Analysis of Tool Life Functions in Hard Turning

János KUNDRÁK, Zoltán PÁLMAI, Gyula VARGA

Abstract: The relation between the technological parameters of cutting and the tool life is important information that has been described by several functions since the publication of the Taylor formula. Although the $v_c-T$ cutting speed-tool life curve has local extremes, the Taylor formula can describe correctly only its monotone decreasing phase, and this is typical also for the $v_c-T$ function proposed by different researchers. This paper describes the results of studies aimed at analysing the relationship proposed by Kundrak and applicable throughout the interval of the $v_c-T$ curve. It introduces cutting speed ranges, analyses their boundaries and explores the relationship between constants of the new $v_c-T$ formula and the exponent $k$ of the Taylor formula valid for section III. These findings were verified by experiments executed on hardened steel type 100Cr6 (HRC 60 ± 2) by K10 type CBN tool.

Keywords: general tool life function; hard turning; tool life function; optimization; Taylor exponent

1 INTRODUCTION

The cutting tools wear out with intensity depending on the cutting conditions. In order to understand this process researchers are conducting new research [1, 2], using new wear measuring methods [3], describing the process [4, 5] by modelling [1, 6-8] and monitoring tool wear [6], so that they can determine the tool life more and more accurately.

In the economical optimization of machining, the tool life curve is generally required. This gives the tool life as function of technological parameters, particularly cutting speed. Several researchers have attempted to set up a tool life equation, but the exact knowledge of the functions expressing tool life is still a current issue in cutting theory and technique. The practical significance is high, because the economy of cutting depends, to a great extent, on its correct or incorrect choice.

As is well known, in cutting theory studies [9] and even more so in practical technology, mainly the relationship between certain cutting parameters is examined, most often cutting speed $v_c$ and tool life.

Fig. 1 shows types of tool life curves [10] that either show a monotonous (continuous) decrease (curve 1) or a more commonly occurring curve with local extremities (curve 2). Curve 2 can be divided into three distinct sections (Fig. 1). In Fig. 1 $T$ is the tool life and $v_c$ is the cutting speed.

The curve with local extremities makes sense considering that the wear that determines tool life originates during very complex mechanical, chemical, thermal, electrical, etc. processes. If the cutting conditions change, the mechanical and thermal loading of the tool also changes and the ratio of wear components is also modified, which is difficult to handle mathematically. All of this results in a relation between the cutting parameters and tool life that leads to a complicated relationship hardly usable in engineering practice, in a wider range of varying cutting parameters.

Over the years, many suggestions have been made to simplify the function describing the relationship between cutting data and tool life by approximation curves.

F.W. Taylor [11] created his tool life model in 1907, which in its standard form today can be written as follows:

$$v_c^{-1} \cdot T^k = C_v$$

(1)

where: $T$ - tool life, min; $v_c$ - cutting speed; $k$ - tool life exponent; $C_v$ - a constant representing the cutting speed that results in 1 min tool life.

The Taylor equation provides data ($v_c$ or $T$) with satisfactory accuracy in practice for single point cutting tools, when the other two cutting data ($a_p$ and $f$) are held constant [12]. The Taylor model approximates the real relationship between the cutting speed and the tool life in the log$v_c$-log$T$ plane by a straight line (Fig. 2a). The lower the cutting speed range is, within which the relation between $v_c$ and $T$ is examined in the branch of the curve descending after the extreme values, the defects results from approximation. Fig. 2 illustrates the measured/real tool life values with dashed curves and with continuous lines the validity range of the tool life equations.

The tool life model developed by Gilbert [13] is essentially an extension of the Taylor model, considering the effect of feed and depth of cut. Since the Gilbert model is an extended Taylor model, here also the real $T-v_c$ curve is replaced by straight line on the log$v_c$-log$T$. The imaging of the relationship to the plane $T-v_c$, in accordance with the effect of the feed and the depth of cut provides numerous straight lines, each of which represents a feed or depth of cut where, besides the cutting speed, the other cutting data are constant (Fig. 2b).
As regards the cutting of alloyed materials used in increasing quantities by carbide and ceramic tools, the correlation between $T$ and $v_c$ is not linear moreover in log-log planes according to the observations. Therefore, models have been proposed that describe the curved section not as logarithmic line, but more accurately with a higher order function relation.

König and Depiereux [14] started from the point that the correlation between the cutting speed and the tool life is described by a curve in the $\log v_c - \log T$ coordinate system which, by some modification of the simple Taylor equation - adding a constant to the cutting speed - can be approached with the following function:

$$T = \frac{C_{T1}}{(v_c + C_{T2})^{CT3}}$$

(2)

The nature of the model is given in Fig. 2c. The tool life model developed by Kronenberg [15] can be described as follows:

$$T = C_{T1} \cdot e^{CT2 \cdot v_c + CT4 \cdot f^{CT5}}$$

(3)

This model, besides cutting speed $v_c$, considers the effect of feed $f$ on tool life. Due to its exponential shape, it represents well those physical and chemical processes that cause wear of the tool (Fig. 2d).

Some more models are mentioned here. The tool life model of Safonov [16], similarly to the König-Depiereux model, is exponential shaped, and considers the influence of cutting speed only at the constant value of the other cutting data:

$$T = \frac{C_{T1}}{v_c^{CT2 + CT3}}$$

(4)

Temcsin's tool life model [17] is most like the Kronenberg model:

$$T = \frac{C_{T1}}{v_c^{CT2} + C_{T3}}$$

(5)

The Wu type tool life model [18] shows a degree of similarity with the simple Taylor model, the difference being in the complex power output of the cutting speed, which itself includes the cutting speed:

$$T = \frac{C_{T1}}{v_c^{CT2 + CT3 + CT5}}$$

(6)

Even in systematic, in-depth studies, if differences were observed across the entire speed range, these differing values (perhaps because of simplification) were often considered as greater standard deviation, while other researchers have taken the differences into account and attempted to describe relationships mathematically as well.

The tool model developed by Colding [19] belongs to the second order models that follow well the flow of $T-v_c$ curves in a wide cutting speed range, which is realized by considering both the linear and quadratic members of the...
technological data and their multiplication. The method is still used today, as can be seen in the work of Thaker et al. [20] and Busham [21], where the multiplication of cutting factors is also considered in the tool life function without paying attention to the extreme values of this curve. The tool life model of Granovskij [22] shows a difference from the previous formulae in considering the effect of technological data (cutting speed) in its coefficients, too:

\[ T = C_{T1} \cdot v_c^{C_{T2}} \cdot C_{T3}^{C_{T4}} \cdot v_c \]  

(7)

It is interesting that even a large-scale study, such as that of Colding and König [23] on the Taylor equation, is practically confined only to section III shown in Fig. 1 and provides well-established calculations related to it for the determination of optimal cutting parameters. In fact, these less-than-satisfactory equations are still used. For instance, when Galoppi et al. [24] evaluate the results of their hard-turning experiments, the measurement results are reported obviously by reference to the Taylor formula in a specific technological situation. The problem of local extremity is not even raised. Also, Abburi and Dix [25] are still focusing only on the examination of section III. However, today an increasing number of cutting researchers are coming to accept the fact that the \( v_c \cdot T \) tool life relationship, in most concrete cases, is correctly characterized by Curve 2 in Fig. 1 in which three sections must be definitely separated. Benga and Abrao [26], in their study of hard turning, for example, report experimental results falling in the difficult to model border zone of section II and III. Punta and Hryniewicz [27] studied the full range of curve 2 in Fig. 1, analysing in detail the extreme value sites separating the three sections of the curve by mathematical methods. Vasilko's proposal considers the above-mentioned nature of the tool life curve [28].

Fig. 3 shows the \( v_c \cdot T \) curve at three different feed rates. A closer look at the tool life curves reveals that the slopes of the curves do not differ significantly, but the range limit is significantly influenced by the cutting data - here feed rate, but other parameters can have a similar effect [29]. It may occur that for a given feed rate - depth of cut combination a certain speed may be optimal, while in other combinations it results in unfavourable tool life.

In section III of the tool life curve (see Fig. 1), from the local maxima towards higher cutting speeds, the Taylor formula can provide a reasonable approximation. However, within the range of smaller cutting speed values that fall into section II, the Taylor exponent \( k \) significantly differs from the actual tool life value. Taylor's version assumes that, in the case of all tool-workpiece pairs, tool life varies in the same way depending on the speed, which would mean that it changes in the same manner as the temperature. (This is clearly not true - think e.g. of the iron-carbon state diagram.) In section II the Taylor equation cannot be recommended. In the lowest speed range (section I), the difference between values calculated with the Taylor formula and actual values is even more significant.

A manageable tool life function is needed that is suitable for describing all three sections of the tool life curve. In an attempt to meet this need, a general tool life model was first proposed by Kundrák in his 1996 dissertation [29]. It is intended to better describe the tool life of modern tool materials, when there is a local minimum and maximum depending on the cutting parameters. As a function of speed \( v_c \), the tool life \( T \) is:

\[ T = \frac{C_{T1}}{v_c^{3} + C_{T2} \cdot v_c^{2} + C_{T3} \cdot v_c} \]  

(8)

Advantages of the Kundrák formula (Eq. (8)) have been explored in detail in [10, 30] but its favourable features can be summarized as follows: in the full cutting speed range it reflects the physical laws of the cutting process more accurately, by expressing the exact location and value of the extremities at \( v_{12} \) and \( v_{23} \) cutting speeds, considering the joint effect of the cutting parameters (Fig. 3). This means that it has a much broader range of applicability, throughout all sections of the tool life curve, because it:

- provides a more accurate possibility of calculation for the full speed range,
- is a simple calculation,
- helps to determine the validity range for the Taylor formula,
- helps the application of combined tools (rotating, working with division of depth of cut, etc.),
- makes optimization more accurate, since the search scope can be assigned more accurately,
- allows maximum tool life value to be determined with different combinations of parameters.

2 EXAMINATION OF THE TAYLOR EXPONENT

It is common to say that the Taylor exponent is the slope of the \( v_c \cdot T \) tool life curve, although it only appears to be so due to the log-log scale. The slope of the tool life curve, which is ultimately characterized by the \( \text{d}T/\text{d}v \) differential ratio, has an interesting connection with the \(-k\) exponent. The Kundrák formula gives exactly the surplus that follows the curvature of the real \( v_c \cdot T \) function section III.

From Taylor Eq. (1)

\[ \frac{dT}{dv} = k \cdot C^{-k} \cdot v_c^{k-1} = k \cdot T / v_c \]  

(9)
Where

\[ k = \frac{v_c}{T} \frac{dT}{dv} \]  \hspace{1cm} (10)

Eq. (8) is derived according to \( v \)

\[ dT = \left( \frac{3v_c^2 + 2C_{T2}v_c + C_{T3}}{C_{T1}} \right) T^2 \]  \hspace{1cm} (11)

which can be substituted into Eq. (10), so

\[ -k(v) = \left( \frac{3v_c^2 + 2C_{T2}v_c + C_{T3}}{C_{T1}} \right) v_c T \]  \hspace{1cm} (12)

Eq. (10) is interesting because if functions \( dT(v)/dv \) and \( T(v) \) are known, the exponent can be determined at any section of the curve, as is done here by using formula (8). Of course, in every case where the exponent is function of \( k = k(v) \), the "constant" \( C \) is also used as a \( C = C(v) \) function, according to Eq. (1).

### 3 VALIDATION OF THE THEORETICAL FUNCTION \( k(v) \)

The cutting tests performed earlier were done by turning inner cylindrical surfaces made of hardened steel 100Cr6 (HRC 62 ± 2) with \( D = 45, 75 \) or 100-mm diameter. The tool was of K10 quality CBN tool with the edge geometry: \( \gamma = -5^\circ, \alpha = \alpha' = 15^\circ, \lambda = 0^\circ, \kappa = 45^\circ, \kappa' = 15^\circ \). Cutting data: \( v_c = 11-120 \text{ m/min}, \text{ feed rate} f = 0.05 \text{ mm/rev}, \) depth of cut \( a = 0.15 \text{ mm} \). The results of the tool life tests are summarized in Tab. 1. Based on these, constants of the tool life Eq. (8), which are given in Tab. 2, could be determined. Fig. 4a shows the tool life curves \( v_c - T \) calculated with these constants. The curves fit well on the experimental results.

| Tab 1 Results of tool life examinations |
|---|---|---|---|---|---|---|---|---|---|
| \( D \) / mm | 11 | 20 | 29 | 40 | 68 | 92 | 105 | 120 |
| \( v_c \) / m/min | 153 | 145 | 181 | 143 | 65 | 17 | 7 | 4 |
| \( T \) / min | 45 | 75 | 100 |
| \( v_c \) / m/min | 206 | 189 | 221 | 211 | 113 | 33 | 10 | 7 |
| \( v_c \) / m/min | 250 | 222 | 254 | 261 | 159 | 41 | 14 | 7 |
| \( v_c \) / m/min | 250 | 222 | 254 | 261 | 159 | 41 | 14 | 7 |

The range of the cutting speed used in the tests was considerably wider than the range for which the Taylor formula is applicable. This is particularly striking in Figure 4a, which describes the \( v_c - T \) function connection on a linear scale, but also in the curves drawn on the normal log-log plane (Fig. 4b) it can be seen that the Taylor formula can be used only from \( v_c \geq 40 \text{ m/min} \) and this is the lower speed limit of its usability (its value depends on the technological parameters as well).

Here, the diameter \( D \) of the workpiece is a parameter that affects this speed limit, as was already mentioned for Fig. 3. Therefore, in the present case, it is advisable to carry out the examination of Taylor exponent test from \( v_c = 40 \text{ m/min} \).

![Figure 4 Comparison of calculated tool life curve using the Kundrák formula and measured values (a) and the logarithmic shape of the tool life curve (b)]

In Fig. 5, the straight-line \( \log v_c - \log T \) and its regression equation, both measured and calculated by Eq. (8) and Eq. (12) can be seen for \( D = 100 \) mm. The other two equations for the examined diameter and \( R^2 \) are shown in Tab. 3 in order not to disturb the clarity of the figure.

| Tab 2 Constants of the tool life function for Eq. (8) |
|---|---|---|---|
| \( D \) / mm | \( C_{T1} \) | \( C_{T2} \) | \( C_{T3} \) |
| 45 | 1.38×10^9 | −71.3 | 1494 |
| 75 | 2.33×10^9 | −76.9 | 1755 |
| 100 | 3.18×10^9 | −80.45 | 1923 |

| Tab 3 Regression equations \( \log v_c - \log T \), measured and calculated by the Kundrák formula |
|---|---|---|---|
| \( D \) / mm | Measured \( T \) / min | Regression equation \( \log v_c - \log T \) | \( R^2 \) |
| Regression equation \( \log v_c - \log T \) | calculated by Kundrák formula \( T \) / min | \( R^2 \) |
| 45 | \( -3.7947 \log v_c + 8.2399 \) | 0.9960 | \( -4.1401 \log v_c + 9.2054 \) | 0.9970 |
| 75 | \( -3.6694 \log v_c + 8.2386 \) | 0.9978 | \( -4.1892 \log v_c + 9.1445 \) | 0.9985 |
| 100 | \( -3.7679 \log v_c + 8.5188 \) | 0.9938 | \( -4.2564 \log v_c + 9.0152 \) | 0.9992 |
The close fitting of the tool life calculated by the Taylor and the Kundrák formula and the measured tool life can be directly determined from Fig. 6, which convincingly supports the hypothesis that in determining the specific velocity-dependent values of the Taylor exponent, Eq. (12) derived from the two tool life functions can indeed be used.

It is worth noting that the range of the cutting speed to which the Kundrák formula can be applied is significantly higher; the constants summarized in Tab. 2 are defined for this entire speed interval. In the entire investigated cutting speed range of curve \(v_c - T\), which is too complicated to be handled at all with the Taylor formula, as it can be seen in Fig. 4.

4 FUNCTION - \(k(v)\)

The characteristic sections of the curve \(v_c - T\) are described by the function \(dT/dv\) defined by Eq. (5). This is shown in Fig. 7 for the full tool life curve.

The value of the Taylor exponent for the three diameters can be calculated from Eq. (12). The results are summarized in Fig. 8, which shows that \(-k\) really depends on the cutting speed \(v_c\). Conversely, the exponents based on the Taylor formula are constants and, at \(D = 45, 75\) and \(100\) are \(-k = 3.7947\); \(3.6694\) and \(3.7679\), respectively.

Of course, this cutting speed dependent \(-k\) exponent also implies that the \(C\), constant of the Taylor formula is not constant, as shown in Fig. 9. In the first two sections of the full \(v_c - T\) curve (Fig. 1), where there are the extremities (first minimum and then maximum), in principle the function \(C(v_c)\) can be calculated, but it is practically unusable, as it varies between \(\pm\infty\). In section III of the \(v_c - T\) curve, where the function becomes monotonically decreasing, the value of Taylor's "constant" \(C\), is formed as shown in Fig. 9.

Consequently, it is worth checking the usual formulas for calculating optimal cutting parameters using the Taylor formula-based extremum calculation; it is recommended to calculate the optimum cutting parameters from the real \(v_c - T\) curve. These values may occasionally differ significantly from the results of the Taylor constants calculation.

5 APPLICATION

The new tool life function is suitable for all practical tasks that have previously used the Taylor formula and other \(T(v_c)\) functions. In addition, the function has the advantage that it can be used over the whole range

---

**Figures:**
- Fig. 5: Monotone reducing section of tool life curves calculated by Kundrák's formula.
- Fig. 6: The fitting of tool life calculated by the Taylor and Kundrák formulas to measurements.
- Fig. 7: Function \(dT/dv\) at different workpiece diameters.
- Fig. 8: Real change of the Taylor exponent as a function of cutting speed at different workpiece diameters.
- Fig. 9: The values of \(C\) "constant" of the Taylor formula calculated from equation (12) and considering \(-k\) exponents.
(interval, spectrum, ...) of the cutting speed used in practice. First of all, it is useful in planning the optimum technological parameters. Whether the task is to minimize the cost or to maximize the power of chip removal, the search for the optimum is not restricted to Section III only. As an example, for the hard turning of an inner cylindrical surface of diameter \( D = 100 \text{ mm} \) or \( 45 \text{ mm} \), the data of chip cross section can be calculated using the data in Tab. 3. The main data of the economy calculation are the machine’s cost per minute: \( k_m, \text{€/min} \), the edge cost of the tool \( E_{\text{tool}}, \text{€} \); while in the case of calculating the chip removing power, the time needed to exchange tools or reassemble the cutting tool \( t_{\text{ch}} \) is essential.

Using the latter, the chip removing power is

\[
P = a \cdot f \cdot \frac{v_c \cdot T}{t_{\text{ch}} + T}
\]

which, as is known from the calculation based on the Taylor formula, has the maximum tool life

\[
T_{\text{opt}} = (-k - 1) \cdot t_{\text{ch}}
\]

for which the cutting speed \( v_{p_{\text{opt}}} \) can be calculated from the Taylor formula. This method is not applicable any more. The meaning of the letters in Eq. (13): \( P \) - chip removing power; \( a \) - depth of cut; \( f \) - feed, \( t_{\text{ch}} \) - tool exchange time.

Using the parameters given in Tab. 3 the function \( P \) of the power of chip removal is shown in Fig. 10, when diameter is \( 100 \text{ mm} \). It can be seen that the two calculations have their maximums at different speeds, and that the magnitude of these maximums is different for the two calculations. In the case of a longer tool exchange time, the application of Kundrák’s formula, which is closer to the real shape of \( v_c-T \) curve, represents higher productivity, while with decreasing tool exchange time, this difference decreases and even becomes a sign of contradiction. It can be observed that, in the former case, the optimum cutting speed moves downward with increasing replacement time, to the range in which the \( -k \) real Taylor exponent deviates more and more from the average exponent, which is independent from \( v \) of the Taylor formula. From this it can be concluded that in the lower speed range the power function derived from the Kundrák model gives the more realistic result. Of course, here we are talking about relating and comparing two models that work close to the actual cutting conditions. Ultimately, in this application example, the two optimums provide roughly the same result from the point of view of chip removal performance. Fig. 3, however, warns that changing the technological parameters can easily shift the cutting process in a direction where the Taylor formula is no longer valid and strongly distorts the economic calculations.

To determine the cutting speed that is economical from the point of view of tool life, it is expedient to examine the chip removal cost of the unit material volume, which is

\[
E = \frac{1}{a \cdot f \cdot v} \left( k_m + \frac{t_{\text{ch}} \cdot k_m - E_{\text{tool}}}{T} \right)
\]

Calculating by the generalized tool life function, the cost per unit of the removed material volume is shown in Fig. 11 as a function of speed.

Of course, the curves of Fig. 11 are also fundamentally influenced by the technological parameters. The cost of the edge of the tool is particularly important for this calculation, as shown in Fig. 12.

For the workpiece \( D = 45 \text{ mm} \), the optimum cutting speed is now below \( v_c = 40 \text{ m/min} \), which is already the zone where the Taylor formula will produce serious errors (cannot provide an accurate value), as shown in Fig. 1.

6 SUMMARIES

The cutting-edge tool life of the cutting tool is important information for technological design and optimization, so since Taylor’s original formula there have been several proposals for mathematical description of the impact of technological parameters affecting tool life. The
The vast majority of the proposed functions do not take into account that the general \( v_c - T \) curve has local extremes and, like the Taylor formula, apply only to the monotonous decreasing section. In the context of the local extremes of the \( v_c - T \) curve, traditional optimization of the cutting process can lead to incorrect results. Taylor’s exponent is a function of the technological parameters, and in the environment of the extremes in the \( v_c - T \) curve, it even changes its sign, so the optimization is strongly distorted.

The tool life algorithm proposed by Kundrác can describe the entire range of the tool life curve, specifying the local minimum, maximum location and value. These are influenced by technological parameters, and their effect can be described in detail and the validity range for the Taylor function can also be specified. For the analysis of the relationships we also carried out cutting experiments. The machining experiments were performed on hardened steel 100Cr6 with a K10 CBN tool insert at a speed range of \( v_c = 11\text{-}120 \text{ m/min, } f = 0.05 \text{ mm/rev, } a = 0.15 \text{ mm}. \)

Based on the analysis of the tool life measurements performed in the D45-D100 mm hole during hard turning, the following results can be summarized.

a) It is confirmed that the third-order rational function proposed by Kundrác for describing the \( v_c - T \) relationship can be applied over the entire interval (range) of \( v_c \) cutting speeds under the assumptions examined.

b) We defined the boundary coordinates of the three sections of the \( v_c - T \) formula with the constants of the new function. These are influenced by technological parameters in a specific way.

c) We found that there is a close relationship between the constants of the new \( v_c - T \) formula and the \( k \) exponent of the Taylor formula valid in Section III.

d) The experiments showed that the Kundrác formula fits the measurement results well over the full range of the test speed, while the Taylor formula can only be used in section III of the \( v_c - T \) curve.

The theoretical results of the research can be directly applied in practice. The optimum technology parameters for operating costs or cutting performance can be determined over the entire \( v_c \) range.

Acknowledgements

Project no. NKFI-125117 has been implemented with the support provided from the National Research, Development and Innovation Fund of Hungary, financed under the K_17 funding scheme.

In addition, the described study was carried out as part of the EFOF-3.6.1-16-00011 “Younger and Renewing University - Innovative Knowledge City - institutional development of the University of Miskolc aiming at intelligent specialization” project implemented in the framework of the Széchenyi 2020 program. The realization of this project is supported by the European Union, co-financed by the European Social Fund.

7 REFERENCES


Contact information:

János KUNDRÁK, Prof., DSc, (Corresponding author)
University of Miskolc
Institute of Manufacturing Science
H-3515 Miskolc, Egyetemvaros
Hungary
janos.kundrak@uni-miskolc.hu

Zoltán PÁLMAI, Prof. C.Sc.
University of Miskolc
Institute of Manufacturing Science
H-3515 Miskolc, Egyetemvaros
Hungary
palmayz@t-online.hu

Gyula VARGA, Assoc. Prof., PhD
University of Miskolc
Institute of Manufacturing Science
H-3515 Miskolc, Egyetemvaros
Hungary
gyulavarga@uni-miskolc.hu