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High-frequency feedback robust control for flocking of multi-agent system with unknown parameters

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ABSTRACT

In this paper, a kind of high-frequency feedback robust control for flocking of a multi-agent system with unknown parameters is put forward in detail. Firstly, a high-frequency feedback robust control scheme is proposed for the flocking problem of a multi-agent system with unknown parameters. Secondly, through the employment of Lyapunov stability theory, it's proved that velocity error is gradually stabilized and collisions between agents are also avoided. In addition, the high-frequency feedback robust control for flocking of a multi-agent system with unknown parameters is further proved. Finally, it is verified with the help of numerical simulation.

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KEYWORDS High-frequency feedback robust control; multi-agent flocking; unknown parameters; Lyapunov stability theory

1. Introduction

It is widely acknowledge that multi-agent systems are widely employed in the process of formation, flocking, and information fusion. Moreover, the study conducted on the multi-agent flocking system [1] has attracted widespread attention from scholars. For instance, Su et al. [2] once carried out a study on the flocking of multiagents with a virtual leader. Yu et al. [3] conducted a relevant study on the distributed leader-follower flocking control for multi-agent dynamical systems with timevarying velocities. In addition, Atrianfar and Haeri [4] implemented a relevant study on the adaptive flocking control of nonlinear multi-agent systems with directed switching topologies and saturation constraints. Moreover, Li et al. [5] launched a related study on the distributed robust control of linear multi-agent systems with parameter uncertainties.

As for the actual system, the model is mainly established on the basis of the observations over the data, which leads to incomplete models. In the process of actual operation, the parameters of the system will vary according to the external conditions. For instance, the sound speed of a single chip varies along with the change of temperature consequently. In addition, the motor parameters will also experience certain changes during different phases in the process of use, which as a result will have an impact on the normal operation of the motor control system. Furthermore, all these will bring uncertainty to the system. Therefore, it is of great significance to study on the control problem of a multi-agent system with unknown parameters.

The fixed controller is named as the robust controller when the robustness of the closed loop system is set up as the design target. When the robust controller has a high-frequency response around zero point, it is called a high-frequency feedback robust controller. Besides, when the control system is subject to external disturbances and modelling errors in the actual control system, it is quite difficult to obtain the precise model of the actual control system. Therefore, a controller with high-frequency feedback robust control is tailored accordingly, which enables the uncertain objects to satisfy the control properties. In other words, the performance of the control system can be maintained to a certain extent when either the external environment or the parameters of the control system change.

It is known that robustness [6,7] play an important role in the research topic within control theory. The study conducted on robustness focuses mainly on the linear time-invariant control systems [8], including stability [9,10], being free from static error [11], adaptive control [12–15] and so on. Qian et al. [16,17] conducted a relevant study on a robust control method for formation manoeuvres of a multi-agent system and carried out an investigation on the problems related to formation control of multiple agents.

The innovative points reflected in this paper is shown as follows: a high-frequency feedback robust control scheme is designed for the flocking problem of a multi-agent system with unknown parameters, and then both the stability and flocking behaviour are strictly proved by employing the Lyapunov method. Moreover, the results obtained from theory are further verified by employing numerical simulations.

Through the comparison made between this paper and [18–20], the following advantages can be obtained.

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In contrast, high-frequency feedback robust control for flocking of multi-agent with unknown parameters can not only be used to identify unknown parameters, but also has perfect robustness. In comparison with the flocking of a multi-agent system in [18–20], it can be observed that high-frequency feedback robust control for flocking of multi-agent with unknown parameters has better performance in both convergence and stability.

In terms of practical application, Unmanned Aerial Vehicle (UAV) in flight, it can be obtained that the speeds of the UAV are gradually uniform, the direction of the speed is the same, and no collision occurs between the UAVs under the action of the distributed controller. Eventually, the flocking of the UAV system can be formed for further investigation or attack.

Section 2 gives the dynamic equation of the multiagent and some preliminaries. Section 3 designs a controller that has a high-frequency feedback robust control with unknown parameters. In Section 4, it is proved that under the controller, the velocity of the multi-agent is gradually stabilized, and no collision occurs between agents. In Section 5, simulation results are given to verify the proposed control scheme.

2. Preliminaries

A set of N ($N \ge 1$) agents moving in an *n*-dimensional Euclidean space are considered. In the system, the dynamics model of the agent *i* is described by

$$\dot{x}_i(t) = v_i(t),$$

 $\dot{v}_i(t) = f_i(x_i, t) + u_i(t), \quad i = 1, 2..., N,$ (1)

 $x_i(t) \in \mathbb{R}^n$ is the position vector of the agent $i, v_i(t) \in \mathbb{R}^n$ is the velocity vector of the agent i, and $u_i(t) \in \mathbb{R}^n$ is the control input of the agent i. The nonlinear function $f_i(x_i, t) \in \mathbb{R}^n$, which describes the agent i, satisfies Assumptions 2.1 and 2.2.

Assumption 2.1: The nonlinear function $f_i(x_i, t) \in \mathbb{R}^n$ can be linearly parameterized:

$$f_i(x_i, t) = g(x_i, t) + \phi_i^{\mathrm{T}}(x_i, t)\theta_i, \qquad (2)$$

 $g(x_i, t)$ is the known nonlinear function, $\phi_i(x_i, t) \in \mathbb{R}^{m \times n}$ is a known basis vector function, and $\theta_i \in \mathbb{R}^m$ is unknown constant parameter.

Assumption 2.2: For the nonlinear function $f_i(x_i, t) \in \mathbb{R}^n$, there exists an upper bound function $\rho(x)$ which satisfies $||f_i(x_i, t)|| \le \rho(x)$.

The directed graph *G* describes the topology between agents. The directed graph *G* is composed of a vertex set *V* and edge set *E*, $V = \{1, 2, ..., N\}$, E = $\{(i, j) | i, j \in V\}$. If agent *i* can receive the information of agent *j*, then $(j, i) \in E$, otherwise $(j, i) \notin E$. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ of graph *G*, where a_{ij} satisfies

$$a_{ij} = \begin{cases} 1, & \text{follower } i \text{ can receive information} \\ & \text{from follower } j, \\ 0, & \text{follower } i \text{ does not receive information} \\ & \text{from follower } j. \end{cases}$$
(3)

The Laplace matrix of graph *G* is defined as $L = [l_{ij}] \in \mathbb{R}^{n \times n}$. The element in *L* satisfies $l_{ii} = \sum_{j=1}^{N} a_{ij}$, $l_{ij} = -a_{ij}$, $i \neq j$.

Definition 2.3 ([21] (Global Reach)): For *G*, if there is a path in *G* from every node *i* in *G* to node 0, we say that node 0 is globally reachable in *G*, which is much weaker than strong connectedness.

Definition 2.4 ([22] (Flocking)): A group of mobile agents is said to be (asymptotically) flocking, when all agents have the same velocity vector, and collision among each agent is always avoided.

3. Design of controller based on high-frequency feedback robust control with unknown parameters

We define the set of control laws,

$$u_{i}(t) = K_{1} \sum_{j=1}^{N} a_{ij}(v_{j} - v_{i}) - 2 \sum_{k=1}^{N} w_{ik} \sum_{j=1}^{N} a_{kj} \nabla_{x_{k}} V_{kj}$$
$$- \frac{\rho^{2}(x)e_{i}}{\rho(x)\|e\| + \omega} - \phi_{i}^{T}\hat{\theta}_{i}, \quad i, j = 1, 2, \dots, N,$$
(4)

$$\dot{\hat{\theta}}_i = \frac{K_2}{K_1} \phi_i e_i, \quad i, j = 1, 2, \dots, N,$$
 (5)

where control parameters $K_1 > 0$, $K_2 > 0$, ω is the error band width, w_{ik} is the element of the matrix L^{-1} , and $\hat{\theta}_i$ is the estimated value of agent *i* on θ_i . $e_i = \sum_{j=1}^{N} a_{ij}(v_i - v_j)$ is a local consistency error vector.

 $\nabla_{x_i} V_{ij}$ is a direction vector of the negative gradient of an artificial potential function defined by the following equation:

$$V_{ij}(x_{ij}) = \frac{1}{M^2 - \|x_{ij}\|^2} + \frac{1}{\|x_{ij}\|^2}, \quad \|x_{ij}\| \in (0, M),$$
(6)

with $x_{ij} = x_i - x_j$, which allows both collision avoidance and maintaining links in the network. *M* represents the maximum distance within which multi-agents are able to obtain information from other agents.

Based on the definition of V_{ij} ,

$$\nabla_{x_{ij}}V_{ij} = \nabla_{x_i}V_{ij} = -\nabla_{x_j}V_{ij}, \quad i, j = 1, 2, \dots, N,$$
 (7)

For convenience,

$$\begin{aligned} x &= \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}, v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}, \\ e &= \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}, \quad \hat{\Theta} = \begin{pmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \vdots \\ \hat{\theta}_N \end{pmatrix}, \\ f(x,t) &= \begin{pmatrix} f_1(x_1,t) \\ f_2(x_2,t) \\ \vdots \\ f_N(x_N,t) \end{pmatrix}, \quad F = \begin{pmatrix} \sum_{j=1}^N a_{1j} \nabla_{x_1} V_{1j} \\ \sum_{j=1}^N a_{2j} \nabla_{x_2} V_{2j} \\ \vdots \\ \sum_{j=1}^N a_{Nj} \nabla_{x_N} V_{Nj} \end{pmatrix}, \end{aligned}$$

 $\Phi = \text{diag}\{\phi_1, \phi_2, \dots, \phi_N\}.$ System (1) can be written in a vector form

.

$$x = v,$$

$$\dot{v} = f(x, t) - K_1 e - 2L^{-1}F - \frac{\rho^2(x)e}{\rho(x)\|e\| + \omega} - \Phi^T\hat{\Theta}.$$
(8)

By the definition of e_i , we have e = Lv.

4. The main theory results

Theorem 4.1: In terms of multi-agents system (1), the final velocity error variable satisfies $||e|| \le \sqrt{w/K_1}$, the velocity of multi-agent is gradually stabilized, and no collision occurs between agents under control laws (4).

Proof: Construct Lyapunov function $V_G(t)$

$$V_G(t) = \frac{1}{2K_1} e^{\mathrm{T}} L^{-1} e + \frac{1}{2K_2} \sum_{i=1}^{N} \hat{\theta}_i^{\mathrm{T}} \hat{\theta}_i$$
$$+ \frac{1}{K_1} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} V_{ij}.$$
(9)

The generalized time derivative of $V_G(t)$ is

$$\dot{V}_G(t) = \frac{1}{K_1} e^{\mathrm{T}} L^{-1} \dot{e} + \frac{1}{K_2} \dot{\theta}_i^{\mathrm{T}} \hat{\theta}_i + \frac{1}{K_1} \sum_{i=1}^N \sum_{j=1}^N a_{ij} \dot{V}_{ij}.$$
(10)

The first part of (10) can be written as

$$\frac{1}{K_{1}}e^{T}L^{-1}\dot{e} = \frac{1}{K_{1}}e^{T}L^{-1}L\dot{v}
= \frac{1}{K_{1}}e^{T}\dot{v}
= \frac{1}{K_{1}}e^{T}[f(x,t) - K_{1}e - 2L^{-1}F
- \frac{\rho^{2}(x)e}{\rho(x)\|e\| + \omega} - \Phi^{T}\hat{\Theta}]
= \frac{1}{K_{1}}[e^{T}f(x,t) - K_{1}e^{T}e - 2v^{T}F
- \frac{\rho^{2}(x)e^{T}e}{\rho(x)\|e\| + \omega} - (\Phi e)^{T}\hat{\Theta}]. \quad (11)$$

By Assumption 2.2 and taking the norm of the first term,

$$e^{T}f(x,t) - \frac{\rho^{2}(x)e^{T}e}{\rho(x)\|e\| + \omega}$$

$$\leq \rho(x)\|e^{T}\| - \frac{\rho^{2}(x)\|e\|^{2}}{\rho(x)\|e\| + \omega}$$

$$\leq \frac{\rho^{2}(x)\|e\|^{2} + \rho(x)\|e\|\omega - \rho^{2}(x)\|e\|^{2}}{\rho(x)\|e\| + \omega}$$

$$= \frac{\rho(x)\|e\|\omega}{\rho(x)\|e\| + \omega}$$

$$\leq \omega.$$
(12)

By (11) and (12), we have

$$\frac{1}{K_1} e^{\mathrm{T}} L^{-1} \dot{e} \le \frac{1}{K_1} [\omega - K_1 e^{\mathrm{T}} e - 2v^{\mathrm{T}} F - (\Phi e)^{\mathrm{T}} \hat{\Theta}].$$
(13)

For the second item of (10), we have

$$\frac{1}{K_2} \dot{\hat{\theta}}_i^{\mathrm{T}} \hat{\theta}_i = \frac{1}{K_2} \sum_{i=1}^N \left(\frac{K_2}{K_1} \phi_i e_i \right)^{\mathrm{T}} \hat{\theta}_i$$
$$= \frac{1}{K_1} \sum_{i=1}^N (\phi_i e_i)^{\mathrm{T}} \hat{\theta}_i$$
$$= \frac{1}{K_1} (\Phi e)^{\mathrm{T}} \hat{\Theta}.$$
(14)

By (7), the last item of (10) can be written as

$$\begin{split} \frac{1}{K_1} \sum_{i=1}^N \sum_{j=1}^N a_{ij} \dot{V}_{ij} &= \frac{1}{K_1} \sum_{i=1}^N \sum_{j=1}^N a_{ij} \dot{x}_{ij}^T \nabla_{x_{ij}} V_{ij} \\ &= \frac{1}{K_1} \sum_{i=1}^N \sum_{j=1}^N a_{ij} (\dot{x}_i^T - \dot{x}_j^T) \nabla_{x_i} V_{ij} \\ &= \frac{1}{K_1} \sum_{i=1}^N \sum_{j=1}^N a_{ij} (v_i^T - v_j^T) \nabla_{x_i} V_{ij} \end{split}$$

$$= \frac{1}{K_{1}} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} v_{i}^{\mathrm{T}} \nabla_{x_{i}} V_{ij}$$

$$+ \frac{1}{K_{1}} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} v_{j}^{\mathrm{T}} \nabla_{x_{j}} V_{ij}$$

$$= \frac{2}{K_{1}} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} v_{i}^{\mathrm{T}} \nabla_{x_{i}} V_{ij}$$

$$= \frac{2}{K_{1}} v^{\mathrm{T}} F. \qquad (15)$$

Above all, we have

$$\dot{V}_{G}(t) \leq \frac{w}{K_{1}} - e^{\mathrm{T}}e - \frac{2}{K_{1}}v^{\mathrm{T}}F - \frac{1}{K_{1}}(\Phi e)^{\mathrm{T}}\hat{\Theta} + \frac{1}{K_{1}}(\Phi e)^{\mathrm{T}}\hat{\Theta} + \frac{2}{K_{1}}v^{\mathrm{T}}F \leq -\left(e^{\mathrm{T}}e - \frac{w}{K_{1}}\right).$$
(16)

When $e^{T}e > w/K_1$, we have $\dot{V}_G(t) < 0$, then $V_G(t)$ monotonously decreases, contradicting with $e^{T}e > w/K_1$. When $e^{T}e \le w/K_1$, $e^{T}e$ bounded, we can obtain $||e|| \le \sqrt{w/K_1}$.

Hence, the theorem is proved.

5. Simulation results

In this section, an example was further displayed to show the effectiveness of the algorithm proposed by us for the system described by (1).

The unknown nonlinear function $f_i(x_i, t)$ is assumed as

$$f_{i}(x_{i},t) = g(x_{i},t) + \phi_{i}^{\mathrm{T}}\theta_{i} = \begin{pmatrix} 10(v_{y} - v_{x}) \\ -v_{x}v_{z} - v_{y} + 28v_{x} \\ v_{x}v_{y} + \frac{8}{3}v_{z} \end{pmatrix},$$
(17)

where

$$g(x_i, t) = \begin{pmatrix} 0 \\ -v_x v_z - v_y + 28v_x \\ v_x v_y + \frac{8}{3}v_z \end{pmatrix}, \ \phi_i = \begin{pmatrix} v_y - v_x \\ 0 \\ 0 \end{pmatrix},$$

 $\theta_i = 10$, and function $f_i(x_i, t)$ satisfies the condition of the assumption.

Two sets of multi-agents were selected by us for comparison. It was obtained that 10 agents are available in one group and 20 agents are available in the other group. Then, the comparison was made between the two groups of multi-agents in different initial states. The initial velocity of the multi-agent is randomly selected, and error band width $\omega = 3600$, upper bound function $\rho(\cdot) = 60$, control parameter $K_1 = K_2 = 1$, unknown parameter $\theta_i = 10$ can finally be identified. The adjacency matrix composed of 10 agents refers to

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

The adjacency matrix composed of 20 agents refers to

	1	0	0	0	1	1	0	0	1	0	0
		0	0	0	0	0	1	1	0	1	1
		0	0	0	0	0	1	1	1	0	1
		1	0	0	0	0	1	1	0	1	0
		1	0	0	0	0	0	1	1	1	1
		0	1	1	1	0	0	0	1	1	0
		0	1	1	1	1	0	0	0	0	1
		1	0	1	0	1	1	0	0	1	0
		0	1	0	1	1	1	0	1	0	0
1 _		0	1	1	0	1	0	1	0	0	0
1 —		0	0	0	1	1	0	0	1	0	0
		0	0	0	0	0	1	1	0	1	1
		0	0	0	0	0	1	1	1	0	1
		1	0	0	0	0	1	1	0	1	0
		1	0	0	0	0	0	1	1	1	1
		0	1	1	1	0	0	0	1	1	0
		0	1	1	1	1	0	0	0	0	1
		1	0	1	0	1	1	0	0	1	0
		0	1	0	1	1	1	0	1	0	0
	l	0	1	1	0	1	0	1	0	0	0
	0	0	0	1	1	0	0	1	0	0	
	0	0	0	0	0	1	1	0	1	1	
	Δ	~	0	Δ	Δ	1		-			
	U	0	0	0	0	1	1	1	0	1	
	1	0 0	0	0	0	1	1 1	1 0	0 1	1 0	
	1 1	0 0 0	0 0 0	0 0 0	0 0 0	1 1 0	1 1 1	1 0 1	0 1 1	1 0 1	
	1 1 0	0 0 0 1	0 0 1	0 0 1	0 0 0 0	1 1 0 0	1 1 1 0	1 0 1 1	0 1 1 1	1 0 1 0	
	1 1 0 0	0 0 1 1	0 0 1 1	0 0 1 1	0 0 0 1	1 0 0 0	1 1 1 0 0	1 0 1 1 0	0 1 1 1 0	1 0 1 0 1	
	1 1 0 0 1	0 0 1 1 0	0 0 1 1 1	0 0 1 1 0	0 0 0 1 1	1 0 0 0 1	1 1 1 0 0 0	1 0 1 1 0 0	0 1 1 1 0 1	1 0 1 0 1 0	
	1 1 0 0 1 0	0 0 1 1 0 1	0 0 1 1 1 0	0 0 1 1 0 1	0 0 0 1 1 1	1 0 0 0 1 1	1 1 0 0 0 0	1 0 1 1 0 0 1	0 1 1 1 0 1 0	1 0 1 0 1 0 0	
	1 1 0 0 1 0 0	0 0 1 1 0 1 1	0 0 1 1 1 0 1	0 0 1 1 0 1 0	0 0 0 1 1 1 1	1 0 0 1 1 0	1 1 0 0 0 0 1	1 0 1 1 0 0 1 0	0 1 1 0 1 0 0 0	1 0 1 0 1 0 0 0	
	1 1 0 0 1 0 0 0 0	0 0 1 1 0 1 1 0	0 0 1 1 1 0 1 0	0 0 1 1 0 1 0 1 0	0 0 0 1 1 1 1 1 1	1 0 0 1 1 0 0	1 1 0 0 0 0 1 0	1 0 1 1 0 0 1 0 1 0	0 1 1 0 1 0 0 0 0	1 0 1 0 1 0 0 0 0	
	1 1 0 0 1 0 0 0 0 0	0 0 1 1 0 1 1 0 0 0	0 0 1 1 1 0 1 0 0 0	0 0 1 1 0 1 0 1 0	0 0 0 1 1 1 1 1 1 0	1 0 0 1 1 0 0 1	1 1 0 0 0 0 1 0 1	1 0 1 1 0 0 1 0 1 0	0 1 1 0 1 0 0 0 1	1 0 1 0 1 0 0 0 0 1	
	1 1 0 0 1 0 0 0 0 0 0 0	0 0 1 1 0 1 1 0 0 0 0	0 0 1 1 1 0 1 0 0 0 0	0 0 1 1 0 1 0 1 0 0 0	0 0 1 1 1 1 1 0 0	1 0 0 1 1 0 0 1 1 1	1 1 1 0 0 0 1 0 1 1	1 0 1 1 0 0 1 0 1 0 1 0 1	0 1 1 0 1 0 0 0 1 0	1 0 1 0 0 0 0 0 1 1	
	1 1 0 0 1 0 0 0 0 0 0 1	0 0 1 1 0 1 1 0 0 0 0 0	0 0 1 1 1 1 0 0 0 0 0	0 0 1 1 0 1 0 1 0 0 0 0	0 0 0 1 1 1 1 1 1 0 0 0	1 1 0 0 0 1 1 0 0 1 1 1 1	1 1 1 0 0 0 0 1 0 1 1 1 1	1 0 1 1 0 0 1 0 1 0 1 0 1 0	0 1 1 1 0 1 0 0 0 1 0 1	1 0 1 0 0 0 0 0 1 1 0	
	1 1 0 0 1 0 0 0 0 0 0 1 1	0 0 1 1 0 1 1 0 0 0 0 0 0 0	0 0 1 1 1 1 0 0 0 0 0 0 0	0 0 1 1 0 1 0 1 0 0 0 0 0	0 0 0 1 1 1 1 1 1 0 0 0 0 0	1 1 0 0 1 1 1 0 0 1 1 1 1 0	1 1 1 0 0 0 0 0 1 0 1 1 1 1 1	1 0 1 1 0 0 1 0 1 0 1 0 1	0 1 1 1 0 1 0 0 0 1 0 1 1 1	1 0 1 0 0 0 0 0 1 1 0 1	
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Figure 1 shows the initial state of 10 agents. Figure 2 indicates the initial state of 20 agents. The direction



Figure 1. The beginning of N = 10 agent's state.





Figure 3. The final state of N = 10 agent's state.



Figure 4. The final state of N = 20 agent's state.

Figure 2. The beginning of N = 20 agent's state.

of the arrow denotes the movement direction of the agents. It can be obtained from Figure 1 that 10 agents are evenly distributed on a straight line, and the initial movement direction of the agents is inconsistent. Moreover, it can be observed from Figure 2 that 20 agents are evenly distributed on a circle with a radius of 1, and the initial movement direction of the agents is inconsistent as well.

Figure 3 displays the final state of 10 agents when the system is acted by the controller of high-frequency feedback robust control with unknown parameters. It is observed that the velocity directions of the 10 agents are consistent and no collision occurs between the agents. Thus, it is able to form high-frequency feedback robust control for flocking of a multi-agent with unknown parameters. Figure 4 shows the final state of 20 agents when the system is acted by the controller of high-frequency feedback robust control with unknown parameters. It is obtained that the velocity directions of the 20 agents conform to each other and no collision occurs between agents. Therefore, it can also form high-frequency feedback robust control for flocking of a multi-agent with unknown parameters. By comparing Figure 3 with Figure 4, it is easy for us to come to the

conclusion that flocking of a multi-agent system can be formed with different numbers of multi-agents under the action of high-frequency feedback robust control. In addition, we can also obtain the result that the flocking of a multi-agent system is able to be formed in different initial states under the action of high-frequency feedback robust control.

Figure 5 represents the identification of unknown parameters of 10 agents with an initial value of 0, and the unknown parameters of $\theta_i = 10$ can be identified. Figure 6 indicates the identification of unknown parameters of 20 agents with an initial value of 0, and the unknown parameters of $\theta_i = 10$ can be identified. Figure 7 shows the identification of unknown parameters of 10 agents with an initial value of 0, and the unknown parameters of $\theta_i = 10$ can be identified in the process. Figure 8 displays the identification of unknown parameters of 20 agents with an initial value of 2, and the unknown parameters of $\theta_i = 10$ can be identified. From the comparison between Figures 5 and 7, as well as between Figures 6 and 8, it can be obtained that the parameters with different initial values can arrive at 10 when the number of agents is the same. Moreover, the convergence speed and stability obtained from them are also the same when the quantity of agents



Figure 5. Identification of unknown parameters of 10 agents with an initial value of 0.



Figure 6. Identification of unknown parameters of 20 agents with an initial value of 0.

is the same. From the comparison between Figures 5 and 6, as well as between Figures 7 and 8, it can also be obtained that the unknown parameters can reach 10 in multi-agents with different numbers and different initial states. In addition, unknown parameters in different numbers of multi-agents and different initial states of multi-agents with the same convergence speed can also be obtained by us. It is also concluded that unknown parameters can reach 10 in different numbers of multiagents with the same stability.

Figure 9 represents the velocity error of flocking with 10 agents. Figure 11 is known as a local magnification of Figure 9. Figure 10 shows the velocity error of flocking with 20 agents. Figure 12 is known as a local magnification of Figure 9. By comparing Figure 9 with Figure 10, it can be obtained that in terms of different numbers of multi-agent and multi-agent with different states, the velocity error of those multi-agents has the same fluctuation range at zero point and the convergence speed is the same under the action of high-frequency feedback robust control. By comparing Figure 11 with Figure 12,



Figure 7. Identification of unknown parameters of 10 agents with an initial value of 2.



Figure 8. Identification of unknown parameters of 20 agents with an initial value of 2.



Figure 9. Velocity error of flocking with 10 agents.

it can be observed that the speed error of a multiagent with different numbers and different initial states is bounded, and the error width of different numbers of multi-agent and multi-agent with different states is the same under the action of high-frequency feedback robust control.



Figure 10. Velocity error of flocking with 20 agents.



Figure 11. Partially enlarged view of velocity error of flocking with 10 agents.



Figure 12. Partially enlarged view of velocity error of flocking with 20 agents.

6. Conclusions

In this paper, high-frequency feedback robust control for flocking of a multi-agent with unknown parameters is obtained. The velocity error of a multi-agent is bounded, and no collision occurs between agents. Results obtained from simulation helps verify the highfrequency feedback robust control for flocking of a multi-agent with unknown parameters. Then the state diagram, the velocity error diagram and the parameter change diagram of the high-frequency feedback robust control for flocking of a multi-agent with unknown parameters are obtained. Under the action of highfrequency feedback robust control, the number of multi-agents and the initial state of multi-agents will not impose any effect on the formation of the flocking of a multi-agent system. Changes in the initial values of unknown parameters also do not have any effect on the identification of parameters in the system.

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