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MODELING STOCK MARKET VOLATILITY IN CROATIA

ABSTRACT

The aim of this paper is modelling short-term volatility at the main Croatian stock market, Zagreb Stock Exchange. We present GARCH models following the hypotheses that volatility in the short-run depends on the volume of traded stocks and that volatility of the Zagreb Stock Exchange (ZSE) main index CROBEX is influenced by the situation on the international financial markets; NYSE Stock Exchange indices and European Stock indices. We have assessed an influence of the American DJIA and NASDAQ, as well as European DAX and FTSE indices on CROBEX. On the bases of the parameter estimates of the proposed GARCH type models, the objective is to investigate which market – American or European – has a stronger impact on CROBEX index.

Keywords: *volatility, GARCH model, Zagreb Stock Exchange, stock index*

1. Introduction

The objective of our study was to explain the price-volume relationship that exists on the main Croatian stock exchange market and to analyse the interdependency between the given market and other international stock exchange markets. The price-volume relationship implies the correlation between stock prices (indices) and volume of trading. The existence of price-volume relationship has been widely recorded in the economic literature for over four decades. However, there is little theoretical justification for the presence of this phenomenon. Dependence between stock markets in different countries has been tested for years; see for example: King and Wadhwani (1990), Engle and Susmel (1993), Brzeszczynski and Welfe (2004) and Wdowinski and Zglinska-Pietrzak (2005). This relationship has been studied through analysing correlations between individual indices representing different markets.[1]

This paper presents GARCH models of the Zagreb Stock Exchange (ZSE) main index CROBEX. We assume the following hypotheses: 1) the volatility in the short-run depends on the volume of traded stocks and on its realizations in the past, 2) the volatility of the

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[1] The search for causal dependencies existing on any financial market usually focuses on two directions: 1) the price-volume relationship present on the local market and 2) the interdependency between the given market and other international stock exchanges. The price-volume relationship analysed as the correlation between stock prices (indices) and volume of trading is one of the most extensively explored causal dependencies in the field of high frequency data financial econometrics. The reason for that is data availability and congruence of different frequencies of observations for both variables. The globalization of financial market leads to transmitting the "shocks" among major international exchanges and produces a side effect to emerging markets. It has been proved that the behavior of the two USA indices: DJIA and NASDAQ Composite is the primary source of international capital movements and resulting stock price changes in the short-run, (Brzeszczynski and Welfe, 2004).

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CROBEX index is influenced by the situation on the international financial markets. We tried to explain the influence of the NYSE Stock Exchange indices (DJIA and NASDAQ) and European Stock indices (DAX and FTSE) on the Zagreb Stock Exchange index CROBEX. On the bases of the estimates of the proposed GARCH type models we analyse which foreign stock market affects Croatian stock exchange index and to what extent.

The paper is organised as follows. The next section presents the methodology employed in the study. Section 3 gives data description and presents empirical results. The final section concludes.

2. Methodology

Many financial time series exhibit volatility clustering. It implies a strong autocorrelation in squared returns or autoregressive conditional heteroscedascity. As a consequence, the least squares estimators are still unbiased but inefficient. The estimates of the variances are biased, thus invalidating the tests of significance and obtained results are dubious. In order to resolve the problem, as a method of estimation we employed GARCH type of model proposed by Bollerslev (1986).

The dependent variable, the input to the GARCH volatility model, is always a return series. The returns are assumed to be generated by a stochastic process with time-varying volatility. GARCH model consists of two equations, the conditional mean and conditional variance equation. A standard linear generalized autoregressive conditional heteroscedastic model GARCH(p,q) is defined:

$$r_t = \boldsymbol{x}_{(k)t}\alpha_{(k)} + u_t, \qquad u_t \approx N(0,\sigma_t^2) \tag{1}$$

$$u_t = \nu_t\sqrt{h_t}, \qquad \nu_t \approx N(0,1) \tag{2}$$

$$h_t = \gamma_0 + \sum_{i=1}^{p}\gamma_i u_{t-i}^2 + \sum_{j=1}^{q}\phi_j h_{t-j} \tag{3}$$

$\boldsymbol{x}_{(k)t}$ is a vector of k explanatory variables and $\alpha_{(k)}$ is a vector of the structural parameters. In the study the dependent variable is a rate of return, $r_t = \ln(y_t) - \ln(y_{t-1})$ where y_t is the corresponding financial market price, *i.e.* stock or currency price. It follows that:

$$E(r_t) = E(r_t|r_{t-1}) = \boldsymbol{x}_{(k)t}\alpha_{(k)}, \quad \text{and } Var(r_t|r_{t-1}) = h_t \tag{4}$$

The model can be extended by further expansion of the conditional variance function (3) or by expansion of the mean equation (1). The simplest and the most common specification for GARCH volatility models is GARCH(1,1) and it is wildly used in empirical studies conducted for different markets and instruments, (Campbell et al., 1997). The conditional variance function for GARCH(1,1) model is defined:

$$h_t = \gamma_0 + \gamma_1 u_{t-1}^2 + \phi_1 h_{t-1} \tag{5}$$

The deterministic part in the regression equation (1) can be extended, too. Additional explanatory variables, which have causal effects on the dependent variable, can be incorporated in the mean equation. If variables are properly defined, a large proportion of the volatility can be explained in such a way and desired properties of the estimators can be achieved.

The objective of our analysis was to model the returns of the financial instruments and to determine the factors which represent the capital flows within different segments of the financial market. Thus, when defining the models we focus on expanding the mean equation rather then concentrating on the best conditional variance function in the ARCH-class model. In the study, GARCH model was used for correcting the estimators and for obtaining better estimates of the parameters in the mean equation.

In the analysis we employed two types of GARCH models, the factor and the predictive GARCH model, (Brzeszczynski and Welfe, 2004). The factor GARCH model does not posses a dynamic structure. It is assumed that the relationship which the model must capture is simultaneous and the model does not have any lagged variables nor incorporate any intertemporal relationships in its structure. On the other hand, the predictive GARCH model is defined on the bases of lagged variables. However, dynamic or non-dynamic specification of the mean equation does not influence the estimation of the GARCH-type model as long as the ARCH effects are present.[2]

3. Empirical results

The database used in our study consists of daily quotations of indices during the period from Jan 4, 2000 to Dec 31, 2004. It includes the Zagreb Stock Exchange (ZSE) main index CROBEX, the volume of traded stocks at ZSE, two European indices: German main index Deutscher Aktienindex (DAX30) and British Financial Times Stock Exchange index (FTSE100) and two American indices: Dow Jones Industrial Average (DJIA) and NASDAQ Composite. The data for CROBEX and ZSE volume of trade come from the Zagreb Stock Exchange and the values for international indices are obtained from Bloomberg. To make all the time series comparable, the database was adjusted to incorporate non-trading days (such as national holidays) in corresponding countries.

We started the analysis of CROBEX index investigating ARCH effects in the initially proposed models. In all cases very strong ARCH effects were detected with the values of the $TR^2 : \chi^2$ test statistics being significant at the 0.0001 significance level. Thus, the GARCH(1,1) specification has been assumed in all the models proposed.

The mean equation in the factor models includes only those international indices which do not overlap in time with the CROBEX index. The stock exchanges in Germany and England are located in the same geographical region as the Zagreb Stock Exchange and have corresponding trading-day hours. The working hours in Frankfurt and London coincide with those in Zagreb. Thus, only the European indices were included in the mean equation of the factor models. The factor GARCH(1,1) model is defined as follows:

$$r_t^{CROBEX} = \alpha_0^F + \alpha_1^F r_t^{VOL} + \alpha_2^F r_t^{DAX30} + \alpha_3^F r_t^{FTSE100} + u_t \, , \tag{6}$$

where r_t^{VOL} refers to daily returns (daily percentage changes) of the volume of trade. r_t^{DAX30} and $r_t^{FTSE100}$ denote respective international indices.

We started the analysis by exploring the price-volume relationship. In the first model (Model F1), the mean equation includes only the returns of trading volume r_t^{VOL} as an

[2] The difference exists if the model is used for prediction purposes. The factor GARCH model cannot be applied in such cases.

explanatory variable. Different lengths of the rate of return, from one to five days, were tested. The best results were obtained by one-day-long return indicating that the change of the trading volume at ZSE in consecutive days constitutes relevant information in relation to future price changes. Such definition of the variable was used in all the proposed models. For all others variables only one-day-long returns were calculated and used in the analyses.

The initial model was extended to models F2 and F3. The models incorporate r_t^{DAX30} and $r_t^{FTSE100}$, the two European indices DAX30 and FTSE100 being the indicators from the biggest European stock exchanges as measured by their capitalization. Finally, in order to eliminate the influence of any control variable and analyse the impact of the European indices on CROBEX only we additionally estimate Model F4 and Model F5. The models were defined without the volume of trade variable and include only international indices.

The correlation between DAX30 and FTSE100 in investigated sample is relatively high (0,75). To avoid the problem of multicollinearity and its negative consequences it included only one European index in all the proposed models. The parameters' estimates are given in Table 1. From the results we can reach several conclusions.

Table 1.

Estimates of the factor GARCH(1,1) models (sample 4.1.2000.-31.12.2004.)

$$r_t^{CROBEX} = \alpha_0^F + \alpha_1^F r_t^{VOL} + \alpha_2^F r_t^{DAX30} + \alpha_3^F r_t^{FTSE100} + u_t$$

Model	α_0	α_1	α_2	α_3	γ_0	γ_1	ϕ_1
Model F1	0.060 (1.708)	0.001 (2.287)	-	-	0.090 (8.228)	0.063 (6.367)	0.890 (74.118)
Model F2	0.061 (1.792)	0.001 (2.458)	0.102 (4.729)	-	0.094 (8.715)	0.067 (6.286)	0.883 (71.662)
Model F3	0.060 (1.822)	0.001 (2.551)	-	0.181 (6.164)	0.103 (9.551)	0.068 (6.328)	0.876 (71.760)
Model F4	0.061 (1.787)	-	0.100 (4.727)	-	0.093 (8.411)	0.065 (6.206)	0.885 (70.153)
Model F5	0.060 (1.809)	-	-	0.179 (6.106)	0.104 (9.584)	0.067 (6.298)	0.876 (70.520)

(z-statistics in parentheses)

First, the volume of trade proves to be significant explanatory variable. The estimates of its parameter are statistically significant and are stable regardless of the other control variables included in the equation. However, its value is very low (at the 0,001 level). The two European indices DAX30 and FTSE100 significantly influence the variability of CROBEX index. Their parameters are positive and highly significant. This indicates the existence of contemporaneous co-movements effects of the CROBEX and the two main European stock markets over the same trading day. The parameter for FTSE100 is roughly by half higher compared to DAX30. The GARCH parameters are strongly significant in all models while the constant proves to be not significant in all proposed models.

In order to analyse the relationship between CROBEX and the American market we had to acknowledge the fact that the information about the values of the American indices had to be known before the trading starts at the Zagreb Stock Exchange. Therefore, the predictive GARCH models were defined on the basis of explanatory variables lagged by one day. More lags showed to be not significant in almost all the models tested. Thus, the mean equation of the predictive GARCH model is defined:

$$r_t^{CROBEX} = \alpha_0^P + \alpha_1^P r_{t-1}^{CROBEX} + \alpha_2^P r_{t-1}^{VOL} + \alpha_3^P r_{t-1}^{DAX30} + \alpha_4^P r_{t-1}^{FTSE100} + \alpha_5^P r_{t-1}^{DJIA} + \alpha_6^P r_{t-1}^{NASDAQ} + u_t \quad (7)$$

However, the correlation coefficient between the DJIA and NASDAQ of 0,69 is relatively high in the investigated sample. For other pairs of variables, one American and one European index, it equals 0.57 (for DAX30 and DJIA), 0,52 (for DAX30 and NASDAQ), 0,43 (for FTSE100 and DJIA) and 0,36 (for FTSE100 and NASDAQ), which is also relatively high. Therefore, to exclude the possibility of multicollinearity we included only one lagged European index (either DAX30 or FTSE100) or one lagged American index (either DJIA or NASDAQ) in all tested models.

The Table 2 presents the estimation results for eight proposed models, P1 to P8. The first four models include different international index, r_{t-1}^{DAX30}, $r_{t-1}^{FTSE100}$, r_{t-1}^{DJIA} or r_{t-1}^{NASDAQ}. Models P5 to P8 have the same specification extended by adding lagged dependent variable r_{t-1}^{CROBEX} aiming at improving their dynamic structure.

Table 2.

Estimates of the predictive GARCH(1,1) models (sample 4.1.2000.-31.12.2004.)

$$r_t^{CROBEX} = \alpha_0^P + \alpha_1^P r_{t-1}^{CROBEX} + \alpha_2^P r_{t-1}^{VOL} + \alpha_3^P r_{t-1}^{DAX30} + \alpha_4^P r_{t-1}^{FTSE100} + \alpha_5^P r_{t-1}^{DJIA} + \alpha_6^P r_{t-1}^{NASDAQ} + u_t$$

Model	β_0	β_1 CROBEX	β_2 VOL(-1)	β_3 DAX30	β_4 FTSE100	β_5 DJIA	β_6 NASDAQ	γ_0	γ_1	ϕ_1
Model P1	0.061 (1.690)	-	-0,001 (-2,077)	-	0,027 (0,936)	-	-	0.077 (6,630)	0.058 (6.255)	0.902 (74.438)
Model P2	0.061 (1.677)	-	-0,001 (-2,061)	0,024 (1,210)	-	-	-	0.078 (6,820)	0.058 (6.288)	0.901 (74.530)
Model P3	0.058 (1.599)	-	-0,0001 (-2,109)	-	-	0,067 (2,248)	-	0.080 (6,776)	0.057 (6.276)	0.900 (74.146)
Model P4	0.060 (1.646)	-	-0,001 (-2,156)	-	-	-	0,051 (2,847)	0.082 (6,990)	0.059 (6.264)	0.898 (73.632)
Model P5	0.059 (1.611)	-0,014 (-0,453)	-0,001 (-2,063)	-	-	0,068 (2,263)	-	0,080 (6,685)	0,057 (6,183)	0,901 (73,780)
Model P6	0.061 (1.664)	-0,016 (-0,511)	-0,001 (-2,106)	-	-	-	0,051 (2,863)	0,081 (6,693)	0,059 (6,200)	0,900 (73,228)
Model P7	0.062 (1.700)	-0,017 (-0,552)	-0,001 (-2,008)	0,026 (1,275)	-	-	-	0,078 (6,728)	0,058 (6,185)	0,901 (74,036)
Model P8	0.062 (1.710)	-0,017 (-0,545)	-0,001 (-2,024)	-	0,020 (1,014)	-	-	0,076 (6,534)	0,057 (6,137)	0,903 (73,939

(z-statistics in parentheses)

The obtained results show that the lagged explanatory variables representing the returns of American indices are significant while the lagged returns of European indices proved not to be significant. All estimated parameters are positive which means that the direction of the movements of the American stock exchange indices is transmitted from the time (t-1) and provides signals for the direction of change of the CROBEX index in time t.

DJIA has a higher impact on CROBEX with the value of 0,07. The relationship between NASDAQ and CROBEX is also strong with the value of the parameter of 0,05. The lower values possess the parameters related to European indices (from 0,02 to 0,03). This indicates that the signals coming from the American market are stronger than those from the European market.

Although the volume of trade lagged by one day proves to be significant at 5% level its value is negative and almost zero. Lagged dependent variable r_{t-1}^{CROBEX} is not significant in any of the proposed models which indicates that the "internal'' relationships, such as autoregressive dependences *etc*., are not so important compared to "external" signals.[3] The GARCH parameters are significant in all analysed predictive models as in the case of factor models.

4. Conclusion

Analysis of short-term volatility at the Zagreb Stock Exchange was performed on the basis of factor and predictive GARCH class of models. Initially, models of the CROBEX index were defined and tested for ARCH effects. Since in all proposed models very strong ARCH effects were detected, the GARCH(1,1) specification was applied in all models in order to correct the estimators and to obtain better estimates of the parameters in the mean equation.

When defining factor GARCH models it was assumed that the relationship which the model must capture was simultaneous and models were defined without dynamic structure. The volume of trade proves to be significant explanatory variable in all models but its value is almost insignificant. Additionally, the factor models include international indices which do not overlap in the time with the CROBEX index. The two European indices DAX30 and FTSE100 prove to significantly influence the variability of CROBEX index which indicates the existence of contemporaneous co-movements effects of the CROBEX and the two main European stock markets over the same trading day.

The predictive GARCH models include only explanatory variables lagged by one day. Defining the mean equation on the basis of lagged variables enables us to know the information about the values of explanatory variables before the trading starts at the Zagreb Stock Exchange. The lagged explanatory variables representing the returns of American indices are significant in the proposed models and all estimated parameters are positive. This leads to the conclusion that the direction of the movements of the American stock exchange indices is transmitted from the previous day and provides signals for the direction of change of the CROBEX index in the present. Lagged dependent variable is not significant in any of the proposed models and indicates that the "internal" relationships are not so important compared to "external" signals.

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[3] The same conclusion was obtained for the case of Poland in the study of Brzeszczynski and Welfe, (2004).

MODELIRANJE VOLATILNOSTI TRŽIŠTA DIONICA U HRVATSKOJ

SAŽETAK

Cilj ovog rada je modeliranje kratkoročne volatilnosti na glavnom tržištu dionica u Hrvatskoj, Zagrebačkoj burzi. Koristili smo se GARCH modelom slijedeći hipotezu da kratkoročna volatilnost ovisi o volumenu trgovine vrijednosnicama te da na volatilnost CROBEXA, glavnog indeksa Zagrebačke burze, utječe situacija na međunarodnim financijskim tržištima; indeksi NYSE i Europskog tržišta dionica. Ustanovili smo da na CROBEX utječu američki DJIA i NASDAQ te europski DAX i FTSE. Na osnovi procjeniteljskih parametara predloženih tipova GARCH modela, cilj je istražiti koje tržište – američko ili europsko – ima jači utjecaj na CROBEX indeks.

Ključne riječi: *volatilnost, GARCH model, Zagrebačka burza, indeks dionica*