# Sliding Mode Controller for Unstable Systems

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The method proposed by Rojas et al.[1] for the design of sliding mode controllers (SMC) for unstable first order plus time delay systems, is extended for delay-time constant ratio ($\varepsilon$) up to 1.8. The SMC settings obtained for various $\varepsilon$ are fitted by simple equations. Up to $\varepsilon$ = 1.2, the method is found to be more robust than that of latest PID Controller proposed by Padmasree *et al.*[2] There is no method available in literature to stabilize unstable systems using PID controller for $\varepsilon$ > 1.2. Simulation results are also given for a nonlinear bioreactor control problem.

*Key words:*
Sliding Mode Control, First Order plus Time Delay Systems (FOPDT), Unstable Systems

## Introduction

Sliding Mode Control (**SMC**) is a robust and simple procedure[3] to synthesize controllers for linear and nonlinear processes. Conventional controllers, such as **PID,** lead-lag or Smith predictors, are sometimes insufficiently versatile to compensate for the uncertainties in the process parameters. A **SMC** could be designed to control linear and nonlinear systems with the assumption that the robustness of the controller will take care of the variations in process parameters due to noise and other disturbances.

The aim of this paper is to design a **SMC** for unstable first order plus delay time (**FOPDT**) processes where $\varepsilon > 1$. A method for tuning **SMC** for open loop unstable processes has been presented by Rojas *et al.*[1] However the method has some inherent disadvantages. The method does not work for $\varepsilon > 1$. So the method is modified to develop tuning formulae for **SMC** for values of $\varepsilon > 1$. The performance of the closed loop system is compared with that of latest PID Controller settings proposed by Padmasree *et al.*[2]

The paper is organized as follows. Section 2 briefly presents some basic concepts about Sliding Mode Control. Section 3 gives the SMC proposed by Rojas *et al.*[1] and its limitations. Section 4 describes the procedure to overcome the limitation. Tuning equations for the controller are also given in this section. Section 5 shows the simulation studies to compare the resulting SMC performance with that of the latest PID controller proposed by Padmasree *et al.*[2] Section 6 shows the application of SMC to an unstable bioreactor. Finally the conclusions are presented.

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## Basic concepts about sliding mode control

The idea behind **SMC** is to define a surface along which the process can slide to its desired final value. Thus the first step in **SMC** is to define the sliding surface $S(t)$. The $S(t)$ selected in this work[4] is an integral-differential equation acting on the tracking error expression.

$$S(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)^n \int e(t)\,\mathrm{d}t \tag{1}$$

where $e(t)$ is the tracking error, $\lambda$ is a tuning parameter, which helps to define $S(t)$; this term is selected by the designer, and determines the performance of the system on the sliding surface, $n$ is the system order.

Once the reference value is reached, eq. (1) indicates that $S(t)$ at steady state reaches a constant value. To maintain $S(t)$ at this constant value, meaning that $e(t)$ has to be zero at all times, it is desired to make

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = 0 \tag{2}$$

Once the sliding surface has been selected, attention must be turned to design the control law that drives the controlled variable to its reference value and satisfies eq. (2). The **SMC** control law, $U(t)$ consists of two additive parts; a continuous part, $U_C(t)$, and a discontinuous part, $U_D(t)$:

$$U(t) = U_C(t) + U_D(t) \tag{3}$$

The continuous part is given by

$$U_C(t) = f(X(t), R(t)) \tag{4}$$

Where $f(X(t), R(t))$ is a function of the controlled variable, and the reference value.

The discontinuous part, $U_D(t)$, incorporates a nonlinear element that includes the switching element of the control law. This part of the controller is discontinuous across the sliding surface:

$$U_D(t) = K_D \frac{S(t)}{S(t)+\delta} \qquad (5)$$

where $K_D$ is a tuning parameter responsible for the reaching mode, $\delta$ is a tuning parameter used to reduce the chattering problem. Chattering is a high-frequency oscillation around the desired equilibrium point.

## SMC proposed by Rojas *et al.*

Rojas *et al.*[1] have proposed a method for sliding mode control for FOPDT open loop unstable processes, whose open loop transfer function is given by $K_p \frac{e^{-t_0 s}}{\tau s - 1}$.

The control law is given by

$$U(t) = \frac{t_0\tau}{K}\left[\left(\frac{-t_0+\tau}{t_0\tau} - \lambda_1\right)\frac{dX(t)}{dt} - \frac{X(t)}{t_0\tau} + \lambda_0 e(t)\right] + + K_D \frac{S(t)}{S(t)+\delta} \qquad (6a)$$

With

$$S(t) = \operatorname{sign}(K)\left[\frac{de(t)}{dt} + \lambda_1 e(t) + \lambda_0 \int e(t)dt\right] \qquad (6b)$$

By taking

$$\lambda_1 = \frac{-t_0+\tau}{t_0\tau} \qquad (7a)$$

we can simplify $U_C$ as

$$U(t) = \frac{t_0\tau}{K_p}\left[-\frac{X(t)}{t_0\tau} + \lambda_0 e(t)\right] + K_D \frac{S(t)}{S(t)+\delta}$$

To assure that the sliding surfaces behave as a critical or overdamped system, $\lambda_0$ should be

$$\lambda_0 \le \frac{\lambda_1^2}{4} \qquad (7b)$$

Using the Nelder-Mead optimization algorithm, Rojas *et al.*[1] have selected the values of $K_D$ and $\delta$ by minimizing ISE values and have proposed the following tuning equations for the SMC tuning parameters:

$$K_P K_D = 0.8 \cdot \left(\frac{\tau}{t_0}\right)^{0.76}; \qquad (7c)$$

$$\delta = 0.68 + 0.12 \cdot K_P \cdot K_D \cdot \lambda_1. \qquad (7d)$$

Eq. (7a) will cause problems if $\varepsilon = \frac{t_0}{\tau} > 1$, since this will make $\lambda_1$ negative which results in instability on the surface.

## Extension for $\varepsilon$ from 1 to 1.8

We make the term $\frac{-t_0+\tau}{t_0\tau} - \lambda_1$ negative, since this will make $\lambda_1$ positive, which will guarantee stability on the surface. In this work, we make $\frac{-t_0+\tau}{t_0\tau} - \lambda_1 = -1$ and calculate $\lambda_1$ accordingly for various values of $\varepsilon$. The SMC parameters, namely $K_D$ and $\delta$ are estimated by minimizing ISE using matlab least squares method. Up to $\varepsilon = 1$, the tuning formulae given in eqs. (7c) and (7d) are used as the initial guesses. For $\varepsilon > 1$, the tuning formulae for stable processes given by Camacho and Smith[5] are used as initial guesses. The converged parameter values are given in Table 1. Fig. 1 shows the variation of the SMC parameters $K_D$ and $\delta$ with $\varepsilon$. The values obtained for $K_D$ and $\delta$ are fitted by the following simple equations as:

For $0 < \varepsilon < 0.8$:

$$K_P K_D = 1.3889 \cdot \left(\frac{\tau}{t_0}\right)^{0.33}; \qquad (8a)$$

For $0.8 \le \varepsilon \le 1$:

$$K_P K_D = 4.9424 \cdot \left(\frac{\tau}{t_0}\right) - 4.6829 \qquad (8b)$$

For $1 < \varepsilon \le 1.8$:

$$K_P K_D = 0.3531 \cdot \left(\frac{\tau}{t_0}\right)^{2.7535}; \qquad (8c)$$

For $\varepsilon \le 1$:

$$\delta = 2.0494 + 0.2160 \cdot K_P \cdot K_D \cdot \lambda_1. \qquad (9a)$$

For $1 < \varepsilon < 1.5$

$$\delta = 3.2192 - 11.016 \cdot K_P \cdot K_D \cdot \lambda_1 \qquad (9b)$$

For $1.5 \le \varepsilon \le 1.8$

$$\delta = 14. \qquad (9c)$$

The maximum error in the fitted equations is less than 10 %.

Table 1 – *Values obtained for $K_D$ and $\delta$ by optimization method for various values of $\varepsilon$*

| Sl. No. | $\varepsilon$ | $K_D$ | $\delta$ |
|---|---|---|---|
| 1 | 0.1 | 2.9696 | 7.8223 |
| 2 | 0.2 | 2.3624 | 4.0905 |
| 3 | 0.3 | 2.0666 | 3.0910 |
| 4 | 0.4 | 1.8794 | 2.6583 |
| 5 | 0.5 | 1.7460 | 2.4265 |
| 6 | 0.6 | 1.6440 | 2.2861 |
| 7 | 0.7 | 1.5625 | 2.1940 |
| 8 | 0.8 | 1.4951 | 2.1301 |
| 9 | 0.9 | 1.4381 | 2.0839 |
| 10 | 1.0 | 0.2595 | 1.7847 |
| 11 | 1.1 | 0.2399 | 2.1366 |
| 12 | 1.2 | 0.2233 | 2.4032 |
| 13 | 1.3 | 0.2091 | 2.5987 |
| 14 | 1.4 | 0.1967 | 2.7531 |
| 15 | 1.5 | 0.1005 | 14.3531 |
| 16 | 1.6 | 0.0746 | 14.1818 |
| 17 | 1.7 | 0.0565 | 13.9817 |
| 18 | 1.8 | 0.0434 | 13.5619 |

## Simulation results

**Example 1:** Let us consider an unstable FOPDT model with $K_p = 1$, $\tau = 1$ and $t_0 = 0.8$. The SMC parameter settings are $\lambda_1 = 0.25$ min$^{-1}$, $\lambda_0 = 0.015625$ min$^{-2}$, $K_D = 1.4951$, $\delta = 2.1301$ (Table 1). The PID settings given by Padmasree *et al.*[2] are $K_C = 1.5694$, $\tau_I = 7.5336$, $\tau_D = 0.4087$. The SMC settings for the method proposed by Rojas *et al.*[1] are $\lambda_1 = 0.25$ min$^{-1}$, $\lambda_0 = 0.015625$ min$^{-2}$, $K_D = 0.9479$, $\delta = 0.7084$. Fig. 2 shows the comparisons of the servo response of all the three methods. The present method is found to be better than the other two. Under perfect parameter conditions, overshoot is lesser for the present method. The robustness of the controller is evaluated by perturbing the gain as 1.2 in the process whereas the controller settings used are for $K_p = 1$. It is found that the PID controller[2] and SMC[1] take a long time to settle and also show high oscillations (Fig. 2b). Similar robust performances are also obtained for uncertainty in time constant and time delay. It is found that the PID

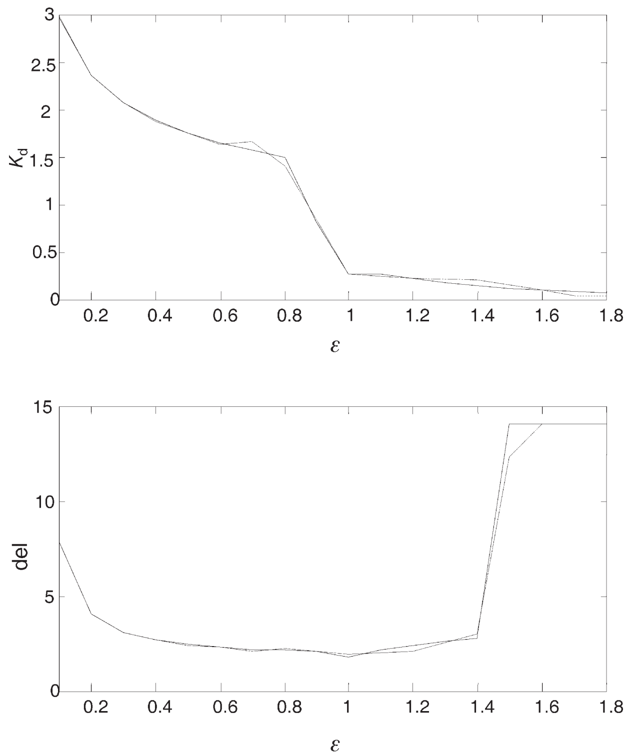


Fig. 1 – *Variation of $K_D$ and $\delta$ with $\varepsilon$. Solid line: Fitted value using eqs. 8a, 8b, 9a, 9b, 10a, 10b. Dotted: actual value*

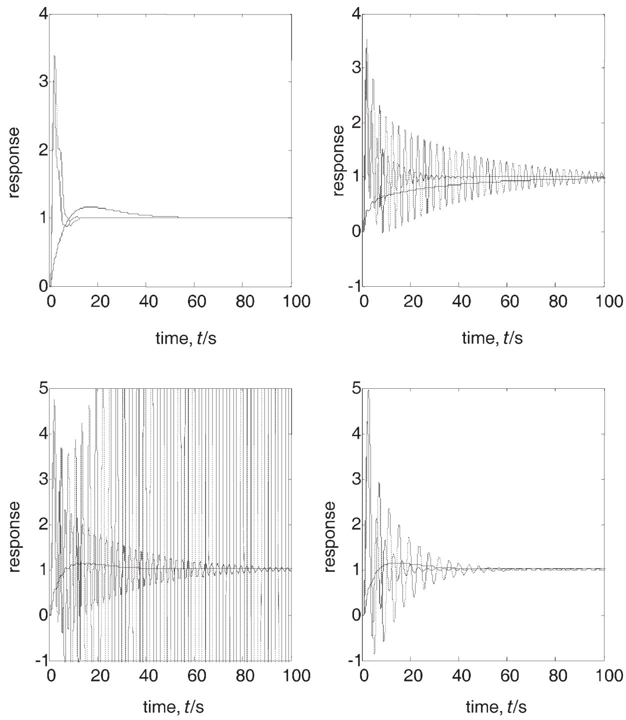


Fig. 2 – *Comparison of servo responses for unstable systems: $K_p = 1$; $\tau = 1$; $t_0 = 0.8$ with PID Controller, SMC controller proposed by Rojas et al.*[1] *Solid Line: Present Method. Dotted: PID, Dashed: SMC proposed by Rojas et al.*[1] *(a) Perfect parameter, (b) Uncertainty of +20 % in $K_p$, (c) Uncertainty of −20 % in $\tau$, (d) Uncertainty of +20 % in $t_0$.*

Controller is unable to stabilize the system with respect to uncertainties in time constant, whereas in the case of time delay, it has a huge overshoot (Figs. 2c and 2d). The performances under model parameter uncertainty are better for the present method. Fig. 3 shows the comparison of the manipulated variables vs. time for the controllers. From the plot, it is obvious that the SMC Controller output of the present method is much smoother than that of the PID Controller and the SMC proposed by Rojas *et al.*[1] which cause highly oscillatory movement of the control valve.

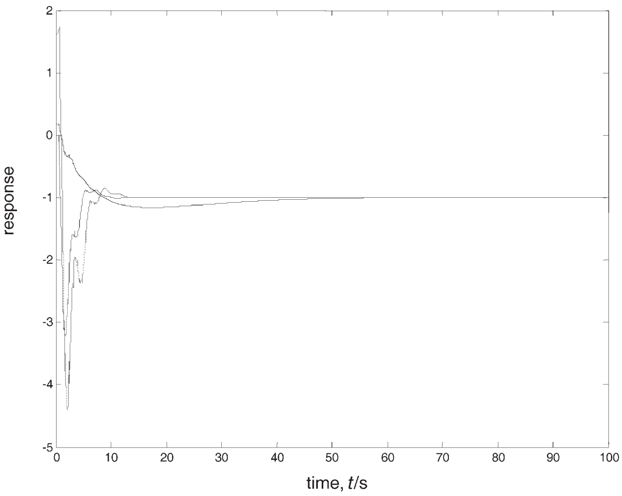


Fig. 3 – *Comparison of manipulated variables. Legend: Same as in Fig. 2*

**Example 2:** Let us consider an unstable FOPDT model with $K_P = 1$, $\tau = 1$ and $t_0 = 1.2$. The SMC parameter settings by the present method are $\lambda_1 = 1/3$ min$^{-1}$, $\lambda_0 = 1/36$ min$^{-2}$, $K_D = 0.2233$, $\delta = 2.4032$ (Table 1). For performance comparison, we use the PID settings given by Padmasree *et al.*[2] The SMC proposed by Rojas *et al.*[1] will not work for $\varepsilon > 1$. Hence we compare the present method with the latest PID controller proposed by Padmasree *et al.*[2] The controller settings given by Padmasree *et al.*[2] are $K_C = 1.2439$, $\tau_I = 23.8115$, $\tau_D = 0.6087$. Fig. 4 shows the servo response for these settings. Regarding perturbations, the PID controller does not stabilize even a 5 % change in $K_p$ whereas the present SMC stabilizes the process. The response is better than that of the PID method in all the cases as can be seen from the figure. The same holds good for the controller output also as shown in Fig. 5.

**Example 3:** The performance of the proposed method is evaluated on the system $e^{-1.4\,s}/(s - 1)$. We obtain the SMC settings for the present method as $\lambda_1 = 3/14$, $\lambda_0 = 9/784$, $K_D = 0.1967$ and $\delta = 2.7531$ (Table 1). Since $\varepsilon > 1.2$, the PID controller fails to

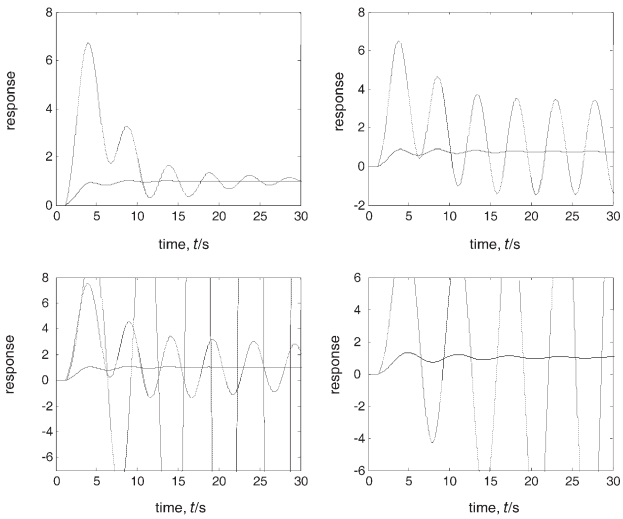


Fig. 4 – *Comparison of servo responses for unstable systems: $K_p=1$; $\tau = 1$; $t_0 = 1.2$ with PID Controller. Solid Line: Present Method. Dotted: PID. (a) Perfect parameter, (b) Uncertainty of +5 % in $K_p$, (c) Uncertainty of −20 % in $\tau$, (d) Uncertainty of +20 % in $t_0$,*

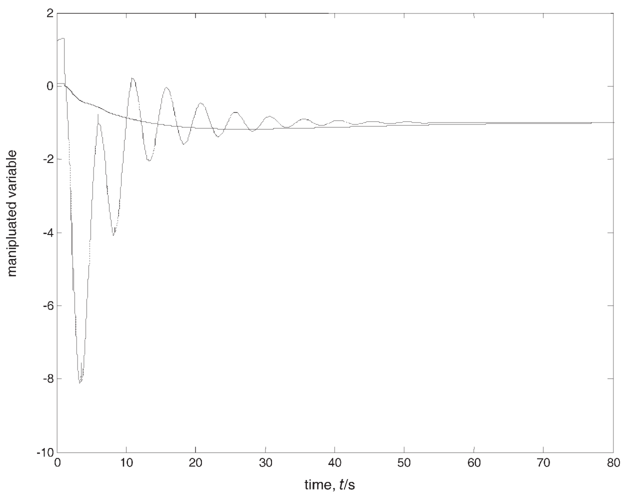


Fig. 5 – *Comparison of controller outputs under perfect parameters condition. Legend: Same as in Fig. 4*

stabilize the system. Fig. 6 shows the performance of the SMC under perfect parameter conditions and variations in the process parameters. The performance of the SMC is found to be very good except for variation in process gain where it takes a lot of time to settle. The manipulated variable vs. time plot under conditions of perfect parameter is given in Fig. 7.

**Example 4:** Let us consider the process $\exp(-1.7\,s)/(s - 1)$ with a larger time delay ($t_0 = 1.7$), which once again cannot be stabilized by PID controller. We obtain the SMC settings for the present method as $\lambda_1 = 3/34$ min$^{-1}$, $\lambda_0 = 9/4624$ min$^{-2}$, $K_D = 0.0565$, $\delta = 13.9817$ (Table 1). Fig. 8a shows the response for servo problem under perfect pa-

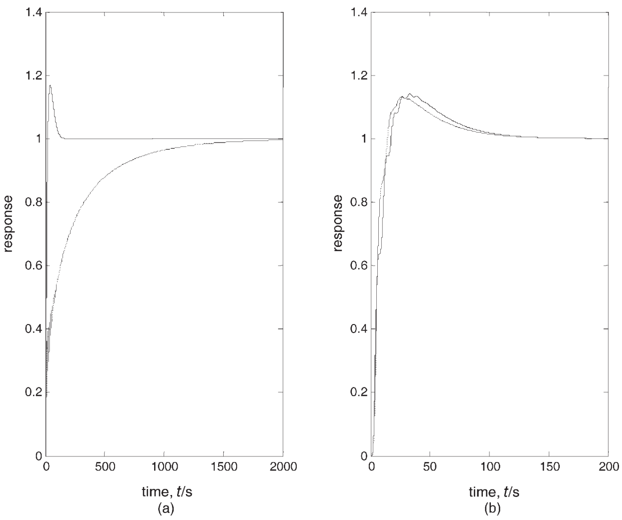


Fig. 6 – *Servo responses for unstable systems: $K_p$=1; $\tau$ = 1; $t_0$ = 1.4 with SMC Controller. (a) Solid Line: Perfect parameter, Dotted: Uncertainty of + 10 % in $K_p$ (b) Solid Line: Uncertainty of −10 % in $\tau$, Dotted: Uncertainty of +10 % in $t_0$*

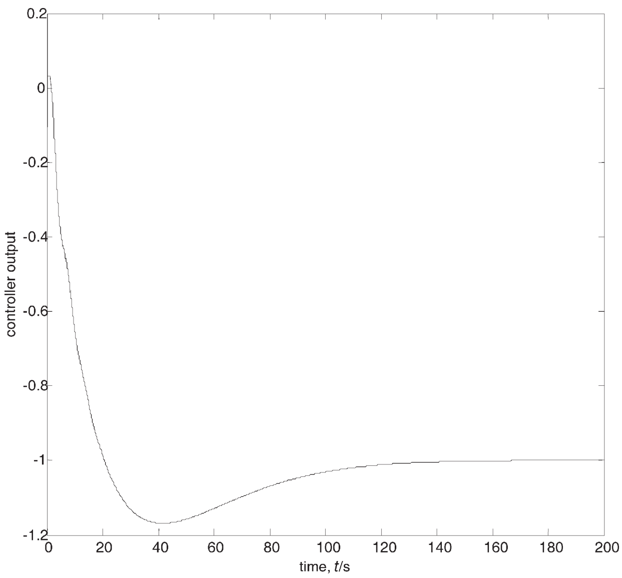


Fig. 7 – *Plot of manipulated variable of SMC vs. time for $K_p$=1; $\tau$ = 1; $t_0$ = 1.4 under perfect parameters condition*

rameters. Fig. 8b shows the response when there is an uncertainty of −10 % in the process time constant.

Table 2 gives the robustness characteristics of the SMC with respect to process parameters separately in process gain, time constant and time delay for various values of $\varepsilon$. From the Table, it is clear that up to $\varepsilon$ = 1.2, the present SMC is robust with respect to gain, time constant and delay. However, as $\varepsilon$ increases further, the robustness with respect to $K_P$, $\tau$ and $t_o$ reduces. At $\varepsilon$ = 1.8, the SMC is found to withstand up to 10 % decrease in $\tau$ and 10 % increase separately in $t_0$ and $K_p$.

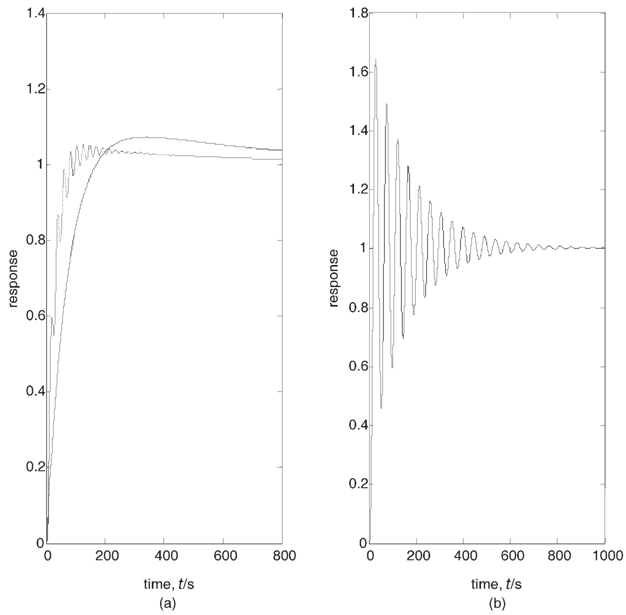


Fig. 8 – *Servo responses for unstable systems: $K_p$=1; $\tau$ = 1; $t_0$ = 1.7 with SMC Controller. (a) Solid Line: Perfect parameter, Dotted: (a) Uncertainty of −6 % in $\tau$, (b) Uncertainty of +5 % in $t_0$*

Table 2 – *Robustness characteristics of SMC with respect to process gain, time constant and time delay for various values of $\varepsilon$*

| Sl. No. | $\varepsilon$ | Robustness in $K_p$ / % | Robustness in $\tau$ / % | Robustness in $t_0$ / % |
|---|---|---|---|---|
| 1 | 0.1 | +20 % | −20 % | +20 % |
| 2 | 0.2 | +20 % | −20 % | +20 % |
| 3 | 0.3 | +20 % | −20 % | +20 % |
| 4 | 0.4 | +20 % | −20 % | +20 % |
| 5 | 0.5 | +20 % | −20 % | +20 % |
| 6 | 0.6 | +20 % | −20 % | +20 % |
| 7 | 0.7 | +20 % | −20 % | +20 % |
| 8 | 0.8 | +20 % | −20 % | +20 % |
| 9 | 0.9 | +20 % | −20 % | +20 % |
| 10 | 1.0 | +20 % | −20 % | +20 % |
| 11 | 1.1 | +20 % | −20 % | +20 % |
| 12 | 1.2 | +20 % | −20 % | +20 % |
| 13 | 1.3 | +17 % | −20 % | +20 % |
| 14 | 1.4 | +11 % | −16 % | +19.29 % |
| 15 | 1.5 | + 3 % | −16 % | +14.67 % |
| 16 | 1.6 | + 1 % | −14 % | +10.63 % |
| 17 | 1.7 | – | −10 % | +7.65 % |
| 18 | 1.8 | – | − 6 % | +5 % |

By simulation, we found that the value of $-1$ for the term $\left[\frac{-t_0+\tau}{t_0\tau}-\lambda_1\right]$ gives satisfactory results. However, if we decrease the value of $\frac{-t_0+\tau}{t_0\tau}-\lambda_1$ to, say $-0.5$, we find that the robustness of the controller decreases considerably for $\varepsilon$ = 1.4. The reduced robustness characteristics are given in Table 3. If we increase the value of $\frac{-t_0+\tau}{t_0\tau}-\lambda_1$ to, say $-1.5$, we find that the performance of the resultant controller is not good; i.e. the controller has larger overshoot and more settling time. Hence, a value of $-1$ is recommended for the term $\left[\frac{-t_0+\tau}{t_0\tau}-\lambda_1\right]$.

Table 3 – *Robustness characteristics of SMC with respect to process gain, time constant and time delay for values of* $\varepsilon$ *> 1.4, up to 1.8 when* $\frac{-t_0+\tau}{t_0\tau}-\lambda_1=-0.5$

| Sl. No. | $\varepsilon$ | Robustness in $K_p$ / % | Robustness in $\tau$/ % | Robustness in $t_0$ / % |
|---|---|---|---|---|
| 1 | 1.5 | +3 % | −16 % | +14.67 % |
| 2 | 1.6 | +1 % | −14 % | +10.63 % |
| 3 | 1.7 | – | −10 % | + 7.65 % |
| 4 | 1.8 | – | − 6 % | + 5 % |

## Application to an unstable bioreactor

A SMC is designed and simulated for an isothermal bioreactor exhibiting multiple steady state solutions. The results are compared with the latest PID controller proposed by Padmasree *et al.*[2] The mathematical model equation of the reactor is given by Liou and Chien[6] as

$$\frac{\mathrm{d}c}{\mathrm{d}t}=\frac{Q}{V}(c_\mathrm{f}-c)-\left[\frac{k_1c}{(k_2c+1)^2}\right] \qquad (10)$$

where $Q$ is the inlet flow rate and $c_\mathrm{f}$ is the feed concentration. The values of the operating conditions are given by $Q$ = 0.03333 L s$^{-1}$; $V$ = 1 L; $k_1$ = 10 s$^{-1}$; and $k_2$ = 10 L mol$^{-1}$. For the nominal value of $c_\mathrm{f}$ = 3.288 mol L$^{-1}$, the steady state solution of the model equation gives the following two stable steady states at $c$ = 1.7673 mol L$^{-1}$ and 0.01424 mol L$^{-1}$. There is one unstable steady state at $c$ = 1.3065 mol L$^{-1}$. Feed concentration is considered as the manipulated variable.

Linearization of the model equation around this operating condition $c$ = 1.3065 mol L$^{-1}$ gives the following unstable transfer function model 3.3226 $e^{-20\,s}$/ (99.69 $s$ – 1). A measurement delay of 20 $s$ is considered. For this unstable FOPDT system, a SMC is designed based on the tuning parameters given in Table 1. The values of the tuning parameters are $\lambda_1$ = 0.04, $\lambda_2$ = 0.0004, $K_\mathrm{D}$ = 0.7103 and $\delta$ = 2.0698. The PID settings given by Padmasree *et al.*[2] are $K_\mathrm{c}$ = 1.616, $\tau_\mathrm{I}$ = 85.73 and $\tau_\mathrm{D}$ = 8.813. The regulatory responses for these two settings for a step disturbance in $Q$ from 0.03333 to 0.03 L s$^{-1}$ are evaluated on the nonlinear model eq. (10) and the responses are shown in Fig. 10. The present method gives a better performance. The regulatory responses for an uncertainty in time delay are also evaluated (24 $s$ delay in the process whereas the controller settings are designed for 20 $s$ delay). The

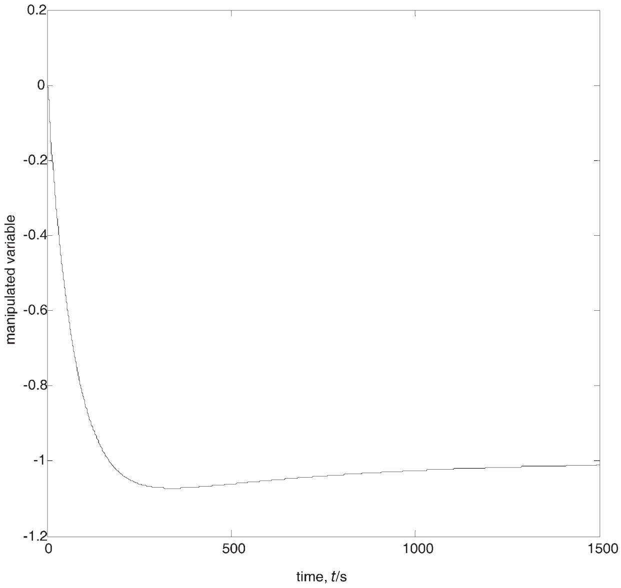


Fig. 9 – *Plot of controller output vs. time for* $K_p$=1; $\tau$ = 1; $t_0$ = 1.7 *under perfect parameters condition*

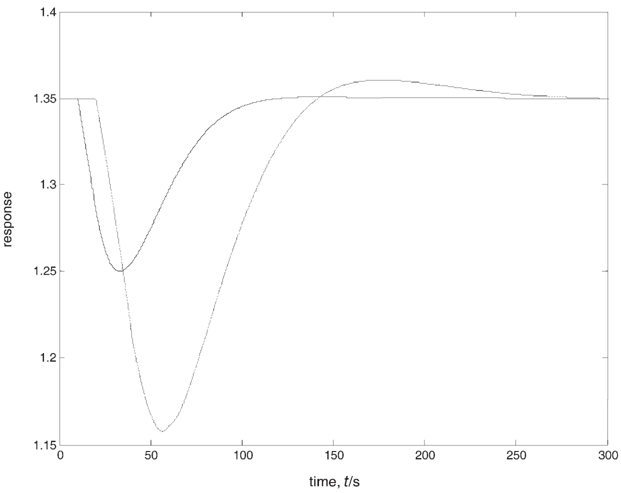


Fig. 10 – *The regulatory response of nonlinear bioreactor for a step change in q from 0.03333 to 0.03* $s^{-1}$ *for delay = 20 s. solid: SMC (Present), dash: PID*

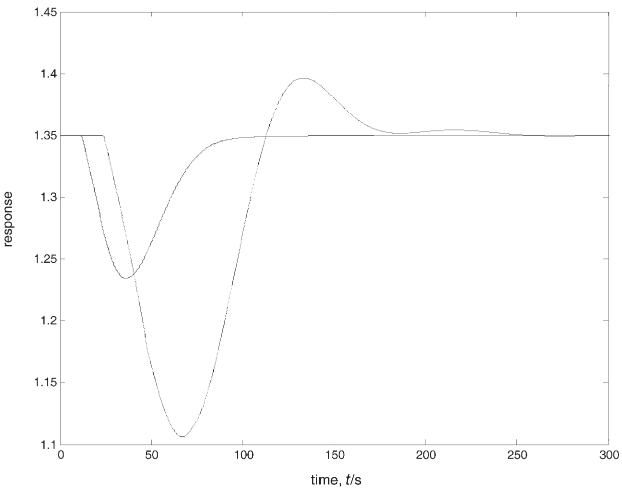


Fig. 11 – *Regulatory response of nonlinear chemical reactor for +20 % uncertainty in time delay. Legend: as in Fig. 10*

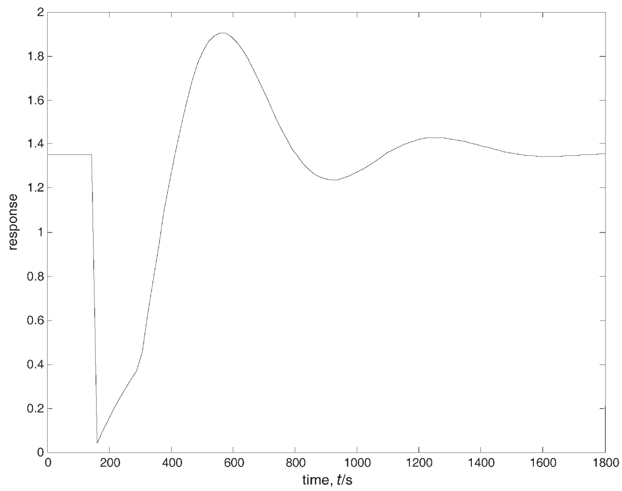


Fig. 12 – *The regulatory response of nonlinear bioreactor for a step change in q from 0.03333 to 0.03233 $s^{-1}$ for delay = 150 s using SMC*

responses are shown in Fig. 11. The present method gives robust performance than that of Padmasree *et al.*[2]

Let us consider the measurement delay of 150 *s* so that the delay time constant ratio ($\varepsilon$) = 1.5. For $\varepsilon$ > 1.2, the PID controller cannot stabilize the system. Hence we design a SMC for stabilizing the system. The values of the tuning parameters are $\lambda_1$ = 0.9967, $\lambda_0$ = 0.2484, $K_D$ = 0.1005, $\delta$ = 14.3531. The regulatory response for a step change in the inlet flow rate from 0.03333 to 0.03233 $s^{-1}$ is shown in Fig. 12. The regulatory response for a +10 % uncertainty in time delay is shown in Fig. 13.

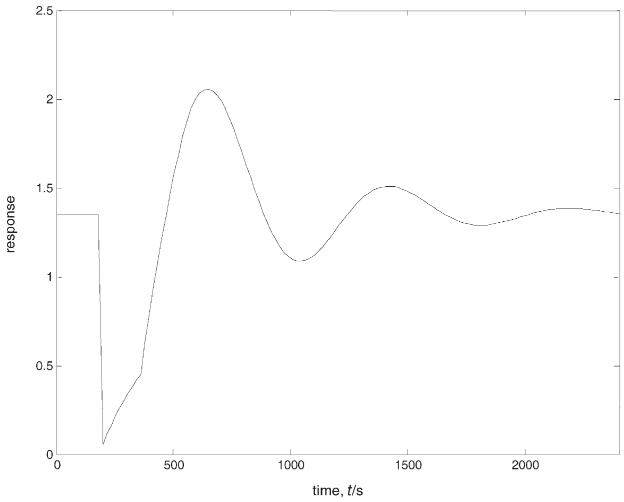


Fig. 13 – *Regulatory response of nonlinear chemical reactor using SMC for +10 % uncertainty in time delay for delay = 150 s*

## Conclusions

A Sliding Mode Controller is synthesized for a FOPDT unstable system for delay to time constant ratio up to 1.8. Up to $\varepsilon$ = 1.2, the performance of the proposed controller is found to be more robust than the latest PID controller proposed by Padmasree *et al.*[2] For $\varepsilon$ beyond 1.2, there is no method available in the literature to stabilize unstable systems using a PID controller. The robustness of the proposed controller with respect to uncertainties in process gain, time constant and time delay is found to decrease with increasing $\varepsilon$. Simulation result on control of a nonlinear bioreactor is also given for the case of $\varepsilon$ = 1.5.

### Nomenclature

$K_p$ – process gain
$\tau$ – process time constant
$t_0$ – process time delay
$\varepsilon$ – delay to time constant ratio

### Abbreviations

SMC – sliding mode controller
FOPDT – first order plus delay time
ISE – integral of square of error

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