Robust Generic Model Control for Dissolved Oxygen in Activated Sludge Wastewater Plant

R. Aguilar-López
Departamento de Biotecnología y Bioingeniería, CINVESTAV-IPN,
Av. Instituto Politécnico Nacional,
No. 2508, San Pedro Zacatenco, D.F. México
E-mail: raguilar@cinvestav.mx

Original scientific paper
Received: September 19, 2006
Accepted: December 10, 2007

The main issue of this paper is the design of a class of SISO robust control law for the regulation of dissolved oxygen (DO) of an industrial Activated Sludge Wastewater Plant via the air flow rate (non-affine control input). The control design is related with an uncertainty estimator (reduced order observer) based Generic Model Control (GMC). The desired dissolved oxygen trajectory is proposed as a Proportional-Integral (PI) form of the regulation error and the modeling error related with the respiration rate and the oxygen mass transfer coefficient are on-line estimated with a reduced order observer, these structures produce a feedback/feed forward regulator which is robust against model uncertainties and disturbances. The system's closed-loop behavior is analyzed via the estimation error and regulation error dynamics. The performance of the proposed control law is illustrated with numerical simulations, comparing the proposed controller with a well-tuned PI controller.

Key words:
Aerobic process, DO regulation, Generic Model Control, uncertainty estimation, robust performance

Introduction

Operating a wastewater treatment plant is not a simple task: raw wastewater varies continuously in quantity and composition and the heart of the process, the biomass, also changes under the influence of internal and external factors. To achieve adequate plant performance, the operational parameters can be adapted in any given situation to meet actual requirements; these changes are based on measuring relevant process parameters using grab or composite samples for further monitoring and control tasks.

The control of non-linear systems has been widely studied over the last 20 years, where several control methodologies have been proposed and employed[1–3] particularly for activated sludge process non-linear predictive control, optimal control, linearizing control, and so on. The model based controllers such as I/O linearizing, and Generic Model Control (GMC) acts by canceling the nonlinearities of the processes and imposing determinate closed-loop behavior to the system, assuming perfect knowledge of the mathematical model, and producing global asymptotic stability.[4] A drawback of exact model based techniques is that they rely on exact cancellation of nonlinearities. In practice, exact knowledge of system dynamics is not possible. A more realistic situation is to know some nominal functions of the corresponding nonlinearities, which are employed in the control design. However, the use of nominal model nonlinearities can lead to performance degradation and even closed-loop instability. In fact, when the systems possess strong nonlinearities, the standard model based controllers (I/O and GMC) cannot cancel completely such nonlinearities and instabilities can be induced. The worst case is when the knowledge of the nonlinearities is very poor or null. Therefore, these conventional techniques are inadequate. In the face of these events, the robust stability problem for uncertain systems arises as a necessary control design approach to supply the controller with the corresponding on-line information and try to realize a satisfactory closed-loop performance. Research on robust control design for linearizable nonlinear systems has been done considering observed-based controllers[5,6] where peaking phenomena, stability issues and robust performance are still topics that deserve further study. To deal with these topics, several control schemes have been proposed. Among them are the neural control strategies, which have been successfully applied, but their main drawback is the over-parameterization when a multiplayer neural-network is employed.

Classical adaptive controllers have shown an adequate performance for a class of uncertain systems, where the parameters appear in a linear way in the model of the plant,[7] but for high non-linear systems (such as the biochemical process) this approach seems inadequate.

In recent papers,[8,9] we have used Luenberger-type observer structures to obtain on-line estimates of uncertain signals. However, the resulting controllers become very sensitive to measurement noise. Since measurement noise is propagated through the control loop, high frequency chattering can induce premature degradation of the actuator (*e.g.* valves, pumps and compressors) components. To avoid some of the drawbacks mentioned above, a Generic Model Control is proposed, in order to add an integral contribution to the controller structure in the effort to compensate disturbances, besides the controller is feed with an estimate of the uncertain signal of the plant (respiration rate and oxygen mass transfer coefficient) related with the corresponding modeling errors; this is estimated by employing a proportional reduced order observer. The proposed methodology is applied to the mathematical model of an activated sludge wastewater plant, which corroborates with COD (Chemical Oxygen Demand) industrial data and industrial operation conditions, with satisfactory results.

The industrial activated sludge wastewater plant

The petrochemical industry under study produces wastewater generated from different chemical processes. The wastewater flow produced is about Q = 7000 m^3 d^{-1} and contains volatile organic carbon substances classified as toxics[10] like 1,2 dichloroethane, chloroform, benzene, among others, and volatile compounds, (VOC's). To comply with the effluent quality required by the Mexican environmental legislation[11] the wastewater is processed in the treatment plant before being discharged into the river. The treatment process comprises oil removal, using a corrugated plates interceptor (CPI), equalization basin, and activated sludge process composed of three independent bioreactors with a V = 5000 m^3 volume each. The residence time in each is about 2 days. The biological sludge produced is concentrated by centrifugation and the treated effluent is subsequently chlorinated. The petrochemical wastewater treatment plant is located near the Mexican coast, where the mean weather temperature in the hottest months (April to August) is nearly T = 35 °C and in extreme conditions it reaches up to T = 38 °C. Such high temperatures affect the air temperature at the compressor exit producing a significant air temperature rise at the diffusers of up to T = 82 °C or more. This provokes an increase in the bioreactor wastewater temperature within the biological reactor. The actual temperature conditions within the bioreactor are T = 32 °C in October-November reaching up to T = 41 °C in August-September. Due to this effect, the microorganism's activity is affected and this must be considered in the dynamic modeling of the system. Some models have been developed to describe the effect of temperature on bacterial growth.[12–14] The authors showed that at high temperatures the maximum specific growth rate (μ_{max}) is reduced.

One of the purposes of this study was to model the sludge activated treatment plant at different temperatures applying a simple carbon removal model. The temperature effects on the maximum specific growth rate, mass transfer coefficient for oxygen (k_La) and death coefficient (k_d), were incorporated into the mass balance equations of the process.

Experimental methodology

Experimental data were obtained from the petrochemical biological wastewater plant. Different samples were taken daily (after 8 h), from the influent and bioreactors during the period from October 2002 to September 2003. The bioreactor capacity was V = 5000 m^3 each. The flow was about Q_r = 2300 to 2600 m^3 d^{-1} and the mean residence time in each bioreactor was about τ_R = 2.0 d. The bioreactors operate with bubble fine diffusers equipment (FBD). The volumetric power level in the chambers with the FBD system is about 0.0298 kW m^{-3}. It was considered that all the bioreactors were mixed flow reactors. The Chemical Oxygen Demand (COD) was measured[15] in the laboratory plant. The mean of three analyses was taken as the daily mean analysis per chamber. In order to obtain the results reported in the figures, the mean of the daily mean analysis was obtained. The kinetic parameters were obtained in laboratory bioreactors following the method by Ramalho.[16] The temperature effect on the maximum specific growth rate was evaluated with eq. (8) where b = 0.05 K^{-1} h$^{-0.5}$ and c = 0.005 K^{-1} (which is a parameter to fit the experimental data to the model),[10] the mass transfer coefficient for the oxygen (k_La) with eq. (10) which is an empirical function of the air flow,[17] the death coefficient (k_d) with eq. (9), the evaporation flux of VOC's ($K_{ev}\,\gamma_S$) is also considered in the COD balance, together with the inactive biomass $(1-f_n)\,\gamma_X$ this term is the biomass part which depends on the metabolic products of other bacteria and cellular lyses, which acts as available organic matter and are quantified as substrate (γ_{COD} mg L^{-1}), the term f_n ($f_n \in [0,1]$) is an effective factor for this inactivate biomass, which is empirically evaluated. The complete description of the parameter estimation methods is giving in.[17]

The activated sludge petrochemical wastewater treatment process is shown in Fig. 1.

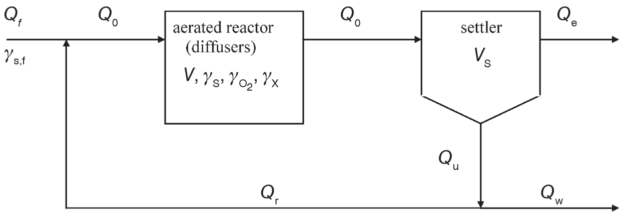


Fig. 1 – *Activated sludge petrochemical wastewater treatment plant*

The process is described by the following mass balance equations.[19] As a first modeling approach, the temperature effect on different quantities is considered introducing an energy balance considering that the metabolic heat generation due to the respiration rate can be neglected in comparison with the other energy flows, the quantities of the energy balance are considered as the same as water. The bioreactor behavior was assumed as a completely mixed flow reactor.

In the reactor:

Substrate (γ_S) concentration mass balance:

$$\frac{d\gamma_S}{dt} = \frac{Q_f}{V}\gamma_{S,r} - \frac{Q_0}{V}\gamma_S - \\ -\frac{\mu_{max}}{Y}\left(\frac{\gamma_S}{K_s+\gamma_S}\right)\left(\frac{\gamma_{O_2}}{K_{OH}+\gamma_{O_2}}\right)\gamma_X + k_d(1-f_n)\gamma_X - k_{ev}\gamma_S \tag{1}$$

Biomass (γ_X) concentration mass balance:

$$\frac{d\gamma_X}{dt} = \frac{Q_r}{V}\gamma_{X,r} - \frac{Q_0}{V}\gamma_X + \\ +\mu_{max}\left(\frac{\gamma_S}{K_s+\gamma_S}\right)\left(\frac{\gamma_{O_2}}{K_{OH}+\gamma_{O_2}}\right)\gamma_X - k_d\gamma_X \tag{2}$$

Oxygen (γ_{O_2}) concentration mass balance:

$$\frac{d\gamma_{O_2}}{dt} = \frac{Q_f}{V}\gamma_{O_2 f} - \frac{Q_0}{V}\gamma_{O_2} - \\ -\frac{\mu_{max}}{Y_{O_2}}\left(\frac{\gamma_S}{K_s+\gamma_S}\right)\left(\frac{\gamma_{O_2}}{K_{OH}+\gamma_{O_2}}\right)\gamma_X + k_L a(\gamma_{O_2 sat} - \gamma_{o_2}) \tag{3}$$

Energy Balance (T):

$$\frac{dT}{dt} = \frac{Q_0}{V}(T_{in} - T) + \frac{Q_{air}\rho_{air}C_{p,air}}{V\rho C_p}T_{air} + \\ +\frac{h_c A}{V\rho C_p}(T - T_\infty) \tag{4}$$

It was assumed that there was no biomass in the overflow of the settler.[15]

In the settler:

$$\frac{d\gamma_{X,r}}{dt} = \frac{Q_U}{V_S}\gamma_{X,r} - \frac{Q_0}{V_S}\gamma_X \tag{5}$$

and

$$Q_0 = Q_f + Q_r \tag{6}$$

$$Q_U = Q_W + Q_r \tag{7}$$

The mathematical model of the wastewater plant and its open loop behavior was previously presented in[17,20] where a local analysis based on Lyapunov criteria showed that all the state variables (substrate, biomass, dissolved oxygen, temperature and biomass of the settler) are stable over a wide range of operation conditions.

Robust control law design

Problem statement

The DO mass concentration in the mixed liquor in biological treatment systems has proved to be an important process parameter. The proper DO control can improve the process performance and give an economic incentive which minimizes excess oxygenation by supplying only the amount of necessary air. An adequate DO mass concentration in the bioreactor must lead to satisfactory biomass growth and a desired consumption of substrate, which is the pollutant of the wastewater to be treated and the real control objective of the plant.

Consider the following dynamic subsystem:

$$\frac{d\gamma_{O_2}}{dt} = \frac{Q_f}{V}\gamma_{O_2 f} - \frac{O_0}{V}\gamma_{O_2} - \\ -\frac{\mu_{max}}{Y_{O_2}}\left(\frac{\gamma_S}{K_s+\gamma_S}\right)\left(\frac{\gamma_{O_2}}{K_{OH}+\gamma_{O_2}}\right)\gamma_X + k_L a(\gamma_{O_2 sat} - \gamma_{O_2}) \tag{8}$$

With:

$$k_L a = 169.32\left(1 - \exp\left(\frac{-Q_{air}}{23040}\right)\right)^{T-20} \tag{9}$$

Note that the oxygen mass transfer coefficient is a nonlinear function of the control input Q_{air} and the reactor temperature T;[18] therefore, the subsystem given by both above equations is non-affine in the control input, this situation leads to a problem for the design and the operation performance of the proposed control methodology. To avoid this situation an alternative representation is proposed of the oxygen mass transfer coefficient via Taylor's series, as follows:

$$k_L a = k_L a(Q^*_{air}, T^*) + \frac{\partial k_L a}{\partial T}(T - T^*) + \\ + \frac{\partial k_L a}{\partial Q_{air}}(Q_{air} - Q^*_{air}) + \Gamma(Q_{air}, T) \tag{10}$$

where $\Gamma(Q_{air}, T)$ are higher order terms. From the above equation, a linear representation of the oxygen mass transfer coefficient as a function of the air flow rate can be obtained as follows:

$$k_L a = \bar{K} Q_{air} + F(Q_{air}, T) \tag{11}$$

here:

$F(Q_{air}, T)$ is the modeling error of the mass transfer coefficient.

On the other hand, the respiration rate is selected as a meaningful biological indicator, as it yields the rate at which the microorganisms utilize oxygen in carrying out their metabolic activities. This variable provides information about the current stage of the biological reactions and can be employed with a number of control strategies, and characterize the DO process and the associate removal and degradation of biodegradable load. Furthermore, a rapid decrease of the respiration rate can be used as a warning that toxic matter has entered the plant. This variable together with the oxygen transfer rate, is needed to monitor the biological activity and assess the performance of the process control system.

The respiration rate (Δ) is generally difficult to evaluate and consequently constitute other modeling error terms for modeling the mass balance of the dissolved oxygen (DO), and can be represented as:

$$\Delta = \frac{\mu_{max}}{Y_{O_2}} \left(\frac{\gamma_S}{K_s + \gamma_S} \right) \left(\frac{\gamma_{O_2}}{K_{OH} + \gamma_{O_2}} \right) \gamma_X \tag{12}$$

Therefore, the subsystem related with the DO mass balance is now represented by:

$$\frac{d\gamma_{O_2}}{dt} = \frac{Q_f}{V}\gamma_{O_{2f}} - \frac{Q_0}{V}\gamma_{O_2} - \Delta + \\ +(\bar{K}Q_{air} + F)(\gamma_{O_{2sat}} - \gamma_{O_2}) \tag{13}$$

defining $\eta = F(\gamma_{O_{2sat}} - \gamma_{O_2}) - \Delta$ as the whole modeling error, the DO mass balance is represented as:

$$\frac{d\gamma_{O_2}}{dt} = \frac{Q_f}{V}\gamma_{O_{2f}} - \frac{Q_0}{V}\gamma_{O_2} + \bar{K}Q_{air}(\gamma_{O_{2sat}} - \gamma_{O_2}) + \eta \\ \frac{d\eta}{dt} = \Phi(T, Q_{air}, \text{etc}) \qquad y = \gamma_{O_2} \tag{14}$$

Note that the uncertain term, $\eta(T, Q_{air}, \text{etc})$ is considered as a new state and $\Phi(T, Q_{air}, \text{etc})$ is a non-linear unknown function that describes the η-dynamics and the measured system output (y) is the dissolved oxygen concentration.

Robust Generic Model Control law

Generic Model Control is employed to utilize the nonlinear dynamics of the system under study in the control algorithm. In GMC, nonlinear process models can be embedded directly into the controller without linearization. GMC is a very simple and robust nonlinear control algorithm in SISO process.[20,21]

Now, considering the following DO state equation:

$$\frac{dy}{dt} = \frac{Q_f}{V}\gamma_{O_{2f}} - \frac{Q_0}{V} y + \bar{K} Q_{air}(\gamma_{O_{2sat}} - y) + \eta$$

Where the desired closed-loop trajectory of the DO (y_d) in the bioreactor is proposed as:

$$\frac{dy_d}{dt} = g_1(y - y_{sp}) + g_2 \int_0^t (y - y_{sp}) d\sigma \tag{15}$$

where y_{sp} is the corresponding set point and g_1, g_2 are the corresponding proportional and integral controller gains; combining the two above equations, the following expression for the control law is generated:

$$Q_{air} = \frac{1}{\bar{K}(\gamma_{O_{2sat}} - y)} \left(g_1(y - y_{sp}) + \\ + g_2 \int_0^t (y - y_{sp}) d\sigma - \frac{Q_f}{V}\gamma_{O_{2f}} + \frac{Q_0}{V} y - \eta \right) \tag{16}$$

In an ideal case with non-modeling errors in the state equation (*i. e.* η known), the corresponding dynamic equation of the regulation error ($\xi = y - y_{sp}$) is:

$$\frac{d\xi}{dt} = g_1 \xi + g_2 \int_0^t \xi(\sigma) d\sigma \tag{17}$$

which provides a stable exponential convergence to zero, as it is well known.

The proposed controller is related with the regulation of the DO levels in the bioreactor (control output) employing the air flow from the compressor system (control input) to develop a single input – single output (SISO) control scheme. Therefore, some other state variables, such as temperature, are not explicitly considered in the controller design, primarily because this kind of bioreactor lacks heat transfer devices; however the temperature affects the dissolved oxygen mass balance, because the oxygen mass transfer coefficient ($k_L a$) and the maximum specific growth rate μ_{max} are explicit functions of the bioreactor temperature.

As it can be noticed, the synthesis of the nominal control law needs accurate knowledge of the mathematical model of the process to be realizable, in order to cancel completely the nonlinear characteristics of the system. However, a perfect model is impossible to be obtained and, consequently, for uncertain systems a conventional GMC controller design is inadequate.

Nevertheless, there is another way to develop a GMC-type controller that is robust against uncertainties. The procedure shown below defines, firstly, a method to estimate the uncertainty term, $\eta(T, Q_{\text{air}}, \text{etc})$. This approach is based on a reduced order estimator design that can provide the corresponding information to the controller to be realizable.

Uncertainty estimator design

One of the major bottlenecks in the application of computer monitoring and control for biological process is the lack of reliable, sterilizable and robust sensors for the on-line measurements of process key variables, such as biomass, precursors, product concentrations and consumption rates. Several attempts to quantify the above variables have been employed, some of them are optical techniques, electrochemical detection and by viscosity, filtration and fluorescence methods,[22] but these approaches frequently do not properly address the most important industrial problems and necessities.

To tackle the problem mentioned above, several state estimation techniques for bioprocess have been developed. These techniques are often named soft-sensors and are based on the balancing technique. Such an approach is adequate for steady-state operation, however it becomes unstable when dynamic and corrupted measurements are presents;[23] and filtering (observing) theory where extended Kalman filters, nonlinear Luenberger observers, sliding-mode, high gain and so on; observers have been successfully employed.[24] Considering our particular case, the state variable to be regulated is directly the measured output of the system, i. e. the DO mass concentration. Therefore, a reduced order observer to infer the uncertain term is proposed. For control purposes, the kinetic terms and the oxygen convective mass transfer are considered as unknown terms which are estimated by the corresponding uncertainty estimator, to feed this information to the control law, consequently the effect of the temperature and other variables are compensated for the proposed controller.

Proposition 1: The following dynamic system is an asymptotic-type reduced order observer for the estimation of the uncertainty defined in the system (14):

$$\frac{\mathrm{d}\hat{\eta}}{\mathrm{d}t} = \tau(\eta - \hat{\eta}) \tag{18}$$

where the observed uncertainty η is obtained by solving the mass balance equation, in accordance with the next equation:

$$\eta = \frac{\mathrm{d}y}{\mathrm{d}t} - \frac{Q_{\mathrm{f}}}{V}\gamma_{\mathrm{O_{2f}}} + \frac{Q_0}{V} y - \bar{K} Q_{air}(\gamma_{\mathrm{O_{2sat}}} - y) \tag{19}$$

As can be seen, the structure of the proposed observer includes the derivative of the DO mass concentration, which must be calculated in order to obtain estimates of the reaction rate. However, the synthesis of derivators is a difficult task; moreover, if the concentration measurements are noisy the synthesis could be impossible. In order to avoid this situation, the following change of variable is proposed:

$$\Theta = \hat{\eta} - \tau y \tag{20}$$

Producing an uncertainty observer with the following structure:

$$\frac{\mathrm{d}\Theta}{\mathrm{d}t} = \tau\left(-\frac{Q_{\mathrm{f}}}{V}\gamma_{\mathrm{O_{2f}}} + \frac{Q_0}{V} y - \bar{K} Q_{\mathrm{air}}(\gamma_{\mathrm{O_{2sat}}} - y) - \hat{\eta}\right) \tag{21}$$

Note that with eqs. (20) and (21) the uncertain term can be expressed finally as:

$$\hat{\eta} = \Theta + \tau y \tag{22}$$

As can be seen, this estimation methodology only depends on measured variables, and is consequently completely realizable. Now, for the realization of the robust (non-ideal) GMC the estimate of the uncertain term determinate above is coupled to the ideal GCM to produce:

$$\begin{aligned} Q_{\mathrm{air}} = \frac{1}{\bar{K}(\gamma_{\mathrm{O_{2sat}}} - y)}\Bigg(& g_1(y - y_{\mathrm{sp}}) + \\ & + g_2\int_0^t (y - y_{\mathrm{sp}})\mathrm{d}\sigma - \frac{Q_{\mathrm{f}}}{V}\gamma_{\mathrm{O_{2f}}} + \frac{Q_0}{V} y - \hat{\eta}\Bigg) \end{aligned} \tag{23}$$

Closed-loop stability analysis

Stability observer comments

First let us consider the convergence analysis of the proposed observer, in accordance with Proposition 1:

$$\frac{d\hat{\eta}}{dt} = \tau(\eta - \hat{\eta}) \tag{24}$$

it is an asymptotic proportional reduced observer for system given by (7), where $\tau > 0$, determines the desired convergence rate of the observer, if the following assumptions are satisfied:

There exist τ and $N \in \Re^+$ as follows:

A1 $\quad ||\Phi(0)|| \leq N$

A2 $\quad \lim_{t \to t_0} \left\| \exp\left(-\int_0^t \tau d\sigma \right) \right\| = 0$ with t_0 large enough

Defining the estimation errors as:

$$e_1 = \eta - \hat{\eta} \tag{25}$$

Considering the above eq. (25), the dynamics of the estimation error are defined as:

$$\dot{e}_1 + \tau e_1 = \Phi(0) \tag{26}$$

Solving it renders:

$$e_1 = e_{10} \exp(-\tau t) + \int_0^t \exp(-\tau t) \exp(\tau\sigma) \Phi(0) d\sigma \tag{27}$$

where e_{10} is the initial condition of the estimation error. Taking norms of the eq. (27) the following inequality arises:

$$0 \leq \lim_{t \to t_0} \| e_1 \| \leq \| e_{10} \| \lim_{t \to t_0} [\| \exp(-\tau \int d\tau) \|] + \\ + \frac{\lim_{t \to t_0} \left[\left\| \int_0^t \exp(\tau\sigma) \Phi(0) d\sigma \right\| \right]}{\lim_{t \to t_0} \| \exp(\int \tau \, dt) \|} \tag{28}$$

From **A1** and **A2**:

$$0 \leq \lim_{t \to t_0} \| e_1(t) \| \leq \frac{\lim_{t \to t_0} \left[N \int_0^t \| \exp(\tau\sigma) d\sigma \| \right]}{\lim_{t \to t_0} \| \exp(\int \tau \, dt) \|}$$

The above equation means that the ∞/∞ case of uniform L'hôpital's rule can be applied as follows:

$$0 \leq \lim_{t \to t_0} \| e_1(t) \| \leq \lim_{t \to t_0} \frac{N \| \exp(\int \tau \, dt) \|}{\| \exp(\int \tau \, dt) \| \, \| \tau \|} = \lim_{t \to t_0} \frac{N}{\| \tau \|}$$

in the limit, when $t \to t_0$:

$$|e_1| \leq \frac{N}{\| \tau \|} \to 0 \tag{29}$$

Besides, the above inequality implies that the estimation error can be as small as desired, if the observer gain τ is chosen large enough.

Note that if the system output is corrupted by additive noise *i. e.* $y = y_{O_2} + \delta$ and the noise is considered bounded so that $\|\delta\| \leq \Pi$, a similar methodology used to analyze the estimation error e_1 can be applied in order to prove that the steady-state estimation error becomes $\frac{(N + \Pi)}{\tau}$ which proves robustness against noisy measurements.

Dissolved oxygen stability comments

The performance of the DO mass concentration trajectory can be done via the analysis of the corresponding closed-loop state equation:

$$\frac{d\xi}{dt} = g_1 \xi + g_2 \int_0^t \xi(\sigma) d\sigma + (\eta - \hat{\eta}) \tag{30}$$

or

$$\frac{d\xi}{dt} = g_1 \xi + g_2 \xi^* + e_1$$

$$\frac{d\xi^*}{dt} = \xi$$

if $E = \begin{bmatrix} \xi \\ \xi^* \end{bmatrix}$; $A = \begin{bmatrix} g_1 & g_2 \\ 1 & 0 \end{bmatrix}$ and $\Omega = \begin{bmatrix} e_1 \\ 0 \end{bmatrix}$, then:

$$\frac{dE}{dt} = AE + \Omega \tag{31}$$

Note that matrix A is Hurwitz stable with an adequate choosing of the control gains, therefore the solution of eq. (31), renders:

$$E = E_0 \exp(At) + \int_0^t \exp(A(t - \sigma)) \Omega(0) d\sigma \tag{32}$$

Here, E_0 is the initial condition of the regulation error.

Assuming two positive constants $j > 0$ and $\lambda > 0$, which satisfy:

$$\| \exp(At) E \| \leq j \exp(-\lambda t) \| E \|$$

Considering the above assumption and taking norms for both sides of eq. (32), the following equation is generated:

$$\| E \| \leq j \exp(-\lambda t) \left[\| E_0 \| - \frac{jN}{\lambda\tau} \right] + \frac{jN}{\lambda\tau} \tag{33}$$

Taking the limit, when $t \to \infty$:

$$\|E\| \leq \frac{jN}{\lambda \tau} \tag{34}$$

Note that the proportional feedback part of the controller provides closed-loop stability to the plant, whereas the integral feedback acts as compensator for disturbances and probable noisy measurements, finally the uncertainty estimator provides a feed forward compensation for modeling errors.

Results and discussion

The model was validated with COD data obtained from the wastewater treatment plant, which was in operation for a year, from October 2002 to September 2003. Table 1 shows the different industrial scenarios used to validate the dynamic modeling:

Table 1 – *Scenarios presented during the wastewater treatment plant operation to validate the dynamic modeling*

Quantity	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
$\gamma_{S,f}$ mg L^{-1}	2500	2800	3100	2000	2600
T °C	32.5	38	36	39	41
Q_f m^3 d^{-1}	7300	7200	7600	7300	7400
Q_r m^3 d^{-1}	1500	2000	1500	2600	2600
Q_w m^3 d^{-1}	900	700	800	800	750

As may be discerned from Fig. 2, the performance of the model to predict the COD (γ_S substrate) concentration from industrial data seems satisfactory. The model tracks the corresponding trajectory under several operational and environmental conditions. On other hand, as mentioned above, for the simulation study, the plant was subject to several disturbances, at day 35 the substrate input mass concentration changed from γ = 3400 mg L^{-1} to 5400 mg L^{-1} and the environmental temperature changed from T = 38 °C to T = 36 °C; at day 36 the substrate input mass concentration

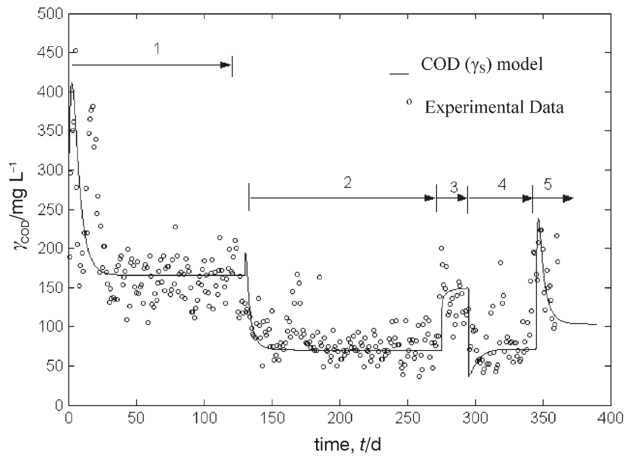


Fig. 2 – *Dynamic model validation with γ_S mass concentration data*

changed again to γ = 4850 mg L^{-1}; at day 125 the environmental temperature changed again to T = 38 °C, at day 190 the corresponding environmental temperature changed to T = 41 °C and finally at day 270 changed to T = 33 °C.

An optimal operating region was previously determinated in order to fix the corresponding set point of the DO mass concentration via steady state analysis, so that y_{sp} = 2.0 mg L^{-1} was chosen because this DO steady state value corresponds to a biomass concentration of γ_X = 2450 mg L^{-1}, temperature of T = 36 °C and the most important to the substrate concentration γ_S = 105 mg L^{-1}, which complies with Mexican regulation; some of these results are presented in Fig. 3. The controller is tuned with a proportional gain g_1 = 20 d^{-1} and the integral gain g_2 = 10 d^{-1}, the observer gain is τ = 1 d^{-1}, with these parameters, the controller is activated and Fig. 4 shows the closed-loop behavior of the DO mass concentration, when the disturbances arrive to the process the controller is able to reject them keeping the DO mass concentration on the desired set point (γ_{O2} ~ 2 mg L^{-1}) without great effort as is shown in Fig. 5. The Fig. 6 refers to the

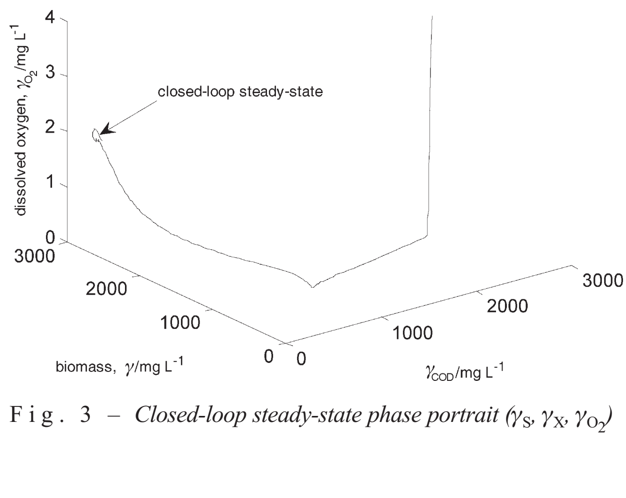
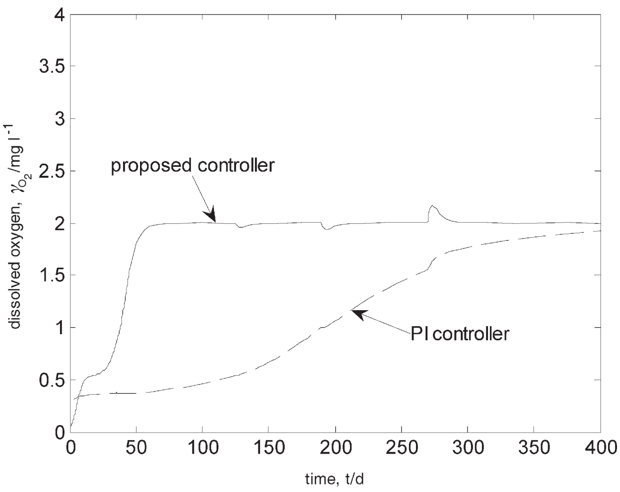
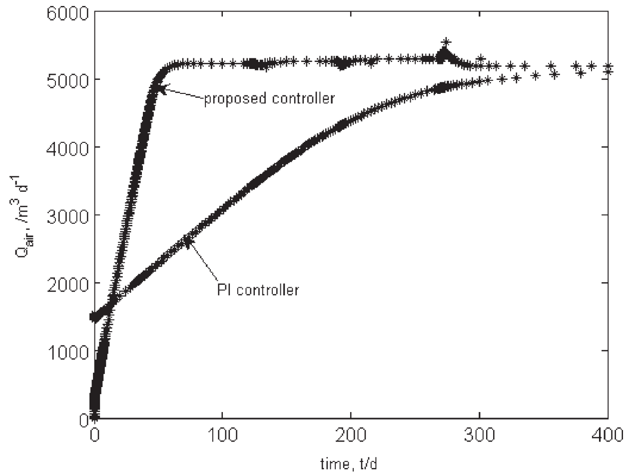


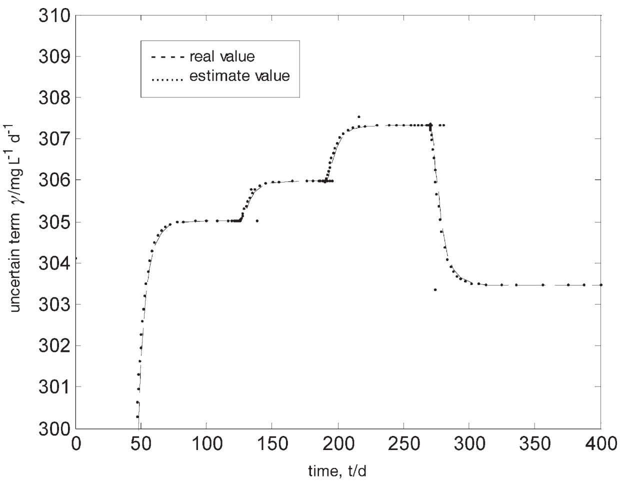
Fig. 3 – *Closed-loop steady-state phase portrait (γ_S, γ_X, γ_{O_2})*



F i g . 4 – *Closed-loop performance of the γ_{O_2} mass concentration with the proposed controller*



F i g . 5 – *Closed-loop performance of the system input (air flow rate, Q_{air}) with proposed controller*



F i g . 6 – *Closed-loop uncertainty observer performance*

closed-loop performance of the uncertainty observer, the performance seems satisfactory and adequate estimation of the uncertainty is reached. The proposed controller is compared with a commercial linear PI controller considering the same control gains, i. e. g_1 = 20 d^{-1} and g_2 = 10 d^{-1} Fig. 4 and 5 contain the corresponding controller performance; note that the standard PI controller has a much slower response than the robust GMC.

Conclusions

A mathematical model of an Activated Sludge Wastewater Plant is developed and corroborates with industrial COD and operating data with good results. This model is employed as a *virtual* process where the respiration and oxygen transfer rates are supposed uncertain. To avoid the problem of modeling errors a reduced order observer is proposed, the information generated by the observer is coupled with a Generic Model Control law, thus achieving a robust structure against modeling error and disturbances. The closed-loop performance of the plant is analyzed with the dynamic equations of the estimation and regulation errors, showing the properties of the proposed methodology. Numerical simulations illustrate the satisfactory performance of the observer based Generic Model Control law, where the plant is led to an optimum operating region and the DO set point chosen (γ = 2 mg L^{-1}) produce a very satisfactory level of COD (γ_S ~ 105 mg L^{-1}) at the plant output which is the main objective for this process.

ACKNOWLEDGEMENTS

The author is very grateful to the Universidad Autónoma Metropolitana, for corresponding support of the sabbatical stay at CINVESTAV-IPN.

N o m e n c l a t u r e

A – transport area, m^2
b – 0.05 K^{-1} d$^{-0.5}$
c – 0.005 K^{-1}
C_p – specific heat capacity, kJ g^{-1} K^{-1}
e_1 – estimation error
F – mass transfer coefficient modeling error
f_n – effectiveness factor for the inactivate biomass
$g_{1,2}$ – control gains, d^{-1}
h_c – heat transfer coefficient, kW m^{-2} K^{-1}
K – number density, m^{-3}
K_s – substrate saturation constant, 30 mg L^{-1}
K_{OH} – substrate saturation constant, 0.2 mg L^{-1}
k_d – death coefficient, d^{-1} = $k_{d\,20}$ $1.05^{(T-293)}$

$k_{d\ 20}$ – death coefficient at 293 K = 0.03 d^{-1}

k_La – mass transfer coefficient, $d^{-1} = k_La_{,20}\ 1.02^{(T-293)}$

$k_La_{,20}$ – $166\left(1-\exp\left(\frac{-Q_{air}}{23040}\right)\right)$ mass transfer coefficient at 293 K d^{-1}

N – uncertainty quota

Q_f – influent flow rate, $m^3\ d^{-1}$

Q_r – recycle flow rate, $m^3\ d^{-1}$

Q_W – waste flow rate, $m^3\ d^{-1}$

Q_{air} – air flow rate, $m^3\ d^{-1}$

T_{ww} – wastewater temperature in the bioreactor °C or K

t – time, d^{-1}

$Y_{x/s}$ – 0.67, yield, mg biomass produced mg^{-1} COD consumed

Y_{O_2} – 2.03, yield oxygen mg biomass produced mg^{-1} O_2 consumed

V – reactor volume = 15000 m^3

V_S – settler volume = 750 m^3

Greek letters

γ_{O_2f} – dissolved oxygen mass concentration in the influent, mg L^{-1}

γ_{O_2} – dissolved oxygen mass concentration in the reactor, mg L^{-1}

γ_{O_2sat} – dissolved oxygen saturation mass concentration, mg L^{-1}

δ – measurement noise, mg L^{-1}

Δ – respiration rate modeling error, mg $L^{-1}d^{-1}$

ξ – regulation error

γ_{Sf} – COD mass concentration in the influent, mg L^{-1}

γ_S – COD mass concentration in the reactor, mg L^{-1}

γ_X – biomass concentration in the reactor, mg L^{-1}

γ_{Xr} – biomass concentration in the settler, mg L^{-1}

η – uncertain term, mg $L^{-1}d^{-1}$

$\hat{\eta}$ – uncertain term estimated, mg $L^{-1}d^{-1}$

μ – specific growth rate, $d^{-1} = \frac{\mu_{max}\gamma_S}{K_s+\gamma_S}$

μ_{max} – maximum specific growth rate, d^{-1} $= b^2\ (T-285)^2\ \{1-\exp\,[c(T-330.5)]\}^2$

ρ – density, mg L^{-1}

τ – observer's gain, d^{-1}

τ_R – residence time, d

References

1. *Chan Kyoo Yoo, In-Beaum Lee*, Environmental Engineering Science **21** (3) (2004) 331.
2. *Chachuat, B., Roche, N., Lafiti, M. A.*, Biochemical Engineering Journal **23** (2005) 277.
3. *Georgieva, P. G., Feyo de Azevedo, S.*, Int. Journal of Robust and Nonlinear Control **9** (1999) 949.
4. *Isidori, A.*, 'Nonlinear Control Theory', Springer-Verlag, New York, 1995.
5. *Krener, A. J., Isidori, A.*, Syst. and Contr. Lett. **3** (1983) 47.
6. *Aguilar, R., Martínez, S. A., Rodríguez, M. G., Soto, G.*, Chemical Engineering Journal **105** (2005) 139.
7. *Soroush, M., Tyner, D., Grady, M.*, Ind. Eng. Chem. Res. **38** (1999) 4337.
8. *Aguilar, R., Martínez-Guerra, R., Poznyak A.,* The Chem. Eng. J. **87** (3) (2002) 351.
9. *Aguilar-López R., Martínez-Guerra R.*, Chemical Engineering Journal **108** (2005) 139.
10. *Eckefelder, W.*, Idustrial water pollution control. Environmental Engineering Series, McGraw Hill, 3rd Ed. 2000.
11. Ministry of the Environment, Natural Resources and Fisheries SEMARNAP (1997). NOM001-ECOL-1996, México D. F., México.
12. *Raltowsky, D. A., Ross, T., McMeekin, T. A., Olley, J.*, Journal of Appl. Bacteorology **71** (1991) 452.
13. *Heitzer, A., Hans-Peter E., Reichert, P., Hamer, G.*, Appl. Environ. Microbiol. **57** (1991) 2656.
14. *Zwieterig, M. H., De Koos, J. T., Hasenack, B. E., De Wit, J. C., Van 'T riet, K.*, Appl. Environ. Microbiol. **57** (1991) 1094.
15. Standard Methods for the Examination of Water and Wastewater, 19th edn, APHA/AWWA/WEFederation, Washington DC, USA, 1995.
16. *Ramalho, R.*, Wastewater Treatment. Ed. Reverté. Spain. 1999.
17. *Maqueda, M. A. M. Martínez Sergio A., Narvaez, D. D., Rodriguez R. M. G., Aguilar L. R., Herrero V. V. M.*, Water Science & Technology **53** (11) (2006) 135.
18. *Dochain, D., Vanrolleghem, P.*, Dynamic modeling and estimation in Wastewater treatment process. IWA Publishing, U. K. 2001.
19. *Olsson, G., Newell, B.*, Wastewater treatment systems. Modeling, Diagnosis and Control. IWA Publishing. U. K. 2001.
20. *Martínez, Sergio A., Rodríguez, Miriam G., Morales, Miguel A.*, Int. J. of Chem. R. Eng. **3** (2005) A62.
21. *Lee, L. P., Sullivan, G. R.*, Computers and Chemical Engineering **124** (1988) 573.
22. *Signal, P. D., Lee, P. L.*, Chemical Engineering Communications **115** (1992) 35.
23. *Daniel R. Omstead* (Editor), Computer Control of Fermentation Process. CRC Press Inc. Florida, 1990.
24. *Schuler, H., Schmidt, C.*, Chem. Eng. Sci. **47** (1992) 899.
25. *Elicabe, G. E., Ozdeger, E., Georgakis, C.*, Ind. Eng. Chem. Res. **34** (1995) 1219.