Prolegomena 18 (2) 2019: 167–180
doi: 10.26362/20190108

Three-Valued Plurivaluationism and the Higher-Order Vagueness

JAN ŠTĚPÁNEK
Department of Philosophy, Faculty of Arts, Masaryk University, Arne Nováka 1, 602 00 Brno, Czech Republic
stepanek@phil.muni.cz

ORIGINAL SCIENTIFIC ARTICLE – RECEIVED: 28/02/19 ACCEPTED: 07/05/19

ABSTRACT: Plurivaluationism is an approach to solving the Sorites Paradox based on the idea that vague discourse has more than one acceptable interpretation. Since plurivaluationism is a framework which enables one to utilize different underlying logics, three basic versions can be identified – bivalent, many-valued and fuzzy. The aim of this paper is to show how three-valued plurivaluationism, as proposed by Wang, fares against its competitors. In the first and second part, some of the traditional approaches to solving the Sorites Paradox are outlined and are confronted with two objections based on the problem of higher-order vagueness – the so-called location problem and the jolt problem. The third part is dedicated to introducing and evaluating different versions of plurivaluationism, with an emphasis on comparing Wang's three-valued plurivaluationism to its competitors. While three-valued plurivaluationism is a promising enterprise, I show that some modifications Wang made to his system cause the location problem in plurivaluationism to re-emerge, and it also fails to provide a satisfactory answer to the jolt problem.

KEY WORDS: Fuzzy logic, many-valued logic, plurivaluationism, sorites paradox, three-valued logic, vagueness.

1. Introduction

Higher-order vagueness is a problem which plagues most approaches to solving the Sorites Paradox. According to Smith (2008), there are two aspects of higher-order vagueness – the *location problem* and the *jolt problem*. Briefly, the location problem consists in our inability to justify the placement of sharp boundaries when dealing with vagueness, and the jolt problem consists in there being a jump in truth-value between some of the members of the soritical sequence. Most of the approaches to solving the Sorites Paradox cannot succeed with at least one of these problems. Plurivaluationism is a framework that is based on a multiplicity of acceptable interpretations, therefore it is equipped to deal with the location problem, as will be shown later. According to Smith, it is only a fuzzy version of plurivaluationism that is capable of successfully dealing with the jolt problem. However, Wang (2016) proposes a three-valued version of plurivaluationism, which can deal with the jolt problem as well.

My goal in this paper is to show the problems connected with Wang's version of plurivaluationism. Namely, I will show his amendments to three-valued logic based plurivaluationism directed towards the jolt problem beg the question as to why one should prefer his three-valued approach to Smith's fuzzy plurivaluationism. I will argue that if one wants to employ a non-fuzzy version of plurivaluationism, one must bite the bullet and approach the jolt problem from another viewpoint.

2. Traditional approaches and the location problem

When dealing with vagueness and the Sorites Paradox, proponents of "traditional approaches", e.g., epistemicism and supervaluationism, failed to see one crucial assumption their approaches are based on – the existence of only one intended interpretation of language. This assumption is the main cause of the higher order vagueness problems, i.e., the location problem and the jolt problem.

First, let us consider the location problem. Vague terms can be contrasted with precise terms based on our inability to sharply draw borderlines to apply to vague terms. With precise terms, we can always unequivocally determine whether the term in question is applicable or not. If we take the term "prime", we know exactly to which entities it can be applied. We know that it is true to say "5 is a prime number", we know that "6 is a prime number" is a false statement, and we also know Julius Caesar is not the right kind of entity for the term "prime" to be applied to.[1] While there are undoubtedly numbers (very large numbers) which we are unable to classify as either primes or composite numbers, this is a problem that can be resolved.

With vague terms, however, the situation is radically different. There is no clear line delineating the applicability of vague terms. Even though the applicability of at least some vague terms may seem to supervene on some precisely expressible value or measure, this does not help us in determining the applicability of the vague terms in question. We may know a pile of sand is composed of exactly 58,438 grains of sand, yet we would still be unable to

[1] For the purposes of this paper, it is inconsequential whether sentences like "Julius Caesar is a prime number" are lacking any truth-value, are false, or have some intermediate truth-value. What matters is that when precise terms are considered, it is clear to which entities these are applicable or not, e.g., it is clear that "prime number" is applicable to numbers.

tell precisely when it becomes a heap of sand. The question of whether some large number is prime or not can be resolved by having more computing power at our disposal. The question of whether a man measuring 1.82 metres is tall cannot be resolved by any empirical measurement or computation. Even if we know that the man actually measures 1.82436738 metres, we are still no closer to a definitive answer to the question whether or not he is tall.

Traditional approaches to solving the Sorites Paradox postulate some borders of vague terms' applicability.[2] According to the epistemicists, there is actually a sharp border between vague terms' applicability and, in this aspect, they are similar to precise terms (cf. Williamson 1994; Sorensen 1988). However, in principle, we are unable to locate this border, since we are unable to distinguish between two entities which are very similar in aspects relevant to the applicability of the vague term in question. Our inability to locate precise boundaries is due to the so-called *margin for error principle* (cf. Williamson 1994: 226–230), which states that if there is no evidence that would enable us to distinguish any two situations, we must judge both situations equally.

Although there are many different many-valued logics (cf. Gottwald 2015), when vagueness and the Sorites Paradox are concerned, there are basically only two kinds that are used – three-valued logics (cf. Körner 1976; Tye 1990, 1994) and degree theories such as fuzzy logic.

The advantages of three-valued logics are their simplicity and truth-functionality. For a proponent of three-valued logic, the range of significance for a vague term cannot be divided into only two sets of entities. Instead, the proponent of three-valued logic uses three sets – a set of entities to which the vague term in question is applicable, a set of entities to which the term is not applicable, and a set of so-called *borderline* or *penumbral* entities, i.e., entities which we cannot surely classify into either of the previous sets. Traditional two truth-values are therefore inadequate in such situations and a third one needs to be introduced in order to incorporate penumbral propositions.

Degree-theoretic and fuzzy approaches share a few common traits – truth comes in degrees, there can be potentially infinite truth-values, and even small changes in aspects relevant to vague term applicability can bring about a small change in truth-value.[3] I will use the term "fuzzy approaches" in its broad sense in which it encompasses what is nowadays called *mathematical fuzzy logic* (cf. Cintula, Fermüller and Noguera 2016; Hájek 1998) and, also, so-called degree-theoretic approaches that often employ the same base logic as many of the fuzzy approaches – Łukasiewicz logic (Ł). It is also worth

[2] Apart from the nihilistic approach, according to which there are no boundaries since there are no heaps, chairs, or no ordinary things (cf. Unger 1979).

[3] Proponents of these approaches therefore do not agree with so called *principle of tolerance* (cf. Wright 1975; Kolář 1994).

mentioning that many fuzzy theoreticians adopted Smith's plurivaluationist framework for fuzzy logic, thereby avoiding both the location and the jolt problem. Naïve fuzzy approaches suffer from the location problem and predate Smith's (2008).

Fuzzy approaches are based on an assumption that vagueness is a matter of degree. If A measures more than B, A should be *taller* than B. This should also be, according to fuzzy theorists, represented by different degrees of truth being ascribed to sentences like “A is tall” and “B is tall”.

Vague property is represented by a fuzzy set, i.e., a function from a universe of discourse to a structure of truth degrees – usually the real unit interval , where stands for absolute truth and for absolute falsity (cf. Běhounek 2014). Truth-values are linearly ordered. When truth-values and are concerned, the connectives behave classically, yet when intermediate degrees of truth are concerned, the connectives are defined using *triangular norms* or *t-norms*. A t-norm is a binary operation on a unit interval , i.e., a function, and is commutative, associative, monotonous and meets boundary conditions (cf. Hájek 1998: 28; Klement, Mesiar and Pap 2000: 4–10). Logic Ł (cf. Hájek 1998: 63–87; Gottwald 2001: 179–266) is the logic often used by both fuzzy logicians and degree theorists alike (cf. Zadeh 1975, 2008; Machina 1976; Edgington 1997; Priest 2003). It is no surprise that Ł is employed even by philosophers that do not proclaim themselves to be fuzzy logicians (cf. Machina 1976; Edgington 1997). Proponents of modern mathematical fuzzy logic, however, even use other logics, such as (Hájek's) Basic fuzzy logic , Pavelka's logic and others (cf. Hájek and Novák 2003).

A distinguishing feature of infinite-valued approaches is that they enable its users to employ *fuzzy modifiers*, like *very true* (cf. Hájek 2001), *fairly true*, or the suffix *-ish*. These modifiers, which are sometimes also called *hedges*, are unary connectives which can modify the sentences' truth-value. If states that somebody is tall, then , , etc. In this way, fuzzy logic can reflect different modifiers by changing the truth-value of a base sentence , which is an option that none of the other approaches discussed here can replicate. Fuzzy modifiers like *very true* or *almost true* are sometimes employed as a tool to model sorites reasoning (cf. Hájek 2001; Hájek and Novák 2003).

The problem all these approaches share is the fact that there are precise boundaries of application in all of them. While it is easier to see these boundaries when epistemicism and three-valued approaches are considered, fuzzy approaches are also based on precise application of limits. While the epistemicist approach only has one precise boundary, three-valued approaches contain two precise boundaries. One can describe fuzzy approaches as having potentially infinite truth-values – and therefore infinitely many boundaries –, yet for our purposes, there are two boundaries of interest. The first delineates the point at which the truth-value increases from 0, and the other one delineates the point at which the truth-value decreases from 1. If we place any sharp boundaries to separate propositions according to their truth-values or entities according to the applicability of some vague term, we must conclude these boundaries are completely arbitrary. The question each of these approaches must answer is therefore why these boundaries are where they are. None of these approaches, however, provides a satisfactory answer.

Trying to avoid the problem of arbitrary boundaries by blurring them is a non-starter since the problem just repeats itself – instead of one penumbra there would be two, and more than three truth-values would be needed to accommodate them. If we add another truth-value, the problem only gets worse, as with more truth-values even more penumbras emerge, so even if we started with a classical bivalent logic, we would end up with something like fuzzy logic. As Sainsbury (1996: 256) eloquently put it, “you do not improve a bad idea by iterating it.”

3. The jolt problem

Another higher-order vagueness-related problem is the jolt problem. According to Smith and other proponents of fuzzy approaches, one of the key features of vague terms’ applicability is their matter of degree. Some people are taller than others, some colours are redder than others, and so on. As was mentioned earlier, the idea behind fuzzy approaches is that small changes in applicability should be reflected in degrees of truth. This is captured by Smith’s (2008: 146) *closeness principle*: “If and are very close in -relevant respects, then ‘’ and ‘’ are very close in respect of truth”. This relation between degrees of truth is not as straightforward as it might seem. If there is an increase in a measure relevant to the applicability of a vague term, there doesn’t have to be an equal increase in truth-value. If A measures more than B, then the sentence “A is tall” must have a truth-value either equal, or greater than the sentence “B is tall”. Even naïve fuzzy approaches are in accordance with this principle since their use of degrees of truth ensures that the difference in truth-value between similar entities should be minimal. So, even though there are many changes in truth-values when moving ahead in the sorites sequence, these changes are always minute.

With epistemicism and three-valued approaches, however, the situation is radically different. Since there are either two or three truth-values, there is always a jolt between these truth-values. There are always entities that are indistinguishable with respect to all aspects relevant to a given vague term’s applicability, yet the truth-values of corresponding sentences’ which mention them would be either polar opposites (in epistemicism), or at least radically different (in three-valued approaches). Consider, e.g., a situation in which we compare two people – A measures 1.86235 metres and B measures 1.86234 metres –, and we have two sentences – "A is tall" and "B is tall". For the sake of the argument, let us assume A is clearly tall, while B is not. In fuzzy approaches, the two statements would have a very similar truth-value, say 1 and 0.9997 respectively. In three valued approaches, the first sentence would have truth-value 1, while the second one would have truth-value 0.5.[4] An epistemicist would say that the first sentence has a truth-value of 1, while the second one would have a truth-value of 0. The change in the truth-value is therefore pretty steep, both in the case of bivalent and three-valued logics.

4. Plurivaluationism

Epistemicism, three-valued approaches, naïve fuzzy approaches, as well as some of the other traditional approaches to solving the Sorites Paradox are limited by an assumption there is one intended interpretation of language. As shown above, this assumption leads to the location problem, since all the aforementioned approaches work with precise boundaries. The idea of there being precise boundaries, however, seems counterintuitive to many. And if we consent to the epistemicist's treatment of vagueness, we are left with a theory that, on the one hand, claims there are precise boundaries but, on the other hand, that there is nothing we can do with this knowledge since we can never know where these boundaries are.

Plurivaluationism is based on an ingenious idea – there are many equally acceptable interpretations of language. The plurivaluationist holds the view that if one is trying to solve the Sorites Paradox only within the bounds set by a single intended interpretation of language, she is fighting a losing battle. By limiting ourselves to a single interpretation of vague discourse, we are bound to become victims of the location objection.

By admitting a multiplicity of acceptable interpretations (none of which are in any way privileged), plurivaluationists successfully solve the location problem by considering many different interpretations, each of which differs in assigning different truth-values to sentences containing vague terms, i.e., they differ in how vague terms are interpreted. In other words, interpretations differ from each other by assigning different objects to the extension and anti-extension of vague terms.

As Smith (2008: §2.5) correctly pointed out, some philosophers (cf. Varzi 2003, 2007) in the past mixed plurivaluationism and supervaluationism together.

[4] In this example, I use numerical representations of truth-values – 1 to designate truth-value True, 0 to designate truth-value False, and 0.5 to designate the third truth-value, i.e., Nonsensical or Indeterminate.

This is an understandable mistake since at a first glance, these approaches look very much alike. Nevertheless, there are important differences between them – the most important being that supervaluationism (cf. van Fraassen 1966; Keefe 2000) works with classical extensions of a non-classical model, while plurivaluationism considers different acceptable interpretations of a language.

One might object that plurivaluationism does not elude the location objection, but it simply moves it to the next level. Even though now we do not have to deal with the location problem when we consider vague terms, we have a problem with acceptable interpretations. If there are acceptable interpretations, there must also be some interpretations which are unacceptable. What if we say, e.g., that an interpretation of language according to which a limit of baldness is having at most 27 hairs is unacceptable, even though interpretations with a smaller number of hairs are acceptable? How did we come to that conclusion? What gives us the reason to draw the line between acceptable and unacceptable interpretations this way?

A proponent of plurivaluationism might propose to treat the notion of acceptable interpretations as vague. This though, would only cause an infinite regress. Another option is to adopt an Unger (1979)-like standpoint. If we take a vague term, then it is acceptable to say that acceptable interpretations range from one in which the term is applicable to all objects, to one in which the term is applicable to no objects.

Though the second option might seem unappealing, I find nothing inherently wrong with it. It is perfectly reasonable to say there is an interpretation of language according to which there are no bald men, i.e., even men with no hair on their heads are not bald. It is also perfectly reasonable to believe that (almost) no one would utilize such an interpretation.

It is also worth mentioning the same objection can be raised against supervaluationism. Supervaluationists, however, cannot employ the solution proposed above, since it would lead to some unwanted consequences. If they were to eliminate arbitrary boundaries in their system by permitting precisifications in which vague terms are applicable from none to all objects in their ranges of significance, this would make supervaluationism practically useless. If such precisifications were permitted, the upshot would be that all atomic sentences' truth-values, which would depend on the applicability of some vague term, would be gappy.[5] So, while plurivaluationists do not have to consider interpretations of vague language that they find unappealing, supervaluationists must take them into consideration, making vague atomic sentences inherently gappy.

[5] Composite sentences could still be either true or false.

4.1. Classical plurivaluationism

It has already been mentioned plurivaluationism is based on a multiplicity of acceptable interpretations, yet there has been no mention of what sort these interpretations can or should be. I therefore believe it is suitable to use the term “plurivaluationism” to denote a framework which employs many interpretations of vague language, independent of the underlying logic. As such, this term should be specified when a particular kind of logic is employed. Some may advocate classical interpretation, i.e., interpretation based on classical logic. I call this approach *classical plurivaluationism*.

As mentioned earlier, by employing a multiplicity of interpretations, plurivaluationists successfully dodge the location objection. Smith (2008: §4.4), however, charges classical plurivaluationism with its inability to deal with the jolt problem. Even though there are many acceptable interpretations, there is a sharp cut-off point in each of them. And since there are sharp cut-off points in every interpretation, the closeness principle is violated and the jolt problem is therefore not resolved. And since every classical interpretation makes a vague term precise, plurivaluationism’s classical version should be, according to Smith, rejected.

4.2. Fuzzy plurivaluationism

Another kind of plurivaluationism employs some kind of fuzzy logic as its underlying logic. It was pioneered by Smith in his seminal book (2008) and developed more recently by, e.g., Běhounek (2014). Unlike its classical version, fuzzy plurivaluationism combines fuzzy models and semantic indeterminacy concerning the existence of different interpretations of language.

> Our vague language has many acceptable fuzzy interpretations simultaneously, and as far as semantics is concerned, that is the end of the story. So we have indeterminacy, or plurality, of meaning. Unlike in the supervaluationist picture, the language is not in a unique (higher-order) semantic state. (Smith 2008: 287)

As Běhounek (2014) points out, fuzzy logics that take only one fuzzy model into consideration and operate with membership function and membership degree are very well suited for practical applications (engineering, fuzzy control, etc.), however, they are not good representations of vague notions. While these logics take gradualness into consideration, they do not incorporate the semantic indeterminacy aspect of vagueness.

A similar approach can also be found in works of some adaptive logics proponents (cf. van Kerkhove and Vanackere 2003; van der Waart, van Gulik and Verdée 2008).

> [I]t is up to a specific person, with specific purposes, in a specific situation, to make specific decisions about the upper and lower bound of the vague stroke of some specific predicate applied to some specific individual. (van Kerkhove and Vanackere 2003: 23)

As it is apparent, adaptive fuzzy logics also work with different interpretations of vague language. There is, however, a significant difference between them and fuzzy plurivaluationism. While adaptive fuzzy logics are only concerned with one fuzzy model at a time (like other application-oriented fuzzy logics), fuzzy plurivaluationism is considered not with particular fuzzy models, but with a fuzzy logic in a consequence relation sense. Not unlike supervaluationism, fuzzy plurivaluationism also prefers supertruth to truth in a particular model. However, for a proponent of fuzzy plurivaluationism, supertrue propositions are those that come out true in all acceptable interpretations.

Naïve fuzzy approaches were the only approaches that successfully dealt with the jolt objection, however, they could not deal with the location objection. By implementing fuzzy logic into the framework of plurivaluationism, we can successfully deal with both jolt and location objection, thereby eliminating higher-order vagueness problems altogether.

4.3. Three-valued plurivaluationism

Another recently proposed version of plurivaluationism employs three-valued logic. This approach was pioneered by Wang (2016), whose goal was to present a three-valued plurivaluationism immune to both the location and the jolt objection, though he also tries to address the penumbral connections.

Wang's three-valued plurivaluationism employs Kleene's strong *K ??* logic as its base logic (cf. Kleene 1938):

p	q	$p \vee_K q$	$p \wedge_K q$	$p \rightarrow_K q$	$p \leftrightarrow_K q$
T	T	T	T	T	T
T	I	T	I	I	I
T	F	T	F	F	F
I	T	T	I	T	I
I	I	I	I	I	I
I	F	I	F	I	I
F	T	T	F	T	F
F	I	I	F	T	I
F	F	F	F	T	T

Table 1: Kleene's *K??* logic (strong three-valued logic)

Unlike Bochvar's (1981) internal logic *B??* (Kleene's weak three-valued logic), which was employed to deal with vagueness by Halldén (1949), *K??* better captures ideas commonly associated with some logical connectives, e.g., if the antecedent of an implication is false, then the implication is true, no matter the consequent's truth-value.

In Wang's system, there are sentences which are true/false *simpliciter*, i.e., true/false in every acceptable interpretation. This is comparable to the notion of supertruth in supervaluationism, however, Wang's plurivaluationist approach has the advantage of being able to cope with *missing witnesses* and *missing counterexamples* problems.[6] While the supervaluationist approach cannot use any of the admissible precisifications as a witness/counterexample, plurivaluationist has no such problems, since all acceptable interpretations are at her disposal.

Wang's approach, however, does not just use Kleene's *K??* logic in the framework of plurivaluationism. Even though using three-valued logic in the framework of plurivaluationism is enough to deal with the location problem, the jolt problem remains an issue, since in all acceptable interpretations there are jumps on the borderlines of the intermediate truth-value. In order to avoid the jolt problem, Wang's approach is supplemented by special truthlikeness metrics and a more fine-grained distinction to different borderline cases.

Even though Wang doesn't want to introduce a system based on degrees of truth, such as Smith's fuzzy plurivaluationism, there is some gradualness introduced by a new classification of borderline cases. While normal three-valued logic-based approaches only consider objects in extension, anti-extension, and penumbral objects of any vague term, Wang (2016: 350) also introduces *unclear borderline cases* which he further classifies into *-unclear* and *-unclear borderline cases*. Propositions that are -unclear borderline are definitely not true, but it is not clear how to classify them further. -unclear borderline propositions are definitely not false, but their further classification is also unclear.

Wang's system is also supplemented with special metrics to measure the degree of closeness to the truth (or falsity) *simpliciter* (cf. Wang 2016: 356). Wang claims that the degree of closeness to the truth (or falsity) *simpliciter*

[6] Missing witness – there are existential statements which are supertrue, yet supervaluationist is unable to specify any particular statement that would confirm it. Missing counterexample – there are universal statements which are superfalse, yet supervaluationist cannot specify any particular statement that would be a counterexample to it. The inductive premise of the mathematical induction sorites, $\forall n\ (Fa_n \rightarrow Fa_{n+1})$, is superfalse, since it is false in all precisifications. Classically, for general claim to be false it takes at least one counterexample. In supervaluationism, however, we have no counterexample, since it is impossible to single out any of the precisifications (cf. Smith 2008: 84–85).

is a way of introducing gradual change into the approach founded on three-valued logic.

I find both these extensions of three-valued plurivaluationism very unappealing, for multiple reasons. First, let us consider notions of -unclear (and -unclear) cases. If a proposition is -unclear, then we know it is not false, however, we don't know how to classify that proposition further. That means we are not sure if it is true, or if it has intermediate truth-value. Therefore, that proposition concerns a borderline borderline case. At this point, one has to ask why normal borderline cases are worthy of special treatment, getting their own truth-value, while -unclear are not. Moreover, is there any reason why one should limit herself to doing only one iteration, instead of continuing and introducing -unclear, -unclear, and so on? Introducing -unclear and -unclear cases reintroduces higher-order vagueness back into the three-valued plurivaluationism and leads to an infinite regress. Moreover, one can wonder whether Wang's system is three-valued at all since -unclear and -unclear cases seem much like an introduction of additional truth-values.

Metrics to measure closeness to truth/falsity *simpliciter* do very little as far as gradualness is concerned. While one may use some metrics to order objects in the range of a vague term's significance and to order propositions mentioning these objects – this, after all, is what the Sorites Paradox is based on –, there is yet no absolutely gradual transition from truth to falsity *simpliciter* (and vice versa), since there are still only three truth-values to choose from. If one wants a transition from truth to falsity to be gradual, one needs to employ degrees of truth.

Although combining three-valued logic with plurivaluationism is a promising enterprise, Wang's version is a system with debatable usefulness. If one is motivated by an effort to create a system which does not suffer from the jolt objection, versions of plurivaluationism based on fuzzy logic are a natural choice.

The question is whether the jolt objection is an objection one should take seriously. While it is indisputable the location problem is a genuine problem concerning higher-order vagueness, the situation is less clear with the jolt problem. It is true there are vague terms which are a matter of degree, such as "red" or "tall". There are also vague terms which may not be taken as a matter of degree. These are *combinatory* vague terms like "religion" or "sport". While some versions of the Sorites Paradox are based on terms such as these, it must be said these versions seem quite artificial. While one surely can compare football to curling, it is questionable if one can actually say that one of these is a sport to a greater degree than the other. While there may be some quantifiable aspects of sports which may be relevant to the applicability of the term "sport", it is far from clear if such aspects can be used to order sports. In these cases, at least, it seems that using a fuzzy logic-based approach would be a bit of a stretch, and either bivalent or three-valued approaches seem to be a more natural choice.

5. Conclusion

While the older approaches must deal with serious difficulties when dealing with the Sorites Paradox and vagueness, different versions of plurivaluationism seem to fare much better. All versions of plurivaluationism can effectively defuse the location problem. Even though Smith makes a strong case for his fuzzy plurivaluationism, I believe it is not the only viable variant of plurivaluationism, quite the contrary – different versions of plurivaluationism have their place. For many philosophers, classical plurivaluationism will be the obvious choice for its simplicity due to it making use of classical logic. For a more detailed analysis and more technical approaches, many-valued versions of plurivaluationism and fuzzy plurivaluationism seem to be better choices. Whatever version of plurivaluationism is employed, it defuses the Sorites Paradox without being exposed to the problem of location.

If one considers the jolt problem to be a genuine problem that a theory needs to address, one is left with only one choice – fuzzy plurivaluationism. While Wang attempted to formulate a version of three-valued plurivaluationism that would account for the gradual change in truth, the enterprise doesn't seem to have succeeded. If, on the other hand, one doesn't consider the jolt to be a problem, then both classical and three-valued plurivaluationism seem to be good choices.[7]

Bibliography

Běhounek, L. 2014. "In Which Sense Is Fuzzy Logic a Logic for Vagueness?", in T. Lukasiewicz, R. Peñaloza and A.-Y. Turhan (eds.), *CEUR Workshop Proceedings Vol. 1205: Logics for Reasoning about Preferences, Uncertainty, and Vagueness*, 26–39.

Bochvar, D. A. 1981. "On a Three-Valued Logical Calculus and Its Application to the Analysis of the Paradoxes of the Classical Extended Functional Calculus", *History and Philosophy of Logic* 2, 87–112.

Cintula, P., Fermüller, C.G. and Noguera, C. 2016. "Fuzzy Logic", *The Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/archives/win2016/entries/logic-fuzzy/ [accessed January 27th 2018].

[7] Acknowledgement: This work was supported by the GAČR *Sémantické pojmy, paradoxy a hyperintenzionální logika založená na moderní rozvětvené teorii typů* (č. GA 16-193955) grant. I would like to thank Jiří Raclavský and Tomáš Ondráček for their invaluable comments and advices.

Edgington, D. 1997. "Vagueness by Degrees", in R. Keefe and P. Smith (eds.), *Vagueness: A Reader* (Cambridge, MA: MIT Press), 294–316.

Fraassen, B. C. van 1966. "Singular Terms, Truth-Value Gaps, and Free Logic", *The Journal of Philosophy* 63(17), 481–495.

Gottwald, S. 2001. *A Treatise on Many-Valued Logic* (Baldock: Research Studies Press).

Gottwald, S. 2015. "Many-Valued Logic", *The Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/archives/spr2015/entries/logic-manyvalued/ [accessed February 1st 2018].

Hájek, P. 1998. *Metamathematics of Fuzzy Logic* (Dordrecht: Kluwer).

Hájek, P. 2001. "On Very True", *Fuzzy Sets and Systems* 124, 329–333.

Hájek, P. and Novák, V. 2003. "The Sorites Paradox and Fuzzy Logic", *International Journal of General Systems* 32(4), 373–383.

Halldén, S. 1949. *The Logic of* Nonsense (Lund: Lundequista Bokhandeln).

Keefe, R. 2000. *Theories of Vagueness* (Cambridge: Cambridge University Press).

Kerkhove, B. van and Vanackere, G. 2003. "Vagueness-Adaptive Logic: A Pragmatical Approach to Sorites Paradoxes", *Studia Logica* 75(3), 383–411.

Kleene, S. C. 1938. "On Notation for Ordinal Numbers", *The Journal of Symbolic Logic* 3(4), 150–155.

Klement, E. P., Mesiar, R. and Pap, E. 2000. *Triangular Norms* (Dordrecht: Springer).

Kolář, P. 1994. "Vagueness without Tolerance", in P. Kolář and V. Svoboda (eds.), *Logica '93* (Prague: Filosofia), 33–44.

Körner, S. 1960. *The Philosophy of Mathematics* (London: Hutchinson).

Machina, K. 1976. "Truth, Belief, and Vagueness", *Journal of Philosophical Logic* 5(1), 47–78.

Priest, G. 2003. "A Site for Sorites", in J. C. Beall (ed.), *Liars and Heaps* (Oxford: Clarendon Press), 9–23.

Sainsbury, R. M. 1990. "Concepts without Boundaries", in R. Keefe and P. Smith (eds.), *Vagueness: A Reader* (Cambridge, MA: MIT Press), 251–264.

Smith, N. J. J. 2008. *Vagueness and Degrees of Truth* (Oxford: Oxford University Press).

Sorensen, R. A. 1988. *Blindspots* (Oxford: Clarendon Press).

Tye, M. 1990. "Vague Objects", *Mind* 99(396), 535–557.

Tye, M. 1994. "Sorites Paradoxes and the Semantics of Vagueness", *Philosophical Perspectives* 8, 189–206.

Unger, P. 1979. "There are No Ordinary Things", *Synthese* 41(2), 117–154.

Varzi, A. C. 2003. "Cut-Offs and their Neighbors", in J. C. Beall (ed.), *Liars and Heaps: New Essays on Paradox* (Oxford: Clarendon Press), 24–38.

Varzi, A. C. 2007. "Supervaluationism and Its Logic", *Mind* 116(463), 633–676.

Waart van Gulik, S. van der and Verdée, P. 2008. "Modeling Sorites Reasoning with Adaptive Fuzzy Logic", *Fuzzy Sets and Systems* 159(14), 1869–1884.

Wang, W.-F. 2016. "Three-Valued Plurivaluationism: A Reply to Williamson's Criticism of the Three-Valued Approach to the Sorites Paradox", *The Philosophical Forum* 1, 341–360.

Williamson, T. 1994. *Vagueness* (London: Routledge).

Wright, C. 1975. "On the Coherence of Vague Predicates", *Synthese* 30(3–4), 325–365.

Zadeh, L. A. 1975. "Fuzzy Logic and Approximate Reasoning", *Synthese* 30(3–4), 407–428.

Zadeh, L. A. 2008. "Is There a Need for Fuzzy Logic?", *Information Sciences* 178(13), 2751–2779.