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Raúl Rascón

Facultad de ingeniería, Universidad Autónoma de Baja California, Mexicali, México

ABSTRACT

In this paper, it is proposed a control structure to solve the tracking problem in a class of uncertain mechanical systems. It is considered that the system is affected by unknown disturbances, discontinuous friction and uncertainties. The proposed control algorithm is based on the twisting control algorithm plus a nested signum term, moreover a disturbance estimator is used as feedback to the controller in order to compensate the non modelled parameters and uncertainties of the plant, also a velocity observer is proposed. Through the usage of Lyapunov tools, it is shown that the closed-loop nonlinear system is globally asymptotically stable and achieves zero steady-state position error, also, it is shown that while being asymptotically stable and homogeneous of degree $q < 0$, these systems approach the equilibrium point in finite time. Numerical simulations and real-time experiments carried out in a mass-spring-damper system show the performance and effectiveness of the control structure.

1. Introduction

The study of mechanical systems under uncertainties for a long time has attracted the interest of researchers. In order to design controllers with better performance and robustness against uncertainties and parametric variation, as well as to achieve a faster convergence time to the reference using a control law with the lowest amplitude possible; the aforementioned control objectives are difficult to improve in real systems performance.

Control of mechanical systems under uncertainties involve different control methodologies such: sliding mode control, fuzzy control or adaptive control. In [1] a novel nonlinear disturbance observer-based sliding mode control approach has been proposed to attenuate uncertainties. However, only numerical simulations have been performed using this approach. In [2] an adaptive sliding-mode controller for the Takagi-Sugeno (T–S) fuzzy system with mismatched uncertainties and exogenous disturbances is designed. In [3,4] a linear matrix inequality (LMI)-based sliding surface design method for integral sliding-mode control of mismatched uncertain systems is proposed. The proposed controller is verified through computer simulations to show the effectiveness of the method. Also, [5] investigates the robust sliding mode control problem for a class of uncertain nonlinear stochastic systems with mixed time delays, a simulation example is given to demonstrate the effectiveness of the proposed scheme. In [6,7] a composite nonlinear feedback method for robust tracking control of uncertain

linear systems with time-varying delays and disturbances is proposed. In [8] an adaptive controller for trajectory control of a class of mechanical systems with unbounded and fast-varying uncertainties is presented, although the algorithm renders good performance, the tracking errors are not guaranteed to converge to zero.

The main goal of this paper is robust control of a class of mechanical systems with uncertainties, where the uncertainties and parametric variations are bounded. It is assumed, that mechanical system has either prismatic or translational links. The proposed control algorithm uses a disturbance estimator and a velocity observer. The integration of these elements in a closed-loop system constitute a robust control structure. The stability of each element and the stability of the control structure is proved using Lyapunov tools.

The proposed controller is based on the twisting algorithm which was one of the first second-order sliding mode controllers presented in the literature [9–12]. The proposed controller can guarantee finite time convergence of the trajectories to the reference and absorb uncertainties and unknown disturbances which necessarily need to be bounded, although this boundary can change trough the time by using self-tuned gains. It is important to note that the amplitude of the discontinuous terms of the controller need to be greater than the sum of all the uncertainties upper bounds, due to this high frequency oscillations that are present in the control signal. Oscillations can be diminished by the usage of the output of the disturbance filter as compensation in the closed-loop system. Other way to mitigate the

CONTACT Raúl Rascón ✉ raul.rascon@uabc.edu.mx 🖾 Facultad de ingeniería, Universidad Autónoma de Baja California, Blvd. Benito Juárez y Calle de la Normal S/N, 21280, Mexicali, Baja California, México

high oscillations in the control signal is using self-tuned gain parameters in the controller, which depends in part on the amplitude of the disturbances present in the closed-loop system. Sliding mode twisting controllers are one of the best choices among other high order sliding mode controllers for the stabilization of nonlinear systems under disturbances or uncertainties [13,14], some previous works about the twisting algorithm can be found in the seminal work of [10], and more recently in [12] and [11]. The proposed controller has a nested signum function which adds robustness to the aforementioned twisting algorithm. A previous work using a nested signum function as a control input to solve the robust tracking and cruise control of a class of robotic systems is presented in [15].

Sliding mode control is the control methodology used in this work, and it constitutes a variable structure control method [16,17]. Some other previous works about mechanical systems under uncertainties using sliding mode control can be found in [18], which addressed force feedback control via position measurement in a spring who collides against a wall. In [19] is addressed the regulation problem in an underactuated mechanical system with an elastic clearance using a sliding mode control with an H_∞ term within the sliding surface, in order to attenuate unmatched perturbations.

The main contribution of the present paper is the control structure proposed, which renders an improved performance through the usage of a disturbance estimator, and a velocity observer along with the control algorithm. Global stability around the equilibria is proved through a strict Lyapunov function; moreover, based on the results of [20], it is shown that the control algorithm while being asymptotically stable and homogeneous of degree $q < 0$ approach the equilibrium point in finite time. It is considered that some parameters of the plant are not perfectly known. Furthermore, it is not necessary to know their upper bounds when the disturbance estimator is used in the control design. Additionally a velocity observer is proposed, which is robust to bounded parameter uncertainties. The validation of the proposed control structure is carried out by means of numerical simulations performed in MATLAB. Furthermore, real-time experiments are made in a mass-spring-damper system to solve the trajectory tracking problem.

The rest of the paper is organized as follows: In Section 2 the statement of the problem in a class of mechanical systems with either rotational or translational links is described. The control design is presented in Section 3. The global asymptotic stability analysis using a strict Lyapunov function, and the global equiuniform finite time stability of the controller using the concept of homogeneity is also presented in the same Section. A velocity observer design and its stability proof is presented in Section 4. In Section 5 is presented

a second order low-pass filter in order to obtain the disturbances affecting the system. In Section 6 the output feedback synthesis considering the controller, the velocity observer, and the disturbance estimator as a whole system is presented. The stability analysis is offered using a Lyapunov function. In Section 7 a comparison is made to twisting controllers [10,21] and the present approach in numerical simulations. Section 8 presents real-time experimental results to solve the tracking problem in a mass-spring-damper platform, from Educational Control Products (ECP-210). Section 9 includes some conclusions. Finally, Section 10 presents some directions to further and improve the work.

2. Statement of the problem

The main concern of this paper is to propose a control structure and its stability proof to solve the regulation and tracking problems in a class of mechanical systems. The system is considered to have uncertainties, besides of unknown disturbances, both uncertainties and disturbances at this stage are considered bounded.

Consider a mechanical system represented by

$$\ddot{x} = c^{-1}\left(-ax - b\dot{x} - f(x,\dot{x}) - \alpha\,\mathrm{sign}(\dot{x}) + \tau + w(t)\right) \tag{1}$$

where $x(t), \dot{x}(t) \in \mathbb{R}$ are the position and velocity of the body, respectively, a, b, c are constants which are different from zero, $f(x,\dot{x})$ is a nonlinear function, $u \in \mathbb{R}$ is the control input. To account for discrepancies in the model, a not completely known Coulomb friction coefficient α has been introduced, such that $0 < \alpha < M$, for some known bound M, also, $w(t)$ is a non completely known non-vanishing disturbance bounded by an upper bound so it satisfies $\sup_t |w(t)| \le N$.

Moreover, the sign is the signum function defined in [22,23] as

$$\mathrm{sign}(\dot{x}) = \begin{cases} 1 & \dot{x} > 0 \\ [-1,1] & \dot{x} = 0 \\ -1 & \dot{x} < 0 \end{cases} \tag{2}$$

since the right hand side of the Equation (1) has discontinuous terms, the solutions of system (1) are understood in the Filippov sense (see [11]). For system (1) the following controller design is proposed

$$\tau = \widetilde{f}(x,\dot{x}) + u. \tag{3}$$

where $\widetilde{f}(x,\dot{x}) = f(x,\dot{x}) + \Delta f(x,\dot{x})$ is an approximate compensation term for the nonlinear function $f(x,\dot{x})$ and Δf represents the error between f and $\widetilde{f}$ which is considered upper bounded by a constant P. Let us denote by x_1 the position x and by x_2 the velocity $\dot{x}$. Then, for a constant force input $u = \bar{u}$ and zero disturbance ($w = 0$), the system (1) has the following equilibrium points:
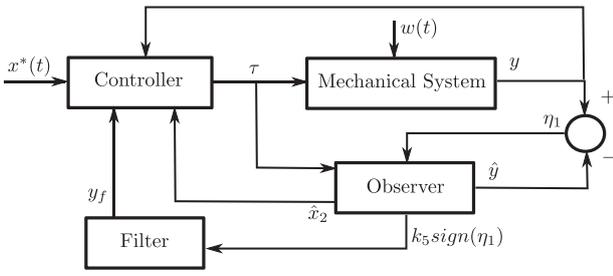
Figure 1. A block diagram of the control structure with disturbance identification.

(1) If $\bar{u} \geq \alpha$, then $\bar{x}_2 = 0$ and $\bar{x}_1 \in [\Delta f/a, (\Delta f + \bar{u} + \alpha)/a)$.

(2) If $|\bar{u}| < \alpha$, then $\bar{x}_2 = 0$ and $\bar{x}_1 \in ((\Delta f + \bar{u} - \alpha)/a, (\Delta f + \bar{u} + \alpha)/a)$.

(3) If $\bar{u} \leq -\alpha$, then $\bar{x}_2 = 0$ and $\bar{x}_1 \in ((\Delta f + \bar{u} - \alpha)/a, \Delta f/a]$.

If it is desired that the equilibrium point to be at the origin, in steady state, then it must be chosen that the force or torque $\bar{u}$ must be equal to the Coulomb friction $\alpha \text{sign}(x_2)$ and $\Delta f = 0$ otherwise if it is chosen $\bar{u}$ to be a constant, the equilibrium region of interest can be considered as $(\bar{x}_1 \in [(\Delta f + \bar{u} - \alpha)/a, (\Delta f + \bar{u} + \alpha)/a], \bar{x}_2 = 0)$.

The proposed control structure is shown in Figure 1. The plant is a second order mechanical system, given by (1). The other components include a nested twisting controller, a discontinuous observer, and a low-pass filter. These components are described in the next sections.

3. Control design

Let us suppose that the disturbance $w(t)$ affecting system (1) satisfies $\sup_t |w(t)| \leq N$, and the Coulomb friction coefficient is such that $0 < \alpha \leq M$, for some known bound M. The control objective is to find a control u, depending on the desired position or trajectory x^*, the generalized displacement x and velocity $\dot{x}$, such that the closed-loop response of system (1) satisfies

$$\lim_{t \to \infty} |x(t) - x^*| = 0. \tag{4}$$

Let us propose a variable structure controller, based on an array of signum functions plus compensation terms, to be applied on system (1). For this purpose, first let us shift the equilibrium point of (1) by defining the following transformation,

$$e_1 = x - x^*,$$
$$e_2 = \dot{x} - \dot{x}^*. \tag{5}$$

Then system (1), can be rewritten as

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = c^{-1}(-a(e_1 + x^*) - b(e_2 + \dot{x}^*)$$
$$- f(e_1 + x^*, e_2 + \dot{x}^*) - \alpha \text{sign}(e_2 + \dot{x}^*)$$
$$+ \tau + w(t)) - \ddot{x}^*. \tag{6}$$

A discontinuous control law with a nested signum function is now proposed for system (6), to the best of our knowledge this simple but functional controller has not been proposed elsewhere

$$\tau = \tilde{f}(e_1 + x^*, e_2 + \dot{x}^*) + u, \tag{7}$$

with

$$u = -k_1 \text{sign}(e_1) - k_2 \text{sign}(e_2) - k_3 \text{sign}(\text{sign}(e_1)$$
$$+ \text{sign}(e_2)) + ax + b\dot{x} + c\ddot{x}^* \tag{8}$$

the parameters k_1, k_2 and $k_3 > 0$ are tunable gains which ensure that the motion of the trajectories will be directed towards the desired trajectory. Necessary conditions for k_1, k_2, and k_3 are given in the following stability sections.

3.1. Stability analysis

The stability of the closed-loop system (6)–(8) will be analyzed in this section using Lyapunov tools, first of all substituting (7) and (8) into (6), the closed-loop system takes the form

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = c^{-1}\left(-\alpha \text{sign}(e_2 + \dot{x}^*) + w(t)\right.$$
$$- k_1 \text{sign}(e_1) - k_2 \text{sign}(e_2)$$
$$\left. - k_3 \text{sign}(\text{sign}(e_1) + \text{sign}(e_2)) + \Delta f\right). \tag{9}$$

An important remark is that $|\Delta f| \leq P$. Now let us find some sufficient conditions to show asymptotic stability, using classical Lyapunov technique. A Lyapunov candidate function for the system is given by

$$V(e_1, e_2) = (k_1 + k_3)|e_1| + \frac{c}{2}e_2^2. \tag{10}$$

In order to keep (10) positive-definite the following condition must be kept $k_1 + k_3 > 0$. Using the properties $\text{sign}(\text{sign}(x)) = \text{sign}(x)$ and $-\text{sign}(x + y) \leq -(\text{sign}(x) + \text{sign}(y) - 1)$ and looking back on the upper bounds $0 < \alpha < M$ and $\sup_t |w(t)| \leq N$, the time derivative of (10) along the solutions of (9) is
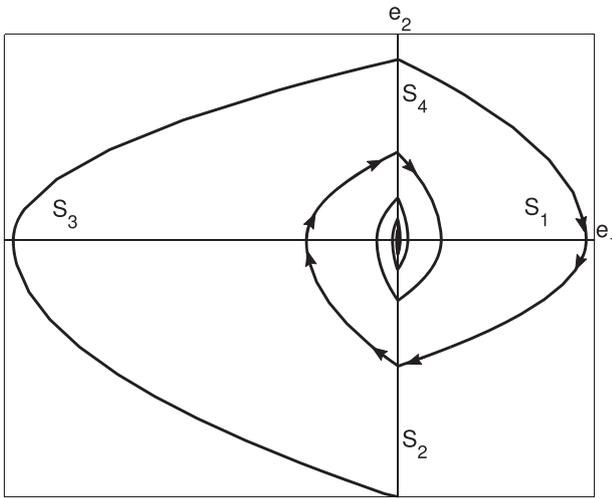
Figure 2. Phase portrait of the second order switched system (9).

given as follows

$$\dot{V}(e_1, e_2) \leq -\left(k_2 - (M + N + P)\right)|e_2| \leq 0 \quad (11)$$

the derivative of $V(e_1, e_2)$ is negative semidefinite under the condition

$$k_2 > M + N + P \quad (12)$$

by this way in the uncertain system (9), the trajectories cross the switching lines $e_1 = 0$ and $e_2 = 0$ everywhere except the origin $e_1 = e_2 = 0$ so that all the system trajectories are uniquely continuable on the right. Hence, the extended version [24,25] of Krasovskii-LaSalle's invariance principle [26–28] is applicable to the switched system (9). Since the equilibrium point $e_1 = e_2 = 0$ is the only trajectory of (9) on the invariance manifold $e_2 = 0$ where $\dot{V}(e_1, e_2) = 0$, this system is globally uniformly asymptotically stable by the aforementioned extension of the invariance principle.

Global equiuniform finite time stability. Let condition (12) be satisfied. Then the uncertain switched system (9) is globally equiuniformly finite time stable around the origin. The qualitative behaviour of system (9) is depicted in Figure 2. Due to the parameter subordination (12), the velocity vectors of (9) point toward the same region in the switching lines

$$S_1 = \{(e_1, e_2) \in \mathbb{R}^2 : e_1 > 0, e_2 = 0\},$$
$$S_2 = \{(e_1, e_2) \in \mathbb{R}^2 : e_1 = 0, e_2 < 0\},$$
$$S_3 = \{(e_1, e_2) \in \mathbb{R}^2 : e_1 < 0, e_2 = 0\},$$
$$S_4 = \{(e_1, e_2) \in \mathbb{R}^2 : e_1 = 0, e_2 > 0\}, \quad (13)$$

regardless of bounded uncertainties affecting the system. Hence, the uncertain system (9) rotate around the origin $e_1 = e_2 = 0$, while approaching the origin in a finite time. Thus, the uncertain system (9) exhibit chattering modes with an infinite number of switches on a finite time interval. These systems do not generate

sliding motions anywhere except the origin. If a trajectory starts there at any given finite time, there appears the so-called sliding mode of the second order (see [10,29,30]).

Due to the upper bounds M, N, and P the piece-wise continuous uncertainties $-\alpha \text{sign}(e_2 + \dot{x}^*) + w(t) + \Delta f$ are locally uniformly bounded, whereas the right-hand side of the nominal model

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = c^{-1}\left(-k_1\text{sign}(e_1) - k_2\text{sign}(e_2)\right.$$
$$\left. -k_3\text{sign}\left(\text{sign}(e_1) + \text{sign}(e_2)\right)\right), \quad (14)$$

is piece-wise continuous and globally homogeneous of degree $q = -1$ with respect to dilation $r = (2, 1)$. Hence, the condition $q + r_2 \leq 0$, required by Theorem 3.2 in [20], is satisfied, and Theorem 3.2 is applicable to the globally equiuniformly asymptotically stable uncertain system (9). By applying Theorem 3.2, the uncertain system (9) is thus globally equiuniformly finite time stable.

By direct integration of (14) the settling times are obtained, depending on the initial states $e_1(0)$ and $e_2(0)$, they are given by

$$e_1(t) = 0 \quad \text{for } t \geq \sqrt{\frac{2e_1(0)}{k_1 + k_2 + k_3}},$$
$$e_2(t) = 0 \quad \text{for } t \geq \frac{e_2(0)}{k_1 + k_2 + k_3} \quad (15)$$

note that the convergence time can be reduced through increasing the gain parameters k_1, k_2, and k_3.

4. Velocity observer design

This section explains a discontinuous velocity observer for the system (1), the following observer design is based on the previous works of [31,32]. Considering $x_1 = x$ and $x_2 = \dot{x}$ system (1) takes the following form

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = c^{-1}\left(-ax_1 - bx_2 - f(x_1, x_2)\right.$$
$$\left. - \alpha\text{sign}(x_2) + \tau + w(t)\right)$$
$$y = x_1 \quad (16)$$

where y is the output of the system and x_2 is not available for measurement. The terms $\alpha\text{sign}(x_2)$ and $w(t)$ are considered as disturbances which are also bounded by some known constants M and N.

The proposed discontinuous observer has the form

$$\dot{\hat{x}}_1 = \hat{x}_2 + k_4\eta_1$$
$$\dot{\hat{x}}_2 = c^{-1}\left(-a\hat{x}_1 - b\hat{x}_2 - \tilde{f}(x_1, \hat{x}_2) + \tau\right)$$
$$+ k_5\text{sign}(\eta_1) + k_6\eta_1 \quad (17)$$

the output of the observer $\hat{y} = [\hat{x}_1, \hat{x}_2]^T \in \mathcal{L}_\infty^2$ is bounded by a saturation function, this is made for stability purposes that will be clear in the following developments. The variables η_1 and η_2 stand for the errors, which are given by $\eta_1 = x_1 - \hat{x}_1$ and $\eta_2 = x_2 - \hat{x}_2$, the primary concern of the present discontinuous observer is the stability analysis of the following dynamical system in terms of the observed errors

$$\dot{\eta}_1 = \eta_2 - k_4 \eta_1$$
$$\dot{\eta}_2 = c^{-1}\left(-a\eta_1 - b\eta_2 + \Delta f - \alpha\operatorname{sign}(\eta_2 + \hat{x}_2)\right.$$
$$\left. + w(t)\right) - k_5\operatorname{sign}(\eta_1) - k_6\eta_1. \tag{18}$$

Let us consider that

$$\Psi(\cdot) = -\alpha\operatorname{sign}(\eta_2 + \hat{x}_2) + w(t) + \Delta f \le M + N + P.$$

Now lets make a change of variables $z_1 = \eta_1$ and $z_2 = \eta_2 - k_4\eta_1$. The dynamics of the system (18) in the new state space representation are given by

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = c^{-1}\left(-(a + bk_4)z_1 - bz_2 + \Psi(\cdot)\right)$$
$$- k_5\operatorname{sign}(z_1) - k_6 z_1 - k_4 z_2. \tag{19}$$

Considering the nominal version of (19) with $c^{-1}\Psi(\cdot) - k_5\operatorname{sign}(z_1) = 0$, the observer can be seen as a second order low pass filter; let us get the cut-off frequency in rad/sec of the observer which is given by

$$\omega_c = \left| \frac{-(c^{-1}b + k_4)}{\pm\sqrt{cb^2 + k_4^2 - 2c^{-1}bk_4 - 4ac^{-1} - 4k_6}} \right|. \tag{20}$$

As it can be seen from (20), the cut-off frequency ω_c depends on both, the parameters of the mechanical systems a, b, c and the observer gains k_4, k_6. By this way in order to modify the range of velocity that can be estimated by the observer, it is enough to tune the gains k_4 and k_6 according to (20).

4.1. Stability analysis

For stability purposes let us consider the following Lyapunov candidate function

$$V(z_1, z_2) = \frac{1}{2}\begin{bmatrix} z_1 & z_2 \end{bmatrix}\underbrace{\begin{bmatrix} \Phi & 1 \\ 1 & 1 \end{bmatrix}}_{P_1}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + k_5|z_1| \tag{21}$$

where $\Phi = k_6 + k_4 + c^{-1}(a + b(k_4 + 1))$. The function $V(z_1, z_2)$ remains positive-definite through keeping $\Phi > 1$. The time derivative of $V(z_1, z_2)$ along the

trajectories of the system (19) is given by

$$\dot{V}(z_1, z_2) = -\left(k_6 + \frac{a + bk_4}{c}\right)z_1^2 - k_5|z_1| + \frac{\Psi(\cdot)}{c}z_1$$
$$- \left(k_4 + \frac{b}{c} - 1\right)z_2^2 + \frac{\Psi(\cdot)}{c}z_2$$
$$\le -\left(k_6 + \frac{a + bk_4}{c}\right)z_1^2 - \left(k_5 - \frac{\Psi(\cdot)}{c}\right)|z_1|$$
$$- \left(k_4 + \frac{b}{c} - 1\right)|z_2|^2 + \frac{\Psi(\cdot)}{c}|z_2|$$
$$= -\left(k_6 + \frac{a + bk_4}{c}\right)z_1^2 - \left(k_5 - \frac{\Psi(\cdot)}{c}\right)|z_1|$$
$$- \left(1 - \theta\right)\rho|z_2|^2 - \theta\rho|z_2|^2 + \frac{\Psi(\cdot)}{c}|z_2|$$
$$\le -\left(k_6 + \frac{a + bk_4}{c}\right)z_1^2 - \left(k_5 - \frac{\Psi(\cdot)}{c}\right)|z_1|$$
$$- \left(1 - \theta\right)\rho|z_2|^2, \ \forall |z_2| \ge \Psi(\cdot)/c\theta\rho \tag{22}$$

where $0 < \theta < 1$ and $\rho = (k_4 + b/c - 1)$, while z_1 decays asymptotically to zero and z_2 remains globally uniformly ultimately bounded, according to Theorem 4.18 and Lemma 9.2 in [33] that bound is given by

$$\Gamma = \frac{\Psi(\cdot)}{c\theta\rho}\sqrt{\frac{\lambda_{\max}(P_1) + k_5}{\lambda_{\min}(P_1) + k_5}} \tag{23}$$

in the case where $\Psi(\cdot) = 0$, the set of equilibria (z_1, z_2) of (19) are uniformly asymptotically stable.

5. Filter

According to [34], the equivalent output injection or equivalent control u_{eq} coincides with the slow component of the discontinuous term in (18) when the state is in the discontinuity surface. The equivalent control provides measurement information about the unmeasured states that can continually move their estimates asymptotically closer to them, this includes uncertainties, disturbances and non well modelled parameters as those contained in $\Psi(\cdot)$. Thus, the equivalent control can be recovered using a low-pass filter with a frequency constant big enough as compared with the slow component response, yet sufficiently small to filter out the high-rate components, for further details see [35]. Given this background, let us propose to use a second order low-pass Butterworth filter to estimate the equivalent control u_{eq}. This filter is described by the following normalized transfer function:

$$\frac{Y(s)}{U(s)} = \frac{w_c^2}{s^2 + \sqrt{2}w_c s + w_c^2} \tag{24}$$

where w_c is the cut-off frequency of the filter. Here, the filter input is the discontinuous term of the observer, $k_5\operatorname{sign}(y - \hat{y})$. Denoting the output of the filter as

$y_f(t) \in \mathbb{R}$, and choosing a constant w_c that minimizes the phase-delay, it is possible to assume that

$$\lim_{t \to \infty} y_f = \tilde{\Psi}(\cdot) \approx \Psi(\cdot), \qquad (25)$$

where $\|\tilde{\Psi}(\cdot) - \Psi(\cdot)\| \le A_0$ for $A_0 \ll M + N + P$.

6. Output feedback synthesis

In this section, it is presented the synthesized control algorithm for the output feedback. For this purpose, let us use the controller (7) and (8) but now we will use as feedback the observed velocity $\hat{x}_2$ and the estimated disturbance y_f, given as follows

$$\tau = \tilde{f}(e_1 + x^*, \tilde{e}_2 + \dot{x}^*) + u, \qquad (26)$$

and

$$
\begin{aligned}
u = &-k_1 \text{sign}(e_1) - k_2 \text{sign}(\tilde{e}_2) \\
&- k_3 \text{sign}(\text{sign}(e_1) + \text{sign}(\tilde{e}_2)) \\
&+ ax + b\dot{x} + c\ddot{x}^* - y_f,
\end{aligned} \qquad (27)
$$

notice that $|\tilde{f}(e_1 + x^*, \tilde{e}_2 + \dot{x}^*) - f(e_1 + x^*, e_2 + \dot{x}^*)| \le L$ and $\tilde{e}_2 = \hat{x}_2 - \dot{x}^*$. Now, the gain parameter k_2 can be greatly reduced by using the output of the filter y_f, just keeping in mind that $k_2 > \|\tilde{\Psi}(\cdot) - \Psi(\cdot)\|$. This gain can help to reduce the amplitude of the high frequency components present in the control signal, which are generated by the *signum* functions. We must keep in mind that the new nominal output error of the filter is $\|\tilde{\Psi}(\cdot) - \Psi(\cdot)\| \le A_1$ for $A_1 \ll L + M + N$.

6.1. Stability analysis of the control structure: controller+observer+filter

The stability proof is developed using the Lyapunov function $V(e_1, \tilde{e}_2) = (k_1 + k_3)|e_1| + (c/2)\tilde{e}_2^2$. The time derivative of $V(e_1, \tilde{e}_2)$ along the trajectories of the closed-loop system (6), (26), and (27) is given by

$$\dot{V}(e_1, \tilde{e}_2) \le -\left(k_2 - A_1\right)|\tilde{e}_2| \le 0 \qquad (28)$$

while keeping $k_2 > A_1$ it can be ensured $\dot{V}(e_1, \tilde{e}_2)$ is a negative semidefinite function, it only remains to apply the extended version of Krasovskii-LaSalle's invariance principle to the system in question to ensure global asymptotic stability.

7. Numerical simulations

This section presents numerical simulations to solve the trajectory tracking problem in a double integrator system. The goal of the numerical simulations is to validate the afore developed control structure and its stability analysis in a closed loop system. For this purpose let us use the controller given in (26) and (27).

Table 1. Desired reference and tuning parameters of the control structure.

Description	Notation	Value
Desired trajectory	x^*	$1.0 \sin(t)$
Controller gain	k_1	1.1
Controller gain	k_2	1
Controller gain	k_3	0.1
Observer gain	k_4	20
Observer gain	k_5	2
Observer gain	k_6	5
Filter cut-off frequency	w_{c1}	6 rad/sec.

The parameters of the controller, observer, and filter are shown in Table 1. In the disturbed case, a disturbance of $1.2 \cos(0.1t)$ is applied to the system.

In order to test the performance and robustness properties of the proposed control structure a comparison has been made to twisting controller [10], where the control gain parameters are shown in Table 1

$$\tau = -k_1 \text{sign}(e_1) - k_2 \text{sign}(e_2) + \ddot{x}^*. \qquad (29)$$

The other controller considered in the comparison is a finite-time dynamic twisting controller recently presented in [21], the control gain parameters are also shown in Table 1

$$
\begin{aligned}
\tau &= -k_1 \text{sign}(e_1) - k_2 \text{sign}(e_2) - 0.5 \text{sign}(x_3) + \ddot{x}^* \\
\dot{x}_3 &= -2e_1 - e_2.
\end{aligned} \qquad (30)
$$

The comparison results can be seen from Figure 3 to Figure 4, for both, undisturbed and disturbed case. In Figure 3, in the undisturbed case (a), the closed-loop system response is quite similar using the proposed control algorithm and the twisting controller, although the convergence time to the reference using the proposed controller is a little bit faster due to the action of the nested signum gain. Moreover, the proposed controller by Luo and Su [21] presents a relatively large convergence time to the reference. The performance of the proposed controller is outstanding in the disturbed case (b); note that the proposed controller in [21] has a better closed-loop performance than the twisting controller, this happens because the amplitude of the disturbance is bigger than the k_1 gain parameter of each one of the three controllers.

In Figure 4, the phase portrait and identified disturbance are shown, for both cases: undisturbed and disturbed. In the phase portrait of the undisturbed case, it can be seen that the proposed controller in [21] does not drive the states of the system to the origin due to its slow time response. In the disturbed case the twisting controller does not drive the states of the system to the origin.

8. Real-time experiments

Performance issues and robustness properties of the proposed control structure have been tested with some
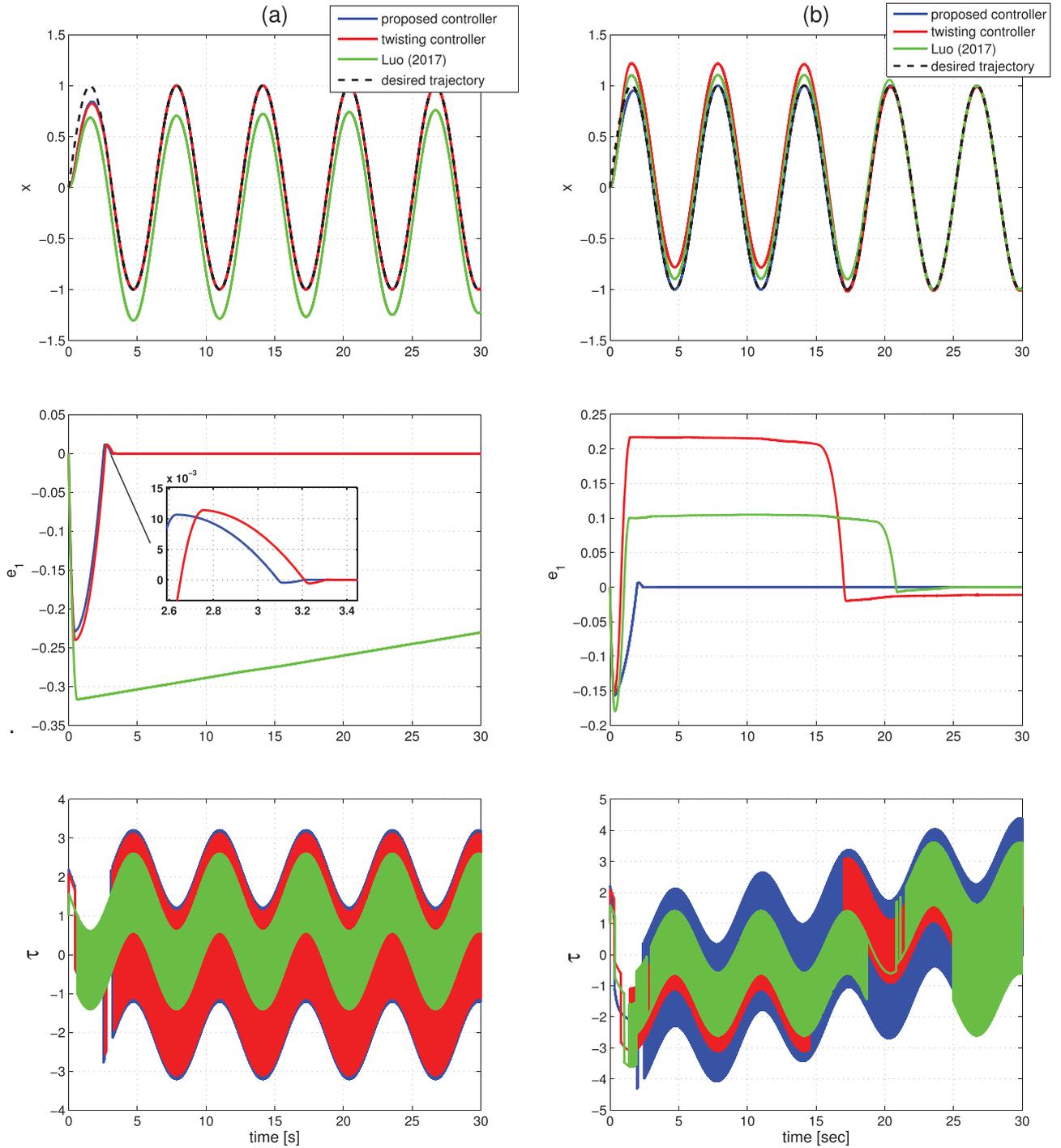
Figure 3. Trajectory tracking, tracking error, and control effort results, for both, (a) undisturbed and (b) disturbed case.

physical experiments performed in the platform ECP-210 see Figure 6. The experiments were implemented with a data acquisition board DSPACE®, running in a computer DELL® XPS 420 Quad-Core 2; the acquisition sampling rate was set to 0.0001 seconds.

8.1. Mass-spring-damper system

Now, let us consider the system shown in Figure 5, described by

$$\ddot{x} = m^{-1}\left(-kx - b\dot{x} - \alpha\mathrm{sign}(\dot{x}) + \tau + w(t)\right) \quad (31)$$

where m is the mass, k is the stiffness coefficient of the spring and b is the damper coefficient. The objective is to design a control τ so that the mass position x converges to a given trajectory or constant x^*. For that purpose the system can be rewritten in terms of the errors $e_1 = x - x^*$ and $e_2 = \dot{x} - \dot{x}^*$ as

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = m^{-1}\left(-k(e_1 + x^*) - b(e_2 + \dot{x}^*)\right.$$
$$\left. -\alpha\mathrm{sign}(e_2 + \dot{x}^*) + \tau + w(t)\right) - \ddot{x}^*. \quad (32)$$
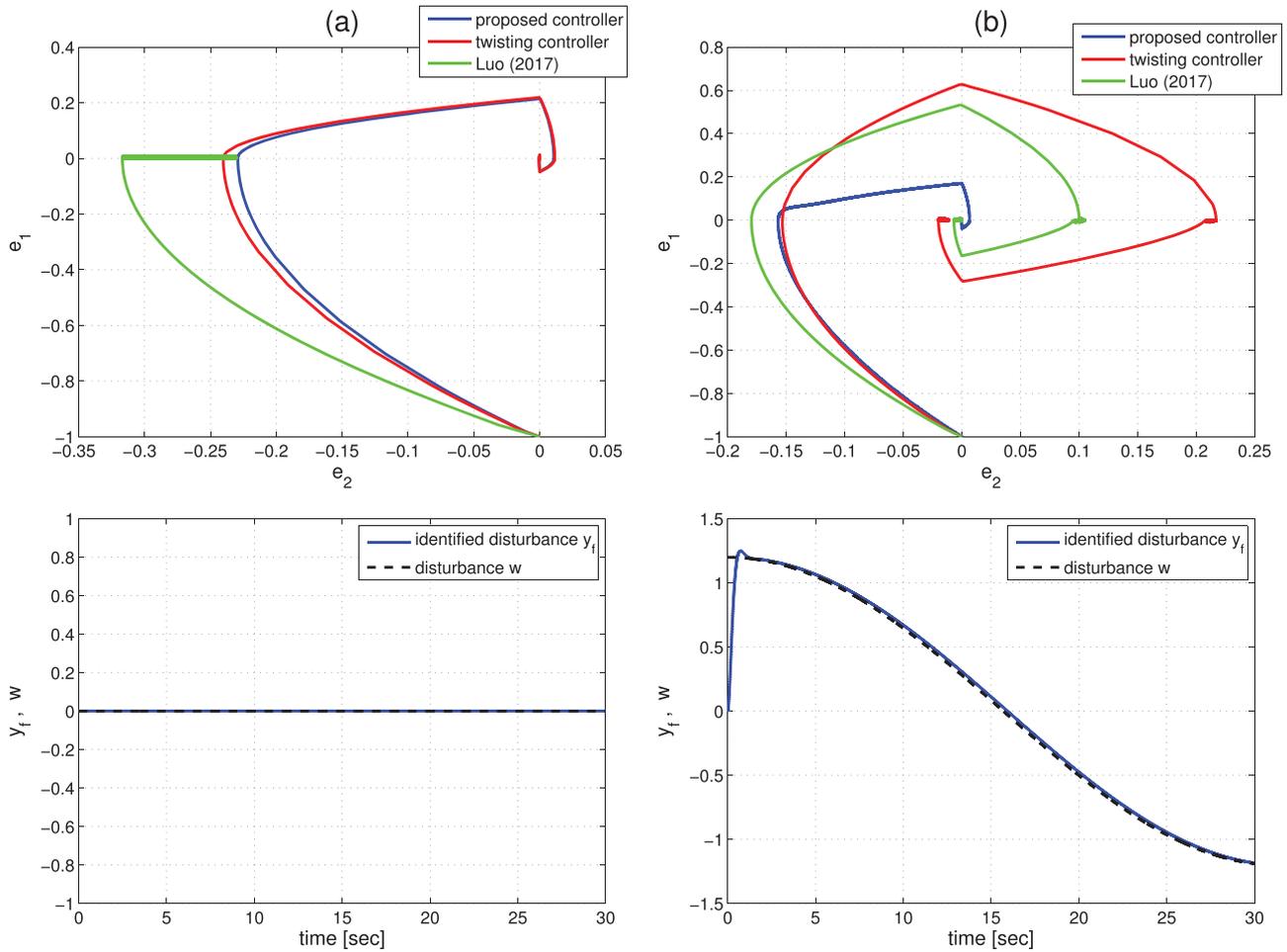
Figure 4. Phase portrait and disturbance, for both, (a) undisturbed and (b) disturbed case.
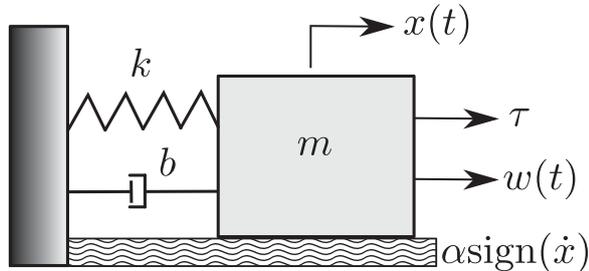

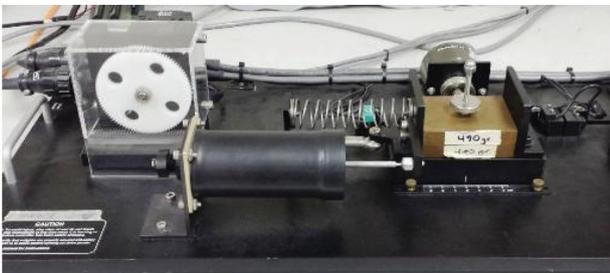**Figure 5.** Mass-spring-damper mechanical system.

**Figure 6.** Mass-spring-damper system, experimental platform.

where τ stands as follows

$$\tau = -k_1 \text{sign}(e_1) - k_2 \text{sign}(e_2) - k_3 \text{sign}(\text{sign}(e_1) + \text{sign}(e_2)) + kx + b\dot{x} + m\ddot{x}^* - y_f. \quad (33)$$

Table 2. Desired trajectory, plant parameters, controller and observer gains (Mass-spring-damper).

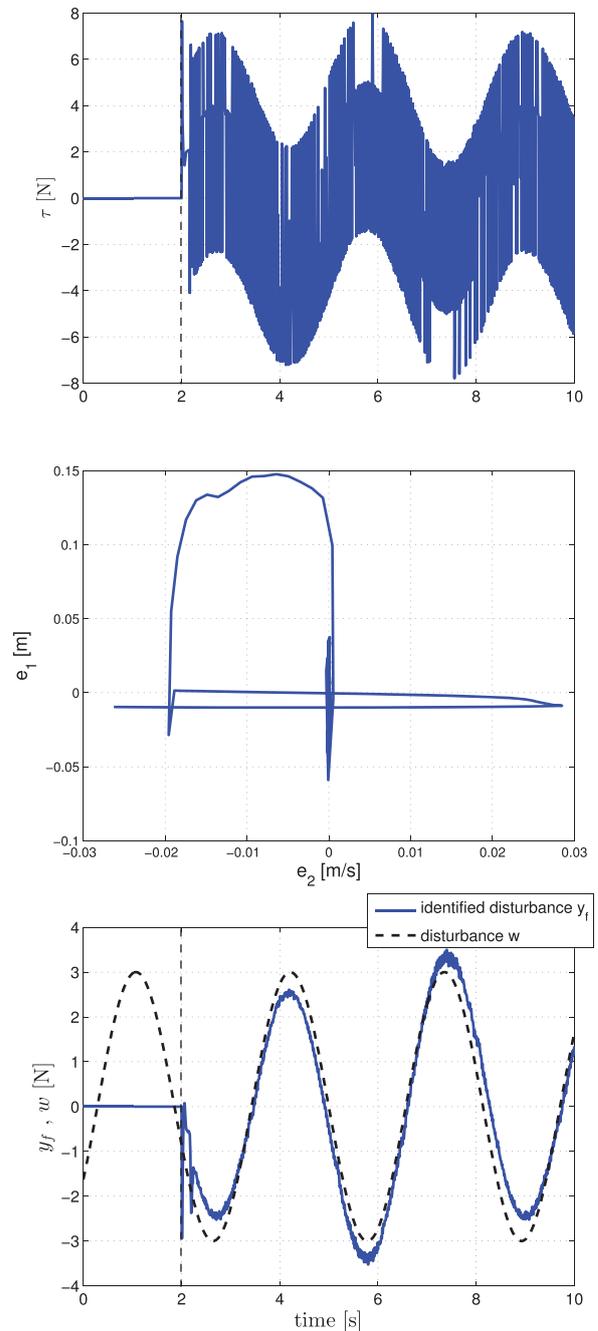| Description	Notation	Value
Desired trajectory	x^*	$1.0 \sin(t)$ cm.
Mass	m	1 kg
Damper coefficient	b	8 kg/s
Spring stiffness	k	127 N/m
Disturbance	$w(t)$	$3\sin(2t)$ N.
Controller gain	k_1	3 N.
Controller gain	k_2	1.5 N.
Controller gain	k_3	0.5 N.
Observer gain	k_4	600
Observer gain	k_5	5
Observer gain	k_6	150
Filter cut-off frequency	w_{c1}	50 rad/sec.

The experiments were performed under the parameters shown in Table 2. In Figure 7 and 8 are shown the results for trajectory tracking, in Figure 7 position, velocity from the observer, and position error are shown; the control signal is activated $t_0 = 2$ seconds after starting the experiment, this is made in order to have a better appreciation of the transient signal and the effect of the disturbance; the nominal stage is reached after 0.2 seconds approximately of being initialized the control signal. In Figure 8 can be seen the control signal, phase portrait, and the identified disturbance using the second order filter (24).
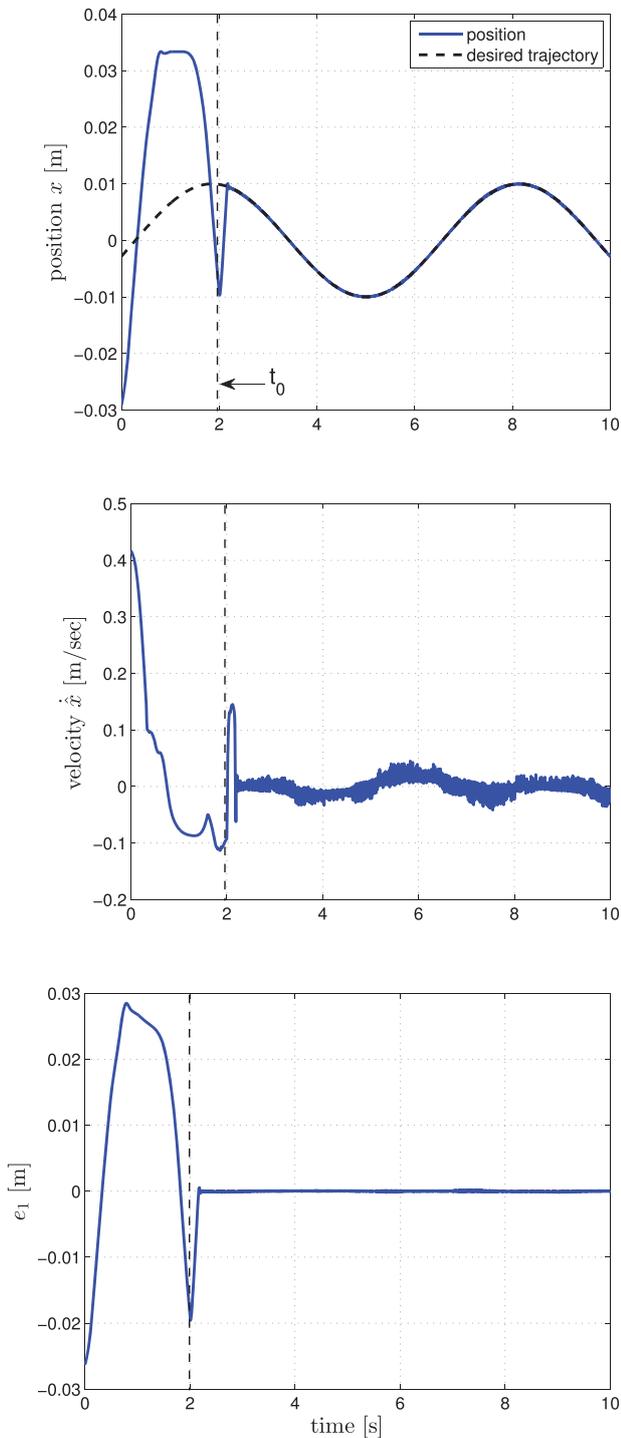
Figure 7. Position regulation, velocity from observer, and position error using the proposed control algorithm. The control signal is activated after $t_0 = 2$ seconds in order to appreciate the effect of the disturbance $w = 3\sin(2t)$ when there is no control action (Mass-spring-damper).

Figure 8. Control signal, phase portrait, and identified disturbance, using the proposed controller. The control signal is activated after the first 2 seconds of the experiment (Mass-spring-damper).

9. Conclusions

A robust control structure to solve the tracking problem is studied for a class of mechanical systems. These systems and their models are rather simple; however, they constitute a basic unit of variable structure systems, from which more complete and complex devices can be addressed. Moreover, they incorporate some real elements like not completely known Coulomb friction, disturbances and parametric variations. A controller, synthesized using a twisting control algorithm and adding a nested signum function, rendered a robust closed-loop system response. It was proved that the controlled system trajectories were globally asymptotically stable using a strict Lyapunov function, also was proved global equiuniform finite time stability using the concept of homogeneity.

High-frequency components in the control signal are typical of this kind of discontinuous controllers. However, the magnitude of high-frequency oscillations can be adjusted by setting an adequate value of the

controller coefficients k_1, k_2 and k_3, always keeping in mind that the parameters should be chosen to satisfy the inequalities $k_1 + k_3 > 0$ and $k_2 > \|\tilde{\Psi}(\cdot) - \Psi(\cdot)\|$. The chattering phenomenon is inherent to this type of controllers, it can be reduced by lowering the controller gain parameters although it is sacrificed the closed-loop system robustness and the convergence time to the reference increases; thus at the time of tuning the controller, it is necessary to prioritize between reducing the chattering or increasing the closed-loop robustness.

Moreover, a velocity observer design is proposed, which can be use in some cases where velocity measurements are not available. The control algorithm along with the observer and the disturbance estimator comprise the proposed control structure which it is considered the main contribution of the present control approach. Stability proofs using Lyapunov tools are given for the control algorithm, the observer and the whole control structure.

Numerical simulations in a double integrator system were carried out using MATLAB, moreover, real-time experiments were performed in a mass-spring-damper system. They showed good agreement and robustness performance against unknown disturbances, Coulomb friction and uncertainties.

10. Directions to further work

The future work lies on the improvement of the controller performance, while the robustness of the closed-loop system is maintained, this can be achieved through:

(1) Design of self-tuned controller gains.
(2) Continuous control signal or at least, high frequencies attenuation in the control signal.
(3) Avoidance of an observer by using only position measurements as feedback to solve the tracking problem.
(4) Achieve finite time convergence to the reference even in the presence of disturbances and uncertainties.

Future efforts will be aimed at achieving these objectives.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Raúl Rascón http://orcid.org/0000-0002-9968-3029

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