Optimized Wave Spectrum Definition for the Adriatic Sea
Optimizirani spektar valova za Jadransko more

Marko Katalinić
University of Split
Faculty of Maritime Studies
e-mail: marko.katalinic@pfst.hr

Maro Ćorak
University of Dubrovnik
Maritime Department
e-mail: maro.corak@unidu.hr

Joško Parunov
University of Zagreb
Faculty of Mechanical Engineering
and Naval Architecture
e-mail: josko.parunov@fsb.hr

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Summary

The wave spectrum provides a short-term sea state description that is used for calculations in various engineering disciplines. Accepted theoretical models are developed for oceans and specific regions and their parameters are determined to best fit the data which is usually gathered in dedicated wave measurement campaigns. Modern numerical wave model simulations of historic periods that are calibrated with available satellite altimetry measurements provide a new data source that is comprehensive in space and time. One such dataset, the WorldWaves database, is used to revise and optimize the standard wave spectrum parameters for the Adriatic Sea. Tabain's, wave spectrum and its modal frequency relation are revised. An optimization methodology is developed to find the parameters to best describe the Adriatic Sea waves. Ultimately, the JONSWAP wave spectrum is chosen and its parameters were optimized together with the parameters of Tabain's modal frequency relation. An improved wave spectrum definition for the Adriatic Sea is proposed named the JONSWAP-Adriatic wave spectrum.

KEY WORDS
wave spectrum
peak frequency
Adriatic Sea
parameter optimization

Sažetak

Spektri valova predstavljaju kratkoročni opis stanja mora koji se koristi u proračunima u različitim inženjerskim disciplinama. Prihvaćeni teoretski modeli su razvijeni za oceane i specifične akvatorije, a njihovi parametri su određeni kako bi najbolje odgovarali podacima koji su obično prikupljeni mjerenjima valova. Simulacije povijesnih razdoblja primjenom modernih numeričkih modela valova, dodatno kalibrirani raspoloživim mjerenjima satelitskom altimetrijom, predstavljaju novi izvor podataka, koji je sustavan i prostorno i vremenski. Jedna takva baza podataka, WorldWaves, je iskorištena kako bi se revidirali i optimizirali parametri spektra valova za Jadransko more. Revidirani su Tabainov spektar valova i njegov izraz za modalnu frekvenciju. Finalno je odabran JONSWAP spektar valova te su njegovi parametri optimizirani zajedno s parametrima Tabainovog izraza za modalnu frekvenciju. Predložena je poboljšana definicija spektra valova za Jadransko more pod nazivom JONSWAP-Adriatic spektar valova.

KLJUČNE RIJEČI
spektar valova
vršna frekvencija
Jadransko more
optimizacija parametra

1. INTRODUCTION / *Uvod*

The Adriatic Sea has a specific wind-wave climate that is widely argued in literature [1, 2, 3]. Considering its importance to surrounding countries in a number of economical and other sectors it is important to always improve the knowledge of its wave climate. Waves represent dominant loads on coastal, near-shore and offshore structures. Sea states are for engineering purposes traditionally described by wave spectrums. Wave measurements in the time-domain are transferred into the frequency domain by decomposing wave components per frequencies while assuming linear superposition. The spectral curve is thus obtained representing the distribution of wave energy over the frequency range. Several theoretical wave spectrum models have been developed and they are usually based on wave measurement campaigns. The most famous are the Pierson-Moskowitz and the JONSWAP spectrum for oceans. Wave spectrum developed specifically for the Adriatic, intended for naval architecture and marine engineering applications, is the Tabain's spectrum [4], called after its author. Aside Tabain's spectrum also Prsic [5] and Smircic-Gacic [6] proposed spectral models for the Adriatic Sea.

Developments in measurements techniques (*in-situ* and remote) as well as in numerical wave model simulation enabled new datasets [7, 8, 9], more systematic in space and time compared to original *in-situ* measurement campaigns. Newly available data allows for revision and improvement of existing spectral definitions. One such dataset is the World Waves (WW) database [10] used in this paper to revise and improve the Tabain's wave spectrum for the Adriatic Sea. In the introductory part, the WW database and the methodology are described. The second chapter reviews Tabain's modal frequency relation by comparing it with WW data and then optimizing its parameters in order to best fit the available measurements. The optimized frequency relation showed to be inapplicable within Tabain's original spectral model, thus, the spectrum optimization was done based on JONSWAP spectrum model. Parameters of the JONSWAP spectrum were modified to best represent the Adriatic Sea waves. The analysis in followed by the results in the third chapter. The results are derived for the entire Adriatic with all sea state conditions included. The paper ends with a conclusion.

1.1. Wave data source / *Izvori podataka za valove*

The WW database is obtained from Fugro-Oceanor, a Norwegian industry leader and scientific partner in the field of wave measurements and analysis. The WW database is constructed as a combination of the WAM numerical wave model run by the ECMWF (European Centre for Medium Range Weather Forecast) and available satellite altimetry measurements. ECMWF used the WAM wave model to performed a hindcast – a numerical reanalysis of a historic period, and its results were calibrated by altimetry wave measurements from satellite missions: ERS-1, ERS-2, TOPEX, Geosat (Follow-On), Jason and Envisat. For the Adriatic Sea, 39 locations are available uniformly distributed at 0.5° latitude/longitude spacing. The Adriatic is considered as a whole, meaning the data from all locations are merged into a single database. The WW database locations and the regional division are presented in Figure 1. Each location in the database contains 12 wind and wave physical parameters at 6-hour intervals. In total, for the period from September 1992 to January 2016, each location contains about 34460 logs. Among the 12 parameters, the significant wave height H_s and the peak (spectral) period, i.e. its corresponding peak (modal) frequency , are extracted and used for the analysis presented in this paper.

Source: Google Earth Pro

Figure 1 Locations and regional division of the WW database (Fugro-Oceanor)

Slika 1. Lokacije i regionalna podjela WW baze podataka (Fugro-Oceanor)

Both numerical wave models and satellite altimetry wave measurements are subject to constant validation and improvement.

1.2. Methodology / *Metodologija*

Based on the data available from the WW database the modal frequency, as defined within the Tabain's wave spectrum, has been reviewed and optimized. The optimization confirmed the possibility of improvement based on data that is more systematic in space and time than the data used by Tabain to derive his model.

The optimized modal frequency model was inapplicable with the Tabain's spectral definition curve even with modifying the numerical constants. Thus, the JONSWAP wave spectrum model has been employed. Its numerical constants were optimized to develop, in conjunction with the optimized Tabain's modal frequency model, a new single-parameter spectral model for the Adriatic Sea, called the JONSWAP-Adriatic wave spectrum. The underlying WW database has been manipulated in order to derive the parameters for the Adriatic as a whole and with all sea states taken account.

2. ANALYSIS / ANALIZA

2.1. Review of Tabain's modal frequency equation / *Revizija jednadžbe Tabainove modalne frekvencije*

Tabain's wave spectrum, developed specifically for the Adriatic Sea, is a single parameter formulation.

$$S(\omega) = 0.862\,\frac{0.0135g^2}{\omega^5}\,exp\left[-\frac{5.186}{\omega^4 H_S^2}\right]1.63^p \qquad (1)$$

where

$$p = exp\left[-\frac{(\omega-\omega_m)^2}{2\sigma^2\omega_m^2}\right] \qquad (2)$$

$$\omega_m = 0.32 + \frac{1.8}{H_S+0.6} \qquad (3)$$

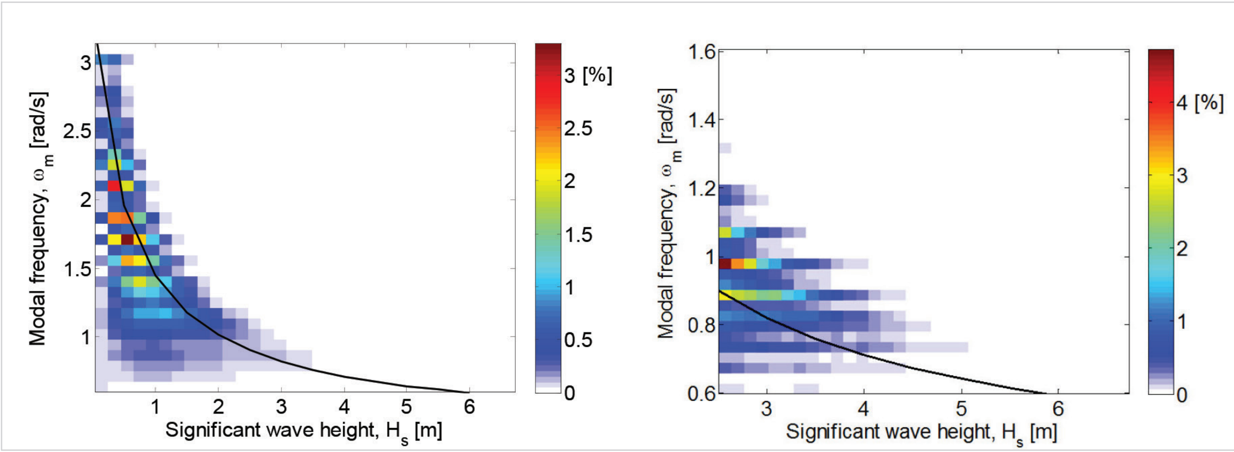
while $\sigma = 0.08\ za\ \omega \leq \omega_m$ and $\sigma = 0.1\ za\ \omega > \omega_m$.

The only free parameter is the significant wave height H_s. More often wave spectrum is defined by two variables, the significant wave height and the peak (modal) frequency of the spectrum. This is the case with the well-known ITTC and JONSWAP spectrums. In the Tabain's wave spectrum the modal frequency is defined as a variable dependent on the significant wave height by a specific relation Eq.(3) and this equation is reviewed compared with the newly available data from the WW database. The available peak period is recalculated to the angular (modal) frequency by the know relation:

$$\omega_m = \frac{2\pi}{T_p} \qquad (4)$$

The two datasets, the WW database modal frequencies and the modal frequency as predicted by Tabain's formulation are compared in Figure 2.

Figure 2 shows large scatter of modal frequency logs for a specific significant wave height according to WW data. When the WW data and the Tabain's modal frequency results are compared it can be noticed that Tabain's expression slightly underestimates the most frequent modal frequencies for moderate significant wave height and underestimates for larger. Furthermore, statistical coefficients of determination between the two datasets are rather low, varying from 0.33 to 0.45 depending on the location. This suggests a possibility for improvement of the modal frequency definition in the single parameter wave spectrum formulation.



a) All data

b) Data H_s > 2.5 meters

Figure 2 Frequency of occurrence distribution of significant wave height and modal frequency. WW scattered data compared with Tabain's formulation (black line)

Slika 2. Frekvencija učestalosti pojavljivanja značajnih valnih visina i modalne frekvencije. Usporedba WW podataka s Tabainovom formulacijom (crna linija)

2.2. Optimization of the modal frequency equation / *Optimizacija jednadžbe modalne frekvencije*

Initially, to improve the Tabain's expression for modal frequency, its numerical constants were defined as variables and their values optimized in order to obtain the best possible agreement with WW data. The optimization is performed within the Matlab software environment by a nonlinear least square method guided by the "trust-region-reflective" algorithm [11]. The original and optimized results overlaid on WW data are shown in Figure 3.

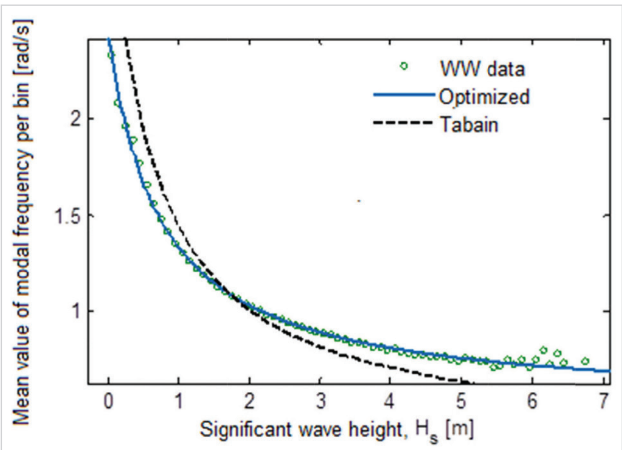


Figure 3 Parameter optimization of Tabain's modal frequency relation

Slika 3. Optimizacija parametara Tabainove jednadžbe za modalnu frekvenciju

The database frequencies were sorted in bins keeping only the mean value per bin in order to obtain a representative result for the entire range and not to be dominated by the bulk quantity of lower points. The optimized relation provides a better representation of - relation for the Adriatic and takes the following form

$$\omega_m = 0.52 + \frac{1{,}4}{H_s + 0{,}7} \tag{5}$$

2.3. Review of the Tabain's wave spectrum equation / *Revizija jednadžbe Tabainova spektra vala*

Initial attempts to apply the optimized Tabain's modal frequency model within the original Tabain's wave spectrum formulation showed inconsistency, i.e. its inapplicability, as shown in Figure 4. Namely, Figure 4 clearly shows that the spectrum loses its characteristic one-peaked shape, but also the peak frequency and significant wave height, when checked, do not coincide with the input values. Modifications of the model parameters were attempted, same as for the frequency model, to provide a viable result but with no success.

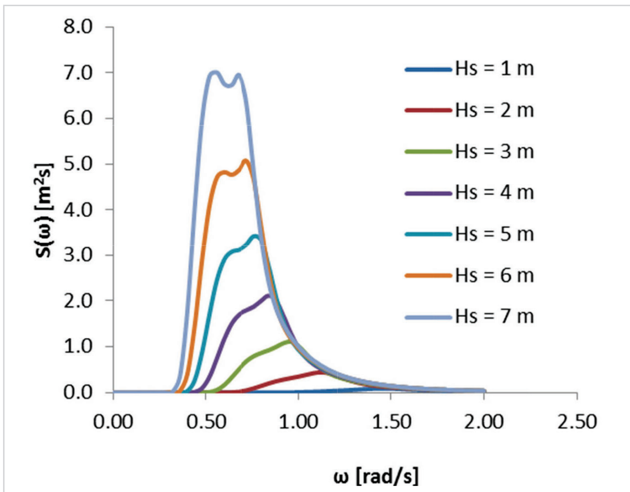


Figure 4 Failure of Tabain's wave spectrum in combination with the optimized modal frequency model

Slika 4. Neprikladnost Tabainovog spektra u kombinaciji s optimiranim izrazom za modalnu frekvenciju

2.4. Optimization of JONSWAP wave spectral equation for the Adriatic Sea / *Optimizacija JONSWAP jednadžbe spektra valova za Jadransko more*

Since the optimized modal frequency model provided and excellent description of WW database measured (and simulated) peak spectral period, but was inconsistent with Tabain's main spectral equation, a different wave spectrum description model needed to be considered. A single-parameter description was kept for its practicality. The JONSWAP spectral model was examined instead of Tabain's considering it is a well-known, world-wide accepted wave spectrum for partially developed sea state conditions. Moreover, Tabain initially derived his spectrum from JONSWAP formulation. Today, JONSWAP is also a practical choice for naval architecture and maritime applications as it is a standardized wave input in most calculation software that consider wave loading.

The JONSWAP formulation given in a generalized form (constants are replaced with parameters) reads

$$S(\omega) = A_\gamma \frac{5}{16}\frac{H_s^2\omega_m^4}{\omega^5} exp\left[-\frac{5}{4}\left(\frac{\omega_m}{\omega}\right)^4\right] \gamma_{ADR}^{\;exp\left[-\frac{1}{2}\left(\frac{\omega-\omega_m}{\sigma\,\omega_m}\right)^2\right]} \quad (6)$$

where: γ_{ADR} is the peak enhancement factor
A_γ is the normalization factor

While the peak enhancement factor raises the spectral curve maximum, which is characteristic for partially developed sea states, the normalization factor acts inversely making the spectrum narrower in order to keep the total amount of wave energy under the spectral curve the same. The DNV normalization factor relation is chosen. Its numerical constant e_0 is defined as a variable that is to be optimized simultaneously with other JONSWAP variables. The model is expressed as

$$A_\gamma = 1 - e_0 \, ln(\gamma_{ADR}) \quad (7)$$

The parameters of the spectral equation are optimized by a numerical optimization procedure. Each optimization procedure needs to define the following:

- objective (one or more) to be achieved (a variable to be maximized, minimized or set to a certain value),
- constraints that need to be satisfied – these are usually upper or lower limits of certain variables or calculated quantities,
- variables that will be changed by the algorithm during the optimization procedure in order to achieve the objective and satisfy the constraints.

The objective, constraints and variables used to optimize the JONSWAP spectrum definition for the Adriatic Sea are briefly reviewed in the following sections. .

2.4.1. Objective - minimum of the sum of errors / *Cilj – minimum zbroja pogrešaka*

The ultimate objective was to define the JONSWAP spectral energy function parameters, modified and optimized for the Adriatic Sea. Several assumptions and criteria have been defined in order to estimate the error, i.e. the deviation between the obtained and desired results. The objective was therefore defined as a minimum of the sum of all errors. However, as individual errors represented different physical or statistical quantities, with different units, they cannot be summed meaningfully in a straightforward way. Ponders were introduced for each error to balance (or emphasize) one error compared to another. Furthermore, the algorithm was set to calculate and optimize the spectrum for a range of significant wave heights H_s = 0,5 – 7,0 m (with a step ΔH_s = 0,5 m) for a range of frequencies 0.2 – 6.28 rad/s (with the step of $\Delta\omega$ = 0,01 rad/s).

To ensure that the results are consistent throughout the entire H_s range, first the errors of each individual quantity are summed together for the complete range and afterwards these sums are multiplied with selected ponders P_i and again summed together to represent one single objective. The minimization of that value is the objective of the optimization problem.

The following errors are defined:

1. Difference squared of the "input H_s" from "calculated H_s". The input H_{s_input} is the quantity given to the spectral model equation to calculate and visualise the wave spectrum curve. The calculated $H_{s\_calculated}$ is the significant wave height recalculated from the spectrum curve. Obviously for the spectrum to represent a physically meaningful result these two quantities, i.e. H_{s_input} and $H_{s\_calculated}$ need to be the same.
2. Difference squared of the "input peak frequency", as calculated by the chosen modal frequency model based on given H_s, from its position on the calculated and visualized spectral curve.
3. Difference squared of the maximum discrete energy value of the original Tabain's spectrum from the same value for the optimized spectrum for a given H_s. This assumption was introduced as an initial test which showed a large number of possible solutions that satisfy the equality of H_{s_input} and $H_{s\_calculated}$ by inversely modifying the spectral width and peak height. Most of these solutions, especially the extreme ones, evidently are not physically meaningful considering wave energy generation, redistribution and dissipation processes. Thus a reference had to be chosen and this error assumes that the energy distribution throughout the frequencies is well described in the original Tabain's spectrum.
4. Difference squared of the width factor κ [12] between the optimized and original Tabain's spectrum. With lack of more detailed data, this demand imposes, similar as with $error_3$, an assumption that the Tabain's spectrum described well the energy distribution around the spectral peak.
5. Sum of difference squared of secants *s* between successive points *S(ω)* on the spectral curve. Such defined error influences resulting spectrum in a somewhat aesthetic manner. Previously defined errors still allow multiple solutions, all of which satisfying physical requirements. Most possible solutions offered a wide base spectrum with a high "jump" around the peak frequency. Governing physical wave processes suggest a gradual transfer of energy between frequencies and a smoother spectrum curve. The smoothness criterion, disabling sudden "jumps" on the spectral curve, is defined mathematically by requiring uniform secants of successive points.

2.4.2. Constraints / *Ograničenja*

Constraints are set as upper and lower boundaries on all variables. Considering specific solutions, aside variable bounds, it was necessary to define the difference between spectral width on the left σ_a and right side σ_b of the spectral peak. The chosen difference is σ_b - σ_a = 0,02 that is the same as in the original Tabain's spectrum.

2.4.3. Variables / *Varijable*

The defined variables for the optimization are:

- γ_{ADR} spectral peak enhancement factor
- e_0 numerical constant of the spectral normalization factor A_γ which is dependent on
- σ_a measure of spectrum width left from the peak

- σ_b measure of spectrum width on the right side from the peak

With the described procedure of finding the appropriate spectrum shape and its parameters, the so called "affine transform" is performed with a goal to relocate the spectral peak while retaining other relevant physical properties.

3. RESULTS / *Rezultati*

After several iterations of the optimization procedure and manual tuning of the error ponders P_i, finally the modified/optimized wave spectrum for the Adriatic Sea has been obtained and is presented in Figure 5.

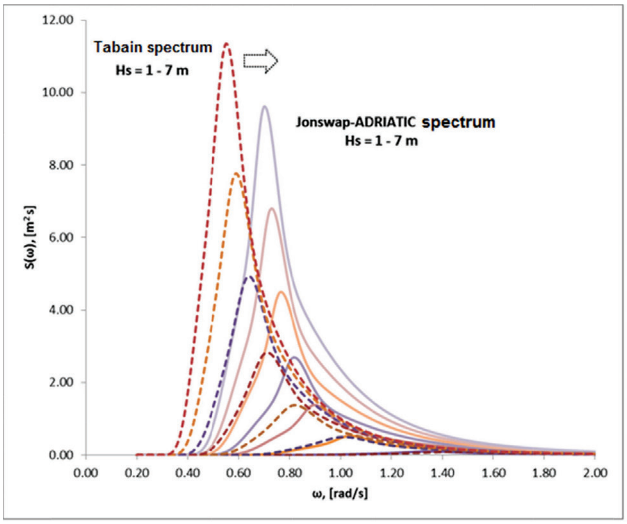


Figure 5 Transformation of Tabain's wave spectrum into the JONSWAP-Adriatic wave spectrum
Slika 5. Transformacija Tabainova spektra vala u JONSWAP-Adriatic spektru vala

The proposed, one-parameter JONSWAP-Adriatic wave spectrum, derived based on the JONSWAP spectrum and Tabain's modal frequency models, with parameters optimized for the Adriatic Sea as a whole and for all sea state conditions is:

$$S(\omega) = 0{,}8626\,\frac{5}{16}\frac{H_S^2\omega_m^4}{\omega^5}\,exp\left[-\frac{5}{4}\left(\frac{\omega_m}{\omega}\right)^4\right]\,1{,}78^{\,exp\left[-\frac{1}{2}\left(\frac{\omega-\omega_m}{\sigma\,\omega_m}\right)^2\right]} \quad (8)$$

$$\omega_m = 0{,}52 + \frac{1{,}4}{H_S + 0{,}7} \quad (9)$$

$$\sigma \begin{cases} \sigma_a = 0{,}06 \; za \; \omega \le \omega_m \\ \sigma_b = 0{,}08 \; za \; \omega > \omega_m \end{cases} \quad (10)$$

4. CONCLUSION / *Zaključak*

Based on the newly available WorldWaves (WW) wave parameter dataset that contains data for period 1992-2016 at 39 uniformly distributed location across the Adriatic Sea, a revision of the existing Tabain's wave spectrum has been made. Initially, it was verified how well the modal frequency model of the Tabain's model describes the measurements available from the WW database. Both large scatter of peak frequencies per specific significant wave height and a deviation of Tabain's model from the most frequently occurring values are noticed. Tabain's modal frequency parameters were optimized in order to provide a better fit to the WW data.

Afterwards, it was concluded that the modified frequency model gives inconsistent results within the original Tabains's spectral model. Therefore the JONSWAP spectral model is chosen to proceed in combination with the DNV-GL proposed calculation of the spectrum normalization factor. The normalization factor counteracts the peak enhancement factor within the JONSWAP model in order to keep the area under the curve, i.e the wave energy, constant. JONSWAP parameters were defined as variables and optimized in order to develop a modification of the JONSWAP spectrum for the Adriatic Sea. The proposed spectrum, in combination with the modified Tabain's peak frequency model, is named the JONSWAP-Adriatic spectrum. To optimize its parameters a set of pondered errors was defined and the minimization of its sum, simultaneously across the entire significant wave height range, presented the objective of the optimization process. The parameters of the JONSWAP-Adriatic parameters, both for the spectrum curve and modal frequency calculation, are presented for the entire Adriatic Sea basin (whole WW database was merged), with all sea states considered.

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Note: Some of the research results were presented at the Naše more 2019 conference.