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Numerical Modelling of Stiffness of RTA Furniture with New Externally Invisible and Dismountable Joints

Numeričko modeliranje krutosti RTA namještaja s novim, izvana nevidljivim vezovima

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ABSTRACT • *The distribution of furniture sold in the form of flat packages requires the use of appropriate design solutions. These include joints, which need to facilitate self-assembly with no need to use tools. Such joints should be functional, aesthetically attractive, durable and safe to use. It was decided in this study to manufacture prototypes of innovative furniture joints and evaluate the quality of furniture assembled using these joints.*
For this purpose, the finite element method and the Abaqus program were used. Joints were modelled as objects made of polylactide (PLA). Surface to surface contact and assembly forces resulting from the construction of joints were introduced between the elements of the joint. The furniture case was subjected to torsional loads. The rigidity of furniture and stress distribution in joints were calculated. On the basis of numerical calculations, the joints were positively validated.

Key words: *furniture; FEM; invisible joints; RTA; stiffness*

SAŽETAK • *Prodaja namještaja koji se isporučuje u obliku ravnih paketa zahtijeva primjenu odgovarajućih dizajnerskih rješenja. U takva se rješenje ubrajaju i vezovi koji trebaju olakšati samostalno sastavljanje namještaja bez primjene alata. Ti vezovi trebaju biti funkcionalni, estetski privlačni, izdržljivi i sigurni za upotrebu. Za ovo su istraživanje autori odlučili proizvesti prototipove inovativnih vezova namještaja te procijeniti kvalitetu namještaja sastavljenoga uz pomoć takvih vezova. Za tu je namjenu primijenjena metoda konačnih elemenata i program Abaqus. Vezovi su modelirani kao predmeti izrađeni od polilaktida (PLA). Između elemenata veza djeluju površinske i kontaktne sile koje nastaju konstrukcijom vezova. Korpus namještaja bio je podvrgnut torzijskim opterećenjima. Izračunana je krutost namještaja i raspodjela naprezanja u vezovima. Na temelju brojčanih izračuna vezovi su pozitivno ocijenjeni.*

Ključne riječi: *namještaj; FEM; nevidljivi vezovi; RTA; krutost*

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1 INTRODUCTION
1. UVOD

Furniture design and construction is an applied art and, as such, it must take into consideration not only aesthetic and functional preferences and fashion trends, but also rigidity and strength requirements. This, in turn, is associated with the development of appropriate numerical models of furniture joints in the environment of software calculating with the assistance of the finite elements method (FEM). A different method of joint rigidity was analysed by Tankut and Tankut (2011). A realistic representation of the examined structure in the FEM environment is very labour-consuming, and requires numerous corrections of mesh geometry and meticulous determination of linear elastic properties of applied materials (Dzięgielewski and Smardzewski, 1996; Kasal, 2008, Kasal *et al*., 2008a, b; Smardzewski, 2004a, b; Smardzewski, 2005). However, it is more practical, but equally effective, to replace joints with semi-rigid joints (Kłos, Smardzewski, 2004; Nicholls, Crisan, 2002; Smardzewski, Kłos, 2004; Smardzewski, Ożarska, 2005; Smardzewski, Prekrat, 2002, 2005). Smardzewski *et al.* (2013) determined the effect of creeping on changes in the rigidity of selected joints used in the structures of upholstered furniture, expressed as the substitute modulus of elasticity. For this reason, an attempt was made to elaborate a method for simplified modelling of furniture joint stiffness for the needs of numerical calculations (Smardzewski and Kłos, 2011; Smardzewski *et al.,* 2013). In the proposed method, joint stiffness was expressed by means of a modulus of elasticity in the form of a load and deflection function.

In this paper, an attempt was made to present alternative methods of numerical rigidity modelling for newly designed cabinet furniture joints using nodes of substitute linear elasticity modulus.

The aim of this research project was to determine the values of the substitute linear elasticity modulus for newly designed furniture joints to compare the obtained results and to select the model most favourable for the virtual prototyping of furniture.

2 MATERIALS AND METHOD
2. MATERIJALI I METODE

2.1 Type of new furniture joints
2.1. Novi tip vezova za namještaj

In this study, two new and original joints (Figure 1a, b) were designed so that the connection of furniture elements was externally invisible and easy to assemble with no need to use tools (Krzyżaniak and Smardzewski, 2019). The minifix joint, commonly used in the furniture industry, was used as a reference joint (Figure 1c). In view of the use of one selected type of material: particleboard (PB), three types of joints S, M, E, two mechanical tests consisting of compression and tension of joints, a total of six models were prepared for the analysis.

The experimental tests were performed on L type angle joints. Their shape and dimensions were present-

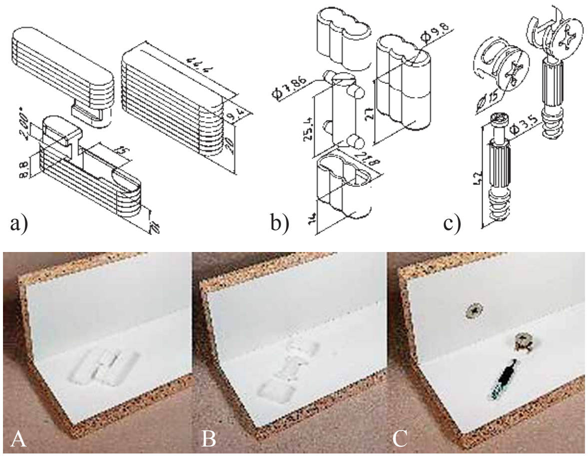

Figure 1 Joints used in tests: a) slide catch (S), b) tension twist (M), c) eccentric (E)
Slika 1. Vezovi obuhvaćeni ispitivanjima: a) klizna kopča (S), b) *tension twist* (M), c) svornjak sa zakretnim klinom

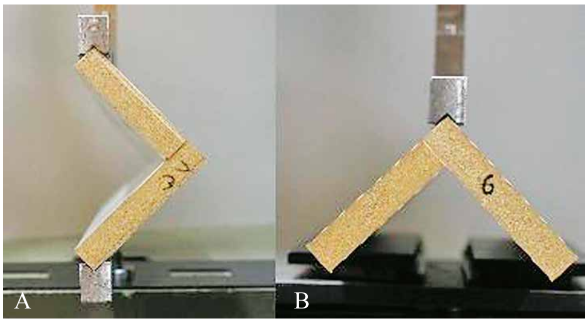

Figure 2 Joints subjected to: a) compression, b) tension
Slika 2. Vezovi izloženi: a) tlačnoj sili, b) vlačnoj sili

ed in a study by Krzyżaniak and Smardzewski (2019) and in Figure 2. The methodology applied to measure force and sample deflection were described in detail by Krzyżaniak and Smardzewski (2019). Based on their results, substitute moduli of elasticity E_s were calculated for the joints.

2.2 Substitute modulus of elasticity
2.2. Zamjenski modul elastičnosti

In order to estimate joint elastic moduli, the authors employed the calculation model presented in Figure 3. In this model, L = 100 mm designates the length of the joint arms, L_2 = 18 mm – height of the arm cross-section, $L = L_1 + L_2$, x_1, x_2 – ranges of integration, E – linear modulus of elasticity of particle board, and E_s – substitute modulus of elasticity.

If the joint is subjected to compression, the constitutive equation describing the deflection DP in the direction of force P assumes the following form (Fig. 3a):

$$DP = 2\left(\int_0^{L_1} \frac{P\cos^2\alpha}{EJ} x_1^2 dx_1 + \int_{L_1}^{L=L_1+L_2} \frac{P\cos^2\alpha}{E_s J} x_2^2 dx_2 \right), \quad (1)$$

$$J = bL_2^3 / 12, \quad (2)$$

where: b – denotes the width of the joint cross-section. It was decided that for the selected joints this width would be equal to 30 mm. The solution of this equation is as follows:

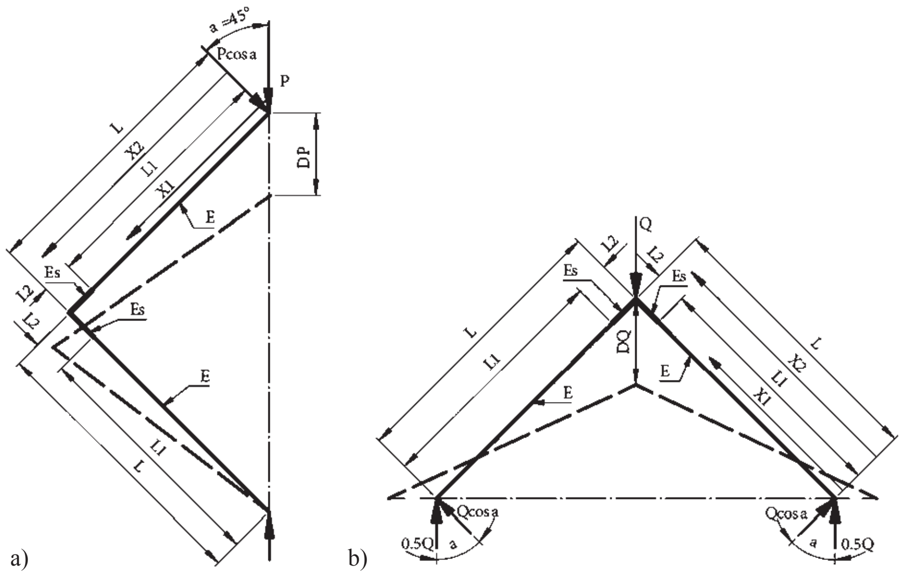

Figure 3 A geometrical model for analytical calculation of substitute moduli of elasticity of joints
Slika 3. Geometrijski model za analitički izračun zamjenskog modula elastičnosti vezova

$$DP = \frac{2P\cos^2\alpha}{3J}\left(\frac{L_1^3}{E} + \frac{L^3 - L_1^3}{E_s}\right). \qquad (3)$$

Therefore, the elastic modulus of the joint assumed the following form:

$$E_s = \frac{2PE\cos^2\alpha(L^3 - L_1^3)}{3EJ\,DP - 2P\cos^2\alpha L_1^3}. \qquad (4)$$

For the joint subjected to tension, the constitutive equation describing the deflection DQ in the direction of action of force Q assumes the following form (Fig. 3b):

$$DQ = 2\left(\int_0^{L_1}\frac{Q\cos^2\alpha}{2EJ}x_1^2dx_1 + \int_{L_1}^{L=L_1+L_2}\frac{Q\cos^2\alpha}{2E_sJ}x_2^2dx_2\right), \qquad (5)$$

hence,

$$DQ = \frac{Q\cos^2\alpha}{6J}\left(\frac{L_1^3}{E} + \frac{L^3 - L_1^3)}{E_s}\right), \qquad (6)$$

and

$$E_s = \frac{QEcos^2\alpha\left(L^3 - L_1^3\right)}{6EJDQ - Qcos^2\alpha\,L_1^3}. \qquad (7)$$

It was further decided to determine physical and mechanical properties of the applied particle board species. The linear modulus of elasticity E, static bending strength or modulus of rupture (*MOR*) and density were determined in accordance with the EN 310 standard. Using equations 4 and 7, E_s was calculated for each of the examined joints and subsequently the obtained results were used for numerical calculations.

2.3 FEM models
2.3. FEM modeli

Tests were conducted on case furniture items with dimensions as specified in Figure 4. All the furniture body elements were manufactured applying an identical technology and using the same particle boards.

Furniture bodies were supported in three corners (Figure 4) and loaded with a horizontal force P (N) in the right upper corner. The value of the loading force was selected so that the deflection was 80 mm. Figure 5 presents an example model for numerical calculations of the furniture. All the furniture elements were modelled using 10–node finite tetrahedral C3D8R elements. Between the narrow surfaces of the furniture structural elements, the points of contact were established for hard surfaces with no friction exerted. The side walls, top and bottom elements were made from particle board (E = 2500 MPa, Poisson ratio 0.3). The back wall made from a HDF board (E = 3500 MPa, Poisson ratio 0.3) was fixed permanently in the rabbets and connected with bonded type joints. The joints were modelled as cuboids with the base of 18 mm x 18 mm and length of 30 mm, which were connected with the side walls and the tops of the furniture case using an elastic bonded type joint. The joints were ascribed respective substitute moduli of elasticity depending on the character of deformation (compression or tension).

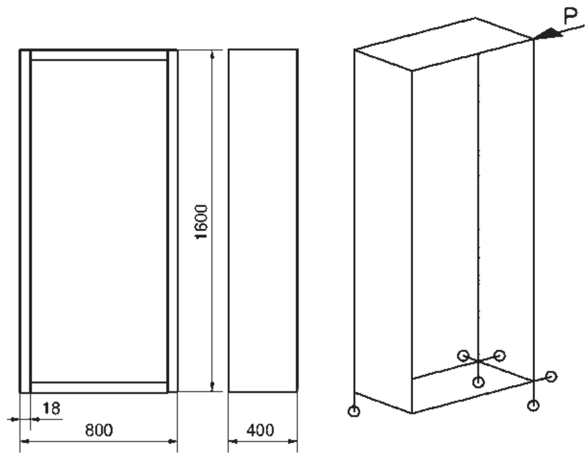

Figure 4 Case furniture used in tests
Slika 4. Ispitivani korpusni namještaj

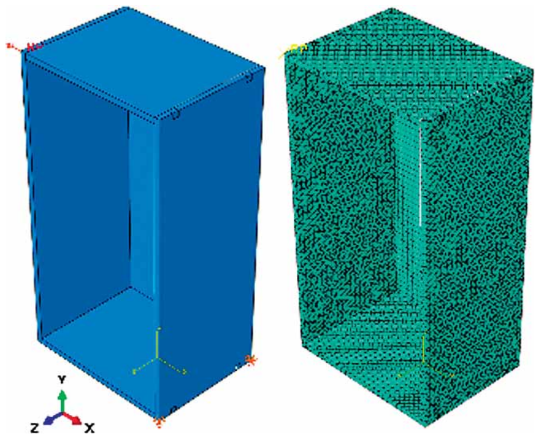

Figure 5 A mesh of finite elements on furniture model
Slika 5. Mreža konačnih elemenata na modelu namještaja

Based on the results of numeric calculations, the dependencies between force and displacement were established for furniture bodies connected using the tested joints.

Moreover, stiffness coefficients were calculated for furniture cases based on formula (8):

$$K = \frac{F}{\delta} \text{ (N/mm)} \tag{8}$$

where: F – force loading the furniture body, δ – displacement in the direction of the force.

The numerical calculations were performed using the Abaqus v.6.16 programme (Dassault Systemes Simulia Corp., Waltham, Ma, USA) at the Poznań Supercomputing and Networking Centre in the Eagle cluster.

3 RESULTS
3. REZULTATI

First, the variation in the substitute modulus of elasticity was calculated in the function of joint deflection (Figure 6).

It results from Figure 6 that the greatest values of the modulus of elasticity for the joint are reached for a deflection of approx. 1 mm. Then these values drop rapidly and for a deflection of 3 mm they amount to approx. 40 % of the maximum value. Thus values of E_s presented in Table 1 were selected for numerical calculations.

Figure 7 presents a characteristic mode of torsional deformation of the furniture body. We may observe clearances and gaps between elements of the furniture body caused by bending of joints and pressure of panel edges.

Table 1 Substitute moduli of elasticity for the joints
Tablica 1. Zamjenski modul elastičnosti za vezove

Type of joint *Tip veza*	**Deflection, mm** *Progib,* mm	***Es*, MPa**		
		S	M	E
Compression *tlačno naprezanje*	1	2201	1554	1815
	3	847	660	1219
Tension *vlačno naprezanje*	1	1211	918	821
	3	550	439	786

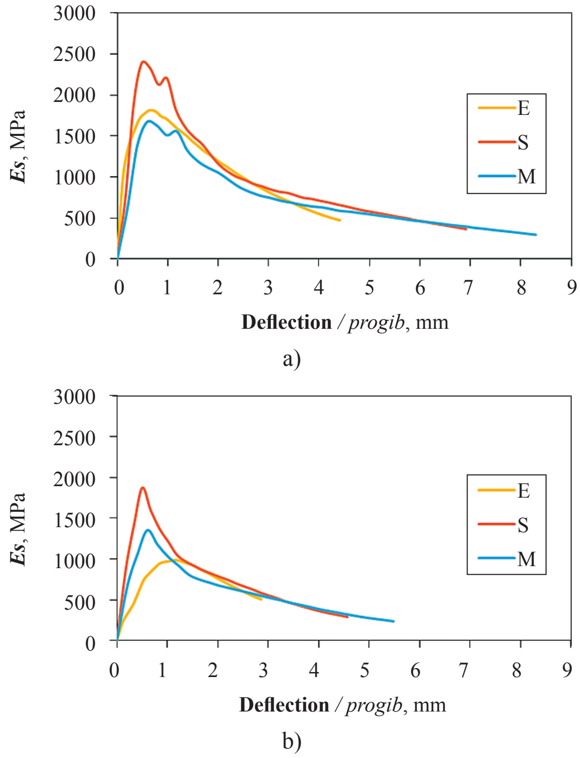

Figure 6 Dependence E_s of deflection in the test of: a) joint compression, and b) joint tension
Slika 6. Ovisnost E_s o progibu u ispitivanjima: a) tlačnog naprezanja vezova, b) vlačnog naprezanja vezova

Stiffness of furniture depending on the type of used joints is presented in Figure 8. It results from this figure that stiffness characteristics are non-linear and progressive. This means that the value of loads increases with an increase in displacements. It is also evident that a change in E_s values within the same joint has no significant effect on changes in stiffness of the furniture body. No significant difference in stiffness coefficients for the furniture bodies is observed between the used joints (Figure 9).

This means that the newly designed joints for case furniture provide this furniture with stiffness com-

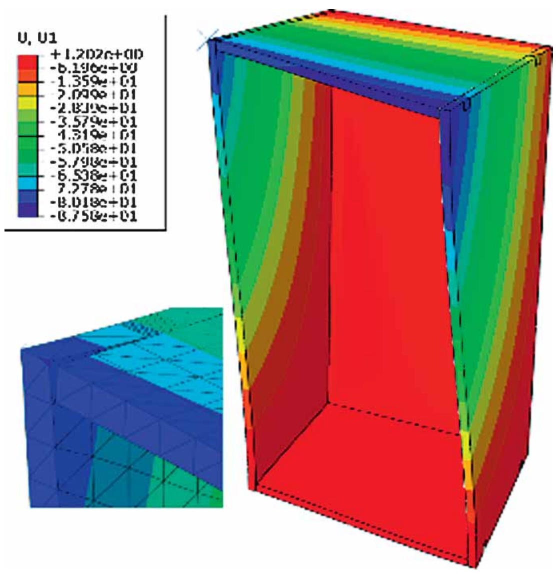

Figure 7 Deformation of furniture (deflection in mm)
Slika 7. Deformacije namještaja (progib u mm)

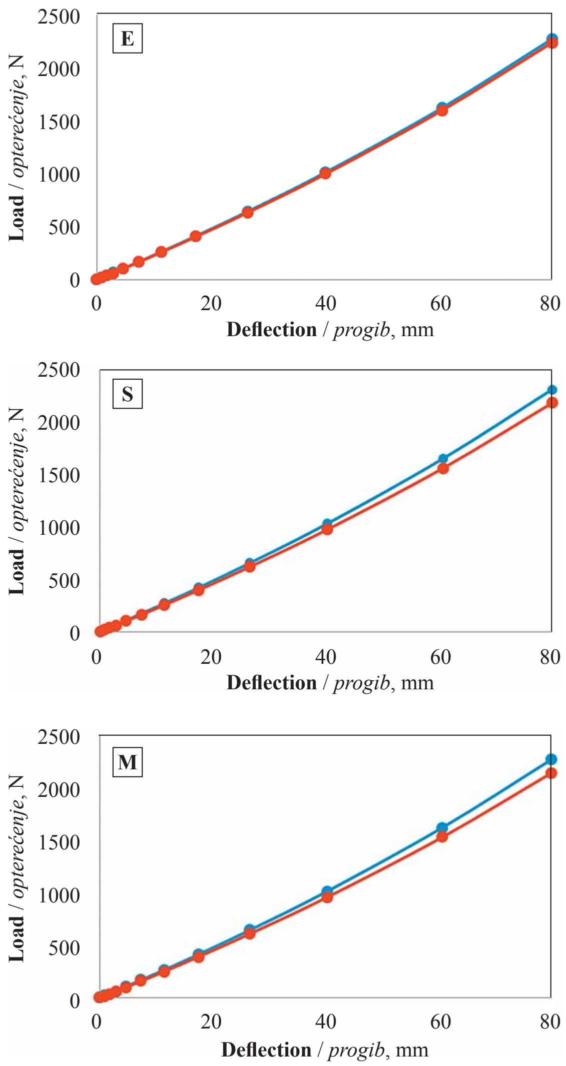

Figure 8 Stiffness of furniture with different joints: eccentric (E), slide catch (S), tension twist (M) (blue: for deflection 1 mm, orange: for deflection 3 mm)
Slika 8. Krutost namještaja pri različitim vezovima: sa svornjakom sa zakretnim klinom (E), s kliznom kopčom (S), s *tension twist* vezom (M) (plavo: za progib 1 mm, narančasto: za progib 3 mm)

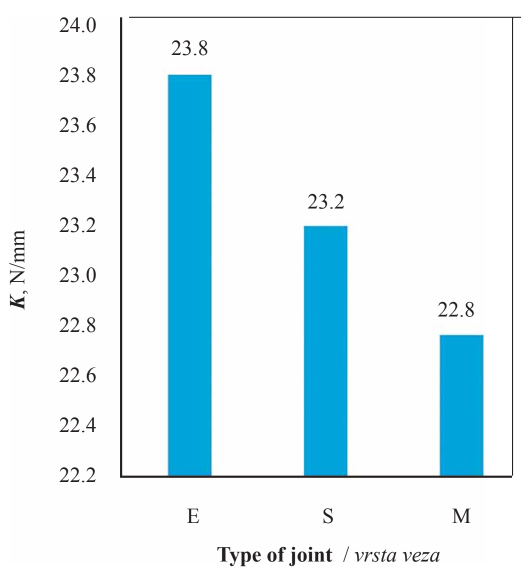

Figure 9 Stiffness coefficient of cabinet furniture
Slika 9. Koeficijent krutosti korpusnog namještaja

parable to that which may be obtained using traditional eccentric joints (E).

4 CONCLUSIONS
4. ZAKLJUČCI

Results obtained based on numerical calculations and their analysis gave grounds for the following conclusions and remarks:

1. The greatest values of the substitute moduli of elasticity are recorded for the slide catch joint. Rigidity of these joints is very close to the rigidity of eccentric joints. For this reason, we can recommend this connection as a good substitute to eccentric connections.
2. The tension twist joints also ensure very good mechanical properties of case furniture. So, it can also be used in furniture industry.
3. Generally, no significant difference was shown between stiffness of case furniture items with traditional eccentric joints and stiffness of furniture items with newly designed joints. This is due to the fact that each of the connections generates internal montage forces. These forces guarantee high rigidity of the case furniture.
4. Relationship between force and displacement of furniture are non-linear. This is due to the interaction of internal montage forces in the connectors and the increase of the contact surface between the elements. It was described in detail in the work by Krzyżaniak and Smardzewski (2019).

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