $\left(\tilde{x}_i,\tilde{t}_i,\tilde{f}_k^{od},\tilde{c}_k^{od}\right)$ are non-negative triangular fuzzy numbers, write *Problem 8* as

$$\operatorname{Min}\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i) \tag{9}$$

subject to

$$\begin{aligned}&\sum_k\left(f_{k1}^{od},f_{k2}^{od},f_{k3}^{od}\right)+(S_{r1},S_{r2},S_{r3})=\left(F_1^{od},F_2^{od},F_3^{od}\right)\\ &+\left(S_{r1}^{'},S_{r2}^{'},S_{r3}^{'}\right)\quad\forall(o,d)\in OD,\ \ r=1,\dots,\eta\end{aligned} \tag{9a}$$

$$\begin{aligned}&(x_{i1},x_{i2},x_{i3})+(S_{r1},S_{r2},S_{r3})=\sum_k\left(f_{k1}^{od},f_{k2}^{od},f_{k3}^{od}\right)\delta_{i,k}^{od}\\ &+\left(\tilde{S}_{r1}^{'},\tilde{S}_{r2}^{'},\tilde{S}_{r3}^{'}\right)\ \forall(o,d)\in OD,\ \forall i\in I,\ r=\eta+1,\dots,\mu\end{aligned} \tag{9b}$$

$$\begin{aligned}&\left(c_{k1}^{od},c_{k2}^{od},c_{k3}^{od}\right)+(S_{r1},S_{r2},S_{r3})=\sum_i(t_{i1},t_{i2},t_{i3})\delta_{i,k}^{od}\\ &+\left(\tilde{S}_{r1}^{'},\tilde{S}_{r2}^{'},\tilde{S}_{r3}^{'}\right)\ \forall(o,d)\in OD,\ \ \forall k\in K,\ \ r=\mu+1,\dots,\theta\end{aligned} \tag{9c}$$

Step 7: Convert *Problem 9* into the following crisp QP problem by using arithmetic operations:

$$\operatorname{Min}\left\{\mathrm{R}\left(\sum_i \tilde{x}_i\tilde{t}_i(\tilde{x}_i)\right)+M\left[\mathrm{R}\left(\sum_{r=1}^{\theta}\left(\tilde{S}_r+\tilde{S}_r^{'}\right)\right)\right]\right\} \tag{10}$$

subject to

$$\left.\begin{aligned}\sum_k f_{k1}^{od}+S_{r1}&=F_1^{od}+\tilde{S}'_{r1}\\ \sum_k f_{k2}^{od}+S_{r2}&=F_2^{od}+\tilde{S}'_{r2}\\ \sum_k f_{k3}^{od}+S_{r3}&=F_3^{od}+\tilde{S}'_{r3}\end{aligned}\right\}\forall(o,d)\in OD,\ r=1,\dots,\eta \tag{10a}$$

$$\left.\begin{aligned}x_{i1}+S_{r1}&=\sum_k f_{k1}^{od}\delta_{i,k}^{od}+\tilde{S}'_{r1}\\ x_{i2}+S_{r2}&=\sum_k f_{k2}^{od}\delta_{i,k}^{od}+\tilde{S}'_{r2}\\ x_{i3}+S_{r3}&=\sum_k f_{k3}^{od}\delta_{i,k}^{od}+\tilde{S}'_{r3}\end{aligned}\right\}\begin{aligned}&\forall(o,d)\in OD,\ \forall i\in I,\\ &r=\eta+1,\dots,\mu\end{aligned} \tag{10b}$$

$$\left.\begin{aligned}c_{k1}^{od}+S_{r1}&=\sum_i t_{i1}\delta_{i,k}^{od}+\tilde{S}'_{r1}\\ c_{k2}^{od}+S_{r2}&=\sum_i t_{i2}\delta_{i,k}^{od}+\tilde{S}'_{r2}\\ c_{k3}^{od}+S_{r3}&=\sum_i t_{i3}\delta_{i,k}^{od}+\tilde{S}'_{r3}\end{aligned}\right\}\begin{aligned}&\forall(o,d)\in OD,\ \forall k\in K,\\ &r=\mu+1,\dots,\theta\end{aligned} \tag{10c}$$

$$\left.\begin{aligned}f_{k2}^{od}-f_{k1}^{od}&\geq 0\\ f_{k3}^{od}-f_{k2}^{od}&\geq 0\end{aligned}\right\}\forall(o,d)\in OD,\ \forall k\in K \tag{10d}$$

$$\left.\begin{aligned}x_{i2}-x_{i1}&\geq 0\\ x_{i3}-x_{i2}&\geq 0\end{aligned}\right\}\forall i\in I \tag{10e}$$

$$\left.\begin{aligned}c_{k2}^{od}-c_{k1}^{od}&\geq 0\\ c_{k3}^{od}-c_{k2}^{od}&\geq 0\end{aligned}\right\}\forall(o,d)\in OD,\ \forall k\in K \tag{10f}$$

$$\left.\begin{aligned}S_{r2}-S_{r1}\geq 0,&\ S_{r3}-S_{r2}\geq 0\\ \tilde{S}'_{r2}-\tilde{S}'_{r1}\geq 0,&\ \tilde{S}'_{r3}-\tilde{S}'_{r2}\geq 0\end{aligned}\right\}r=1,\dots,\theta \tag{10g}$$

where M is a sufficiently positive large constant and the ranking function of the objective function is given in 7. Here, the reason for taking a sufficiently large constant M is to prevent new defined variables $\tilde{S}_r,\tilde{S}'_r\,(r=1,\dots,\theta)$ from being non-zero.

Step 8: Find the optimal solution x_{i1}, x_{i2} and x_{i3} by solving *Problem 10*, and find the approximate fuzzy optimal solution by substituting the values of x_{i1}, x_{i2} and x_{i3} in $\tilde{x}_i=(x_{i1},x_{i2},x_{i3})$. Determine the fuzzy approximate optimal solution by substituting $\tilde{x}_i$ in the *objective function 4*.

5. NUMERICAL EXPERIMENT

This Section provides a numerical experiment which is taken from [23] to show the proposed solution procedure of SO-FTAP.

In this paper, the origin and destination points of the specified network are considered as Ataturk Airport (Node A) and Atasehir Junction (Node B), respectively. Ataturk Airport is located on the European land whereas Atasehir Junction is on the Asian land. Intermediate nodes of the network are Okmeydani Junction (Node C) and Hasdal Junction (Node D), and these nodes are also on the European land. *Figure 2* shows these nodes on the map taken from Google Maps.

Figure 2 – Illustration of Node A, Node C, Node D and Node B on the map [23]

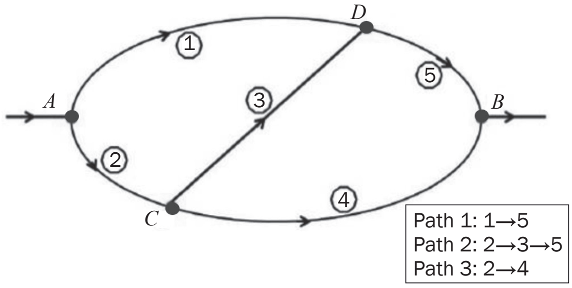

Figure 3 – The specified network

Using *Figure 2*, the network representation of our experiment is given in *Figure 3*, and the data are presented in *Table 1*.

Table 1 – Data of the specified network

Link	1	2	3	4	5
Nodes	AD	AC	CD	CB	DB
Length of link [km]	23.3	15.3	5	19.3	22.7
Number of lanes	2	3	2	3	4

In this paper, it is assumed that in the case of low traffic density, the average high speed is $\tilde{u}_1=(50,60,70)$ km/h; and in the case of high traffic density, the average low speed is $\tilde{u}_2=(30,40,50)$ km/h. Furthermore, the average length of a vehicle is taken as $\tilde{l}=(5,6,7)$ metres. Using $\tilde{u}_2$, the clearance and spacing are determined as $\tilde{c}=(15,20,25)$ and $\tilde{s}=(20,26,32)$ metres, respectively. Utilizing these parameters, other parameters are calculated which are given in *Table 2*. Also, the fuzzy inflow is taken as $\tilde{F}=(100,125,150)$ vehicles.

Since $\tilde{t}_i$ and $\tilde{x}_i$ are triangular fuzzy variables, fuzzy link travel time functions are formulated as

$$\tilde{t}_1(\tilde{x}_1)=(0,0.23,0.85)\tilde{x}_1+(19.97,23.3,27.96),$$
$$\tilde{t}_2(\tilde{x}_2)=(0,0.1,0.37)\tilde{x}_2+(13.11,15.3,18.36),$$
$$\tilde{t}_3(\tilde{x}_3)=(0,0.05,0.18)\tilde{x}_3+(4.29,5,6),$$
$$\tilde{t}_4(\tilde{x}_4)=(0,0.13,0.47)\tilde{x}_4+(16.54,19.3,23.16),$$

and

$$\tilde{t}_5(\tilde{x}_5)=(0,0.11,0.42)\tilde{x}_5+(19.46,22.7,27.24).$$

Table 2 – Fuzzy parameters for constructing fuzzy link travel time functions

Link	$\tilde{T}_i^1$ [min]	$\tilde{T}_i^2$ [min]	$\tilde{q}_i$ [veh]
1	(19.97, 23.3, 27.96)	(27.96, 34.95, 46.6)	(31.25, 51.28, 83.33)
2	(13.11, 15.3, 18.36)	(18.36, 22.95, 30.6)	(46.88, 76.92, 125)
3	(4.29, 5, 6)	(6, 7.5, 10)	(31.25, 51.28, 83.33)
4	(16.54, 19.3, 23.16)	(23.16, 28.95, 38.6)	(46.88, 76.92, 125)
5	(19.46, 22.7, 27.24)	(27.24, 34.05, 45.4)	(62.5, 102.56, 166.67)

The mathematical model of this experiment is

$$\text{Min} \sum_{i=1}^{5} \tilde{x}_i \tilde{t}_i(\tilde{x}_i)$$

subject to

$$\sum_{k=1}^{3} \tilde{f}_k = (100, 125, 150)$$

$$\tilde{x}_i = \sum_{k=1}^{3} \tilde{f}_k \delta_{i,k}, \quad i = 1,2,3,4,5$$

$$\tilde{c}_k = \sum_{i=1}^{5} \tilde{t}_i \delta_{ik}, \quad k = 1,2,3$$

$$\tilde{f}_k \geq (0,0,0), \quad k = 1,2,3$$

To solve the numerical experiment, we adapted the proposed method given in Section 4.

Steps 1-2:

$$\text{Min} \sum_{i=1}^{5} (x_{i1}, x_{i2}, x_{i3})(t_{i1}, t_{i2}, t_{i3})$$

subject to

$$(f_{11}, f_{12}, f_{13}) + (f_{21}, f_{22}, f_{23}) + (f_{31}, f_{32}, f_{33}) = (100, 125, 150),$$
$$(x_{11}, x_{12}, x_{13}) = (f_{11}, f_{12}, f_{13}),$$
$$(x_{21}, x_{22}, x_{23}) = (f_{21}, f_{22}, f_{23}) + (f_{31}, f_{32}, f_{33}),$$
$$(x_{31}, x_{32}, x_{33}) = (f_{21}, f_{22}, f_{23}), (x_{41}, x_{42}, x_{43}) = (f_{31}, f_{32}, f_{33}),$$
$$(x_{51}, x_{52}, x_{53}) = (f_{11}, f_{12}, f_{13}) + (f_{21}, f_{22}, f_{23}),$$
$$(c_{11}, c_{12}, c_{13}) = (t_{11}, t_{12}, t_{13}) + (t_{51}, t_{52}, t_{53}),$$
$$(c_{21}, c_{22}, c_{23}) = (t_{21}, t_{22}, t_{23}) + (t_{31}, t_{32}, t_{33}) + (t_{51}, t_{52}, t_{53}),$$
$$(c_{31}, c_{32}, c_{33}) = (t_{21}, t_{22}, t_{23}) + (t_{41}, t_{42}, t_{43})$$

Steps 3-4:

$$\text{Min R}\left(\sum_{i=1}^{5} (x_{i1}, x_{i2}, x_{i3})(t_{i1}, t_{i2}, t_{i3})\right)$$

subject to

$$f_{11} + f_{21} + f_{31} = 100, \quad f_{12} + f_{22} + f_{32} = 125,$$
$$f_{13} + f_{23} + f_{33} = 150, \quad x_{11} = f_{11}, \quad x_{12} = f_{12},$$
$$x_{13} = f_{13}, \quad x_{21} = f_{21} + f_{31}, \quad x_{22} = f_{22} + f_{32},$$
$$x_{23} = f_{23} + f_{33}, \quad x_{31} = f_{21}, \quad x_{32} = f_{22}, \quad x_{33} = f_{23},$$
$$x_{41} = f_{31}, \quad x_{42} = f_{32}, \quad x_{43} = f_{33}, \quad x_{51} = f_{11} + f_{21},$$
$$x_{52} = f_{12} + f_{22}, \quad x_{53} = f_{13} + f_{23},$$
$$c_{11} = t_{11} + t_{51}, \quad c_{12} = t_{12} + t_{52}, \quad c_{13} = t_{13} + t_{53},$$
$$c_{21} = t_{21} + t_{31} + t_{51}, \quad c_{22} = t_{22} + t_{32} + t_{52}, \quad c_{23} = t_{23} + t_{33} + t_{53},$$
$$c_{31} = t_{21} + t_{41}, \quad c_{32} = t_{22} + t_{42}, \quad c_{33} = t_{23} + t_{43},$$
$$f_{12} - f_{11} \geq 0, \quad f_{13} - f_{12} \geq 0, \quad f_{22} - f_{21} \geq 0, \quad f_{23} - f_{22} \geq 0,$$
$$f_{32} - f_{31} \geq 0, \quad f_{33} - f_{32} \geq 0,$$
$$x_{12} - x_{11} \geq 0, \quad x_{13} - x_{12} \geq 0, \quad x_{22} - x_{21} \geq 0, \quad x_{23} - x_{22} \geq 0,$$
$$x_{32} - x_{31} \geq 0, \quad x_{33} - x_{32} \geq 0, \quad x_{42} - x_{41} \geq 0, \quad x_{43} - x_{42} \geq 0,$$
$$x_{52} - x_{51} \geq 0, \quad x_{53} - x_{52} \geq 0,$$
$$c_{12} - c_{11} \geq 0, \quad c_{13} - c_{12} \geq 0, \quad c_{22} - c_{21} \geq 0, \quad c_{23} - c_{22} \geq 0,$$
$$c_{32} - c_{31} \geq 0, \quad c_{33} - c_{32} \geq 0$$

By solving this constructed crisp QP problem, the fuzzy optimal solution is found, which is given in *Table 3*.

It is observed from the solution that the assignment of vehicles changes with the increment of inflow. When the inflow is between 100 and 125, the usage of Link 2, Link 3 and Link 4 does not change; it remains about 92, 10 and 81 vehicles, respectively, and other vehicles are assigned to Link 1 and Link 5. When the inflow increases and becomes between 125 and 150, the usages of all links except Link 4 increases. These vehicles are assigned to minimize the total system travel time.

It is also seen that when there is a low traffic density on the specified network, the drivers traverse each link in the free flow travel time notwithstanding vehicle length. These free flow travel time values correspond to the left bounds of the link travel times of the solution obtained. For example, to traverse Link 2, the drivers spend at least 13.11 minutes on that link. Accordingly, in the case of low traffic density, the drivers use Path 3 in which Link 2 and Link 4 exist, and the arrival time from Ataturk Airport to Atasehir Junction

Table 3 – The fuzzy optimal solution

Link	Fuzzy link flows [veh]	Fuzzy link travel times [min]	Path	Fuzzy path travel times [min]
1	(7.916, 32.916, 47.102)	(19.97, 30.871, 67.997)	1	(39.43, 58.302, 123.803)
2	(92.084, 92.084, 92.084)	(13.11, 24.508, 56.432)	2	(36.86, 57.445, 122.003)
3	(10.099, 10.099, 20.913)	(4.29, 5.505, 9.764)	3	(29.65, 54.466, 118.125)
4	(81.985, 81.985, 81.985)	(16.54, 29.958, 61.693)		
5	(18.015, 43.015, 68.015)	(19.46, 27.432, 55.806)		

is minimum 29.65 minutes. On the contrary, if high traffic density is observed, the drivers spend much more time on the traffic, which is the maximum travel time. Right bounds of the link travel times of the solution obtained refer to the maximum travel time. In the case of congestion, the drivers use Path 1 in which Link 1 and Link 5 exist, and the arrival time is 123.803 minutes (about 2 hours).

6. CONCLUSION

This paper considers the SO-TAP which has been extensively studied in the literature. Since the data in real life problems are not exact and stable, the problem of SO-FTAP was dealt with in a more realistic way by taking the parameters as fuzzy numbers. Thus, the focus was on solving SO-FTAP by minimizing the total system travel time. All the parameters of the problem are assumed as triangular fuzzy numbers. The fuzzy link travel time function is constructed in terms of flow and defined as linear. Modifying the proposed method in [20], a solution method is presented that converts the SO-FTAP to a QP problem and this method is illustrated with a real life experiment taken from the literature. The results show that the method has the ability to generate a fuzzy optimal solution or an approximate fuzzy optimal solution. It is concluded that the left and right bounds of the solution obtained, which is a triangular fuzzy number, refer to the free flow travel time and the maximum travel time traversing on the specified link, respectively. As future work, the fuzzy analysis of TAP can be extended by taking short time-intervals, which is more complex.

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SİSTEM OPTİMUM BULANIK TRAFİK ATAMA PROBLEMİ

ÖZET

Bu çalışmada, Sistem Optimum Trafik Atama Problemi'nin (SO-TAP) Sistem Optimum Bulanık Trafik Atama Problemi'ne (SO-BTAP) dönüştürülmesi üzerine odaklanılmıştır. SO-TAP, belirtilen başlangıç-bitiş noktaları arasında yol ağı üzerindeki toplam sistem yolculuk süresini en aza indirmeyi amaçlamaktadır. Bağlantı yolculuk süresi bulanık bağlantı akışına bağlı doğrusal bir fonksiyon olarak alınmıştır. Her bir bağlantı yolculuk süresi üçgensel bulanık sayı olarak yapılandırılmıştır. Model, akış korunumu kısıtları altında olup her bir bağlantı akışı ile o bağlantıya ait yolculuk süresi fonksiyonunun çarpımlarının toplamlarından oluşan doğrusal olmayan bir amaç fonksiyonuna sahiptir. Ele alınan problemin parametreleri bağlantı yolu uzunluğu, bağlantıdaki şerit sayısı, araç ortalama hızı ve araç uzunluğudur. Bu parametreler ile hesaplanan diğer parametreler ise iki araç arası boşluk, takip mesafesi, bağlantı kapasitesi ve serbest yolculuk süresidir. Bir yol ağı göz önüne alınarak bağlantı yolu uzunlukları ve şerit sayısı bulanık olmayan sayı olarak alınmıştır. Bir aracın ortalama hızı ve araç uzunluğu belirli bir değere sahip olmadıklarından bu parametreler üçgensel bulanık sayılar olarak ele alınmıştır. Böylece, bilinen SO-TAP, bulanık kuadratik (doğrusal olmayan) bir programlama problemine dönüştürülmüştür. Bu yeni bulanık problem, SO-BTAP, literatürde var olan bir çözüm yönteminin uyarlanması ile çözülmüştür ve bu çözüm bir örnek üzerinde gösterilmiştir.

ANAHTAR KELİMELER

Bulanık trafik atama problemi; ortalama hız; araç uzunluğu; bağlantı kapasitesi; yolculuk süresi;

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