

STEADILY CUBIC: A SQUARED ODYSSEY

Wittgenstein's cube–examples

Kristijan Krkač

<https://doi.org/10.32701/dp.21.1.5>

1. Life in a day *of a cube*

This paper is about cubes. We live in a world overcrowded with cubes and cuboid forms. Cubes are a natural phenomenon (in the primitive cubic crystal systems like pyrite, and in the multi–element compounds like caesium chloride and rock–salt structure) and a man–made thing as well (stone–cubes are still present in houses and other architectural designs, and it seems illustrative to mention that Haus Wittgenstein is also cubic or at least cuboid in its basic form, as shown in Illustration 1). Take a look at cities, streets, buildings, houses, rooms, windows, furniture, home appliances and packaging. Cubes are the topic in poetry, novels, paintings, sculptures, films, TV series and computer games. Given that cubes and cuboid forms are more frequent as a man–made than as a natural structures, besides many other symbolic uses, they are often taken as a signal of order, rationality and intelligence.



Illustration 1: Left: Pyrite cubic crystals on marlstone, from Navajún, Rioja, Spain. Size: 95 mm x 78 mm. Main crystal: 31 mm on edge. Mass: 512 g. (2009, URL: <https://commons.wikimedia.org/wiki/File:2780M-pyrite1.jpg>), Right: Haus Wittgenstein (2003, URL: <https://www.nextroom.at/building.php?id=2338>)

* Prof. Kristijan Krkač, PhD, Zagreb School of Economics and Management, Email: kristijan.krkač@gmail.com

This paper is about cubes that Wittgenstein was drawing and describing in his works. Especially about two cubes, the one in TLP and the other in PPF (ex PI II). He never referred either to Necker’s paper (Necker 1832), to the Necker’s cube, nor did the likely sources in which he may have found the illustration of the cube (Jastrow 1900). So, it is perhaps misleading to call his TLP cube the Necker’s cube. Since we have no evidence of the source of the TLP cube, while the probable source of the PPF cube is his own TLP cube and Jastrow’s cube, it also seems possible that he *reinvented* the TLP cube.

- (1) All we can be sure of concerning Wittgenstein’s cubes is that he knew of puzzle pictures (NB 1979 p. 28, 9.11.14.), that drawings of cubes appear in many works (BT, BB, Z, PI and RPP I), that there are many mentions and remarks on such multi-aspectual cubes, that the drawing of the Necker’s cube model appears in PT (PT 1971 6.0051) and in TLP (TLP 1974 5.5432) with small differences in drawings, that the different drawing of the cube appears in PPF (PPF 2009 116) and in RPP I (RPP I 1980 9) with some small differences in texts and which seems to refer to the TLP cube.

Concerning actual illustrations TLP has 9 tables, schemes and drawings on 89 pages, while PI has 18 of them on 240 pages (7 in PI and 11 in PPF). PI has much more textual examples of visual phenomena, scenes, and images then TLP which means that PI is more pictorial then TLP. However, contrary to this, TLP is actually more illustrated then PI which doesn’t seem to be the first impression. Before we turn to Wittgenstein’s cubes it seems appropriate to describe the Necker’s cube and to supply a note on recent literature on comparison between cubes in the TLP and the PPF. The general structure of the argument will be the following (as shown Table 1).

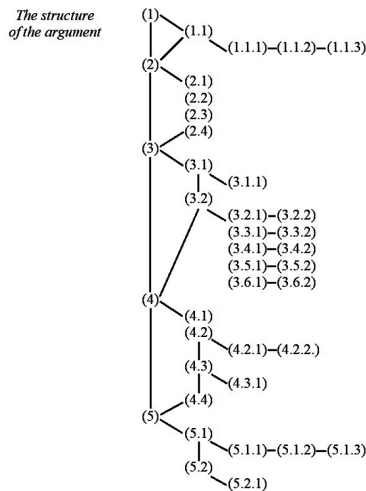


Table 1: The general structure of the argument given by the relations of the major and auxiliary remarks listed here from (1) to (5).

1.1. *A note on scientific observations and research of the Necker's cube*

A short description of the Necker's cube will be supplied here as a kind of prescript. A part of the description and the explanation by Necker from his 1832 paper is the following.

“The object I have now to call your attention to, is an observation which is also of an optical nature, and which has often occurred to me while examining figures and engraved plates of crystalline forms: I mean a sudden and involuntary change in the apparent position of a crystal or solid represented in an engraved figure. (...) I have been a long time at a loss to understand the, reason of the apparently accidental and involuntary change which I always witnessed in all sorts of forms in books of crystallography. The only thing I could observe was, that at the time the change took place, a particular sensation was felt in the eye (for it takes place as well when seen with only one eye, as with both eyes), which proved to me that it was an optical, and not merely as I had at first thought a mental, operation which was performed. After, however, a more attentive analysis of the fact, it occurred to me, that it was owing to all involuntary change in the adjustment of the eye for obtaining distinct vision.” (Necker 1832, pp. 336–337)

The phenomenon of this optical illusion was known for centuries before the Necker's observations. For instance, this illusion was found as a mosaic on the floor of one of the houses in the Pompeii archeological site that was preserved after the AD 79 eruption of the Mount Vesuvius. However, Necker's paper seems to be the first recorded scientific publication as an attempt of a scientific description and explanation of the phenomenon. Perhaps there were earlier similar descriptions and explanations, but they are unknown.

The Necker's cube is an optical illusion, i.e. an illusion caused by vision characterized by a visual percept that 'looks' different from reality. It is a simple wireframe drawing of a cube with no sensory cues (e.g. depth, contrast or color). Therefore, the Necker's cube (as shown in Illustration 2) is interpreted by normally sighted people to have either the lower-left as its front side and the upper-right as its back side or *vice versa*.

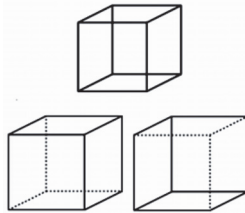


Illustration 2: The Necker cube. Upper cube is the actual drawing, while lower cubes are two possible ways of seeing the drawing, i.e. either the lower-left side or the upper-right side as being its front side, but not both simultaneously (illustration by the author).

The Necker's cube is also an ambiguous drawing that can remain in one stable state (of perception) and then change to another stable state (lower cubes in Illustration 2). This phenomenon is called bistable or multistable perception. The opposite of such ambiguous drawings (like the Müller-Lyer arrows) are impossible objects (such as the impossible cube that represents both stable states of the Necker's cube). The lower-left state of perception in Illustration 2 seems to be more frequent because people view objects mostly from above (Troje and McAdam 2010) and because they view objects from left to right (Khan and Crawford 2001). They tend to see the upper cube as the lower left cube with the lower-left side as its front side first. People can naturally switch from seeing one to another state (of perception) spontaneously or by focusing on particular parts of the drawing (mostly to the lower-left or to the upper-right parts of it).

Scientific explanations of the phenomenon of change of interpretation/perception of the Necker's cube roughly fall in two groups; bottom-up and top-down explanations. (Kornmeier and Bach 2005, 2012) Authors write about a 'mental switch' between seeing different appearances of the drawing and this is reminiscent of Wittgenstein's 'aspect-change' (in his early philosophy perhaps taken from Hertz 1956 and Boltzmann 1974 and in his later from Jastrow 1900 and mostly from Köhler 1929).

"Most hypothetical explanations about the neural processes underlying spontaneous perceptual reversals of ambiguous figures fall into two classes, emphasizing either bottom-up, or top-down factors. Bottom-up approaches assume the perceptual reversal results from passive adaptation early in the visual stream. (...) Top-down approaches emphasize attentional or expectational factors, acting in a centrally governed active decision process near awareness and consequently later in the visual hierarchy." (Kornmeier and Bach 2005, p. 955)

Both approaches have their well-known defenders and are backed by measurements, studies, and effects of experience, learning and cognitive states of the subjects. What is interesting is that experiments showed that "disambiguation of ambiguous figures seems to occur already 200–300ms before perceptual awareness is established". (Kornmeier and Bach 2005, p. 959)

What seems important here is that there is no reliable scientific explanation that is backed by solid evidence and that would dismiss all other rival explanations. In other words, analyzing Wittgenstein's standpoint on this matter is still puzzling from scientific and from conceptual point of view.

1.2. *A note on previous research of Wittgenstein's TLP and PPF cubes*

Concerning the topic of the present research there are only few texts that analyze Wittgenstein's drawings of the cube and accompanied remarks from TLP (TLP 1974 5.5432) and even fewer that do this in comparison with the cube from PPF (PPF 2009 116).

Among older works there is the paper by J. Hintikka's and M. B. Hintikka's 'Ludwig looks at the Necker Cube: The Problem of 'Seeing As' as a Clue to Wittgenstein's Philosophy' (Hintikka and Hintikka 1996, pp. 179–191). Their paper is important mostly because it suggests the comparison of the TLP-cube and the PI-cube on the same grounds.

"In his belatedly (1984) published book *Theory of Knowledge*, which Russell wrote in 1913 but left unpublished because of Wittgenstein's criticisms, Russell extends this semantical theory to complex propositions and more generally to questions of logical form. In order to understand, e.g., the unanalyzable relational proposition 'aRb', one has to be acquainted with the entities a, R, and b. But this is not enough, for it does not, e.g., distinguish 'aRb' semantically. Hence, according to Russell's onetime theory, one must also be acquainted with a fourth entity, the logical form of 'aRb'. (...) The operative feature of the situation Wittgenstein envisages here [5.5423] is that one and the same configuration of physical objects can give rise to two different perceptual experiences. Wittgenstein tries to explain this by saying that the two experiences are perceptions of two different facts. Since facts are configurations of objects and since there is only one physical configuration here, Wittgenstein is obviously assuming that the facts he is talking about are configurations of phenomenological (perceptual) objects. Thus 5.5423 is unmistakably predicated on the assumption that Wittgenstein's objects in the *Tractatus* are phenomenological objects, just as we have argued. This simple and perhaps rather obvious analysis of *Tractatus* 5.5423 also provides a key to Wittgenstein's discussion of 'seeing as' in such later writings as the *Philosophical Investigations*." (Hintikka and Hintikka 1996, pp. 179, 181–2)

Among newer works there is M. ter Hark's paper 'Aspect perception in the *Tractatus* and the *Philosophical Investigations*' (ter Hark 2015, pp. 165–182). Ter Hark discusses the paper by Hintikka and Hintikka and few older texts and comes up with a solution that resolves the supposed paradox of seeing the same and the different.

"It is this intersection of two uses of words, two language games, which Wittgenstein failed to recognize in the *Tractatus*. It is also this failure to see that two different uses of concepts, e.g. 'organization' in the transitive and intransitive sense, intertwine which Wittgenstein recognizes in Köhler's theory of perception." (ter Hark 2015, p. 179)

Since there are only few works that discuss TLP-cube and TLP-cube compared to PPF-cube and since such comparison seems intriguing for several philosophical reasons, in what follows some remarks on these illustrations and texts will be made. Since there are significantly more comments of PPF-cube connected to other illustrations, mostly to *The Duck-rabbit head*, and with broader issues, such as seeing-as and perception, these texts will not be discussed, but mentioned if necessary.

(1.1) The limited number of analyses comparing the TLP and the PPF cubes in philosophical papers and books on Wittgenstein seems at least strange if the PPF-cube belongs to Wittgenstein's corrections

that he made in PI concerning ‘grave mistakes’ of TLP in this case of the TLP–cube (PI 2009, p. 4).

- (1.1.1) The probable cause of this lack of literature may lie in the fact that there is a general standpoint which says the two cubes are connected with the two different issues, namely, the TLP–cube to the issue of Russell’s standpoints on complexes and facts, and the PPF–cube to the issue of interpretation of ambiguous drawings.
- (1.1.2) As opposed minority opinion to such major standpoint, some philosophers (among which are Hintikkas and ter Hark) think there is a common ground between these illustrations, accompanied texts and their places in the contexts of the basic ideas of both Wittgenstein’s works. Remark (1) supplies the reference for the second opinion, since it seems that Wittgenstein knew and was interested in ‘puzzle pictures’ during his whole philosophical career, and in the PT, TLP as well as in the PI, PPF and RPP.
- (1.1.3) Nevertheless, if one analyses them together in the contexts of the works they belong to and compares them, this doesn’t contradict the possibility that both cubes are more closely connected than it seems. Remarks supplied hereafter generally aim at supporting the exegetical hypothesis mentioned in (1.1.2).

2. Cubes *and fascination*

- (2) Wittgenstein, as a thinker sensitive to pictorial aspects of thought and philosophy, frequently used tables, figures, schemes and drawings in his writings in order to illustrate examples and/or ideas or ‘visual images that occurred to him’ (PPO 2003 123).

Wittgenstein was fond of images, drawings, illustrations and pictures perhaps partly because he was thinking mostly in pictures (which one can only speculate about), partly because of his education as engineer (Hamilton 2001) and partly because his concepts of image and picture were aesthetically and philosophically important in both his early and later philosophy (Janik and Toulmin 1973). Some of his illustrations were borrowed from others (like Necker’s Cube or Jastrow’s Duck–rabbit head), while others he re/invented (like truth tables, smiling faces, drawing of patents or drawings of various mechanical models). While he used the majority of such images only in some periods of his philosophy, he used only a few throughout the whole career. The most frequent illustrations are dots, lines, arrows, various grids, circles, squares, pentagrams, hexagrams, pyramids, cubes etc.

- (2.1) Wittgenstein used an illustration of a cube continuously from his early to his later works, sometimes in different ways in order to illustrate (some argue radically) different ideas. These illustrations

are perhaps more significant than others frequently used since they seem to illustrate his foremost early and later ideas.

Additionally, illustrations of a cube are the only illustrations that were and still are of similar interest to scientists, artists, and philosophers (the scientific research on Necker's cube includes dozens of experiments with the drawing itself, light, eye movement, seeing, neural processes etc. and cannot even be summarized here, but it needs to be taken into account concerning all of Wittgenstein's cubes, see section 1.1).

(2.2) The TLP-cube (TLP 1974 5.5432) seems to illustrate early ideas about meaning, proposition and fact, while the PPF-cube (PPF 2009 116) as the later correction seems to criticize the earlier idea illustrated by the TLP-cube and to introduce new one. However, this is only a hypothesis since there is no conclusive evidence.

The following remarks from PI seem to confirm this point, since we cannot find any other illustration so continuously and purposely used in two different ways concerning significant ideas of his philosophy (we can find other frequently used illustrations and even some descriptions and explications, but these are not as closely connected to his major ideas as the example of a cube accompanied with the illustration).

I have purposely so chosen the example [of a cube] that it is quite easy to imagine a method of projection according to which the picture does fit after all. The picture of the cube did indeed suggest a certain use to us, but it was also possible for me to use it differently. (...) Then what was the nature of my mistake — the mistake one would like to express by saying 'I thought the picture forced a particular use on me?' How could I think that? What *did* I think? Is there a picture, or something like a picture, that forces a particular application on us; so that my mistake amounted to a confusion? (...) What was the effect of my argument? It called our attention to (reminded us of) the fact that there are other processes, besides the one we originally thought of, which we should sometimes be prepared to call 'applying the picture of a cube'. So our 'belief that the picture forced a particular application upon us' consisted in the fact that only the one case and no other occurred to us. 'There is another solution as well' means: there is something else that I'm also prepared to call a 'solution', to which I'm prepared to apply such-and-such a picture, such-and-such an analogy, and so on. (...) Can't I now imagine different applications of this schema too? — Well, yes, but can't an application come before my mind? It can: only we need to become clearer about our application of this expression. (PI 2009 139–41)

(2.3) So, it seems that he used the same example in both cases purposefully, since the TLP-cube is closely connected to the idea of a complex and the form aRb which he criticized in the middle period and later on in PI and PPF by use of the PPF-cube. (The issue of the difference between applications of a picture given in advance and applications of a picture determining the picture itself will be discussed later on.)

However, things are more complicated than this, since it seems important to differentiate *between a mentioning the word and discussing it* on the one hand and *illustrating the example and discussing it* on the other.

First, the word cube is mentioned more than 100 times in his works and discussed in detail at least 20 times. A cube as an example is connected to the TLP concepts of a proposition and of a complex, and to the PI concepts of a language–game and of an aspect–change. An illustration of a cube appears in different works besides the most famous illustrations in the TLP (1974 5.5432) and the PPF (2009 116), e.g. in BT (2005 39), Z (1981 249), BBB (1969 163), and the one from PPF (2009 116) repeats in RPP I (1980 9). In Z there is besides two cubes the only attempt to draw a four–dimensional cube (as shown in Illustration 3).

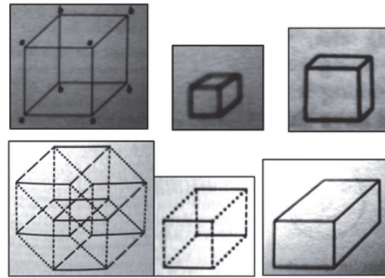


Illustration 3: Five examples of Wittgenstein's cubes. Upper row from the left to the right: TLP, BT, BB Lower row: Z, and PI (which repeats in RPP I).

Finally, the word cube and the discussion accompanied with an illustration appear only 10 times given that some remarks and illustrations are repeated in various works (e.g. in the PPF and the RPP).

(2.4) These *remarks with illustrations* are important because illustrations aren't just visual aspects of his textual description of examples. They add something to the text, namely, one can actually see what he meant by the text (and that the text was perhaps only an afterword to the 'image that occurred to him'). Now, what, if anything, does an illustration add to the description of an example of a cube?

In the following sections, the TLP–cube and the PPF–cube texts will be described in the context of the basic ideas of the TLP and the PPF (PI included), analyzed and compared. This procedure should establish solid grounds for drawing conclusions concerning the importance of a cube–example not only in itself, but also for understanding the differences and similarities between the TLP and the PI (PPF) in terms of the object and the method in both works.

An additional reason for discussing the TLP and the PPF–cube together isn't just the fact that the PPF–cube is a purposeful later response to his own

previous TLP–cube, but also the fact that he wanted PI and TLP to be published together in order for readers to see ‘the new ideas in the light of the old ones’, i.e. his earlier ‘grave mistakes’ (PI 2009, p. 4).

Furthermore, perhaps the conclusion of this research could contribute not only to putting the same weight on Wittgenstein’s texts as well as to his drawings and sketches, but also on understanding him as a uniquely pictorially sensitive thinker and philosopher.

3. T–cube *pleasantly disturbed* by P–cube

The most famous examples are cubes in TLP and PPF (ex PI II). While in TLP (1974 5.5423, see Z 1981 249) he seems to have used Necker’s Cube from some unknown source in order to illustrate the relation of parts and complexes concerning the structure of a proposition and its relation to a fact, in PPF he used the drawing of an ordinary non–Necker cube, in fact a rectangular cuboid in order to illustrate the variety of its uses and its relation to its interpretation that follows, with a quite important note about the interpretation, i.e. ‘So we interpret it, and *see* it as we *interpret* it.’ (PPF 2009 116).

- (3) There are two cubes, the one in TLP (TLP 1974 5.5.423), and the other in PPF (PPF 2009 116) with accompanied texts (both shown in Table 2). (In the following text TLP 1974 5.5423 will be abbreviated as the T–cube and PPF 2009 116 will be abbreviated as the P–cube.)

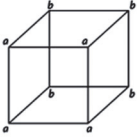
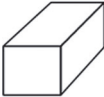
TLP 1974 5.5423	PPF 2009 116
<p>To <u>perceive a complex</u> means to perceive that its constituents are related to one another in such and such a way.</p> <p>This no doubt also explains why there are <u>two possible ways of seeing the figure</u></p>  <p>as a <u>cube</u>; and all similar phenomena. For we really see <u>two different facts</u>.</p> <p>(If I look in the first place at the corners marked <u>a</u> and only glance at the <u>b</u>'s, then the <u>a</u>'s <u>appear to be in front</u>, and vice versa).</p>	<p>One could <u>imagine the illustration</u></p>  <p>appearing in <u>several places</u> in a book, a textbook for instance. In the accompanying text, something different is in question every time: here a <u>glass cube</u>, there an <u>upturned open box</u>, there a <u>wire frame</u> of that shape, there <u>three boards forming a solid angle</u>. <u>Each time</u> the text supplies the <u>interpretation</u> of the illustration.</p> <p>But we can also <u>see</u> the illustration now as one thing, now as another. — So we interpret it, <u>and see it as we interpret it</u>.</p>

Table 2: Parallel texts of TLP 1974 5.5423 and PPF 2009 116 with underlined words and expressions that seem to be important for the illustrations, texts, and analysis of their relations.

- (3.1) It is a speculation to call the T–cube the Necker’s cube, but it is justified to call it Wittgenstein’s drawing of the Necker’s cube model, and also to call the P–cube a cube at all, because it seems to be more like a cuboid form than a cube.

(3.1.1) Concerning the context and relevance of the cited places from the TLP and the PI (as shown in Table 2), it should be mentioned that the TLP text and illustration are related to complexes (aRb), propositions, and facts (see TLP 1974 2.0201, 3.1432, 3.24, 3.3442, 4.2211), while the PPF text is related to aspect–change seeing/describing, to the morphology and grammar of use, and PPF illustration is related to many examples, descriptions and remarks on the use of the word ‘cube’ (see PI 2009 74, 139; PPF 2009 135, 173, 218–9, 236, 258). So far as the TLP (philosophical) logic become a “chapter” in PI (philosophical), it can be assumed that the T–cube and its two aspects become parts of P–cube and its grammar of many aspects.

(3.2) In PI 2009 141 (quoted in the 2nd section) it seems that an important difference is set concerning the T–cube and the P–cube, namely the one between:

(3.2.1) ‘The *picture* [of a cube] that we *visualize*’ and

(3.2.2) ‘The *application* [or *use*] that we make of this *image* [of a cube]’ (*Italics* are added; see Baker and Hacker 2009, pp. 297–302).

It seems that (3.2.1) applies to the T–cube, while (3.2.2) applies to the P–cube. So, (3.2.1) and (3.2.2) can serve as a criterion of differentiation between T–cube and P–cube. Concerning the analysis of the T–cube and the P–cube, the following seems to be clear. There is an obvious difference at the beginning of the both texts.

(3.3.1) ‘Seeing the figure’ (the T–cube) is a quite specific expression (PPF 2009 135),

(3.3.2) ‘Imagining the illustration’ is quite different from (3.2.1).

There is also very important difference between the drawings themselves.

(3.4.1) The T–cube is a geometrical drawing of cube with six square sides and two opposite sides differed by letters on the corners of each side (vertices).

(3.4.2) The P–cube is a sketch not even of a cube, rather of a cube–like body, a rectangular cuboid in fact or something like a brick or a slab. Further on, there is a difference in possibilities of the T and P–cubes.

(3.5.1) For the T–cube it is said that it has ‘two possible ways’ of *appearing*.

(3.5.2) While for the P–cube it is said that it can *appear* ‘in several places’.

Therefore, the range of possibilities is in a way broadened from the logical possibilities, which are only two in the case of T–cube, to the possibilities of real practical and imagined situations which are numerous in the case of the P–cube. Namely, in the T–cube case there is only the possibility that ‘a–side’ is in front while ‘b–side’ is in the back and *vice versa*, and in the P–cube there are much more possibilities, e.g. a glass cube, a wire frame etc.

(3.6.1) In the case of the T-cube the logic behind it and previous to it ‘explains’ two possibilities of *seeing* ‘two different facts.’ i.e. two configurations of objects (Hintikka and Hintikka 1996, p. 182).

(3.6.2) In the case of the P-cube the ‘interpretation’ of it and after it (or simultaneously with the seeing of it) *interprets* what we *imagine*. There are no previously given possible facts (states of affairs), rather many interpretations.

In fact, there are no just two possible facts concerning the T-cube, i.e. seeing the T-cube in two 3D ways, but also the third way of seeing T-cube in a 2D way and in 3D way (as shown in Illustration 4).



Illustration 4: Necker's cube seen in 3 dimensions and in only 2 dimensions, i.e. as a cube (or an open box and as a hexagon in which vertices are connected by lines.

These are some basic differences between the T-cube and the P-cube examples. They both seem to be examples of much more basic ideas of both works they belong to (TLP and PPF, PI). However, in order to explicate both examples in more detail, additional information on their origin is needed.

4. *Glittering cube cubify* yourself

Both the T-cube and the P-cube are ambiguous but for different reasons. Let us start with the T-cube. The source of Wittgenstein's T-cube is uncertain, i.e. there is no clear derivation from Necker's text and drawing (Necker 1832, pp. 329–337) to Wittgenstein's text and drawing, and that is the problem of its historical origin.

The P-cube is a “purposely chosen” drawing (PI 2009 139). It is similar to cubes and slabs of his builders from the beginning of PI (PI 2009 2–21), to other previous cube remarks in PPF, PI and earlier works after TLP. The motivation for the P-cube is not clear, but one can hypothesize that it is related to his personal architectural and engineering expertise and results. The nature of the P-cube is explicated by Wittgenstein himself. It is a drawing chosen ‘purposely’ in order to refer to the T-cube and to the cubic representation of isomorphism, formalized correspondence, or picturing between propositions and facts. (TLP 1974 2.1–2.225) However, among cubes in the later

works, especially in the PPF, there is no explicit reference to the T-Cube. (ter Hark 2015, p. 165)

- (4) So far we can hypothesize the nature and purpose of the P-cube, and its possible relation to the T-cube. However, the nature, purpose and origin of the T-cube are even less certain. Its nature and purpose are a little clearer if one relates it to the nature of complexes (aRb), and to the relation of propositions and facts (see previously 3.1.1). However, its name and origin are unclear. For one thing, in Wittgenstein's published works, Necker or Necker's cube aren't mentioned at all. However, the T-cube is an obvious illustration and description of Necker's cube model as given in Jastrow (Jastrow 1899, 1900).

So, the question is where from did Wittgenstein find the idea for T-cube, or from which author? There's no mention of such cube, of Necker or of Necker's cube in W. James's *The Principles of Psychology* (James [1890]1983). There are mentions in Köhler's *Gestalt psychology* (Köhler 1929) which was published after the TLP. It should be mentioned that James cited Jastrow in his *Principles* a lot, but it doesn't seem that any quote is related to the Necker's cube. However, in Jastrow's *Fact and Fable in Psychology* (Jastrow 1990, p. 289) the Necker cube is explained and there are illustrations. It is remarkably similar to the illustration by Necker (1832, p. 336) (as shown in Illustration 5).

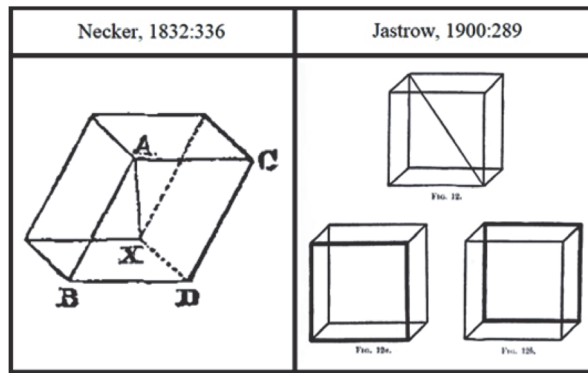


Illustration 5: Necker's cube by Necker (1832) and by Jastrow (1990).

There is obvious similarity between Necker and Jastrow's illustrations and descriptions of the phenomenon and the illusion. However, Jastrow doesn't mention Necker while supplying the illustration. Further on, there is a difference between illustrating tools in Jastrow and Wittgenstein, because Jastrow, as well as Necker, uses a diagonal line, while Wittgenstein uses letters a and b to point to different ways of seeing.

- (4.1) Wittgenstein was almost certainly influenced by Jastrow (1990), James ([1890], 1983), and by Köhler (1929) in his later works from

1929 onwards, but where does the T-cube comes from? He finished the TLP in 1918 and the illustration was already there. Even in 1914 in NB he writes ‘Puzzle pictures and seeing of situations.’ (NB 1979 p. 28, 9.11.14.). Besides the fact that we don’t know what kind of puzzle picture he had in mind, he was obviously acquainted with the concept and at least with some examples of puzzle pictures.

We also don’t know whether he read James and Jastrow in the period of studying with Russell (1911–1914). We know that in this period he read other James’ books but we don’t know whether he read James’s *Principles* or Jastrow’s *Fact and Fable* (it would be interesting to research the Jastrow’s book at the Trinity’s Library in the period 1911–1914). His first mentions of James and Jastrow are from the early 1930s, not before. However, as a student of Russell and friend of Moore, given that they discussed James before 1918 and that Russell mentions James in his works, it is certain that he heard of James and read at least *Varieties of Religious Experience* (as he wrote to Russell in a letter on 26.12.1912). (LD 2008, p. 30) For example, Wittgenstein criticized Russell’s manuscript of *Theory of Knowledge* in 1913 in which Russell discusses James, so it is clear that Wittgenstein heard of James and read some James’s works. (Russell [1913], 1992, pp. vii–xlvii)

Perhaps this search for influences on the TLP in psychology is generally mistaken, because Wittgenstein’s major influences in those days and years before the period of study were in physics, namely, in works of Hertz and Boltzmann. Wittgenstein was reading Boltzmann on some occasions previous to 1911 and during his turn to philosophy, and he even planned to study physics under Boltzmann’s supervision (after 1906). Concerning the influence of Boltzmann, J. M. Preston writes the following.

“One aspect of Boltzmann’s scientific work that has often been thought to leave its mark on the *Tractatus* is his development of the idea of phase-space. A phase-space is a multidimensional geometrical representation of a given system, each axis of which represents a degree of freedom of that system. The resulting ‘space’ charts every possible state of the system, each such state corresponding to a single point in the phase-space, and each change of state corresponding to a trajectory there.” (Preston 2017, p. 117, see Janik and Toulmin 1973, pp. 120–202)

The idea of the phase-space is well illustrated by the T-cube and its connection to complexes and logical space. However, the point of the use of the T-cube isn’t clear. Concerning this matter, Boltzmann writes the following, and the following citation is the best what can be found in Boltzmann’s writings that could influence Wittgenstein’s drawing of the T-cube.

“He [the person who is philosophically trained] is convinced that these concepts are taken straight from experience and not explicable further, so that here the now irresistible mental habit of asking for the cause and definition overshoots the mark, but still he cannot overcome a certain residual dissatisfaction that such important concepts as number or causality defy all attempts at definition.

It is as when an optical illusion fails to vanish even after one has clarified its mechanical cause. (Boltzmann 1974, p. 137) It is exactly as with ordinary sense illusions, which continue to exist even after their cause has been recognized. Hence the feeling of insecurity, the lack of satisfaction that grips the scientist when he philosophizes. Only very slowly and gradually will all these illusions recede and I regard it as a central task of philosophy to give a clear account of the inappropriateness of this overshooting the mark on the part of our thinking habits; and further, in choosing and linking concepts and words, to aim only at the most appropriate expression of the given, irrespective of our inherited habits. Then, gradually, these tangles and contradictions must disappear.” (Boltzmann 1974, p. 167)

The fact here is that neither in Hertz, nor in Boltzmann is there any direct evidence for the introduction of the T-cube. So we do not know the source of the T-cube. Since such optical illusions and multi-aspectual drawings were published in various popular magazines, it wouldn't be surprising that Wittgenstein saw something similar in a humor magazine like *Fliegende Blätter* where the *Duck-rabbit* was originally published in 1892, the same year in *Harper's Weekly* magazine, and from that illustration it ended in Jastrow's paper (Jastrow 1899) and later in his book (Jastrow 1990) where from Wittgenstein quoted it.

However, these quotes from Boltzmann seem to be connected to the purpose of introducing not only the T-cube in TLP, but of the P-cube in PPF too.

(4.2) The 'grave mistake' with the T-cube is resolved by introduction of the P-cube. It is not just that the T-cube has been seen—as differently, but the very concept of seeing—as has been seen from the new aspect.

(4.2.1) It was thought that T-cube was the answer to the question about full representation of facts by a configuration of objects in the logical space (similar to phase-space), but it was the problem, because not only was a proposition a picture and a model, but a configuration was also a picture and a model.

(4.2.2) The solution to the problem of the T-cube is the P-cube in terms that there is an illustration of the P-cube, but there is also a modeling and perspicuous presentation of many interpretations of the illustration of the P-cube which determines how we can see the illustration for various practical applications or purposes (here enters the whole story from psychology connected to the texts of Jastrow and Köhler from the early 1930s).

In order to see the T-cube not as a solution to some problem, rather as a problem in itself (leading to the basic issue of the complex aRb), one needs to see that there are not only two ways of seeing T-cube (i.e. a-a-a-a and b-b-b-b) because that was presupposed. One needs to see that if a cube is simply drawn differently, e.g. as the P-cube, this expands the space for

more than two interpretations, as well as the philosophical logic expands the space for the philosophical grammar (with logic being only a chapter of it), cases or facts for forms of life, propositions for language-games, pictures for perspicuous representations and models for an activity of modeling.

(4.3) Conceptions of the interpretation and perspicuous presentation of the whole illustration is determined not by anything within the illustration or for that matter by anything in our minds, but by an actual practice of application of the illustration.

(4.3.1) Not the problem of ‘seeing-as’ should be viewed ‘as a clue to Wittgenstein’s philosophy’ (Hintikka and Hintikka, 1996), rather the ‘lighting up of an aspect’ (PPF 2008 118). The aspect-change from the T-cube to the P-cube can be viewed as one of the symbols of Wittgenstein’s whole philosophy as being at least partly pictorial, since, after all, symbols are looked at, recognized and used for practical purposes.

Here remarks from (4) to (4.3.1) aim at the further description of the relation of the T-cube and the P-cube that was basically described in remarks from (1) to (3.6.2). Concerning the remarks (4.3 and 4.3.1), there is no interpretative dependence of the remarks in this section on the remarks of the 3rd section. It is only a possible interpretation of both cubes and their relations and is this interpretation more probable than the opposite one it is hard to decide. Further on, if the P-cube is one among many parallel issues in PI and TLP by which Wittgenstein wanted not just to point out his previous ‘grave mistake’ with the T-cube, but also what is the correct answer in the P-cube, then the whole issue can be made structurally parallel to many others in PI and PPF that seem to respond to those of TLP.

Finally, if the P-cube was the response to the T-cube as the question (or as an incorrect answer) influenced by new sources from psychology, then the same cannot be said about the T-cube, because even if general sources were recognized, the particular source of it isn’t known. We simply do not know what the actual source of the T-cube was. Any further conclusion from the given evidence and premises simply cannot be drawn. Before supplying an unscientific postscript in the absence of a solid conclusion it seems appropriate to at least ask the question that was implied throughout the whole text.

(4.4) If for a moment we suppose that all previously said is correct and important, then what does it say about not only the nature of Wittgenstein’s philosophy, but also about the nature of philosophy generally speaking, and perhaps about human thinking too?

By asking (4.4) we aren’t asking about measurable features of human or philosophical thinking, or about Wittgenstein’s philosophy, rather about the originality and uniqueness of ‘the pictorial’ that cannot be fully trans-

lated into words and *vice versa* (being entirely aware that there is no a sharp boundary between words and pictures, and that words are primarily acoustic not visual phenomena). Much cannot be said here based solely on two Wittgenstein's drawings and short quotes. However, one could imagine the T and the P-cube quotes done only via (perhaps slightly modified) drawings and only by words as two radical cases (and not mixed). Perhaps such experiment on the understanding, interpretation, and application of them (pure text and pure illustration) could throw some light and in part show us the direction in which we should look for the answer to the question in (4.4).

5. T-cube: *Don't you P-cube forget about me*

This strange philosophical issue about the T-cube and the P-cube is that just by observing them one can easily understand the basic subject matter of the TLP and of the PPF (PI), the methods used and goals aimed at in both works.

(5) In short and in rather poetic jargon it can be said that the T-cube is a *symbol of a static model* (the irony of calling it dynamic by following Hertz in TLP 1974 4.04) of a formalized language concerning the issue of the meaning and sense of a proposition, while the P-cube is a *symbol of dynamic activity of modeling* (an application of a model) and morphological perspicuous (re)presenting of the grammar of our ordinary language (echoing Goethe and Spengler) concerning the issue of the meaning as use of our ordinary words and sentences.

(5.1) By putting these two illustrations one next to the other and by supplying some basic descriptions and questions about them one can create the symbol of the larger part of Wittgenstein's complete philosophy. The point is not just that each cube perhaps (re)presents at least a part of different philosophical standpoint, but also that the second cube if put next to the first one (re)present at least a part of the difference between two philosophical standpoints. Perhaps the P-cube is the 'visible evidence of the limitations of an application of the T-cube' (noted by Jim Hamlyn in private correspondence)?

It is often claimed with a series of illustrated and illustrative examples that sketches, drawings and pictures comprise a huge amount of data via use of more or less simple images. This is mostly claimed in the field of engineering design (Petroski 1989, 2011), but also in connecting engineering and philosophy (Christiansen et. al. 2012) and engineering, information and art (Tufte 1990, Allen 2008). To some extent different source, namely semiotics, often claims that besides the quantity of information, a picture or a sketch if chosen properly can symbolize, i.e. do something quite complicated in a rather simple manner (Sebeok 1994, Cobley 2010).

- (5.1.1) Are these two aspects presented by Wittgenstein's sketches and drawings presented intentionally or not we don't know? Example of the T— and the P—cube seem to show exactly this point. The P—cube is made 'purposely' and it seems as a reaction to the T—cube even if this isn't stated explicitly, but this is a hypothesis. If there are "grave mistakes" in TLP that he tried to correct in PI (PPF), and if T— and P— cubes are related to grounding ideas of logic and grammar, given that TLP logic become a part of PI grammar, then P—cube at least looks like one of such corrections, in this case of T—cube. This issue isn't completely marginal if it is dependent on the basic ideas of the relation of propositions with facts and/or language—games to forms of life.
- (5.1.2) In other words, if one looks at the cubes, what one could see? Perhaps propositions in contrast to language—games, facts in contrast to forms of life, and by that (after a reflection) pictures in contrast to surveyable representations, a philosophical logic in contrast to a philosophical grammar, and a model in contrast to a modeling (as an application of a model, because 'The different conceptions must correspond to different applications.' RFM 1989 VII 5)? Again, we don't know for sure. We can only guess.

Nonetheless, if (5.1.2) is the case, then at least to some degree Wittgenstein's philosophy cannot be fully introduced without such illustrations actually being shown or being created as drawings on a board, and if this is the case, then the pictorial aspect of his philosophy is important as well as the textual. Surely there are not many similar examples in the history of philosophy, not even in Wittgenstein's philosophy. Philosophy is still mainly done by words and not by drawings or illustrations, even Wittgenstein's philosophy given the quantity of illustrations in his works, as mentioned in the remark (1).

Nonetheless, Wittgenstein's case, if correctly described, seems to show that not just general human thinking is in some part pictorial, but also that philosophy is in some part pictorial and that it can at least sometimes be presented by carefully chosen illustrations (not necessarily in comic book form like in *Wittgenstein for beginners* or similar). It looks that the pictorial aspect of philosophy is sometimes as important as the textual aspect.

- (5.1.3) The pictorial aspect of philosophy seems to be important in Wittgenstein's case, in which illustrative examples don't play some marginal role, i.e. as illustrations of some 'more philosophical' textual parts of his philosophy. They themselves are the 'pictorial texts'.

Given that he adopted a morphological method (mainly derived from Goethe and Spengler), examples are all there is (or jokes for that matter). One could paraphrase Goethe (Goethe 1998:§575) by saying — There is nothing behind examples. They themselves are effectively picturing the doctrine.

A pictorial example exemplifies itself as an application. What an *example* (*exemplum*) is *taking out* (*eximere*) was out there all the time; it is just (re) presented in a clear or perspicuous manner.

(5.2) On the other hand, points made in remarks (5) — (5.1.5) cannot be proven with sufficient evidence. That he was using many illustrations, drawings and sketches in his philosophical works, in fact much more than other philosopher of his times, is a fact. That these illustrations are integral parts of his examples which stand on their own by manifesting the whole point on particular issue is also a fact. The fact is too that only few drawings could be found in early and later works both concerning a similarly important issues, and that the drawing of a cube is such example. Everything beyond that, even his own standpoint on the T-cube itself and on the T-cube from the point of view of the P-cube is a highly interpretative matter and little if anything can be proven with certainty. Therefore, it is permissible to talk only about possible interpretations. In the same time the issue should be considered open for the future research, biographical, historical, and philosophical as well.

(5.2.1) The sheer possibility that complicated philosophical issues can be presented by simple illustrations and short descriptions contradicts not only the technical nature of the contemporary philosophy, but also the public's general opinion about it as being something completely detached or at least semi-detached from our daily realities. The irony is that Wittgenstein as a philosopher sensitive to the pictorial aspect of thought and philosophy is as such rediscovered in the 21st century in which the greatest amount of visual philosophy is available (for example in comics, animated, short, art and documentary films, etc.) only as a methodical tool, not as the form of philosophical thinking.

Perhaps a paradox emerges here. While being one of the most important philosophies of the 20th century, at the same time interpreted in many (sometimes radically) different ways even by other great philosophers and by being often utterly hard to understand even by his own students, Wittgenstein's philosophy manifests the basic feature of contemporary popular philosophy and even culture that he despised, namely, the possibility of philosophy being *represented* by a single illustration, a few words of description, and a series of puzzling questions for which there are no obvious answers.

Acknowledgments

I would like to thank Jim Hamlyn, Josip Lukin and Nikolina Krbot for discussing the topic and supplying valuable comments and objections.

References

- Allen, B. (2008). *Artifice and Design* (Ithaca and London: Cornell University Press)
- Baker, G. P. and Hacker, P. M. S. (2009). *Wittgenstein: Understanding and Meaning*, Volume 1 of *An Analytical Commentary on the Philosophical Investigations*, Part II: Exegesis §§1–184 (Oxford: Wiley–Blackwell)
- Boltzmann, L. (1974). *Theoretical Physics and Philosophical Problems*, Selected writings (Dordrecht: D. Reidel Publishing Company).
- Christiansen, S. L. *et. al.* (eds.) (2012). *Engineering, Development, and Philosophy* (Dordrecht: Springer)
- Cobley, P. (2010). *The Routledge Companion to Semiotics* (London: Routledge)
- Goethe, J. W. von (1998). *Maxims and Reflections* (London: Penguin)
- Hamilton, K. (2001). Wittgenstein and the Mind's Eye, in Klagge, J. C. (ed.) *Wittgenstein, Biography and Philosophy* (Cambridge: Cambridge University Press) pp. 53–98
- Hertz, H. (1956). *The Principles of Mechanics in a New Form* (New York: Dover)
- Hintikka, J. and Hintikka, M. B. (1996). Ludwig Looks at the Necker Cube: The Problem of 'Seeing As' as a Clue to Wittgenstein's Philosophy, in Hintikka, J. 1996 *Ludwig Wittgenstein: Half-Truths and One-and-a-Half-Truths* (Dordrecht: Springer) pp. 179–191.
- James, W. (1983). *The Principles of Psychology*, Volumes I and II (Cambridge, MA: Harvard University Press)
- Janik, A. and Toulmin, S. (1973). *Wittgenstein's Vienna* (New York: Simon & Schuster)
- Jastrow, J. (1899). The mind's eye, *Popular Science Monthly*, 54, pp. 299–312
- (1900). *Fact and Fable in Psychology* (Houghton: Mifflin and Co.)
- Khan, A., Crawford, J. D. (2001). Ocular dominance reverses as a function of horizontal gaze angle, *Vision Research*. 41(14):1743–8
- Köhler, W. (1929). *Gestalt psychology* (New York: Liveright)
- Kornmeier, J., Bach, M. (2005). The Necker cube — an ambiguous figure disambiguated in early visual processing, *Vision research*, 45(8), pp. 955–960
- (2012). Ambiguous figures—what happen in the brain when perception changes but not the stimulus, *Frontiers in human neuroscience*, 6, pp. 1–23
- Necker, L. A. (1832). Observations on some remarkable optical phenomena seen in Switzerland; and on an optical phenomenon which occurs on viewing a figure of a crystal or geometrical solid, *Philosophical Magazine Series 3*, 1:5, pp. 329–337
- Petroski, H. (1996). *Invention by Design, How Engineers Get from Thought to Thing*, Harvard University Press.
- (2010). *The Pencil* (New York: Henry A. Knopf)
- (2011). *An Engineer's Alphabet* (Cambridge: Cambridge University Press)
- Preston, J. M. (2017). Wittgenstein, Hertz, and Boltzmann, in Hans-Johann Glock and John Hyman (eds.) 2017 *A Companion to Wittgenstein*, (Oxford: Wiley–Blackwell) pp. 110–125
- Russell, B. (1992). *Theory of Knowledge 1913 Manuscript* (London: Routledge)

- ter Hark, M. (2015). Aspect perception in the Tractatus and the Philosophical Investigations, in Campbell M., O'Sullivan M. (eds.) (2015). *Wittgenstein and Perception* (London: Routledge) pp.165–182
- Sebeok, T. A. (1994). *An Introduction to Semiotics* (London: University of Toronto Press)
- Troje, N, F., McAdam, M. (2010). The viewing–from–above bias and the silhouette illusion, *i-Perception*, 1(3) pp. 143–148
- Tufte, E. R. (1990). *Envisioning Information* (Cheshire: Graphic Press)
- Wittgenstein, L. (1971). *Prototractatus. An Early Version of Tractatus Logico-Philosophicus* (London: Routledge & Kegan Paul) (PT)
- (1974). *Tractatus Logico-Philosophicus* (London: Routledge) (TLP)
- (1979). *Notebooks 1914–1916* (Oxford: Basil Blackwell) (NB)
- (1980). *Remarks on the Philosophy of Psychology, vol. 1* (Oxford: Basil Blackwell) (RPP I)
- (1989). *Remarks on the Foundations of Mathematics* (Oxford: Blackwell) (RFM)
- (2003). *Public and Private Occasions*, (Lanham: Rowman & Littlefield) (PPO)
- (2008). *Wittgenstein in Cambridge, Letters and Documents 1911–1951* (Oxford: Blackwell) (LD)
- (2009). *Philosophical Investigations* (Oxford: Wiley–Blackwell) (PI, PPF)

Summary

STEADILY CUBIC: A SQUARED ODYSSEY

Wittgenstein's cube–examples

In the paper, the author analyzes and compares cube examples in Wittgenstein's works, especially the cube from TLP (TLP 1974 5.5432) and the cube from PPF (ex PI II) (PPF 2009 116). There is no direct evidence that the PPF–cube is a reaction to the “grave mistake” of the TLP–cube. Also, there is no evidence that the TLP–cube is a representation of the Necker's cube (1832), although it resembles the Jastrow's cube (1900). These are negative results. Both cubes present important ideas of TLP and PPF (PI). Also, Wittgenstein writes that he “purposely chose” the example of the cube in PI and PPF, perhaps to show one of the “grave mistakes” of the TLP–cube. By analysis of the Necker's cube and comparison of drawings and accompanied text of the TLP–cube and the PPF–cube the author tries to explicate some possible implicit pictorial aspects of Wittgenstein's thought and of the general nature philosophical thinking as pictorial.

KEY WORDS: Boltzmann, Jastrow, Necker, PPF, Russell, TLP, Wittgenstein.

ABBREVIATIONS: T–cube = TLP–cube, P–cube = PPF–cube (ex PI II).