



Selective harmonic elimination method for unequal DC sources of multilevel inverters

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ABSTRACT

This paper presented an analytical and closed-form solution for the harmonic contents that were injected in the five-level inverters from asymmetrical DC sources or batteries which were suitable for renewable energy sources. In the five-level inverters, two DC sources V_1 and V_2 were used to synthesize the output voltage. Therefore, two transcendental equations were formulated for the fundamental and the third harmonic orders based on the unequal DC sources V_1 and V_2 . These transcendental equations were solved analytically for low switching control technique. The proposed solution included the modulation index change which resulted in changing the fundamental component. Therefore, two switching angles are formulated and solved analytically to control the amplitude of the fundamental voltage and cancel the third order harmonic. A well-known third order equation was generated. Thus, a closed-form solution for the above two switching angles will be generated in terms of the modulation index. The proposed solution was tested for a wide range of V_1/V_2 ratio. Some selected simulation and experimental results were provided to validate the proposed method for wide ranges of both modulation index and V_1/V_2 ratio.

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Multilevel inverters (MLIs); selective harmonic elimination (SHE); DC-AC power conversion; renewable energy sources; total harmonic distortion

1. Introduction

Multilevel inverters (MLIs) [1–4] are preferred compared to the conventional two-level H-bridge inverter for many reasons. MLIs are able to process high power at a low switching frequency, low switching frequency is an integer multiple of the fundamental frequency [5,6], and high output voltage resolution (low total harmonic distortion (THD)). Therefore MLIs have lower switches stresses with higher efficiency. One more important advantage of the MLIs is the modularity which enables MLIs to operate even if under faulty condition [7,8]. This can be done by modifying the control algorithm to bypass the faulty section without changing the MLI structure. There are three main MLIs categories (1) neutral point clamped MLI; (2) cascaded DC source inverter, (3) flying capacitor MLI inverter. MLIs can be divided into two main categories based on the DC sources available, symmetrical and asymmetrical MLIs. In symmetrical MLI, all DC sources are equal while in asymmetrical MLIs DC sources are not equal. The most common ratio among DC sources is 1:2 and 1:3. It can be noted that the 1:3 ratio can develop the highest output voltage resolution and thus lowest THD. Recently, hybrid MLI was introduced [9–12] to reduce

the number of switches used and thus increase their reliability. In this MLI type different structure with different DC sources ratio.

Lower order harmonics have the highest impact on the THD and the FFT analysis in the case of low switching frequency [5,6], thus they need to be removed. Either symmetrical or asymmetrical DC sources can be used for the MLI. However, MLI based asymmetrical DC sources are the most general and practical case as it generates a higher number of levels and so lower THD. In low switching control scheme, selective harmonic elimination (SHE) [13–15] technique is well-known in success to cancel lower order harmonics. A set of nonlinear equations is formulated to meet the targeted objective functions. Therefore, many articles solved these harmonic equations of the MLI using different algorithms. Some of these attempts use the well-known Newton Raphson technique, the theory of symmetric polynomials, genetic algorithms (GA) and artificial neural network (ANN) [16–20]. In these techniques, the solution of harmonic equations is done in the offline state using a computer, and then the solution of switching angles is stored in a lookup table which leads to low-resolution results. Also, the solution can be

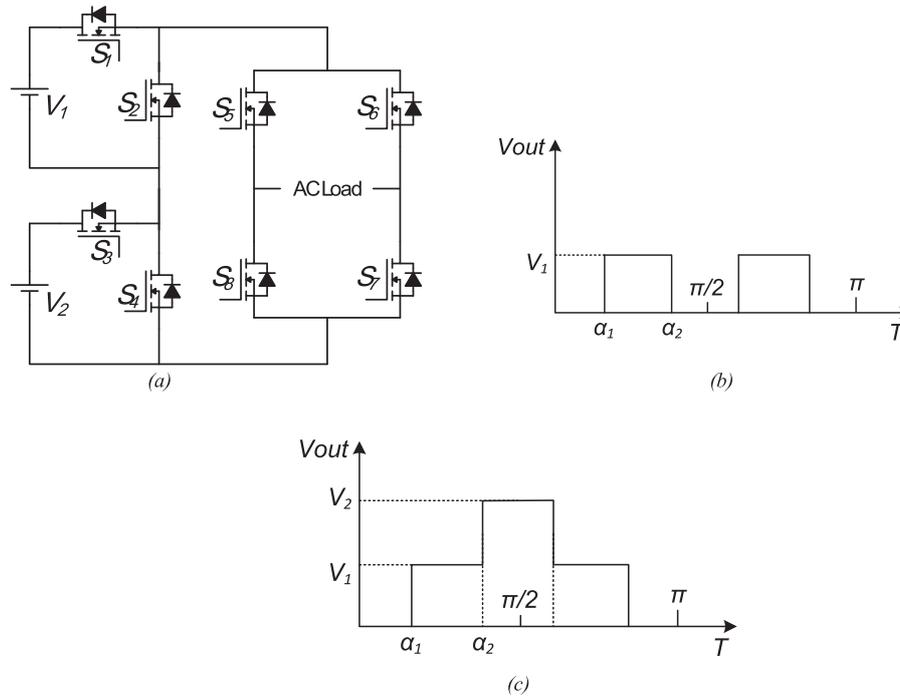


Figure 1. (a) Asymmetric cascaded single-phase inverter; (b) output waveform at low mi ; (c) output waveform at high mi with $V_1 \neq V_2$.

carried out in real time, but the solution is very complicated with a discontinuous range solution over the full modulation index range.

Other solutions are proposed in [21,22]. A simple closed-form analytical solution is proposed in [21] which is fast, accurate and it can guarantee harmonic cancellation as stipulated. Also in [22], a real-time solution was done using classical proportional–integral (PI) control. Both solutions proposed in [21,22] are limited only for MLI using symmetrical DC sources which is not the practical case. Therefore, this letter extends the solution proposed in [21] for MLI using asymmetrical DC sources. It is worth to note that asymmetrical DC sources can be resulted from designing different input DC sources with arbitrary functions or due to mismatch usually introduced by renewable energy sources due to environmental conditions such as partial shadow in Photovoltaic case [22].

2. Proposed system model

In this case study, the Fourier transforms for the well-known two switching angles waveform shown in Figure 1(b,c) respectively are given below:

$$V(n\omega) = \frac{4V_1}{n\pi} [\cos(n\alpha_1) - \cos(n\alpha_2)], \quad (1)$$

$$V(n\omega) = \frac{4}{n\pi} [V_1 \cos(n\alpha_1) + V_2 \cos(n\alpha_2)], \quad (2)$$

$$mi = \frac{h_1}{V_1 + V_2}, \quad (3)$$

where mi is the modulation index and h_1 is the fundamental amplitude.

It is worth noting that for the low modulation index (mi) as in (3), only a single DC V_1 source is utilized to generate a three-level inverter waveform as shown in Figure 1(a). Thus, there is no meaning for symmetrical or asymmetrical DC sources and solution of this system of (1) is given in [21]. In addition, when higher h_1 required, both DC sources V_1 and V_2 must be used, and the solution of (2) for symmetrical DC sources; i.e. $V_1 = V_2 = V_{dc}$, is also given in [21]. However, such a solution becomes inapplicable when V_1 and V_2 are not equal, as shown in Figure 1(b). Therefore, the target of this case study is to find a closed-form solution for the system defined by (2). The solution presented α_1 and α_2 that can fix the fundamental output voltage component and at the same time, it can cancel the lowest order harmonic, which is the third order harmonic. This can be driven from (2) and (3) as follow:

$$\begin{aligned} V_1 \cos(\alpha_1) + V_2 \cos(\alpha_2) &= k_1 = \frac{\pi}{4} h_1 \\ &= \frac{\pi}{4} (V_1 + V_2) mi, \end{aligned} \quad (4)$$

$$V_1 \cos(3\alpha_1) + V_2 \cos(3\alpha_2) = 0, \quad (5)$$

where $k_1 = (\pi/4)h_1$.

Using the cosine of the triple angle identity: $\cos(3\alpha_i) = -3 \cos(\alpha_i) + 4 \cos^3(\alpha_i)$, $i = 1, 2$. Then the third harmonic Equation (5) can be manipulated to become:

$$V_1 \cos(\alpha_1) + V_2 \cos(\alpha_2) = k_1 = \frac{\pi}{4} h_1, \quad (6)$$

$$V_1 \cos^3(\alpha_1) + V_2 \cos^3(\alpha_2) = \frac{3\pi}{16} h_1 = k_2. \quad (7)$$

From (6), $\cos(\alpha_1)$ can be expressed in terms of $\cos(\alpha_2)$

$$\cos(\alpha_1) = \frac{k_1}{V_1} - \frac{V_2}{V_1} \cos(\alpha_2). \tag{8}$$

Substituting (8) into (7) results in

$$V_1 \left[\frac{k_1^3}{V_1^3} - 3 \frac{k_1^2 V_2}{V_1^3} \cos(\alpha_2) + 3 \frac{k_1 V_2^2}{V_1^3} \cos^2(\alpha_2) - \frac{V_2^3}{V_1^3} \cos^3(\alpha_2) \right] + V_2 \cos^3(\alpha_2) = k_2. \tag{9}$$

This equation can be rearranged as follows:

$$\begin{aligned} & \left(V_2 - \frac{V_2^3}{V_1^2} \right) \cos^3(\alpha_2) + \left(3 \frac{k_1 V_2^2}{V_1^2} \right) \cos^2(\alpha_2) \\ & - \left(3 \frac{k_1^2 V_2}{V_1^2} \right) \cos(\alpha_2) + \frac{k_1^3}{V_1^2} - k_2 = 0 \end{aligned} \tag{10}$$

$$A \cos^3(\alpha_2) + B \cos^2(\alpha_2) + C \cos(\alpha_2) + D = 0, \tag{11}$$

where $A = (V_2 - V_2^3/V_1^2)$, $B = (3 k_1 V_2^2/V_1^2)$, $C = -(3 k_1^2 V_2/V_1^2)$, $D = k_1^3/V_1^2 - k_2$, $mi = h_1/(V_1 + V_2)$.

By solving (11), can be found, then can be calculated from (8). Equation (11) gives a closed-form expression in a third order polynomial, which can be solved directly by any digital controller. It calculates the optimum switching angles at high mi over a mismatch between V_1 and V_2 . Figure 2 illustrates the area where the solutions are feasible. It could be noticed that the proposed solutions depend on two main factors, they are mi which varies from 0.6 to 1.1 and ratio of (V_1/V_2) , which varies from 0.6 to 1.7. The resulted solutions are switching angles and α_2 , which varies from . By noticing Figure 2, it can be concluded that the proposed solution is valid for almost the whole ranges of mi (V_1/V_2) except very low scattered points (the white spot points in the curve). This is attributed to the solution at these white spot points do not satisfy ranges of switching angles. A representation of the areas where a satisfying solution exists is done by plotting percentage between V_1/V_2 and modulation index mi . The solution is almost continuous for V_1/V_2 range from 0.6 to 1.6. Note that for mi less than 0.5, the system is not 5-level and so there is no meaning for solving it. Figure 3(a,b) shows the different values of θ_1 and θ_2 respectively, based on the solution of (10) versus V_1/V_2 ratio at different mi , within the area defined by the rectangle in Figure 2.

3. Simulation results

A MATLAB/SIMULINK model has been built to check the validity of the proposed solution. The model is based on a single-phase cascaded full bridge multi-level inverter as shown in Figure 1(a). Figure 4 shows

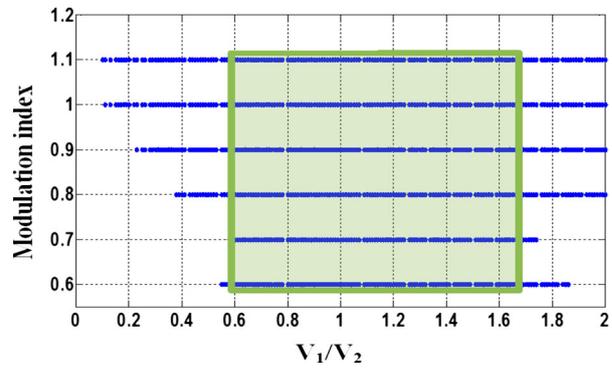


Figure 2. Modulation index (mi) versus the ratio V_1/V_2 .

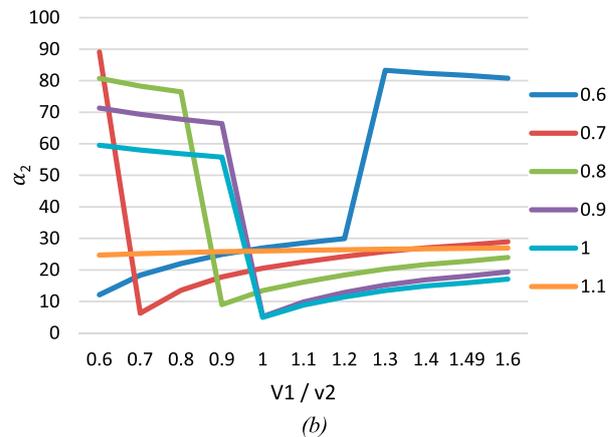
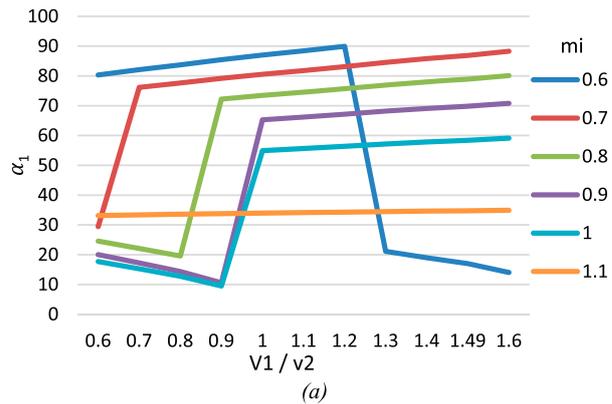


Figure 3. For $mi = 0.6$ to 1.1 , (a) α_1 versus V_1/V_2 (b) α_2 versus V_1/V_2 .

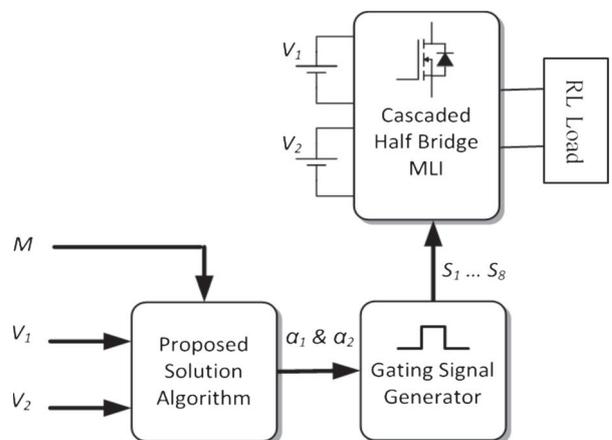


Figure 4. System block diagram.

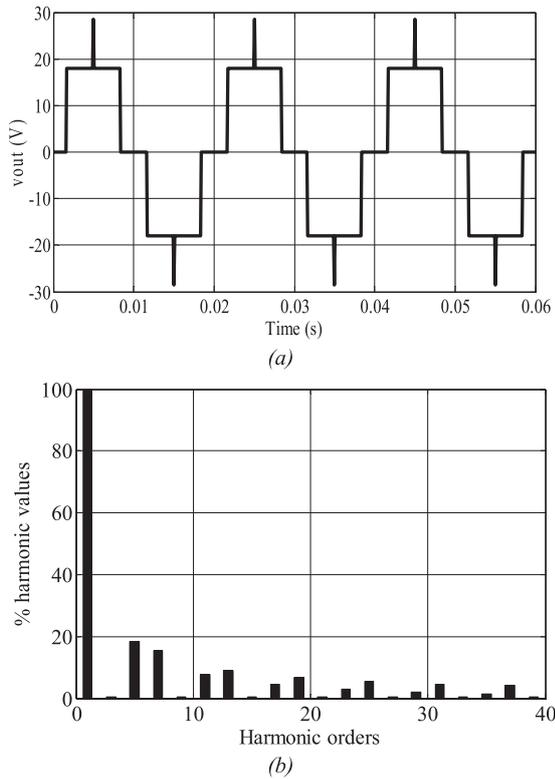


Figure 5. (a) Output voltage for $V_1 = 10.8\text{ V}$, $V_2 = 18\text{ V}$, $V_1/V_2 = 0.6$, $mi = 0.7$, $\alpha_1 = 29.48^\circ$ and $\alpha_2 = 89.13^\circ$, (b) output voltage spectrum.

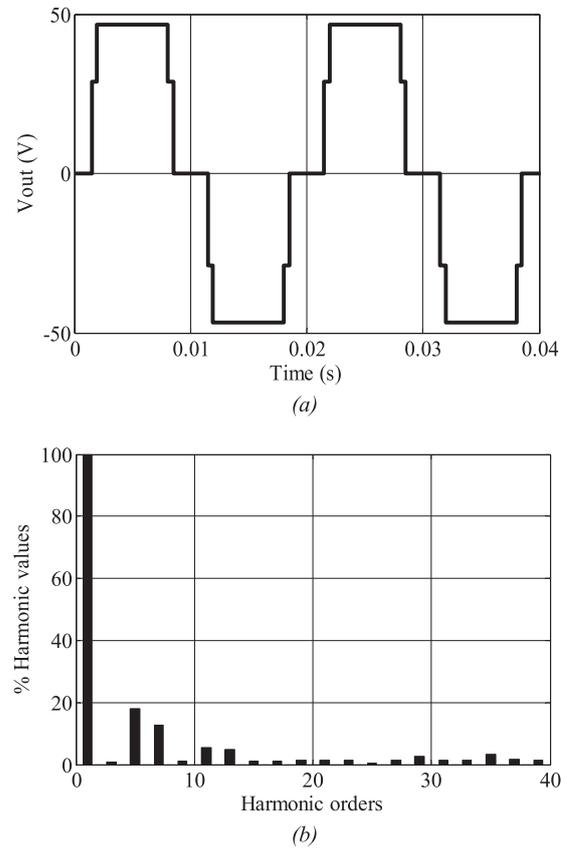


Figure 7. (a) Output voltage for $V_1 = 28.8\text{ V}$, $V_2 = 18\text{ V}$, $V_1/V_2 = 1.6$, $mi = 1.1$, $\alpha_1 = 26.94^\circ$ and $\alpha_2 = 34.92^\circ$, (b) output voltage spectrum.

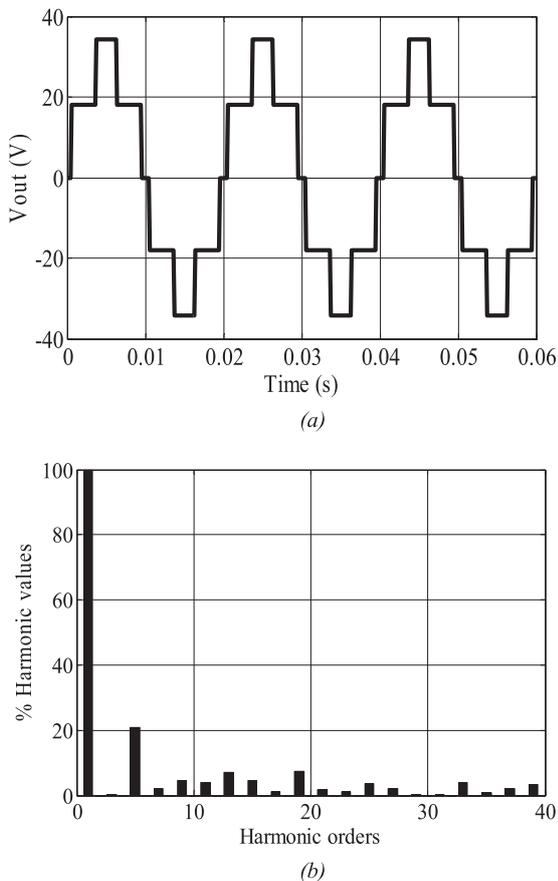


Figure 6. (a) Output voltage for $V_1 = 16.2\text{ V}$, $V_2 = 18\text{ V}$, $V_1/V_2 = 0.9$, $mi = 0.9$, $\alpha_1 = 10.61^\circ$ and $\alpha_2 = 66.41^\circ$, (b) output voltage spectrum.

the block diagram of the MATLAB/SIMULINK model, its gating signals are generated by a proposed solution that finds the optimal switching angles α_1 and α_2 to eliminate the third harmonic component, taking into consideration values V_1 , V_2 and modulation index mi . Two unequal DC sources V_1 and V_2 are connected to the inverter with ratio of V_1/V_2 . Both inverter's modulation index mi and DC voltage ratio can be changed. Based on the aforementioned analyses, mi can be changed from 0.6 to 1.1 while DC voltage sources ratio can be changed from 0.6 to 1.6. Three different cases have been selected to show and represent the proposed analyses. Figures 5–7 show these three different cases. Each figure includes output voltage time response and its harmonic spectrum. It is obvious that the third harmonic is cancelled in these three cases, in addition to; multiples of the third harmonics are cancelled too in some cases especially for mi around 1.

Figure 8(a–d) shows the absolute values for the fundamental voltage component and the harmonic contents for the fundamental frequency 50 Hz versus DC voltage ratio at $V_1 = 18\text{ V}$ for a wide range of modulation index. It proves the validity of the proposed solution by successfully eliminating the third harmonic content at every modulation index step change. Around modulation index equals 1, the third harmonic and its

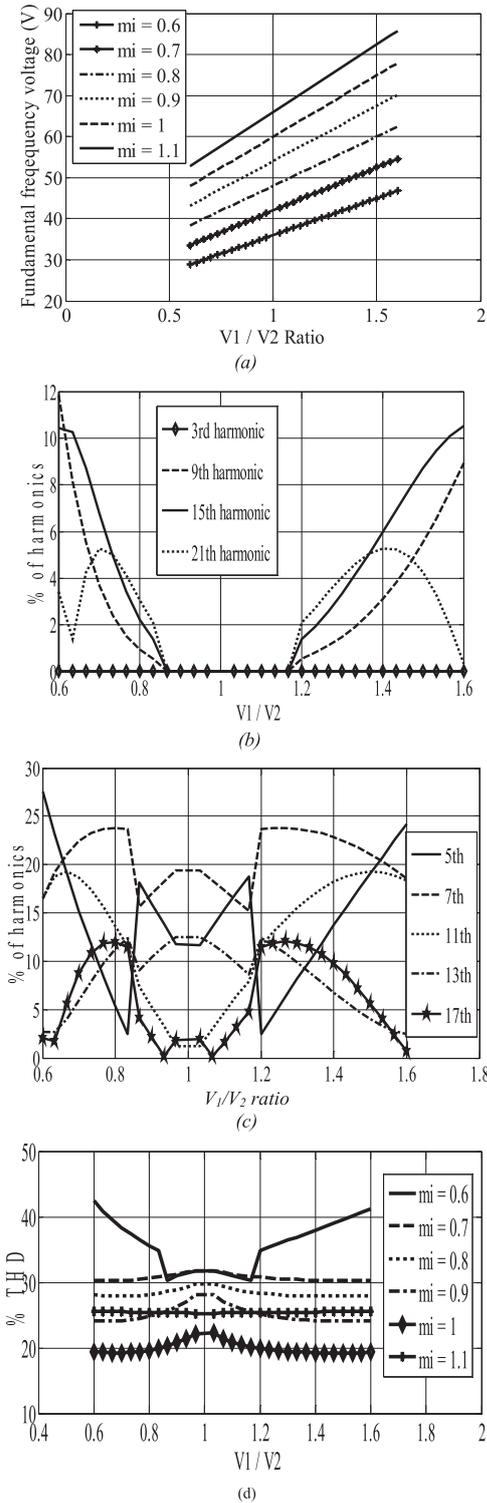
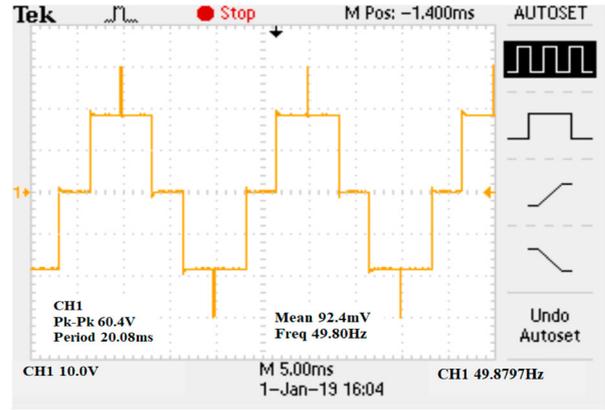


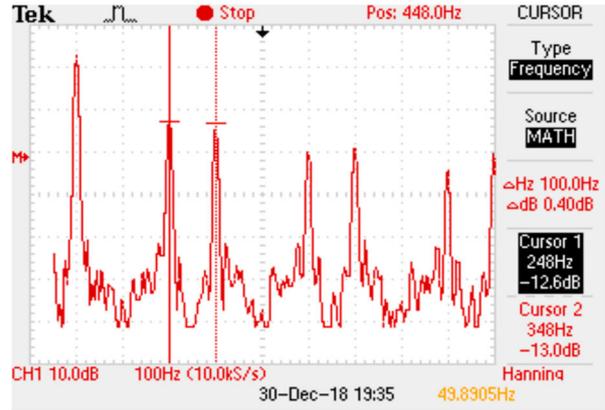
Figure 8. Harmonic analyses versus V_1/V_2 ratio for $V_1 = 18\text{V}$; (a) Value of fundamental component; (b) Values of third harmonic component and its multiples; (c) Other lower harmonic components. (d) THD versus V_1/V_2 ratio for different values of modulation index.

multiple are vanished leads to lower THD as shown in Figure 8(d).

Therefore, it is clear from the simulation results that the proposed solution outcomes matched with the theoretical analysis expectations within the predefined



(a)



(b)

Figure 9. (a) Output voltage for $V_1 = 10.8\text{V}$, $V_2 = 18\text{V}$, $V_1/V_2 = 0.6$, $mi = 0.7$, $\alpha_1 = 29.48$ and $\alpha_2 = 89.13^\circ$, (b) output voltage spectrum where cursor X1 at 250 Hz and cursor X2 at 350 Hz.

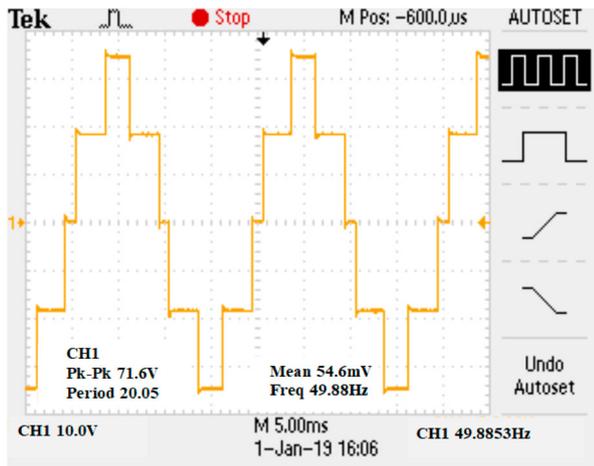
range of DC sources mismatch ratio V_1/V_2 versus different values of modulation index mi .

4. Experimental results

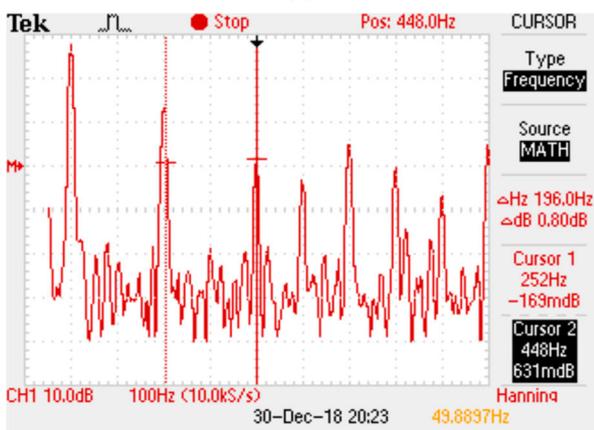
A small inverter prototype was built to experimentally prove the validity of the proposed analyses with the same circuit and model that was used in the simulation. It consists of 2 regular DC power supplies, 8 power MOSFET switches IRF740 and microcontroller PIC16F917.

In order to validate the proposed solution and its simulation results, similar test cases were applied to the experiment setup as shown in Figures 9–11, they show the inverter’s output waveforms and its harmonic spectrums for the three cases similar to simulation results. It can be noticed that the third harmonic is vanished in these figures as stipulated in the aforementioned analysis.

In each case, the harmonic spectrum proves the validity of the proposed solution to eliminate the third harmonic component in single-phase five-level inverter. The proposed solution produces values of



(a)



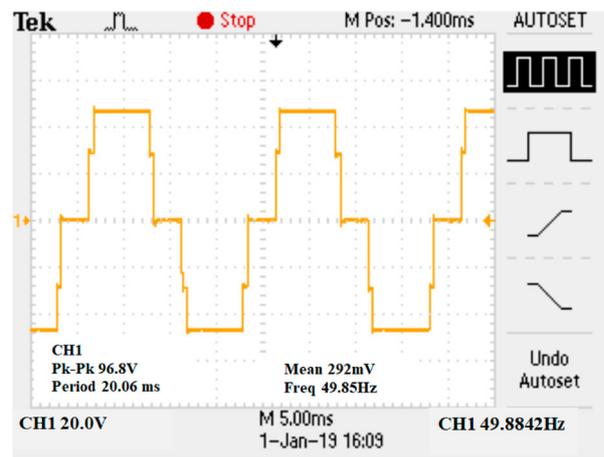
(b)

Figure 10. (a) Output voltage for $V_1 = 16.2\text{V}$, $V_2 = 18\text{V}$, $V_1/V_2 = 0.9$, $m_i = 0.9$, $\alpha_1 = 10.61^\circ$ and $\alpha_2 = 66.41^\circ$, (b) output voltage spectrum where cursor X1 at 250 Hz and cursor X2 at 450 Hz.

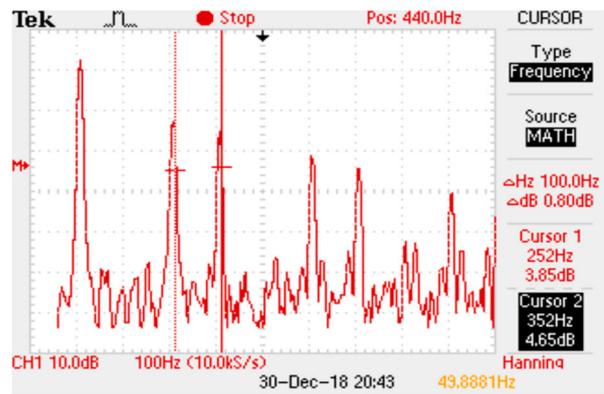
switching angles (α_i) in corresponding to input DC voltage mismatch and targeted modulation index.

5. Conclusions

For asymmetrical cascaded single-phase, five-level inverter, a low switching control scheme employing SHE technique was successfully done. The provided solution was simple and it was a closed-form analytical solution type. The proposed solution was generated from a third order equation which has a well-known solution. The main idea was that the harmonic equations have been solved after replacing the $\cos(n\alpha)$ to its $\cos(\alpha)$, then a mathematical processing was done to form the third order equation in a single variable. The proposed solution modulation index (m_i) range was found to be wide which was from $m_i = 0.6$ to 1.1. Noting that for lower m_i range less than 0.6 there was no meaning for asymmetrical DC sources where single DC source was used. The selected simulation and experimental results were well matched with the analytical provided which proved the proposed idea.



(a)



(b)

Figure 11. (a) Output voltage for $V_1 = 28.8\text{V}$, $V_2 = 18\text{V}$, $V_1/V_2 = 1.6$, $m_i = 1.1$, $\alpha_1 = 26.94^\circ$ and $\alpha_2 = 34.92^\circ$, (b) output voltage spectrum where cursor X1 at 250 Hz and cursor X2 at 350 Hz.

Disclosure statement

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