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An improved adaptive fuzzy backstepping control for nonlinear mechanical systems with mismatched uncertainties

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ABSTRACT
When the nonlinear mechanical system has mismatched uncertainties, it is difficult to design control algorithm to achieve high precision trajectory tracking control. Traditional backstepping control is an effective control method for uncertain, mismatched nonlinear systems, but there is inherent problem of "complexity due to the explosion of terms". In this paper, based on the backstepping method, only one fuzzy system is used to approximate the unknown nonlinear functions, the unknown control gain and the differential of virtual control law of each subsystem. In order to reduce the influence of fuzzy approximation error and external interference, the above control scheme is further improved by designing special adjustable control parameters and first order low pass filter. The improved control scheme not only improves the control precision of the system obviously, but also solves the problem of "explosion of terms", and greatly reduces the initial control input, and provides the conditions for the practical application. The simulation results show the effectiveness of the proposed methods.

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Mismatched uncertainty; mechanical system; nonlinear; fuzzy system; backstepping

1. Introduction

In mechanical system modelling, due to modelling error, unknown physical phenomena (friction in mechanical system), load variation and random disturbance, system uncertainty is unavoidable. The mechanical system is mainly affected by the following two kinds of uncertainties, the structured uncertainty, that is, the parameter uncertainty, and the unstructured uncertainty, such as the modelling error and the external disturbance. Because of the existence of high nonlinearity and uncertainty of the mechanical system, it is very difficult for the design of the controller [1–4].

In the past few decades, the adaptive control technology for feedback linearization for nonlinear systems has made remarkable progress [4–6]. The feedback linearization requires the uncertainty to satisfy the linear parameterized condition. However, most of the systems in practice can't be linearized. Robust control strategy has good control effect for structured uncertain systems, but the premise is that the boundary of uncertainty is known [1–2]. But in actual control, it is often impossible to know the boundary of its structured uncertainty.

Wang Li-Xin proposed adaptive fuzzy control by using the fuzzy system to approximate the unknown control law or the unknown nonlinear function [7]. In literatures [8–12], an adaptive neural network controller is designed by using a neural network to approximate uncertain continuous nonlinear functions. The advantage of sliding mode controller is that it has strong robustness to disturbances and unmodeled dynamics, so it has also been widely applied [13–18]. However, all the above methods require that the system meet an important condition, that is, the unknown nonlinearity and the control input appear in the same equation of the state space model, which is usually regarded as the matching condition.

In the actual system, there is a large class of nonlinear systems that do not meet the matching condition, such as the system of the mechanical hand which driven by the motor. For mismatched uncertainty nonlinear systems, the Backstepping control is very effective and has achieved great success [19–24]. Traditional backstepping control needs to repeated differentiations of the virtual control law of the former subsystem. If there is a nonlinear function in the virtual control, repeated differentiations will lead to the problem of "explosion of complexity" with the increase of the order of the system. This makes high order systems face great difficulties in controller implementation. If the system has parameters or structural uncertainties and external disturbances, it will further lead to the difficulty in the application of backstepping control.

Hedrick et al. proposed a dynamic surface control method (DSC), which can avoid the problem of repeated differentiations by using n first order low pass filters, has been widely used [25–27]. But DSC can't deal with the uncertainty problem. Since fuzzy systems

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and neural networks can approximate arbitrary nonlinear functions with arbitrary precision, many literatures have combined them with DSC or backstepping control in recent years [27–34].

Furthermore, in literatures [35–36], fuzzy system and backstepping control are used to control the nonstrict-feedback stochastic nonlinear systems. In [37] both adaptive fuzzy state feedback and observer-based output feedback control design schemes are proposed. Qi Zhou et al. proposed adaptive fuzzy tracking control for a class of pure-feedback nonlinear systems with time-varying delay and unknown dead zone [38].

Shaocheng Tong et al. proposed an adaptive fuzzy output feedback control for a class of switched nonstrict-feedback nonlinear systems in literatures [39,40]. Yongming Li et al. proposed an adaptive fuzzy fault-tolerant control of non-triangular structure nonlinear systems with error-constraint in [41], and robust adaptive output feedback control to a class of non-triangular stochastic nonlinear systems was designed in [42].

Based on the above results, a novel adaptive fuzzy backstepping control method is studied in this paper, which is suitable for nonlinear mechanical systems with mismatched uncertainties. In each subsystem, only one fuzzy system is used to approximate the unknown control gain, the unknown nonlinear function, and the differential of the virtual control of the former subsystem, and the control of the system is realized by the backstepping method. The advantage of the proposed control method is to compensate all the uncertainties at the same time and avoid the inherent "explosion of complexity" problem. By introducing special adjustable control parameters, not only the control precision of the system is greatly improved, and the initial control input is significantly reduced.

This paper is organized as follows: Problem statement and preliminaries is described in Section 2. In Section 3 fuzzy system and its approximation is presented. Control design is presented in Section 4. Stability analysis and adaptive law design are proposed in Section 5. The simulation results and conclusion are given in Sections 6 and 7, respectively.

2. Problem statement and preliminaries

Consider the following class of uncertain, mismatched nonlinear mechanical systems, which has the strict-feedback structure:

$$\begin{cases} \dot{x}_1(t) = b_1 x_2(t) + f_1(x_1(t)) + \omega_1(t) \\ \dot{x}_2(t) = b_2 x_3(t) + f_2(x_1(t), x_2(t)) + \omega_2(t) \\ \quad \vdots \\ \dot{x}_i(t) = b_i x_{i+1}(t) + f_i(x_1(t), x_2(t), \cdots, x_i(t)) + \omega_i(t) \\ \dot{x}_n(t) = b_n u(t) + f_n(x_1(t), x_2(t), \cdots, x_n(t)) + \omega_n(t) \\ y(t) = x_1(t) \end{cases} \tag{1}$$

where $1 \le i \le n-1$, $x_1, x_2, \cdots, x_n$ are the state variables, $u \in R$, $y \in R$ are input and output of the system, respectively; $\omega_i(t)$ is the unknown, but bounded external disturbance, f_i is the unknown smooth function,b_i is the unknown control gain.

Assumption 2.1: *There exist positive constants b_{im} and b_{iM} such that $0 < b_{im} \le |b_i| \le b_{iM}$, $i = 1, 2, \ldots, n$.*

Assumption 2.2: *For smooth nonlinear function $f(x)$ and fuzzy logic system, there exists optimal parameter $\boldsymbol{\theta}^*$ which defined as $\boldsymbol{\theta}^* = \arg\min_{\theta \in \Omega_0}[\sup_{x \in \Omega} |f(x) - \boldsymbol{\theta}^T \boldsymbol{\xi}(x)|]$, where Ω_0 and Ω are the sets of boundedness of $\boldsymbol{\theta}$ and $\boldsymbol{x}$ respectively.*

Assumption 2.3: *The reference signal $y_d(t)$ is a sufficiently smooth function of t, and $y_d(t)$, $\dot{y}_d(t)$ are bounded.*

Control Objective. The control objective is to design an adaptive control scheme such that the output $y(t)$ tracks the desired trajectory $y_d(t)$ while ensuring the boundedness of all the closed-loop signals.

For convenience, symbols $\bar{\boldsymbol{x}}_i$ are introduced, where $\bar{\boldsymbol{x}}_i = [x_1, x_2, \cdots, x_i]^T \in R^i$, $i = 1, 2, \cdots, n$.

The system (1) does not satisfy the so-called matching condition, and there are uncertain nonlinear functions and unknown control gains, and at the same time, it is affected by external interference, so it is difficult to achieve the above control purposes.

3. Fuzzy system and its approximation

Fuzzy system with product inference, singleton fuzzifier and center-average defuzzifier is a universal approximator. If the fuzzy rule has the following form:

IF x_1 is F_1^j, and x_2 is F_2^j, and ... and x_n is F_n^j, THEN y is $B^j (j = 1, 2, \cdots, N)$, where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the system input, y represents the output of the system, $F_i^j (i = 1, 2, \cdots, n)$ and B^j stand for fuzzy sets, N stands for the number of fuzzy rules, the output of fuzzy system can be expressed as:

$$y(x) = \frac{\sum_{j=1}^{N} \theta_j \prod_{i=1}^{n} \mu_i^j(x_i)}{\sum_{j=1}^{N} \prod_{i=1}^{n} \mu_i^j(x_i)} \tag{2}$$

where $\theta_j = \max_{y \in R} B^j(y)$, $\mu_i^j(x_i)$ and $B^j(y)$ denote Gaussian membership functions with respect to fuzzy sets F_i^j and B^j.

Define fuzzy base functions as:

$$\xi_j(x) = \frac{\prod_{i=1}^{n} \mu_i^j(x_i)}{\sum_{j=1}^{N} \prod_{i=1}^{n} \mu_i^j(x_i)}, j = 1, 2, \cdots, N \tag{3}$$

$\boldsymbol{\xi}(x) = [\xi_1(x), \xi_2(x), \cdots, \xi_N(x)]^T$ is the fuzzy basis function vector, and $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_N]^T$ is the weight parameter vector, thus the final output of the fuzzy system (2) can be rewritten as:

$$y(x) = \boldsymbol{\xi}^T(x)\boldsymbol{\theta} \tag{4}$$

According to the universal approximation theorem of fuzzy system, if $f(x)$ is a continuous function defined on the compact set Ω, and using fuzzy system $y(x) = \boldsymbol{\xi}^T(x)\boldsymbol{\theta}$ to approximate $f(x)$, there exist optimal parameter vector $\boldsymbol{\theta}^*$ such that $\sup_{x\in\Omega} |f(x) - \boldsymbol{\theta}^*\boldsymbol{\xi}(x)| \le \varepsilon$, for any given small constant $\varepsilon > 0$ [7].

4. Adaptive fuzzy backstepping control design

4.1. Reconstruction of system equation

The mismatched nonlinear mechanical system (1) can be reconstructed as follows:

For subsystem 1: $\dot{x}_1(t) = b_1x_2(t) + f_1(x_1) + \omega_1(t)$, we can define $e_1 = y - y_d$. A virtual control signal α_1 is introduced, then we can have

$$\begin{aligned} \dot{e}_1 &= \dot{y} - \dot{y}_d \\ &= b_1x_2 - b_1\alpha_1 + b_1\alpha_1 + f_1(x_1) + \omega_1(t) - \dot{y}_d \\ &= b_1e_2 + b_1\alpha_1 + f_1(x_1) + \omega_1(t) - \dot{y}_d \end{aligned} \tag{5}$$

where $e_2 = x_2 - \alpha_1$.

For subsystem 2, a virtual control signal α_2 is introduced, and then we can have

$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\ &= b_2x_3 - b_2\alpha_2 + b_2\alpha_2 - \dot{\alpha}_1 + f_2(\bar{x}_2) + \omega_2 \\ &= b_2e_3 + b_2\alpha_2 - \dot{\alpha}_1 + f_2(\bar{x}_2) + \omega_2 \end{aligned} \tag{6}$$

where $e_3 = x_3 - \alpha_2$.

For subsystem k, a virtual control signal α_k is introduced, and then we can have

$$\begin{aligned} \dot{e}_k &= \dot{x}_k - \dot{\alpha}_{k-1} \\ &= b_kx_{k+1} - b_k\alpha_k + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\bar{x}_k) + \omega_k \\ &= b_ke_{k+1} + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\bar{x}_k) + \omega_k \end{aligned} \tag{7}$$

where $e_{k+1} = x_{k+1} - \alpha_k$.

Define $e_n = x_n - \alpha_{n-1}$ for the final subsystem, and then we can have

$$\dot{e}_n = \dot{x}_n - \dot{\alpha}_{n-1} = b_nu + f_n(\bar{x}_n) + \omega_n - \dot{\alpha}_{n-1} \tag{8}$$

As a result, (1) can be rewritten as the following form:

$$\begin{cases} \dot{e}_k = b_ke_{k+1} + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\bar{x}_k) + \omega_k \\ \dot{e}_n = b_nu - \dot{\alpha}_{n-1} + f_n(\bar{x}_n) + \omega_n \end{cases} \tag{9}$$

where $\alpha_0 = y_d$, $1 \le k \le n-1$.

Equation (9) shows that the control object can be achieved as long as the appropriate virtual control law α_k and control law u are designed [31]. However, Because of the existence of unknown nonlinear function f_k and f_n, unknown virtual control gain b_k and b_n, and external disturbance ω_k and ω_n, it is very difficult to design the control law. Moreover, Equation (9) containing $\dot{\alpha}_{k-1}$ and $\dot{\alpha}_{n-1}$, it shows that the design of virtual control law needs differentiation of the virtual control of the former subsystem. Such repeated differentiations will greatly increase the complexity of the whole control system, resulting in the problem of "explosion of complexity". In order to deal with the above problems, in this paper adaptive fuzzy system is applied to approximate the unknown nonlinear functions, unknown control gains and the differentiation of virtual control. The problems of the inherent problem of "explosion of complexity" and the mismatched uncertainties can be solved at the same time.

4.2. Adaptive fuzzy backstepping control (AFBC)

Step 1, the Lyapunov function can be chosen as

$$V_1 = \frac{1}{2b_1}e_1^2 \tag{10}$$

The time derivative of V_1 is equal to

$$\begin{aligned} \dot{V}_1 &= \frac{1}{b_1}e_1\dot{e}_1 \\ &= e_1(e_2 + \alpha_1 + \frac{f_1(\bar{x}_1) - \dot{y}_d}{b_1}) + \frac{1}{b_1}e_1\omega_1 \\ &= e_1(e_2 + \alpha_1 + \hat{f}_1) + \frac{1}{b_1}e_1\omega_1 \end{aligned} \tag{11}$$

where $\hat{f}_1 = \frac{f_1(\bar{x}_1) - \dot{y}_d}{b_1}$.

Define $\alpha_1 = -\lambda_1e_1 - \varphi_1, \lambda_1 > 0$, where φ_1 is a fuzzy system for approximating nonlinear function $\hat{f}_1$, then we can have

$$\dot{V}_1 = -\lambda_1e_1^2 + e_1e_2 + e_1(\hat{f}_1 - \varphi_1) + \frac{1}{b_1}e_1\omega_1 \tag{12}$$

Step 2, the Lyapunov function can be chosen as

$$V_2 = V_1 + \frac{1}{2b_2}e_2^2 \tag{13}$$

Define $\alpha_2 = -\lambda_2e_2 - e_1 - \varphi_2, \lambda_2 > 0$, where φ_2 is a fuzzy system for approximating nonlinear function $\hat{f}_2$, then we can have

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \frac{1}{b_2}e_2\dot{e}_2 \\ &= \dot{V}_1 + e_2\left(e_3 + \alpha_2 + \frac{f_2(\bar{x}_2) - \dot{\alpha}_1}{b_2}\right) + \frac{1}{b_2}e_2\omega_2 \\ &= \dot{V}_1 + e_2(e_3 + \alpha_2 + \hat{f}_2) + \frac{1}{b_2}e_2\omega_2 \\ &= -\sum_{i=1}^{2}\lambda_i e_i^2 + e_2e_3 + \sum_{i=1}^{2}e_i(\hat{f}_i - \varphi_i) + \sum_{i=1}^{2}\frac{1}{b_i}e_i\omega_i \end{aligned} \tag{14}$$

where $\hat{f}_2 = \frac{f_2 - \dot{\alpha}_1}{b_2}$.

Step $k(k = 3, 4, \dots n-1)$, we can define

$$\alpha_k = -\lambda_k e_k - e_{k-1} - \varphi_k \tag{15}$$

where $\lambda_k > 0$ and φ_k is a fuzzy system for approximating nonlinear function $\hat{f}_k$, then we can have

$$\begin{aligned}\dot{V}_k = &-\sum_{i=1}^{k}\lambda_i e_i^2 + e_k e_{k+1} + \sum_{i=1}^{k}e_i(\hat{f}_i - \varphi_i) \\ &+ \sum_{i=1}^{k}\frac{1}{b_i}e_i\omega_i \end{aligned} \tag{16}$$

Step n, the Lyapunov function can be chosen as

$$V_n = V_{n-1} + \frac{1}{2b_n}e_n^2 \tag{17}$$

The time derivative of V_n is equal to

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \frac{1}{b_n}e_n\dot{e}_n \\ &= \dot{V}_{n-1} + e_n\left(u + \frac{f_n(\bar{x}_n) - \dot{\alpha}_{n-1}}{b_n}\right) + \frac{1}{b_n}e_n\omega_n \\ &= -\sum_{i=1}^{n-1}\lambda_i e_i^2 + \sum_{i=1}^{n-1}e_i(\hat{f}_i - \varphi_i) \\ &\quad + e_n(u + e_{n-1} + \hat{f}_n) + \sum_{i=1}^{n}\frac{1}{b_i}e_i\omega_i \end{aligned} \tag{18}$$

where $\hat{f}_n = \frac{f_n(\bar{x}_n) - \dot{\alpha}_{n-1}}{b_n}$.

Define fuzzy system φ_n for approximating nonlinear function $\hat{f}_n$, then we can design the control law as follows:

$$u = -\lambda_n e_n - e_{n-1} - \varphi_n, \lambda_n > 0 \tag{19}$$

Equation (18) can be rewritten as:

$$\dot{V}_n = -\sum_{i=1}^{n}\lambda_i e_i^2 + \sum_{i=1}^{n}e_i(\hat{f}_i - \varphi_i) + \sum_{i=1}^{n}\frac{1}{b_i}e_i\omega_i \tag{20}$$

According to the virtual control (15) and the control law (19), not only all the uncertainties are compensated by fuzzy systems, but also there are no repeated differentiations problems [43].

4.3. Improved adaptive fuzzy backstepping control (IAFBC)

Equation (20) illustrates the influence of fuzzy approximation error $(\hat{f}_i - \varphi_i)$ and external disturbance ω_i on system stability. In order to improve the stability of the system and reduce the initial control input, an improved adaptive fuzzy backstepping control method is proposed in this paper.

Step 1, we redesign the Lyapunov function as follows:

$$V_1 = \frac{1}{2b_1\beta_1}e_1^2,\ \beta_1 > 1 \tag{21}$$

The derivative of Equation (21) is given by the following expression:

$$\begin{aligned}\dot{V}_1 &= \frac{1}{b_1\beta_1}e_1\dot{e}_1 \\ &= \frac{1}{\beta_1}e_1(e_2 + \alpha_1 + \hat{f}_1) + \frac{1}{b_1\beta_1}e_1\omega_1 \end{aligned} \tag{22}$$

Define $\alpha_1 = -\lambda_1\beta_1 e_1 - \varphi_1$, we can have

$$\dot{V}_1 = -\lambda_1 e_1^2 + \frac{1}{\beta_1}e_1e_2 + \frac{1}{\beta_1}e_1(\hat{f}_1 - \varphi_1) + \frac{1}{b_1\beta_1}e_1\omega_1 \tag{23}$$

Step 2, the Lyapunov function is redesigned as

$$V_2 = V_1 + \frac{1}{2b_2\beta_2}e_2^2, \beta_2 > 1 \tag{24}$$

Define $\alpha_2 = -\lambda_2\beta_2 e_2 - \frac{\beta_2}{\beta_1}e_1 - \varphi_2$, we can have

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \frac{1}{b_2\beta_2}e_2\dot{e}_2 \\ &= \dot{V}_1 + \frac{1}{\beta_2}e_2(e_3 + \alpha_2 + \hat{f}_2) + \frac{1}{b_2\beta_2}e_2\omega_2 \\ &= -\sum_{i=1}^{2}\lambda_i e_i^2 + \frac{1}{\beta_2}e_2e_3 + \sum_{i=1}^{2}\frac{1}{\beta_i}e_i(\hat{f}_i - \varphi_i) \\ &\quad + \sum_{i=1}^{2}\frac{1}{b_i\beta_i}e_i\omega_i \end{aligned} \tag{25}$$

Step $k(k = 3, 4 \cdots n-1)$, define

$$\alpha_k = -\lambda_k\beta_k e_k - \frac{\beta_k}{\beta_{k-1}}e_{k-1} - \varphi_k \tag{26}$$

We can obtain

$$\begin{aligned}\dot{V}_k = &-\sum_{i=1}^{k}\lambda_i e_i^2 + \frac{1}{\beta_k}e_k e_{k+1} + \sum_{i=1}^{k}\frac{1}{\beta_i}e_i(\hat{f}_i - \varphi_i) \\ &+ \sum_{i=1}^{k}\frac{1}{b_i\beta_i}e_i\omega_i \end{aligned} \tag{27}$$

Step n, the Lyapunov function is redesigned as follows:

$$V_n = V_{n-1} + \frac{1}{2b_n\beta_n}e_n^2, \beta_n > 1 \tag{28}$$

The derivative of Equation (28) is given by the following expression:

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \frac{1}{b_n\beta_n}e_n\dot{e}_n \\ &= -\sum_{i=1}^{n-1}\lambda_i e_i^2 + \sum_{i=1}^{n-1}\frac{1}{\beta_i}e_i(\hat{f}_i - \varphi_i) \\ &\quad + \frac{1}{\beta_n}e_n\left(u + \frac{\beta_n}{\beta_{n-1}}e_{n-1} + \frac{f_n(\bar{x}_n) - \dot{\alpha}_{n-1}}{b_n}\right) \\ &\quad + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i \\ &= -\sum_{i=1}^{n-1}\lambda_i e_i^2 + \sum_{i=1}^{n-1}\frac{1}{\beta_i}e_i(\hat{f}_i - \varphi_i) \\ &\quad + \frac{1}{\beta_n}e_n\left(u + \frac{\beta_n}{\beta_{n-1}}e_{n-1} + \hat{f}_n\right) + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i\end{aligned} \tag{29}$$

The control law is redesigned as follows:

$$u = -\lambda_n\beta_n e_n - \frac{\beta_n}{\beta_{n-1}}e_{n-1} - \varphi_n, \lambda_n > 0 \tag{30}$$

Then (29) can be rewritten as:

$$\dot{V}_n = -\sum_{i=1}^{n}\lambda_i e_i^2 + \sum_{i=1}^{n}\frac{1}{\beta_i}e_i(\hat{f}_i - \varphi_i) + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i \tag{31}$$

From Equation (31), we can see that by introducing the control parameters β_i ($\beta_i > 1$) in virtual control law (26) and control law(30), the influence of fuzzy approximation error and external disturbance in each subsystem can be reduced by β_i times, thus the stability and control precision of the system are improved.

From Equations (26) and (30), we can see that because of introducing of β_i, the initial control input will increase, such that the control method is difficult to implement in practice. In order to decrease the initial control input, a low pass filter is designed in this paper such that $\beta_i = \beta_{i0} + K_i(1 - e^{-\frac{1}{T_i}t})$, where β_{i0} is a constant far less than 1, K_i is amplification coefficient, T_i is time constant. If T_i is very small, β_i will increase from β_{i0} to $K_i + \beta_{i0}$ at a very fast rate. Through the above control scheme, not only the initial control input can be obviously reduced, but also the system can achieve higher control accuracy.

5. Stability analysis and adaptive law design

Now, consider the following Lyapunov candidate function:

$$V = V_n + \sum_{i=1}^{n}\frac{1}{2r_i\beta_i}\tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i \tag{32}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i$, $r_i > 0$.

The time derivative of Equation (32) is given by the following expression:

$$\begin{aligned}\dot{V} &= \dot{V}_n + \sum_{i=1}^{n}\frac{1}{r_i\beta_i}\tilde{\boldsymbol{\theta}}_i^T\dot{\tilde{\boldsymbol{\theta}}}_i \\ &= -\sum_{i=1}^{n}\lambda_i e_i^2 + \sum_{i=1}^{n}\frac{1}{\beta_i}e_i(\hat{f}_i - \boldsymbol{\theta}_i^{*T}\boldsymbol{\xi}_i(\bar{x}_i)) \\ &\quad + \sum_{i=1}^{n}\frac{1}{\beta_i}e_i\tilde{\boldsymbol{\theta}}_i^T\boldsymbol{\xi}_i(\bar{x}_i) - \sum_{i=1}^{n}\frac{1}{r_i\beta_i}\tilde{\boldsymbol{\theta}}_i^T\dot{\boldsymbol{\theta}}_i + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i \\ &\le -\sum_{i=1}^{n}\lambda_i e_i^2 + \sum_{i=1}^{n}\frac{1}{\beta_i}|e_i\varepsilon_i| + \sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^T\frac{1}{\beta_i}(e_i\boldsymbol{\xi}_i(\bar{x}_i) \\ &\quad - \frac{1}{r_i}\dot{\boldsymbol{\theta}}_i) + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i\end{aligned} \tag{33}$$

Equation (33) shows that for the same control parameters λ_i, as long as the designed parameters $\beta_i > 1$, the influence of fuzzy approximation error and external disturbance on system stability can be reduced. For convenience, symbols S is introduced, where

$$\begin{aligned}S &= -\sum_{i=1}^{n}\lambda_i e_i^2 + \sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^T\frac{1}{\beta_i}\left(e_i\boldsymbol{\xi}_i(\bar{x}_i) - \frac{1}{r_i}\dot{\boldsymbol{\theta}}_i\right) \\ &\quad + \sum_{i=1}^{n}\frac{1}{\beta_i}|e_i\varepsilon_i| + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i\end{aligned} \tag{34}$$

Select the positive coefficients λ_i as

$$\lambda_i = \alpha_i + \frac{1}{2} + \frac{1}{2\rho^2(b_i\beta_i)^2} \tag{35}$$

where α_i and ρ are positive constants, so $\lambda_i > 0.5$.

Substituting (35) into (34) results in

$$\begin{aligned}S &= -\sum_{i=1}^{n}\alpha_i e_i^2 - \frac{1}{2}\sum_{i=1}^{n}e_i^2 - \sum_{i=1}^{n}\frac{1}{2\rho^2(b_i\beta_i)^2}e_i^2 \\ &\quad + \sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^T\frac{1}{\beta_i}(e_i\boldsymbol{\xi}_i(\bar{x}_i) - \frac{1}{r_i}\dot{\boldsymbol{\theta}}_i) + \sum_{i=1}^{n}\frac{1}{\beta_i}|e_i\varepsilon_i| \\ &\quad + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i\end{aligned} \tag{36}$$

Because $\beta_i \ge 1$, we can have

$$-\frac{1}{2}\sum_{i=1}^{n}e_i^2 + \sum_{i=1}^{n}\frac{1}{\beta_i}|e_i\varepsilon_i| \le \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 \tag{37}$$

$$-\sum_{i=1}^{n}\frac{1}{2\rho^2(b_i\beta_i)^2}e_i^2 + \sum_{i=1}^{n}\frac{1}{b_i\beta_i}e_i\omega_i \le \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \tag{38}$$

Then, the inequality can be obtained as follows:

$$S \leq -\sum_{i=1}^{n}\alpha_i e_i^2 + \sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^T \frac{1}{\beta_i}(e_i\boldsymbol{\xi}_i(\bar{x}_i) - \frac{1}{r_i}\dot{\boldsymbol{\theta}}_i) + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \tag{39}$$

Define the adaptive updating law as:

$$\dot{\boldsymbol{\theta}}_i = r_i\mathbf{e}_i\boldsymbol{\xi}_i(\bar{x}_i) - 2k_i\theta_i \tag{40}$$

where $i = 1, 2, \cdots, n, k_i > 0$.

Substituting (40) into (39) results in

$$\begin{aligned} S &\leq -\sum_{i=1}^{n}\alpha_i e_i^2 + \sum_{i=1}^{n}\frac{2k_i}{\beta_i r_i}(\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i)^T\boldsymbol{\theta}_i \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \\ &= -\sum_{i=1}^{n}\alpha_i e_i^2 + \sum_{i=1}^{n}\frac{k_i}{\beta_i r_i}(2\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i - 2\boldsymbol{\theta}_i{}^T\boldsymbol{\theta}_i) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \\ &\leq -\sum_{i=1}^{n}\alpha_i e_i^2 + \sum_{i=1}^{n}\frac{k_i}{\beta_i r_i}(\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i{}^T\boldsymbol{\theta}_i) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \\ &= -\sum_{i=1}^{n}\alpha_i e_i^2 + \sum_{i=1}^{n}\frac{k_i}{\beta_i r_i}(-\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i{}^T\boldsymbol{\theta}_i) \\ &\quad + \sum_{i=1}^{n}\frac{2k_i}{\beta_i r_i}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \end{aligned} \tag{41}$$

Since

$$\begin{aligned} \tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i &= (\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i)^T(\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i) = \boldsymbol{\theta}_i{}^*T\boldsymbol{\theta}_i^* - 2\boldsymbol{\theta}_i{}^*T\boldsymbol{\theta}_i \\ &\quad + \boldsymbol{\theta}_i^T\boldsymbol{\theta}_i \leq 2\boldsymbol{\theta}_i{}^*T\boldsymbol{\theta}_i^* + 2\boldsymbol{\theta}_i^T\boldsymbol{\theta}_i \end{aligned} \tag{42}$$

We have

$$-\frac{1}{2}\tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i \geq -\boldsymbol{\theta}_i{}^*T\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i^T\boldsymbol{\theta}_i \tag{43}$$

Substituting (43) into (41) results in

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^{n}\alpha_i e_i^2 - \sum_{i=1}^{n}\frac{k_i}{2\beta_i r_i}\tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i + \sum_{i=1}^{n}\frac{2k_i}{\beta_i r_i}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \\ &\leq -\sum_{i=1}^{n}\alpha_i\frac{2b_{im}\beta_i}{2b_i\beta_i}e_i^2 - \sum_{i=1}^{n}\frac{k_i}{2\beta_i r_i}\tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i \\ &\quad + \sum_{i=1}^{n}\frac{2k_i}{\beta_i r_i}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\frac{1}{2}\rho^2\omega_i^2 \end{aligned} \tag{44}$$

Define $A = \min\{2b_{im}a_i\beta_i, k_i, i = 1, 2, \cdots, n\}$ and $B = \sum_{i=1}^{n}\frac{2k_i}{\beta_i r_i}\boldsymbol{\theta}_i^{*T}\boldsymbol{\theta}_i^* + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2$. ω_i are bounded disturbances, so we have $\omega_i^2 \leq c_i$. Define $C = \sum_{i=1}^{n}\frac{1}{2}\rho^2c_i^2$.

So (44) can be rearranged as:

$$\begin{aligned} \dot{V} &\leq -A\left(\sum_{i=1}^{n}\frac{1}{2b_i\beta_i}e_i^2 + \sum_{i=1}^{n}\frac{1}{2\beta_i r_i}\tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i\right) + B + C \\ &\leq -AV + B + C \end{aligned} \tag{45}$$

where A, B, C are positive constants.

Then, integrating the inequality (45), one has

$$V(t) \leq V(0) + \frac{B + C}{A}, \forall t \geq 0 \tag{46}$$

From (46), we can prove that all the signals of the designed control system are semi-globally uniformly ultimately bounded.

6. Simulations

A motor driven manipulator used in the simulations, the dynamic equation can be written as follows [43]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{B}{M_t}x_2 + \frac{N}{M_t}f_2(x_1, x_2) + \frac{K_t}{M_t}x_3 \\ \dot{x}_3 = f_3(x_1, x_2, x_3) + \frac{1}{L}u - \frac{1}{L}\omega \\ y = x_1 \end{cases} \tag{47}$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = I$, $N = mgl + Mgl$, $M_t = J + \frac{1}{3}ml^2 + \frac{1}{10}Ml^2D$, g is the gravity acceleration constant,f_2 and f_3 are unknown nonlinear functions, ω is the external disturbance, θ is connecting rod angle, I is electric current, K_t is torque constant, K_b is back EMF coefficient, B is viscous friction coefficient of bearing, D is load diameter, l is connecting rod length, M is load quality, m is connecting rod weight, L is reactance, R is resistance, u is the control voltage of the motor, J is actuator torque.

The desired trajectory is $y_d = 5\sin(2\pi t)$, $f_2 = \sin\theta$, $f_3 = -\frac{R}{L}x_3 - \frac{K_b}{L}x_2$, $\omega(t) = 4\sin(t)$.

The parameters of the manipulator: $B = 0.015$, $L = 0.0008$, $D = 0.05$, $R = 0.075$, $m = 0.01$, $J = 0.05$, $l = 0.6$, $K_b = 0.085$, $M = 0.05$, $K_t = 1$, $g = 9.8$. The initial state of the manipulator is $\mathbf{x}(0) = [0, 0, 0]^T$.

The two kinds of control schemes described above are summed up as (48) and (49), respectively. In order to verify the effectiveness, we compare their control performance with that of DSC control (50). The Three schemes use the same fuzzy system (4) and the adaptive law (40).

Control Scheme 1: AFBC

$$\begin{cases} \alpha_1 = -\lambda_1(x_1 - y_d) + \dot{y}_d \\ \alpha_2 = -\lambda_2(x_2 - \alpha_1) - (x_1 - y_d) - \varphi_2 \\ u = -\lambda_3(x_3 - \alpha_2) - (x_2 - \alpha_1) - \varphi_3 \\ e_1 = x_1 - y_d \\ e_2 = x_2 - \alpha_1 \\ e_3 = x_3 - \alpha_2 \end{cases} \tag{48}$$

Control Scheme 2: IAFBC

$$\begin{cases} \alpha_1 = -\lambda_1\beta_1(x_1 - y_d) + \dot{y}_d \\ \alpha_2 = -\lambda_2\beta_2(x_2 - \alpha_1) - \dfrac{\beta_2}{\beta_1}(x_1 - y_d) - \varphi_2 \\ u = -\lambda_3\beta_3(x_3 - \alpha_2) - \dfrac{\beta_3}{\beta_2}(x_2 - \alpha_1) - \varphi_3 \\ e_1 = x_1 - y_d \\ e_2 = x_2 - \alpha_1 \\ e_3 = x_3 - \alpha_2 \end{cases} \tag{49}$$

Control Scheme 3: DSC

$$\begin{cases} \alpha_1 = -\lambda_1(x_1 - y_d) + \dot{y}_d \\ \alpha_2 = -\lambda_2(x_2 - s_1) - (x_1 - y_d) - \varphi_2 + \dfrac{a_1 - s_1}{\tau_1} \\ u = -\lambda_3(x_3 - a_2) - (x_2 - a_1) - \varphi_3 + \dfrac{a_2 - s_2}{\tau_2} \\ \tau_1\dot{s}_1 + s_1 = \alpha_1, s_1(0) = \alpha_1(0) \\ \tau_2\dot{s}_2 + s_2 = \alpha_2, s_2(0) = \alpha_2(0) \\ e_1 = x_1 - y_d \\ e_2 = x_2 - s_1 \\ e_3 = x_3 - s_2 \end{cases} \tag{50}$$

The control parameters in all the three control methods are designed as follows: $\lambda_1 = 3,\ \lambda_2 = 8.5,\ \lambda_3 = 8.5,\ K = 10,\ T = 0.1s,\ \beta_1 = \beta_2 = \beta_3 = 0.1 + K(1 - e^{-\frac{1}{T}t}),\ \tau = 0.01s,\ r_1 = r_2 = r_3 = 2,\ k_1 = k_2 = k_3 = 1.5$.

Figures 1–3 show the performance of the position tracking of the three control methods. From the simulation results, we can see that the positon tracking accuracy of AFBC and IAFBC is much higher than that of DSC which is widely used in the existing literatures. Due to the addition of control parameters β_i, the influence of fuzzy approximation error and external disturbance is reduced, thus the control accuracy of IAFBC can be further improved compared with AFBC.

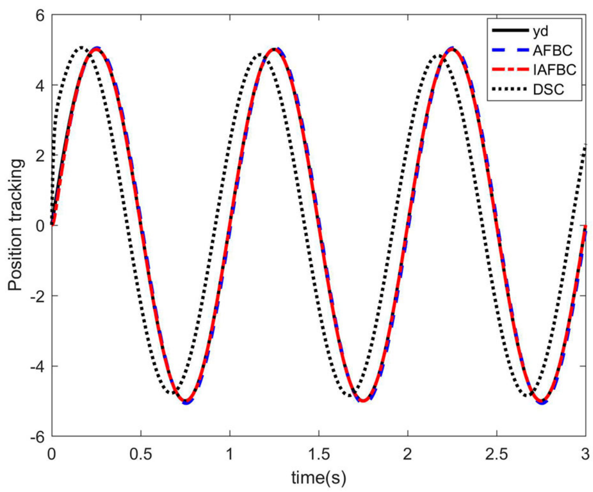

Figure 1. The position tracking.

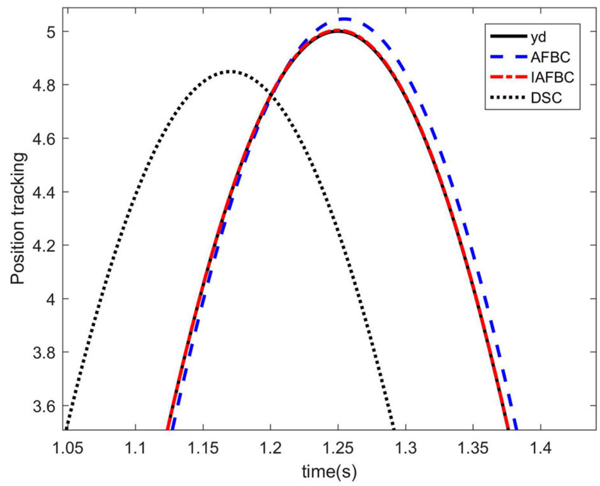

Figure 2. Part of details of Figure 1.

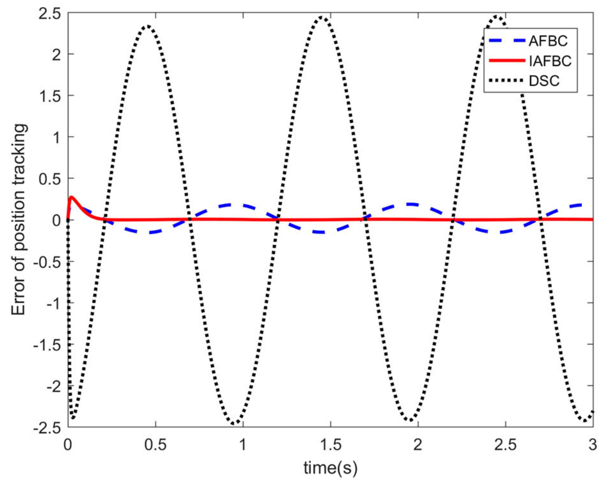

Figure 3. The error of position tracking.

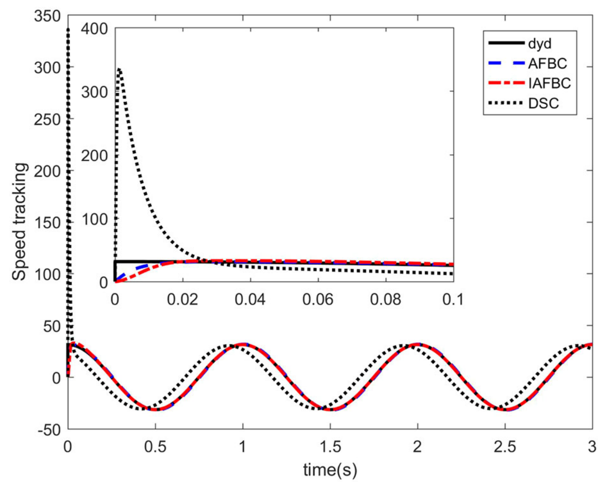

Figure 4. The speed tracking.

Figures 4–6 show the performance of the speed tracking of the three control methods. From the simulation results, we can see that the initial error and steady-state tracking error under DSC control are much larger than those of AFBC and IAFBC. In the three methods, the speed tracking error of IAFBC is the smallest.

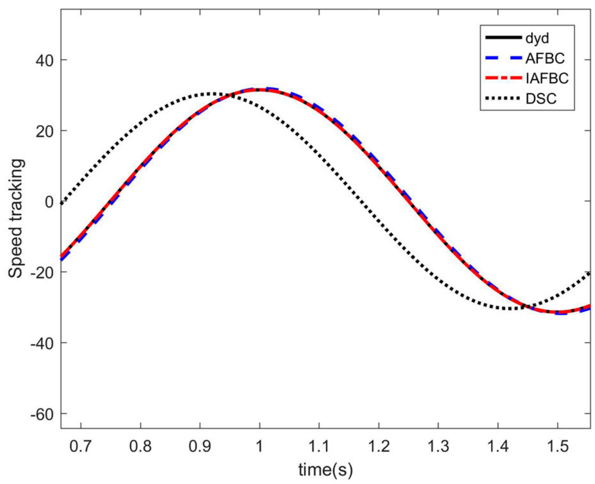

Figure 5. Part of details of Figure 4.

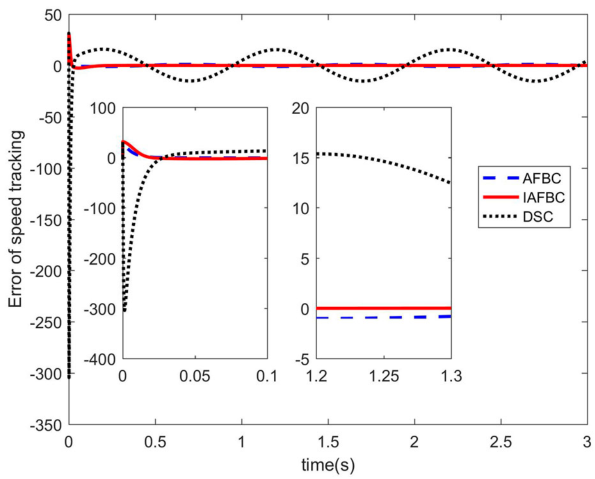

Figure 6. The error of speed tracking.

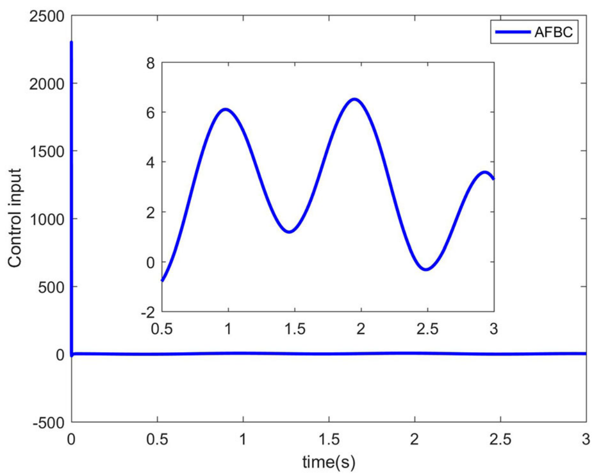

Figure 7. The control input of AFBC.

Figures 7–9 show the control input signal of the three control methods respectively. From Figure 7, we can see that the initial control input of AFBC is 2300. Figure 8 shows the initial control input of IAFBC is 54. Figure 9 shows the initial control input of DSC is 368210. The initial control input signal is significantly reduced by IAFBC, this benefit is very conducive to the actual engineering application.

Finally, for a better illustration, all the simulation results are summarized in Table 1. From the Table 1, it is clearly shown that the proposed control method IAFBC indeed can have much better performance than DSC.

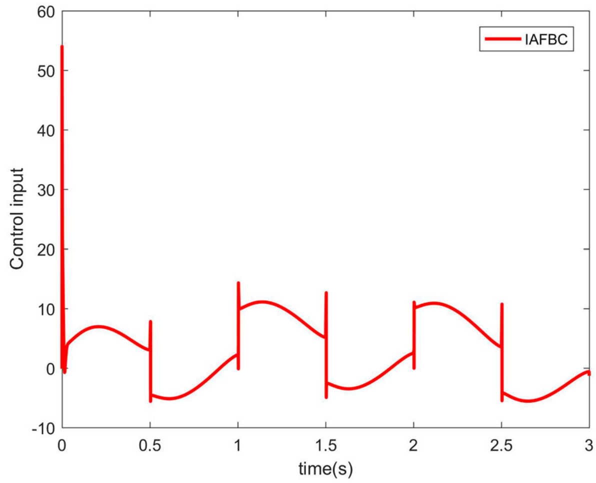

Figure 8. The control input of IAFBC.

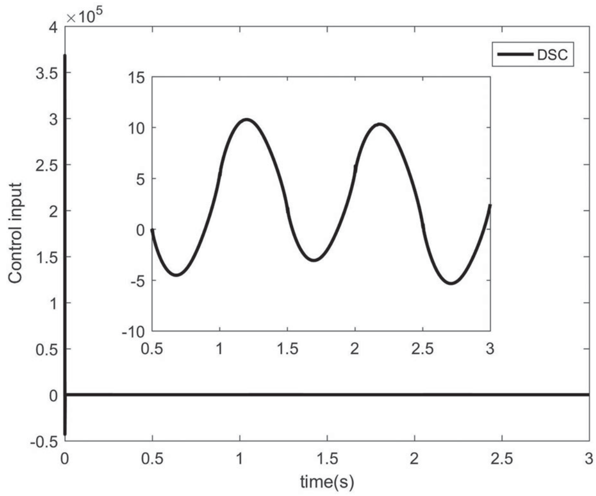

Figure 9. The control input of DSC.

Table 1. Performance comparison of the three control schemes.

Performance comparison	AFBC	IAFBC	DSC
$\int_0^t e^2 dt$ of position tracking	1.8×10^3	5.8×10^1	3.9×10^5
$\int_0^t e^2 dt$ of speed tracking	1.1×10^5	4.9×10^4	1.9×10^7
u_{max}	2.3×10^3	5.4×10^1	3.6×10^5

7. Conclusion

In this paper, an improved adaptive fuzzy backstepping control scheme has been proposed for a class of nonlinear systems with mismatched uncertainties. By using fuzzy systems and backstepping control, not only mismatched uncertainties in systems, such as unknown functions and unknown control gains, are identified, but also the problem of "explosion of terms" inherent in traditional backstepping control is avoided. Moreover, through the introduction of adjustable control parameters and low pass filters, the influence of fuzzy approximation error and external disturbances can be further reduced, thus the control error convergence is faster, the control precision is obviously improved, and the initial control input signal is greatly reduced. Finally, the stability of the control system is proved and all the signals are guaranteed to be bounded. Some simulation results have demonstrated the effectiveness of the proposed results. Future research directions are the extension of the results to nonstrict-feedback stochastic MIMO nonlinear systems with uncertainties and unmeasured states.

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