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Preliminary communication

Application of Semi-Analytical Methods in Production Systems Engineering: Serial Lines

Viktor Ložar, Filip Abdulaj, Tihomir Opetuk*, Neven Hadžić, Hrvoje Cajner

Abstract: Production lines can be designed by an analytical, semi-analytical, or numerical approach. This paper gives a brief introduction to the analytical approach of a single buffer line, the aggregation method, and the analytical approach of a multi-buffer line. An automotive paint shop production system will be used as a figurative example to compare the aggregation method and the recently developed analytical approach for a multi-buffer line. A discussion at the end will show the advantages and disadvantages of the analytical approach.

Keywords: aggregation method; analytical approach; multi-buffer lines

1 INTRODUCTION

Production lines have a big influence on our lives nowadays. Since Henry Ford, they developed rapidly and helped shape the modern economy. The main advantages are an increase in productivity, while at the same time, they lower costs. The result of this constellation is mass production and products with a higher profit. That is the reason why there are various efforts to optimize them. At the early beginnings, the organization was simple as the factories were simple and small. Foremen dominated the shop floors, they decided what would be manufactured and where. The company's growth and products got more sophisticated. The organization of production at the beginning was founded on the experience of foremen. Later on, organization was based on numerical software, whereas today, it is based on big data.

Production lines can be described by various approaches, which can be summarized into analytical, semi-analytical, or numerical. Throughout the decades, since the first manufacturing systems got modelled, up until now, a lot of different subtypes were discovered.

In today's industries, it is convenient to design manufacturing processes [1]. Such an approach allows the operator to decide which machine he can turn off to save energy without losing the required performance [2]. Another benefit is the ability to test various scenarios of investing in new machines while minimizing the risk of investment failures [3]. In the end, the benefits of designing the manufacturing processes can be simulated and presented to the decision-makers in a company in order to ensure a better acceptance toward the Industry 4.0 [4].

An analytical approach of a steady-state series Bernoulli production line with one buffer and two machines was published for the first time in 1962 [5]. For a long time, the problem could not be solved for an arbitrary number of machines and for the buffers with arbitrary capacity because of the complexity to define the transition matrix. The generalized transition matrix was formulated recently [6]. Methods for the evaluation, analysis, and control of the system's continuous random variables were developed by using the analytical approach [7].

The semi-analytical approach can be divided into the aggregation and decomposition methods. The semi-analytical approach dominates because the analytical approach was only developed recently. The aggregation method will be further described. This method has a wide application; it can be used to simulate the setup time of a manufacturing line [8]. One of the main benefits of the aggregation method is the short processor time. This makes it a quick tool in the designing and optimization of a production system.

2 THE ANALYTICAL APPROACH – A SINGLE BUFFER LINE

The Bernoulli line with two machines and one buffer was described by Markov chains in 1962. The sample space of the random variable is 0 and 1. If the machine state is up, the number is 1, if the machine is down, the number is 0.

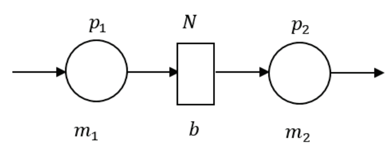

Figure 1 Two-machine Bernoulli production line [9]

The following conventions must be fulfilled [9]:

- blocked before service,
- the first machine is never starved; the last machine is never blocked,
- the status of the machines is determined at the beginning and the state of the buffer at the end of each time slot,
- each machine status is determined independently from the other,
- time-dependent failures.

2.1 State Transition Diagram for a System with One Buffer

Various buffer conditions can be shown in a transition diagram. The transition diagram is built up from circles and arrows. The circles describe the buffer condition and the arrows, called trajectories, show the direction of a possible change of the buffer status. The values of the arrows are called the transition probability and depend on the conditional probability of the machines (machine up or machine down). The system with one buffer has two trajectories at the zero and the end status of the buffer condition. Between them, there are three trajectories from each buffer condition. The characteristics of the machines and the buffer can be shown in a matrix called transition matrix, where the sum of the probabilities in a column must be one.

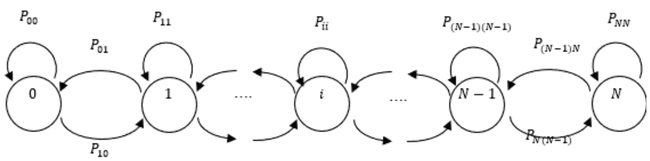

Figure 2 Transition diagram of a two-machine Bernoulli production line [9]

When the transition probability after a time circle $n + 1$is not changing, the whole line reaches the steady-state environment. In such case, the transition matrix can be multiplied with the buffer conditional vector and the result will be the same buffer conditional vector. Following some mathematical operations, it is possible to define each state of the buffer conditional vector.

3 THE AGGREGATION METHOD

This method belongs to the class of semi-analytical solutions. The aggregation method was developed because of the multi-machine and -buffer problems with the transition matrix. Some authors claimed that it is not possible or even necessary to solve these issues [9]. However, this problem was finally solved in 2018 [6].

The aggregation approach has three steps. The first step is the backward aggregation, the second step is the forward aggregation and the third step is the iteration of both aggregations.

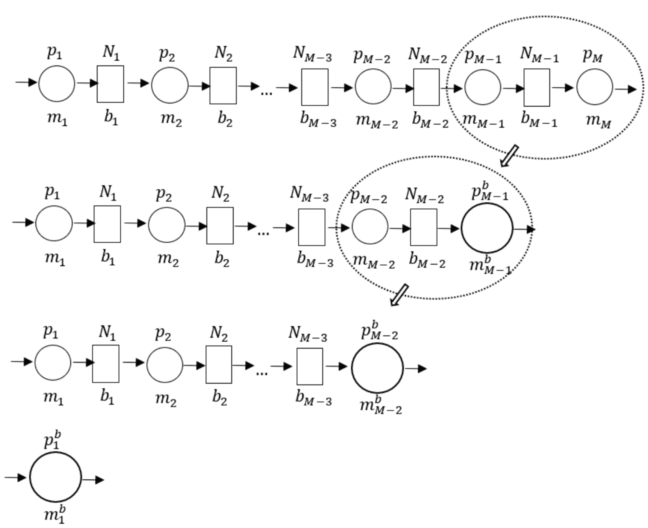

Figure 3 Backward aggregation [9]

The first step starts with the substitutions from the end of the production line. The first substitution consists of the last two machines and the buffer between them. This new composition creates a machine denoted by m^b_{M-1}, where b describes backward aggregation. This aggregated machine represent a two-machine single buffer line. Corresponding to that, the probability of this new machine p^b_{M-1} is calculated by the production rate of this two-machine single buffer line. The next substitution consists of this aggregated machine m^b_{M-1}, machine m_{M-2} and the buffer b_{M-2} which will be denoted as m^b_{M-2}. These substitutions are repeated until the whole line is aggregated into one machine p^b_1. This is the end of the first step.

The forward aggregation starts with the substitution of the first machine m_1, first buffer b_1 and the backward aggregate rest of the line m^b_2 into the machine m^f_2. This aggregated machine represents a two-machine single buffer line. Corresponding to that, the probability of this new machine p^f_2 can be calculated as the production rate of the two-machine single buffer line. The next substitution consists of this aggregated machine m^f_2, the next buffer b_2 and the backward aggregated rest of the line m^b_3. This combination will be denoted as m^f_{M-1}.

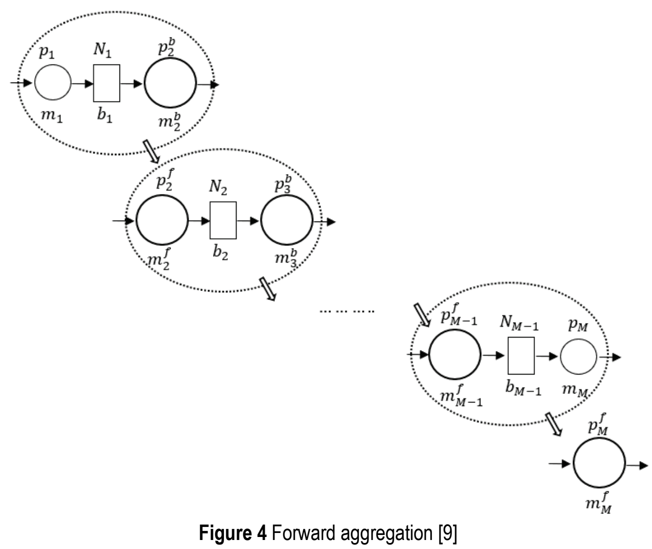

Figure 4 Forward aggregation [9]

Such aggregations will be repeated until the whole line is substituted into a single machine m^f_M which is built up from the last machine, last buffer, and the before aggregated machine.

During the third step, the backward and forward aggregation are repeated by using the results of each cycle. After three or four circles, the results will not change anymore and they can be used to calculate the following parameters: *PR* -Production Rate, *WIP* - Work in Process, *BL* - Blockages and *ST* - Starvations, and the *RT* - Residence time.

PR defines the average number of parts that are produced by the last machine per cycle time.

The *WIP* defines the average number of parts contained in all process buffers.

The *BL* defines the probability of a blocked machine. This case happens when the machine in front of the blocked one is up, the buffer in front of it is full and the machine after the blocked machine did not take an object.

The *ST* parameter defines the probability when a machine is running out of parts. This case happens when the machine is up, but the buffer in front of the machine is empty.

The *RT* residence time can be calculated out of the *WIP* and *PR*. In some literature, it is called flow time or system cycle time.

$$
\begin{aligned}
&PR = p_1^b = p_M^f \\
&PR = p_{i+1}^b\left[1 - Q\left(p_i^f, p_{i+1}^b, N_i\right)\right] \\
&PR = p_i^f\left[1 - Q\left(p_{i+1}^b, p_i^f, N_i\right)\right] \quad i = 1, \dots, M-1
\end{aligned} \tag{1}
$$

$$
WIP = \begin{cases} \dfrac{p_i^f}{p_{i+1}^b - p_i^f \alpha^{N_i}\left(p_i^f, p_{i+1}^b\right)} \left[\dfrac{1-\alpha^{N_i}\left(p_i^f, p_{i+1}^b\right)}{1-\alpha\left(p_i^f, p_{i+1}^b\right)} - N_i \alpha^{N_i}\left(p_i^f, p_{i+1}^b\right)\right] & \text{if } p_i^f \neq p_{i+1}^b \\ \dfrac{N_i\left(N_i+1\right)}{2\left(N_i + 1 - p_i^f\right)} \quad \text{if } p_i^f = p_{i+1}^b \end{cases} \tag{2}
$$

$$BL_i = p_i Q\left(p_{i+1}^b, p_i^f, N_i\right) \quad i = 1, \dots, M-1 \tag{3}$$

$$ST_i = p_i Q\left(p_{i+1}^b, p_i^f, N_i\right) \quad i = 2, \dots, M \tag{4}$$

$$RT = \frac{WIP}{PR} \tag{5}$$

4 THE ANALYTICAL APPROACH – MULTI-BUFFER LINE

The multi-machine and -buffer problem was solved recently [6]. The first step is the creation of the transient matrix which is built up form constitutive matrices $P_i(p_i)$ (6). There can be three different types of constitutive matrices. The first matrix, the last matrix and the matrices between the first and the last matrix. For each type, there is a set of four different boundary conditions, which defines the elements $P_{h_1^n h_2^n h_1^{n+1} h_2^{n+1}}$ of these matrices [6]. The number of constitutive matrices is equal to the number of machines in the line. Each matrix has the same structure of elements which depends on the system state of the whole multi-machine line.

Fig. 5 shows an example of the structure of a constitutive matrix with three machines and two buffers.

$$
\begin{aligned}
&\left[P(p_1, p_2, \dots, p_M)\right] = \left[P_1(p_1)\right]\left[P_2(p_2)\right] \dots \left[P_M(p_M)\right] \\
&\left[P(p_1, p_2, \dots, p_M)\right] = \prod_{i=1}^{M}\left[P_i(p_i)\right]
\end{aligned} \tag{6}
$$

All matrices have the dimension $d \times d$. The number of elements d depends on the number of buffers as shown in the Eq. (7).

$$
\begin{aligned}
&d = (N_1+1)(N_2+1)(N_3+1)\cdots(N_{M-1}+1) \\
&d = \prod_{i=1}^{M-1}(N_i+1)
\end{aligned} \tag{7}
$$

The transient matrix is a stochastic matrix where the sum of each column equals one, the maximum eigenvalue equals one and all the elements of the matrix are smaller than one. These properties are crucial in the solution of the eigenvalue problem, which is the next step.

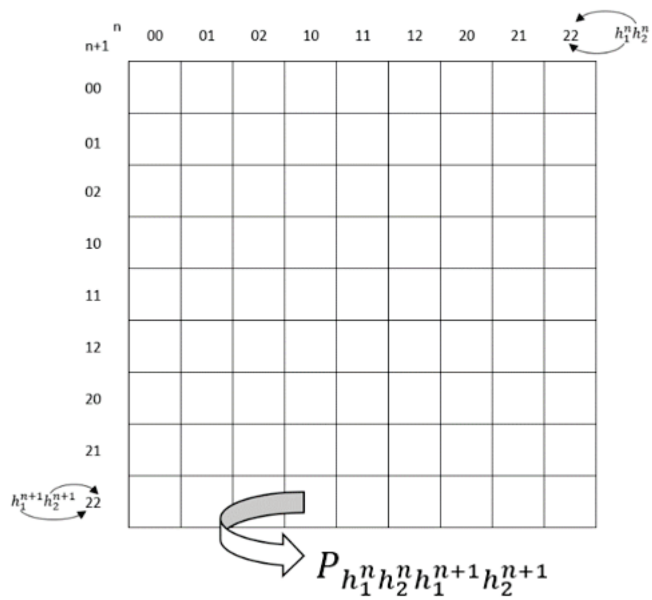

Figure 5 Matrix structure

The solution of the eigenvalue problem depends on the steady-state of the production line. In that case, it can be written as

$$\left(\left[P\right] - \Omega_1\left[I\right]\right)\left\{P_1\right\} = \left\{0\right\} \tag{8}$$

where Ω_1 is the eigenvalue for the steady-state, P_1 is the unknown eigenvector which is built up of probability elements $P_{h_1 h_2 \dots h_{M-1}}$. These elements can be used to calculate the following parameters: *PR* - Production Rate, *WIP* - Work in Process, *BL* - Blockages and *ST* - Starvations. Formulas are listed in the paper [6].

5 ILLUSTRATIVE EXAMPLE

The illustrative example will be an automotive paint shop production system [9] with 11 operations. During these operations, the car bodies are cleaned, sealed, painted and finally finessed [9]. Parts are moving on carriers along the operational and accumulator conveyors. The operational conveyor enables the stopping of carriers without stopping the whole line. The initial layout of the automotive paint shop is simplified to ensure the application of the aggregation method and the analytical approach, see Fig. 6.

In this illustrative example, the machine parameters from month 5 will be taken into consideration, Tab. 1. The effect of a closed loop is considered with the factor p_{st}.

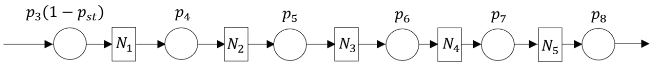

Figure 6 Simplified structural model of a paint shop system

Table 1 Machine probability parameters month 5

$p_3(1-p_{st})$	p_4	p_5	p_6	p_7	p_8
0.8832	0.9587	0.9740	0.9938	0.9675	0.9935

Table 2 Buffer capacities

N_1 (pcs)	N_2 (pcs)	N_3 (pcs)	N_4 (pcs)	N_5 (pcs)
3	4	7	60	5

These input parameters are used for the aggregation method and for the analytical approach.

Table 3 Comparison of the aggregation method with the analytical approach

Parameters	Aggregation	Analytical
PR (pieces/cycle time)	0.88069	0.88125
WIP 1 (pieces)	1.26520	1.26526
WIP 2 (pieces)	1.12095	1.11791
WIP 3 (pieces)	0.92897	0.92819
WIP 4 (pieces)	1.21042	1.23578
WIP 5 (pieces)	0.93144	0.93256
Sum *WIP* (pieces)	5.45698	5.47969
BL 1	0.00251	0.00251
BL 2	0.00014	0.00014
BL 3	0.00000	0.00000
BL 4	0.00000	0.00000
BL 5	0.00000	0.00000
ST 1	0.07788	0.07788
ST 2	0.09331	0.09331
ST 3	0.11311	0.11311
ST 4	0.08681	0.08627
ST 5	0.11281	0.11225
RT (cycle time)	6.19623	6.21807

Tab. 2 shows that the results between the aggregation method and the analytical approach for this figurative example are almost equal. The advantage of the aggregation method is the lower CPU load which makes the calculation much faster than the calculation of the analytical approach. The analytical approach is still necessary to validate the aggregation method.

It can be recommended to first calculate with the faster aggregation method and at the same time to start the analytical calculation, which will take some time, but in the end, the user will know if the first results are good enough or not. The calculation of the aggregation method will take just a second on an average PC. The calculation time for the analytical approach needs approximately 64 h on an average PC.

6 CONCLUSION

The analytical, semi-analytical and numerical approaches in the production system engineering are valuable tools to describe and improve the production. The figurative example shows the importance of a double approach concept to validate the results. The result of the aggregation method alone is not necessarily the best. After the application of the analytical approach, the results get validated.

Further investigation of the analytical approach may result in a speed-up of the calculation time. The numerical approach should be validated in further comparison. Measurements in the industry should be provided to validate all three approaches.

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Notice

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Authors' contacts:

Viktor Ložar, univ. mag. ing. naval architect
University of Zagreb,
Faculty of Mechanical Engineering and Naval Architecture,
Ivana Lučića 5, 10000 Zagreb, Croatia
E-mail: viktor.lozar@fsb.hr

Filip Abdulaj, univ. bacc. ing. naval architect
University of Zagreb,
Faculty of Mechanical Engineering and Naval Architecture,
Ivana Lučića 5, 10000 Zagreb, Croatia
E-mail: fa201754@fsbhr.onmicrosoft.com

Tihomir Opetuk, PhD, Assistant Professor
(Corresponding author)
University of Zagreb,
Faculty of Mechanical Engineering and Naval Architecture,
Ivana Lučića 5, 10000 Zagreb, Croatia
E-mail: tihomir.opetuk@fsb.hr

Neven Hadžić, PhD, Assistant Professor
University of Zagreb,
Faculty of Mechanical Engineering and Naval Architecture,
Ivana Lučića 5, 10000 Zagreb, Croatia
E-mail: neven.hadzic@fsb.hr

Hrvoje Cajner, PhD, Assistant Professor
University of Zagreb,
Faculty of Mechanical Engineering and Naval Architecture,
Ivana Lučića 5, 10000 Zagreb, Croatia
E-mail: hrvoje.cajner@fsb.hr