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Realized density estimation using intraday prices

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Abstract

Availability of high-frequency data, in line with IT developments, enables the use of
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more information to estimate not only the variance (volatility), but also higher realized moments and the entire realized distribution of returns. Old-fashioned approaches use only closing prices and assume that underlying distribution is time-invariant, which makes traditional forecasting models unreliable. Moreover, time-varying realized moments support findings that returns are not identically distributed across trading days. The objective of the paper is to find an appropriate data-driven distribution of returns using high-frequency data. The kernel estimation method is applied to DAX intraday prices, which balances between the bias and the variance of the realized moments with respect to the bandwidth selection as well as the sampling frequency selection. The main finding is that the kernel bandwidth is strongly related to the sampling frequency at the slow-time-time scale when applying a two-scale estimator, while the fast-time-time scale sampling frequency is held fixed. The realized kernel density estimation enriches the literature by providing the best data-driven proxy of the true but unknown probability density function of returns, which can be used as a benchmark in comparison against ex-ante or implied driven moments.

Keywords: bandwidth selection, intraday prices, Kernel density, realized moments, sampling frequency selection, two-time scale estimator.

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Introduction

In the field of quantitative finance, it is very common to assume a probability density function (pdf) of underlying asset returns with unknown moments such as variance, skewness and kurtosis. These moments are parameters which are estimated frequently by many practitioners and academics using different approaches. Forecasting of these moments is of special interest to market participants as their future expectations are embedded in the current trading activities. Most of existing studies have focused on parametric models to compute ex-ante measures of variance, skewness and kurtosis of returns. However, not only moments are unknown, but distribution itself is also unobservable, making parametric models partially incoherent. True probability density function of underlying asset returns was and will remain one of the major issues. The main goal is to find a "true" distribution of returns, i.e. realized ex-post distribution, which is data-driven. Appropriate realized distribution is later used to derive realized moments of returns, which can be compared to the ex-ante counterparts. Even most studies have used simulation techniques to generate the true probability density function (Neuberger, 2012); it still lacks attempts for finding appropriate benchmark of the true density for comparison purpose.

Motivation for finding a realized ex-post distribution, which is closest to the true distribution of returns, is availability of the high-frequency data. These intraday observations enable us to use as much information as possible to obtain probability density function for each trading day separately. Thus, one would expect that pdf varies across trading days as its moments are time-varying. Two key issues emerge here. First issue refers to the sampling frequency selection and the second issue is about density estimation method. The most popular method for density estimation among users is Kernel method, especially when dealing with big data such as high-frequency observations. Kernel method is nonparametric and it does not require assumption imposed on the data generating process (DGP). Moreover, the Kernel estimator converges to the true DGP with probability one in a certain conditions. This brings us to new issues about the choice of the bandwidth and Kernel function. Luckily, the choice of the Kernel function has no effect on the results, and therefore a main issue remains on bandwidth selection (Marron, Nolan, 1988; Grith, Härdle, Schienle, 2012). Usually, an optimal bandwidth is chosen to minimize both the bias and the variance of the estimator. In line with this minimization problem, different rules where proposed in the literature, but many of them give oversmoothed densities (Terrell, Scott, 1992; Wand, Jones, 1995; Racine, 2008,).

In this study the bandwidth selection is related to the problem of sampling frequency selection in obtaining bias adjustment of the realized variance, the first issue outlined above. The most well-known high-frequency estimator is the realized variance proposed by Andersen and Bollerslev (1998). This estimator is formulated as the sum of squared intraday returns, which are equally spaced. Studies showed that this estimator is biased when sampling frequency is large (Hansen, Lunde, 2005; Bandi, Russell, 2008). This bias is induced by the autocorrelation resulting from non-synchronous trading, discrete price observations, bid-ask bounce and the influence is collectively regarded as a market microstructure noise. Higher sampling frequency will lead to a more significant noise problem (Ait-Sahalia et al. 2011). Many bias-corrected estimators of realized variance which are robust to microstructure noise where proposed (Barndorff-Nielsen et al., 2002; Ait-Sahalia et al., 2005; Oomen, 2006). In this paper we utilize two scale realized variance estimator (TSRV) proposed by Zhang et al. 2005. Thus, a main objective is to determine if there is a realized density from which we can derive consistent and asymptotically unbiased estimate of integrated variance such as TSRV. If such density exists, obtained by Kernel estimation method, then it can be taken as the benchmark of the true but unknown density of returns. Basically, this means that choice of Kernel bandwidth depends on the slow-time-time scale sampling frequency, while the fast-time-time scale sampling frequency is held fixed when computing TSRV.

This paper contributes to the existing literature in a several segments. Previous research has not considered benchmarking the true but unknown probability density function of returns using intraday prices. This study also enriches the literature on data-driven approaches for realized density estimation which is free of microstructure noise by usage of Kernel. The obtained findings offer valuable information to market participants by pinpointing the appropriate Kernel bandwidth as well as optimal slow-time-time scale sampling frequency.

The rest of the paper is organized into four sections. Section 2 comprehensively describes realized measures of moments and highlights their adjustment in the presence of microstructure noise. Section 3 presents Kernel density estimation method and dependence between bandwidth and slow-time-time scale frequency. Section 4 presents the obtained empirical results using intraday prices of DAX index. Directions for future research as well as final concluding remarks and limitations of present research are given in Section 5.

Realized measures of moments

The concept of realized variance was introduced among the first by Andersen and Bollerslev (1998) who have computed the ex-post measure of volatility at a lower frequency using data sampled at a higher frequency. Realized variance is defined as the sum of squared equidistant intraday returns:

$$RV_t^{\Delta} = \sum_{j=1}^{J} r_{t,j}^2 = \sum_{j=1}^{J} \left(p_{t,j} - p_{t,j-1}\right)^2, \tag{1}$$

where $p_{t,j}$ is the natural logarithm of the closing price observed at interval j for a given trading day t. The length of time interval Δ measures how frequently data are sampled. As the sampling frequency Δ increases realized variance RV_t converges to the quadratic variation of the semi-martingale process, known as integrated volatility (Andersen, Bollerslev, Diebold, Labys, 2001; Barndorff-Nielsen, Shephard, 2002, 2006). Realized skewness RS_t and realized kurtosis RK_t are obtained in a similar way and additionally standardized by the same realized variance:

$$RS_t^{\Delta} = \frac{\sum_{j=2}^{J} r_{t,j}^3}{\left(\sqrt{RV_t}\right)^3} = \frac{\sum_{j=2}^{J} r_{t,j}^3}{\left(\sqrt{\sum_{j=2}^{J} r_{t,j}^2}\right)^3}, \tag{2}$$

$$RK_t^{\Delta} = \frac{\sum_{j=2}^{J} r_{t,j}^4}{{RV_t}^2} = \frac{\sum_{j=2}^{J} r_{t,j}^4}{\left(\sum_{j=2}^{J} r_{t,j}^2\right)^2}. \tag{3}$$

Abovementioned realized measures are all contaminated by the microstructure noise, i.e. estimators become biased as sampling frequency increases (Hansen, Lunde, 2005; Bandi, Russell, 2008). This bias is induced by the autocorrelation resulting from non-synchronous trading, discrete price observations, bid-ask bounce (Aït-Sahalia et al., 2011). Higher sampling frequency will lead to a more significant noise problem. Thus, a reduction of the noise is required. For this reason, a two scale estimator of realized variance is proposed and comprehensively discussed by Zhang et al. (2005) and Zhang (2011). The major advantage of the two scale realized variance TSRV is ability to keep all intraday returns, observed at very high frequency, and still having unbiased and consistent estimator of integrated volatility IV. The background of TSRV relies on the subsampling and averaging techniques, which are also applicable to higher realized moments.

Following Amaya et al. (2013) and Shen et al. (2018), two scale estimators of realized moments are:

$$TSRV_t^{\Delta,S} = \frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^2 - \frac{\bar{n}}{J}\sum_{j=1}^{J} r_{t,j}^2 \ , \tag{4}$$

$$TSRS_t^{\Delta,S} = \frac{\frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^3}{\left(\sqrt{\mathrm{TSRV}_{t,s}}\right)^3} = \frac{\frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^3}{\left(\sqrt{\frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^2 - \frac{\bar{n}}{J}\sum_{j=1}^{J} r_{t,j}^2}\right)^3} \ , \tag{5}$$

$$TSRK_t^{\Delta,S} = \frac{\frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^4}{TSRV_{t,s}^2} = \frac{\frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^4}{\left(\frac{1}{S}\sum_{s=1}^{S}\sum_{j=1}^{n_s} r_{t,j,s}^2 - \frac{\bar{n}}{J}\sum_{j=1}^{J} r_{t,j}^2\right)^2} \ , \tag{6}$$

where S is the number of subsamples and n_s is the number of returns within each subsample (not necessary equal). First term of $TSRV_t^{\Delta,S}$ expression gives the average realized variance over S subsamples known as average sparse realized variance. The second term removes the bias from the average sparse realized variance, where $\bar{n}/J$ is the proportion of average subsample size in total sample size. This way $TSRV_t^{\Delta,S}$ becomes microstructure free, i.e. it is consistent and asymptotically unbiased estimate of integrated variance IV (Zhang et al., 2005; Zhang, 2011).

Prior to the calculation of $TSRV_t^{\Delta,S}$ as well as $TSRS_t^{\Delta,S}$ and $TSRK_t^{\Delta,S}$ respectively, one should select Δ and S. Time interval Δ is the fast-time-time scale, i.e. the highest sampling frequency available at which equally spaced intervals are non-empty. According to Arnerić et al. (2019 a) 1 minute fast-time-time scale eliminates zero prices and transaction gaps, while the fast-time time scale less than 1 minute is not reliable due to not so frequent trading. On the other hand, S determines the size of slow-time-time scale, i.e. sparse sampling frequency (Aït-Sahalia et al., 2005). The optimal slow-time-time scale can be found by minimizing the mean squared error (MSE) of the average RV sampled sparsely. As we know that MSE of an estimator is the sum of the squared bias and its own variance, we can use this criterion to balance between the bias and the variance, suggested by Zhang et al. (2005). In the study of Zhang et al. (2005) it is proposed to search for both optimal time scales when using thick-by-thick data. Opposite to that, it is more convenient to keep fast-time-time scale fixed at the best available sampling frequency and to search for slow-time-time scale only according to Arnerić et al. (2019 b). This approach has practical significance, as it is not so computational demanding, and the selection of fast-time-time scale by the researchers give themselves opportunity to control the quality of high-frequency data due to cleaning and filtering process prior to the analysis.

Two important conclusions arise here. Firstly, finding optimal slow-time-time scale sampling frequency enables bias adjustment of realized measures of moments. Secondly, if such moments exist then there should also exist realized density function from which the same moments can be recovered.

As already mentioned, Kernel method is utilized in this paper to obtain realized density estimation. The Kernel method is data-driven with many useful properties, explained in the next section. However, as the role of the Kernel bandwidth is the same as the role of slow-time-time scale we believe that those two are strongly related, i.e. selection of the bandwidth depends on the optimal slow-time-time scale.

Kernel density estimation

Contrary to parametric methods, nonparametric ones, such as Kernel, do not require functional form specification. That's why these methods became popular among users, especially when dealing with large data. The univariate Kernel density of intraday returns is given as:

$$\hat{f}(r_{t,j}) = \frac{1}{Jh}\sum_{j=1}^{J} K\left(\frac{r_{t,j-}r_t}{h}\right), \tag{7}$$

where h is the bandwidth or smoothing parameter and $K(\cdot)$ is a kernel function. This type of estimator in the literature is known as the Rosenblatt-Parzen estimator according to Rosenblatt (1956) and Parzen (1962). Assuming that kernel function $K(\cdot)$ is nonnegative and that lower and upper limits of integration are -∞ and +∞, it has following properties:

$$\int K(z)dz = 1, \tag{8}$$

$$\int zK(z)dz = 0, \tag{9}$$

$$\int z^2K(z)dz < \infty \tag{10}$$

Appropriate choice of bandwidth is most important for density estimation and it's not so straightforward in practice. Other parameters, like kernel function $K(\cdot)$ have negligible influence on the final result in finite samples (Marron, Nolan, 1988). Thus, a great attention should be given to bandwidth selection as both the bias and the variance depend on the same bandwidth. As bandwidth decreases the bias also decreases but the variance increases and an optimal bandwidth should minimize both bias and variance of the estimator. To achieve this integrated criterion is usually used, i.e. integrated mean square error IMSE. Minimizing the IMSE with respect to the bandwidth provides a basis for data-driven bandwidth selection. Consequently, different approaches exist in the literature. Many approaches use a reference rule-of-thumb suggested by Silverman (1986) or Scott (2015). The rule-of-thumbs, even appealing among users, tend to over-smooth and hide important properties of the data (Scott, 2015). Some academics prefer plug-in bandwidths suggested by Sheather and Jones (1991). Plug-in bandwidths support the idea of "plugging in" estimates of the unknown quantities that appear in formulae for the asymptotically optimal bandwidth (Chu et al., 2015). Nevertheless, plug-in rules are not fully automatic as they depend on the pilot bandwidth.

In small samples, better choices can be made by cross-validation methods, which are computationally intensive (Park, Marron, 1992).

It is now obvious that all proposed bandwidth selectors have advantages and disadvantages and if we take a chance to apply them, new issues will always emerge. However, the purpose of this paper is not to pinpoint which of them is most appropriate, but to select a bandwidth at which realized density is rescaled to have the variance equal to $TSRV_t^{\Delta,S}$.

Application to DAX index

For the purpose of the analysis four trading days were chosen randomly during year 2018, i.e. July 20, August 17, September 21 and December 21. There is no special reason why this research is restricted to four dates only, but it is enough to illustrate the issue we are dealing with as well as to confirm that entire density function is not time-invariant. Therefore, this is to check if all realized measures of moments are time-varying and to find which of them vary the most. Number of intraday DAX prices across dates ranges from 515 to 519 within official trading hours of Frankfurt Stock Exchange. These prices are observed with the sampling frequency of 1 minute which is the best available after cleaning and filtering process.

To illustrate research problem more clearly one trading day is chosen, i.e. July 20, 2018. For this trading day different Kernel densities are obtained with respect to arbitrarily chosen bandwidths but fixed sampling frequency of 1 minute (left top panel of Figure 1). Contrary to that for every fixed value of the bandwidth, a different Kernel densities are also obtained due to different sampling frequency selection, i.e. 1, 5 or 10 minutes (other panels of Figure 1). The same results are obtained for other trading day but not presented here to preserve the space. Nevertheless, it is more convenient to fix the highest sampling frequency at 1 minute and search for optimal bandwidth then opposite. The main reason for this is to keep all available data and still apply Kernel method to estimate the density of returns, which bring us to the issue of bandwidth selection.

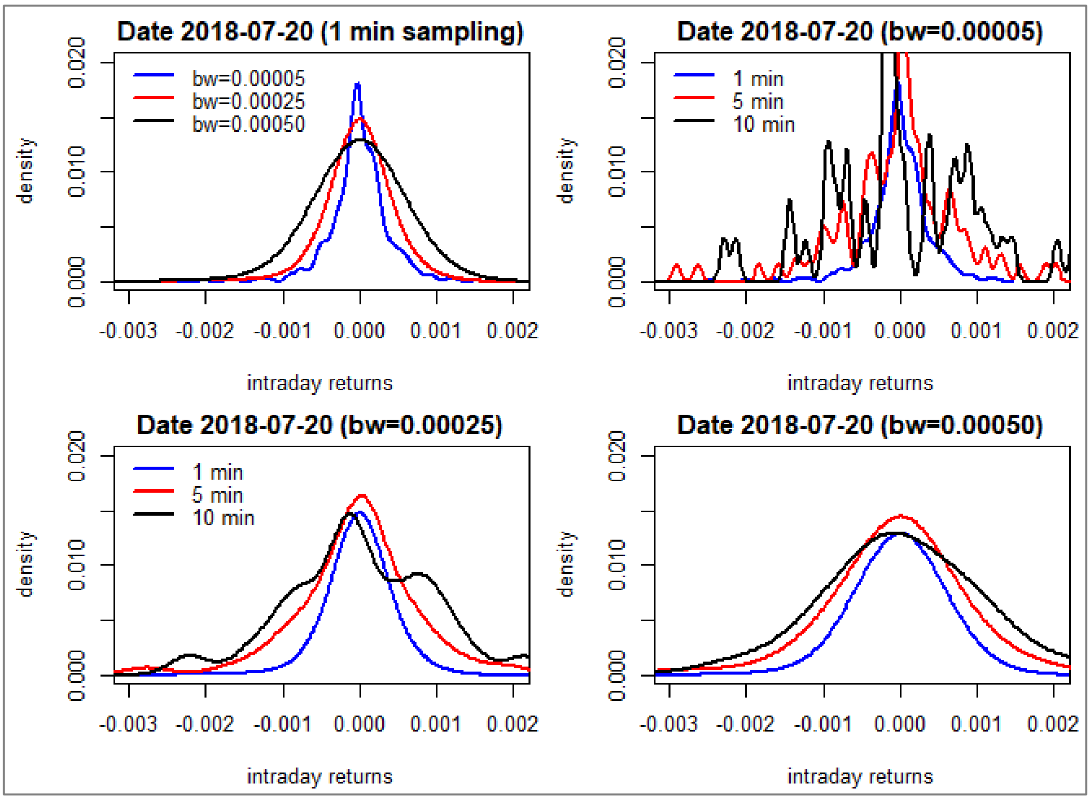

Figure 1 Different Kernel densities with respect to arbitrarily selected bandwidths and different sampling frequencies at July 20, 2018
Source: Author calculation according to data provided by Thomson Reuters Tick History

As discussed in previously two sections, finding an optimal slow-time time scale frequency will lead us to the appropriate bandwidth, i.e. a bandwidth at which realized Kernel density is rescaled to have the variance equal to $TSRV_t^{\Delta,S}$ which is asymptotically unbiased and consistent. The same realized Kernel density is latter used to extract realized measures of higher moments such as realized skewness and realized kurtosis. Results are given in Table 1 and realized densities are plotted on the Figure 2.

Table 1 Selection of slow-time-time scale frequencies, Kernel bandwidths and estimation of realized measures of moments

Dates in 2018	High-scale observations	Slow-time-time scale	Bandwidth	Realized variance	Realized skewness	Realized kurtosis
July 20	515	8 min	0.011016	0.000121	-0.00045	2.986069
August 17	519	9 min	0.006649	0.000044	-0.00122	2.895766
September 21	515	8 min	0.005597	0.000031	-0.19945	3.999873
December 21	517	7 min	0.010423	0.000108	-0.21608	5.007762

Source: Author calculation according to data provided by Thomson Reuters Tick History

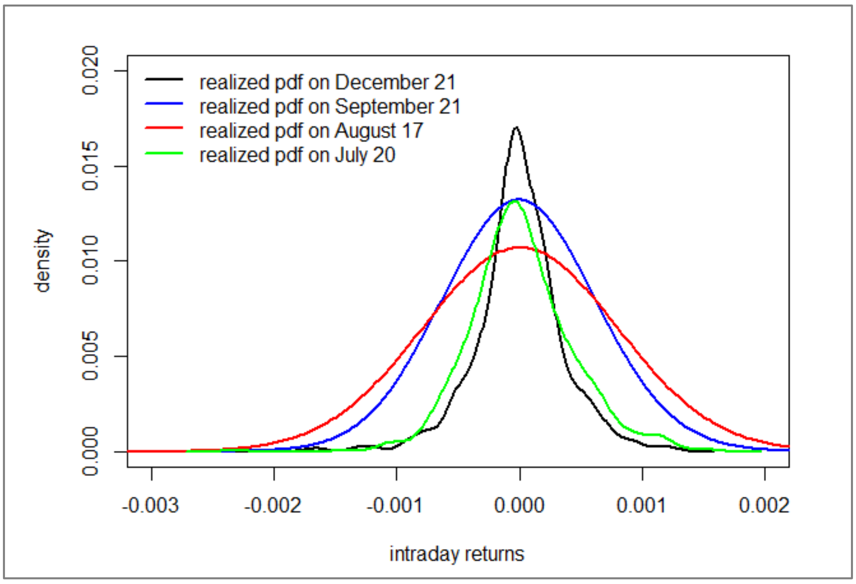

Figure 2 Realized pdf's with bandwidths at which realized Kernel density is rescaled to have the variance equal to two-times scale estimator for each trading day
Source: Author calculation according to data provided by Thomson Reuters Tick History

Results from Table 1 clearly indicate that slow-time-time scale frequency on average is 8 minutes and doesn't vary much across trading days. However, the bandwidths are significantly different and realized measures of moments correspond to plotted densities on the Figure 2. All densities are negatively skewed with kurtosis approximately equal to or greater than 3. These findings support expectation about time-varying realized moments, i.e. returns are not identically distributed across trading day and unlikely Gaussian.

Conclusion

Proposed approach has scientific as well as practical contribution. Namely, scientific contribution is related to the main finding that Kernel bandwidth is strongly related to the sampling frequency at slow-time-time scale when applying TSRV estimator, while the fast-time-time scale sampling frequency is held fixed at 1 minute. Practical contribution is related to the finding that DAX index intraday returns should be sampled approximately every 8 minutes at the slow-time-time scale which is of special interest to market participants. Namely, if returns of DAX index are sampled every 8 minutes then an appropriate realized density can be found and rescaled for each trading day using Kernel method. This approach enables to recover the "true" density of returns along with the realized moments. It is necessary to emphasize that these moments are robust to microstructure noise.

Moreover, it is recommended to keep fast-time-time scale fixed at the best available sampling frequency and to search for slow-time-time scale only. This approach is not so computational demanding and the selection of fast-time-time scale by the researchers give themselves opportunity to control the quality of high-frequency data due to cleaning and filtering process prior to the analysis.

Empirical findings indicate that appropriate bandwidths are significantly different across trading days even slow-time-time scale frequency is stable. All realized measures of moments are time-varying as well as realized densities. Kurtosis varies the most. This can be explained in relation to fat-tails phenomenon of underlying distribution as kurtosis controls the thickness of the left and right tail. This type of information should not be ignored, and it is of special interest to market participants as their future expectations are embedded in the current trading activities.

Limitation of this research is that considers density of returns estimation of a single stock index, but not multiple market indices. DAX index is chosen as a representative one among developed European markets. However, in emerging markets trading within a day is not so frequent and lack of intraday and synchronized observations would be a great challenge. Accordingly, further research will consider more market indices for comparison purpose.

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