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A REVIEW OF FORMULATIONS TO DESIGN AN ADHESIVE SINGLE-LAP JOINT FOR USE IN MARINE APPLICATIONS

Summary

The single adhesive joint has many applications in the shipbuilding industry, where it offers the advantage of joining materials (adherents) with different properties and characteristics using an adhesive. However, one disadvantage of this type of joint is the stress concentration at the ends of the joint, which directly affect the adhesive. Another disadvantage is the possible difference between the coefficients of thermal expansion of the adherents of the joint. Through compilation and classification of the formulas found in various publications, this study presents a state-of-the-art review of an adhesive single-lap joint that can be used in marine applications. It will consider the types of materials used as the adhesive and as the adherents, the possibility of varying the thicknesses of the adherents and the thickness of the adhesive, and the recommended design factors for each proposed methodology. This study proposes formulas to estimate the stresses for joints with balanced thicknesses and extrapolates the results for non-balanced joints; also, an equation is derived to calculate the minimum overlap joint length for ship lengthening, allowing the design process to be simplified. The results are expected to facilitate the design of single-lap joints in marine applications, such as reinforcing composite panels and lengthening of hulls and superstructures.

1. Introduction

The need to produce adhesive joints between two materials with the same or different characteristics has led to multiple investigations into developing equations that allow estimating the stresses in single-lap joints. Single-lap joints can be observed in different applications in the marine industry using composite material, such as the following examples:

− Dominguez [1] made a review of the state of the art presenting different hybrid joints between a steel deck and an FRP (fibre reinforced polymers) superstructure of various sizes. Hybrid adhesive bonding has also been applied by the Kockums shipyard on commercial vessels and military ships.

− In the welding of a fibre reinforced polymers (FRP) beam or the reinforcement of an FRP composite panel, or in a hybrid joint when fixing a metal reinforcement to an FRP composite panel, [1]-[3], as presented in Fig. 1. In these examples, the FRP stiffener laminate is considered the top adherent, the inferior FRP sandwich panel is the bottom adherent, and the polyester resin is the adhesive.

− In the lengthening of FRP hull or superstructure of a vessel, whereby a single-lap joint must be made, which can be balanced or non-balanced joint, as indicated in Fig. 2.
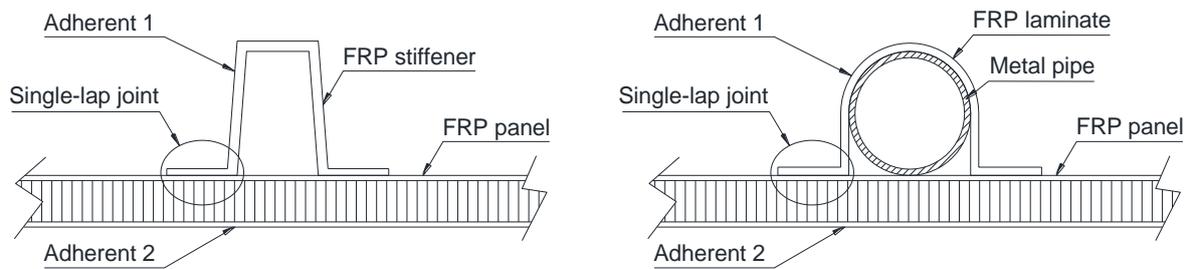

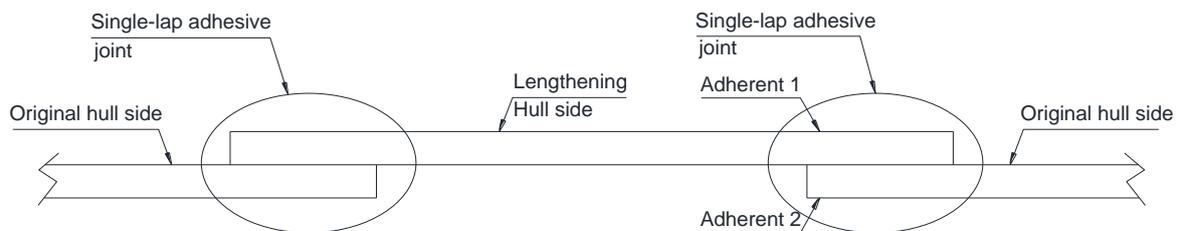**Fig. 1:** Adhesive joint between FRP laminate and FRP stiffener

Fig. 2: Hull shell lengthening of an FRP ship

The adherent materials in these adhesive joints can be steel, stainless steel, aluminium, FRP composite laminate, carbon fibre reinforced polymer (CFRP), bio-composite [4], or a combination of these.

To perform comprehensive literature, we reviewed several types of research, and abstracting databases published from 1938 to 2019 were initially considered. Only peer-reviewed journal articles with novel contributions to the field were critically reviewed.

The first research on single-lap joints was undertaken by Volkersen [5] in 1938. Since then, several authors have continued to improve and propose new methodologies for estimating the shear and normal stresses in the adhesive. Successive investigations have been developed considering the adherents as isotropic, orthotropic, or anisotropic, or considering the adhesive as isotropic. The stress-strain curve is approximated in a linear or non-linear manner, and the resulting stress formulas in the adhesive can be explicit or implicit.

This document aims to review the methods developed and proposed for the analysis of single-lap joints, thereby allowing the reader to select the methodology that is most convenient for the marine application at hand. Table 1 presents a classification of these joints based on their configuration, mathematical model and formulas proposed by each author. Later, an analysis of each mentioned formulation and the involved variable is conducted to provide a general approach for selecting a single adhesive joint.

2. Summary of the reviewed studies of adhesive single-lap joints

2.1. State of the art

This research compiles the most cited investigations that have contributed to the development of the analysis of single adhesive joints. Volkersen [5] and Goland and Reissner [6] were the first to analyse this type of joint, and their assumptions are still taken as a comparative reference in new researches.

Table 1 summarizes the classification of the proposed methods by the different authors based on the main considerations of a single-lap joint:

Table 1 Summary of calculation methods for a single adhesive joint (*); Clark [7].

Year	Author	Reference	Adherent type		Material type			Adhesive material		Adhesive behaviour		Adhesive mode		Adhesive effective length
			Balanced	Not balanced	Isotropic	Orthotropic	Anisotropic	Isotropic	Anisotropic	Elastic	Plastic	Linear	Non-linear	
Methods with the explicit formulation														
1938	Volkersen	[5]	x		x			x		x		x		
1944	Goland and Reissner	[6]	x		x			x		x		x		x[(*)]
1973	Hart Smith	[8]	x	x	x			x		x	x	x	x	
1977	Allman	[9]	x		x			x		x		x		
1989	Bigwood and Crocombe	[10]	x	x	x			x		x		x		
1991	Oplinger	[11]	x	x	x			x		x		x		x
2004	Zou	[12]	x		x			x		x		x		
Methods with the implicit formulation														
1973	Renton and Vinson	[13]	x	x	x	x	x	x		x		x		x
1977	Ojalvo	[14]	x		x			x		x		x		
1981	Delale	[15]	x		x	x		x		x		x		
1992	Adams and Mallick	[16]	x	x			x	x		x		x	x	
1996	Tong	[17]	x		x			x		x		x	x	
2003	Smeltzer	[18]	x	x			x	x	x	x	x	x	x	

The comparative analysis undertaken in this review accounts for the aspects presented in Table 2 to make the proposed methodologies more comprehensive.

Table 2 Description of the specific aspects of the methods for producing single-lap joints.

ASPECTS	TYPE	DESCRIPTION
Method formulation	Explicit	The method develops a closed solution; that is, the authors provide formulas of stresses that can be directly evaluated.
	Implicit	The method is not fully developed; that is, the authors express the formulas or require numerical analysis or implementation of additional programs to apply the method.
Adherent type	Balanced	The upper and lower adherents are of equal thickness and mechanical properties.
	Non-balanced	The upper and lower adherents are of different thicknesses or different mechanical properties.
The material of adherent and adhesive	Isotropic	The method considers the adherent or adhesive as a material that retains the same properties in all directions.
	Orthotropic	The method considers uses the adherent or adhesive as a material that has defined properties in three directions.
	Anisotropic	The method considers the adherent or adhesive as a material that has defined properties in all directions.
Adhesive behaviour	Elastic	For the stress analysis, the adhesive deformation is maintained in the elastic zone of the stress-strain curve.
	Plastic	For the stress analysis, the adhesive deformation is maintained in the plastic zone of the stress-strain curve.
Adhesive model	Lineal	The methodology considers the linear behaviour of the adhesive for its mathematical approach and stress estimation.
	Non-lineal	The methodology considers the non-linear behaviour of the adhesive for its mathematical approach, variables, and assumptions.
Adhesive effective length		Each author proposes a formula to estimate or recommend the length of the adhesive for the single-lap joint geometry.

2.2. Configuration of the adhesive joint

2.2.1. Type of joint

In Fig. 3, the following four configurations for single-lap joints are shown:
- Option a: Classic joint with orthogonal vertices at the ends of the adherents and the adhesive.
- Option b: Joint with rounded vertices at the ends of the adherents and orthogonal at the ends of the adhesive.
- Option c: Joint with short bevelled vertices at the ends of the adherents and orthogonal at the ends of the adhesive.
- Option d: Joint with long bevelled vertices at the ends of the adherents and orthogonal at the ends of the adhesive.

Option a is typically used in most single adhesive joints. This option is described in the methods listed in section 2.3.

Options b and c allow the reduction of the maximum shear and normal stresses generated at the ends of the adhesive, but their mathematical development is complex; therefore, the finite element analysis (FEA) is recommended for designing this type of joint (Calik [19]).
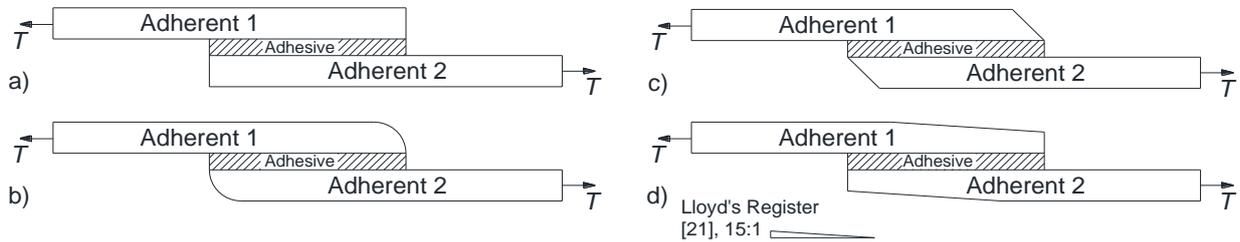

Fig. 3: Adhesive single-lap joint types

Option d is mostly used for the adherents of composite materials with staggered laminate layers at the ends of the adherents. Oterkus [20] investigated this type of overlapping joint, proposing a semi-analytical method taking into account the linear and bilinear elastic behaviour of the adhesive and the linear behaviour of the adherents. As a result of this analysis, he obtained a system of non-linear equations for shear and normal stresses, to be solved by an iterative procedure using the Newton Raphson method together with Broyden's Jacobian matrix [20].

Fig. 4 shows Oterkus's [20] results, whereby it is observed that the shear and normal stresses decrease with the increase in the size of the bevel on the adherent ends. For this case, Lloyd's Register recommends using staggered bevels, as shown in Fig. 8.
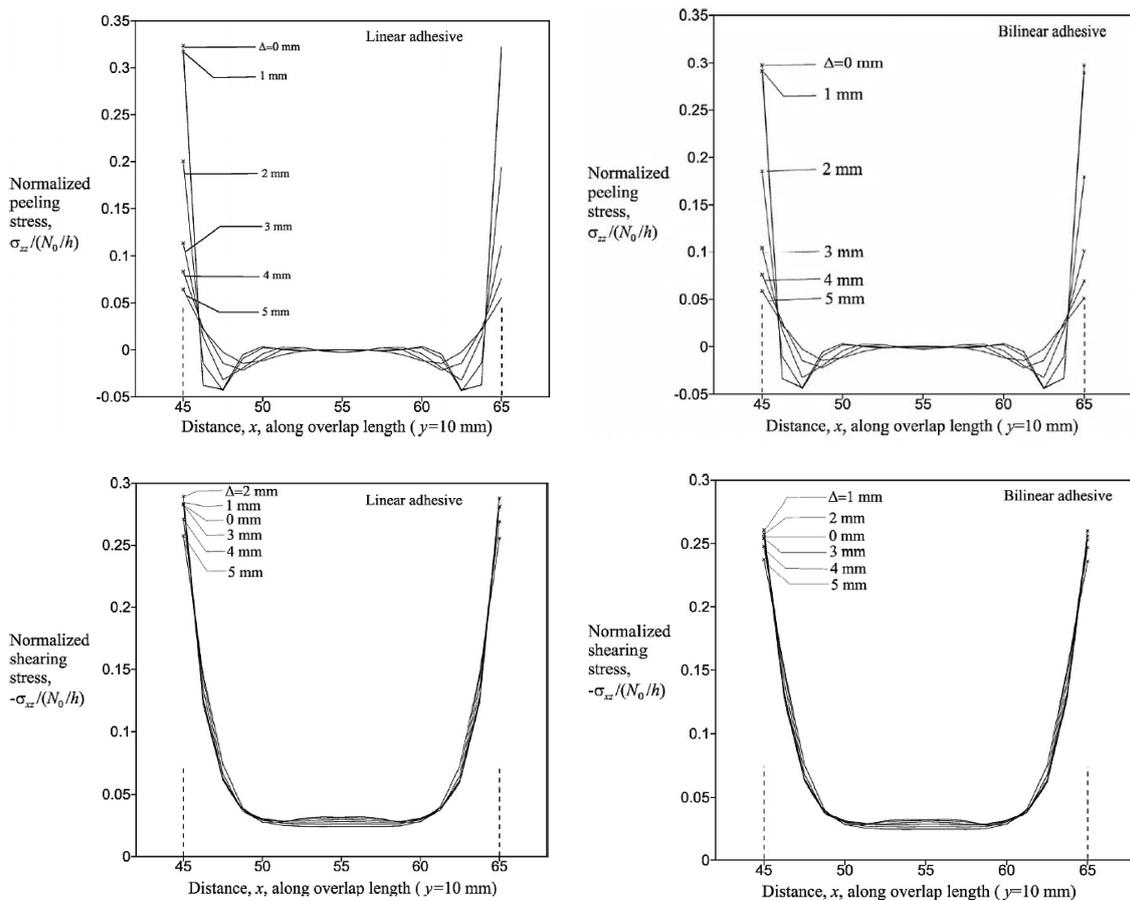

Fig. 4: Stress results, based on the linear and bilinear behaviour of the adhesive; Oterkus [20]

2.2.2. Materials

The materials used in a single-lap joint can vary depending on the intended application. In the case of composite materials, adherents can be considered as isotropic, orthotropic or anisotropic materials, depending on the methodology applied for the analysis.

2.2.2.1. Adherents

The materials, and their combinations, that have been used as adherents are presented in Table 3.

Table 3 Materials used in single-lap joints.

Metals	Composites	Combined Materials
Steel	Fibre-reinforced polymers, FRP	Steel/fibreglass laminate
Aluminium	Carbon fibre reinforced polymers, CFRP	Steel/carbon fibre laminate
Stainless steel		Aluminium/fibreglass laminate
		Aluminium/carbon fibre laminate

Metals are common adherents, and their mechanical properties depend on the type of alloy used. For composite materials, the properties depend on the type of resin (polyester, vinyl ester, or epoxy) and the type of fibre used. Composite materials can be grouped as orthotropic or anisotropic [13] [18] based on their laminate; however, in the explicit methods, adherents are considered isotropic.

In adhesive lap joints where the adherents are considered metallic, the first failure is expected to be generated in the adhesive and then in the adherent. Meanwhile, for joints with laminated composite adherents, the first failure is expected to appear in the adherent [8]; see Fig. 5.

In the case of a joint between FRP composite materials as the adherents with polyester resin as the adhesive, the behaviour of the stress-strain curve of the adherents must be taken into account because after the elastic deformation, the joint will present a failure by delamination [21].
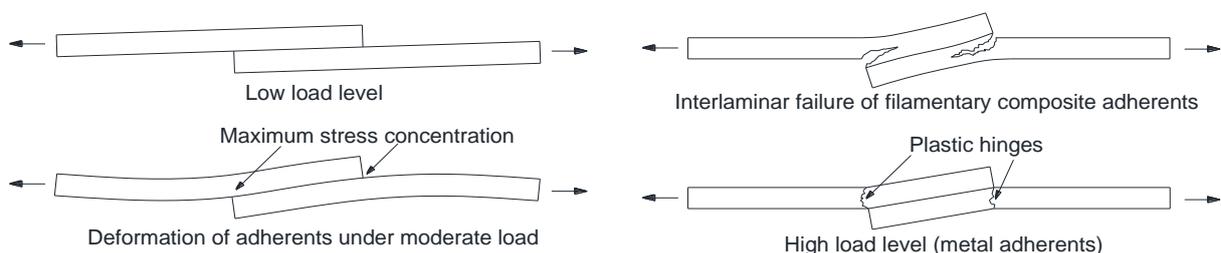

Low load level

Interlaminar failure of filamentary composite adherents

Maximum stress concentration

Plastic hinges

Deformation of adherents under moderate load

High load level (metal adherents)

Fig. 5: Possible failures of a single-lap joint; Hart-Smith [8]

2.2.2.2. Adhesives

In most of the investigated methods for the single-lap joint, the behaviour of the adhesive is approximated as isotropic-elastic. Banea [22] presented a table summarizing the typical properties of the different types of adhesives, which include the epoxy type, anaerobic or silicone type, and polyurethane type, among others.

Hart-Smith [8][23]-[26] emphasized the importance of including, in the calculations, the estimation of the stresses obtained in the plastic area of the adhesive. The typical stress-strain behaviour of an adhesive and the equivalent linear and bilinear curves are presented in Fig. 6. The hatched sections correspond to the proposed method to find the equivalence between the energy density of the typical nonlinear characteristic curve and the linear or bilinear curve. Hart-Smith [23] concluded that the complexity of the bilinear representation of the adhesive leads to fairly approximate results when compared to the results obtained with the linear estimation, provided that the same equivalent adhesive energy density curve is maintained.
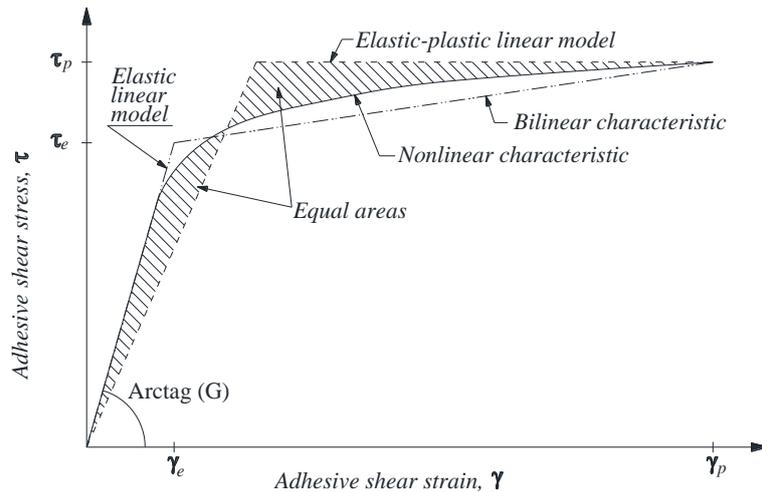

Fig. 6: Typical representation of the equivalence between the characteristics of the adhesive; Hart-Smith [26]

Fig. 7 shows an example of the distribution of shear and normal stresses in the adhesive, considering elastic-plastic behaviour, whereby it is observed that the length of the adhesive is divided into three sections: a central elastic zone and two plastic zones at either end. This detail is important because the larger the plastic zone, the greater the possibility of normal stress increase, which may lead to cracking in the adhesive.
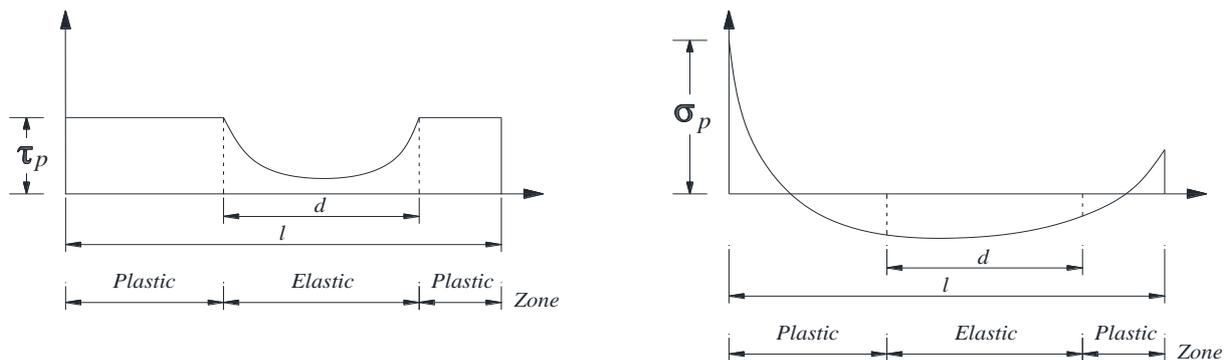

Fig. 7: Distribution of stresses in the adhesive, elastic and plastic zones; Hart-Smith [23]

For length l where the adhesive is considered perfectly plastic ($d=0$), it is true that $\tau_{avg} = \tau_p$. Hart-Smith [8] recommended considering the following:

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$$l \leq \frac{1}{\lambda} \cdot \sqrt{\frac{4 \cdot \left(\dfrac{2 \cdot \gamma_e}{\gamma_p}\right)}{1 + \dfrac{3 \cdot k \cdot \left(1 - \nu^2\right)}{k_b} \cdot \left(1 + \dfrac{n}{t}\right)}}$$

(1)

The formulas for estimating the length of the adhesive are explained in section 2.4.1

2.3. Considerations for modelling joint behaviour

Volkersen [5] investigated the behaviour of the single-lap joint, in which the balanced adherents and the adhesive were considered as elastic and isotropic materials. This investigation did not consider deformations in the adherents or bending moments in the adhesive joint generated by eccentricity. The linear mathematical model initially proposed by Volkersen [5] only considered the shear force on the adhesive, with maximum values at its ends and a minimum at the halfway point.

Goland and Reissner [6] developed formulas to estimate the shear and the normal stresses of an adhesive single-lap joint. They considered that the deformations that occur in the adherents are relatively small in comparison to the deformations produced in the adhesive. Besides, the deformations in the adherents are due to the cylindrical flexion generated by the flexural moment that is formed by the eccentricity of the applied load. Adherents and adhesives are considered perfectly elastic. This study resulted in a linear method that applies only for adhesives with thin thicknesses as well as for balanced joints, that is, those with the same geometry and properties.

Hart-Smith [8] analysed a single-lap joint considering the behaviour of both linear-elastic and plastic adhesives. The plastic zone of the adhesive bond is considered for the range (*l-d*); see Fig A.3. With this assumption, Hart-Smith validated the theoretical results with the experimental results. Furthermore, this analysis found that maximum shear and normal stresses occur at the ends of the adhesive joint, while the lowest stresses occur in the middle, concluding that an exaggerated increase in the length of the adhesive bond does not reduce stress because the load is processed along an effective length.

Allman [9] based his investigation on research by Goland and Reissner [6] that considered the elastic linear behaviour of an adhesive on a balanced joint. This author proposed estimating the stresses based on the non-deformable geometry of the single-lap joint and considering the adherents and the adhesive as isotropic materials, allowing both metallic and composite adherents to be analysed. This method assumes that the shear stresses do not vary across the thickness of the adhesive, while the normal stresses vary.

Bigwood and Crocombe [10] investigated the shear and normal stress estimation of a single-lap joint considering the adhesive as elastic-linear. For their mathematical analysis, they considered the length of the adhesive and its ends subjected to tensile, shear, and moment loads; see Fig. A.6. The adherents and adhesives were considered as isotropic material, and the adherents could be unbalanced.

Oplinger [11] [27] proposed formulas to estimate the stresses of the single-lap joint. Like other authors, Oplinger based this work on that of Goland and Reissner [6], but in these formulas, the adherents work independently (upper and lower). Adherents and adhesives were analysed as an elastic isotropic material. This methodology allows the estimation of the shear and normal stresses for adherents with thin thicknesses. Oplinger [11] obtained similar results to those of Goland and Reissner [6] for thick adherents; however, greater differences were found in the estimations for thin adherent thicknesses.

Lastly, Zou [12], when analysing the single-lap joint, defined the adhesive as having homogeneous, isotropic, and linear elastic behaviour and stated that the adherents must be balanced.

The listed explicit and the implicit methods in Table 1 are published formulas to calculate the adhesive stresses on single-lap joints; other papers are dedicated to validate these methods with FEA or experimentation.

2.4. Summary of formulas

2.4.1. Adhesive length

Clark [7], taking as reference the formulas proposed by Goland and Reissner [6], recommended an adhesive length based on the parameter β/t, adhesive shear stress τ_a and average adherent stress τ_{avg}.

Renton and Vinson [13] recommended estimating the length of the adhesive using the approximate ratio of l/t=10. If this relationship is greater than 10, failures in the adherent are expected; however, if this ratio is less than 10, failure in the adhesive is expected.

Oplinger [27] proposed a detailed formula based on geometric and mechanical properties, using the parameters λ and Λ.

Table 4 Adhesive effective length formulas.

METHOD	EFFECTIVE LENGTH
Clark [7], Goland and Reissner [6]	$l = 2 \cdot c$ $c = \dfrac{\tau_{a-yield}}{\tau_{avg}} \cdot \left(\dfrac{\beta}{\bar{t}} \right)^{-0.9704}$ $\beta^2 = G_a \cdot \dfrac{\bar{t}^2}{t_a} \cdot \left(\dfrac{1}{E_1 \cdot t_1} + \dfrac{1}{E_2 \cdot t_2} \right) \qquad \bar{t} = \dfrac{t_1 + t_2}{2}$
Renton and Vinson [13]	$\dfrac{l}{t} \approx 10$
Oplinger [11]	$l_{eff} = max\left\{ \dfrac{3 \cdot t_u}{\lambda}, \dfrac{3 \cdot t_u}{\Lambda} \right\}$

The variables in Table 4 are indicated in the appendix.

2.4.2. Moment due to the eccentricity of the adherents

In the single-lap joint, applying a load to the adherents generates a moment of eccentricity between the axis of the adhesive and the axis of the applied load [6][8][11]. This moment then generates deformations in the adhesive and the adherent as well as shear and normal stresses. Due to the condition of the single-lap joint for the estimation of the normal and shear stresses in the adhesive, the moment generated by the adherent must be multiplied by the eccentricity factor k.

Goland and Reissner [6] were the first to estimate the eccentricity factor k for a balanced joint involving the properties of isotropic adherents, such as thickness, elasticity modulus, Poisson number and applied load. Their proposed moment is:

$$M_0 = k \cdot \frac{P \cdot t}{2} \cdot \left(1 + \frac{t_a}{t}\right)$$ (2)

Hart-Smith [8] retained the formula proposed by Goland and Reissner [6], modifying the eccentricity factor so that it could be used for unbalanced adhesive joints. Therefore, in this case, two eccentric moments are generated, one at either end of the adherents (k_1 and k_2). Oplinger [11] then modified the eccentricity factor formula to have a more approximate value. His estimation included the thickness t_b and adhesive shear modulus G_b using the parameter R. In contrast to the formula devised by Goland and Reissner [6], in Oplinger's formula [11] the greatest differences in the eccentricity factor k are generated for thin adherent thicknesses. Zhao [28] proposed the estimation of the bending moment generated by the eccentricity, assuming that the adhesive only deforms at its ends. The proposed formula works for single balanced or unbalanced joints with an adhesive length between 25 and 50 mm and thin adherent thicknesses (steel <4 mm, aluminium <6 mm). The formulas proposed by Zhao [28] for the eccentric moment in the upper and lower adherent for unbalanced joints are:

$$M_1 = \frac{0.5 \cdot T \cdot (t_1 + t_2) \cdot \zeta_2}{\zeta_2 + \zeta_1 \cdot \zeta_2 \cdot l + \zeta_1}$$ (3a)

$$M_2 = \frac{0.5 \cdot T \cdot (t_1 + t_2) \cdot \zeta_1}{\zeta_2 + \zeta_1 \cdot \zeta_2 \cdot l + \zeta_1}$$ (3b)

Where:

$$\zeta_i = \sqrt{\frac{T}{D_i}}$$ (3c)

For balanced joints, Zhao's [28] proposed formula is:

$$M = \frac{T \cdot t \cdot k}{2}$$ (4)

Table 5 presents a summary of the eccentricity factors that are proposed in the stated methods.

Table 5 Eccentricity factors, k.

METHOD	ECCENTRICITY FACTORS
Goland and Reissner [6]	$k = \dfrac{1}{1 + 2 \cdot \sqrt{2} \cdot tanh\left[\left(\dfrac{l}{2 \cdot t}\right) \cdot \sqrt{\dfrac{3 \cdot p \cdot (1 - v^2)}{2 \cdot E}}\right]}$
Hart-Smith [8]	$k_i = \dfrac{1}{1 + \xi_i \cdot c + \dfrac{1}{6}(\xi_i \cdot c)^2}$
Oplinger [11]	$k_n = \dfrac{R_1 \cdot (1 + R^2 \cdot C_2) + 8 \cdot R_2 \cdot \dfrac{T_{h21}}{T_{h22}} \cdot R \cdot (C_1 - C_2)}{R_1 + 8 \cdot R_2 \cdot \dfrac{T_{h21}}{T_{h22}} \cdot R \cdot C_1 + \sqrt{8} \cdot (1 + R^2 \cdot C_1) \cdot \dfrac{T_{h21}}{T_{h1}}}$

Zhao [28]	$k = \dfrac{1}{1 + c\zeta}$

2.4.3. Adhesive stresses

Tables 6 and 7 summarize the formulas for calculating the maximum shear and normal stresses at the ends of the adhesive, as indicated.

Renton and Vinson [13] graphically presented the distribution of the normal and shear stresses (transverse and longitudinal) of the adherent in a single-lap joint, the maximum stresses being expected at the ends of the adherents. Their work also showed that the variation of the stresses through the thickness of the adherent are higher and are generated in the upper or lower part of the ends of the adherents.

Table 6 Shear stress formulas.

METHOD	MAXIMUM SHEAR STRESS
Volkersen [5]	$\tau_{max} = \dfrac{\lambda \cdot l}{2 \cdot tanh(0.5 \cdot \lambda \cdot l)} \cdot \tau_{avg}$
Goland and Reissner [6]	$\tau_{max} = \dfrac{p}{b \cdot l} \cdot \left[\left(\dfrac{1 + 3 \cdot k}{4} \right) \cdot \dfrac{\alpha \cdot l}{tanh(\alpha \cdot l)} + \dfrac{3 \cdot (1 - k')}{4} \right]$
Hart-Smith [8][*]	$\tau_{max} = \tau_{avg} \cdot \left\{ 1 + \left[1 + \dfrac{3 \cdot k_1 \cdot (1 - v_1^2)}{k_b} \left(1 + \dfrac{n}{t_1} \right) \right] \cdot \left[\dfrac{\lambda^2}{4 \cdot \lambda'^2} \right] \cdot \left[\dfrac{2 \cdot \lambda' \cdot c}{tanh(2 \cdot \lambda' \cdot c)} - 1 \right] \right\}$
Allman [9]	$\tau_{0max} = \dfrac{3 \cdot T}{8 \cdot a} \left\{ (1 - k) + \dfrac{K \cdot e^{-\omega_2 \cdot \xi}}{\sqrt{\left(1 + (\omega_2 / \omega_1)^2\right)}} \right\}$
Bigwood and Crocombe [10]	$\tau_{max} = \tau_V + \tau_M + \tau_T$ $\tau_V = \dfrac{3 \cdot V}{4 \cdot h_1} \qquad \tau_M = \dfrac{3 \cdot \alpha_1 \cdot M}{h_1 \cdot (\alpha_1 + \alpha_2)^{0.5}}$ $\tau_T = \dfrac{-\alpha_1 \cdot T}{2 \cdot (\alpha_1 + \alpha_2)^{0.5}}$
Oplinger [11]	$\tau_{max} = \sigma_x \cdot \left[\dfrac{\sqrt{8}}{8} \cdot \beta \cdot (1 + 3 \cdot k_n) \cdot \dfrac{1}{T_{h22}} + \dfrac{3}{8 \cdot \sqrt{8}} \cdot U \cdot (1 - k_n) \cdot \dfrac{1}{T_{h21}} \right]$
Zou [12]	$\tau_{max} = \dfrac{G_a \cdot N \cdot (4 \cdot k_{11} + k \cdot k_{14} \cdot h^2)}{2 \cdot \zeta \cdot h_a}$

(*) Hart-Smith [8] proposed their shear stress formula for use only with balanced adherents, while their normal stress formula was developed for balanced and unbalanced adherents.

3. Formulas for ship applications

The adhesive single-lap joint is mostly used in shipbuilding processes to lengthen the hull or superstructure or to reinforce FRP panels. Depending on the case, it is first necessary to

estimate the forces or the eccentricity moment to be applied to each joint, then to estimate the minimum length of the adhesive and the interlaminate stresses. Table 8 shows the recommended values of the interlaminar design stresses of a reinforced laminate panel.

Considering a limiting stress fraction of 0.33 and based on the ultimate strength S_U, the FRP panel design stress S_D is estimated:

$$S_D = 0.33 \cdot S_U \tag{5}$$

A guide to limiting stress fraction is proposed in [29]. Clark [7] suggests using a safety factor between the interval [1.5, 10], depending on the laminate factors.

Table 7 Normal stress formulas.

METHOD	MAXIMUM NORMAL STRESS
Goland and Reissner [6]	$\sigma_{max} = \dfrac{t}{b \cdot l^2 \cdot \Delta} \cdot \left[\dfrac{\lambda^2}{2} \cdot k \cdot (sinh(2 \cdot \lambda) - sin(2\lambda)) - \lambda \cdot k' \cdot (cosh(2 \cdot \lambda) + cos(2 \cdot \lambda)) \right]$
Hart-Smith [8] [(*)]	$\sigma_{max} = \sigma_{avg} \cdot k_1 \sqrt{\dfrac{3 \cdot \left(1 - v_1^2\right) \cdot E_a \cdot t_1 \cdot \left(1 + n/t_1\right)^2}{E_1 \cdot n \cdot \left[1 + \dfrac{k_{b1} \cdot E_1 \cdot t_1^3 \cdot (1 - v_4^2)}{k_{b4} \cdot E_4 \cdot t_4^3 \cdot (1 - v_1^2)}\right]}}$
Allman [9]	$\sigma_{0max} = \dfrac{T \cdot h}{4 \cdot a} \left[2 \cdot (1 - k) \cdot \omega_2 + k \cdot a \cdot \omega_1^2 \cdot \left(1 + \left(\dfrac{\omega_2}{\omega_1}\right)^2\right) \right]$
Bigwood and Crocombe [10]	$\sigma_{max} = \sigma_V + \sigma_M$ $\sigma_V = \dfrac{-(2)^{0.5} \cdot \beta_1 \cdot V}{(\beta_1 + \beta_2)^{0.75}}$ $\qquad$ $\sigma_M = \dfrac{-\beta_1 \cdot M}{(\beta_1 + \beta_2)^{0.5}}$
Oplinger [11]	$\sigma_{b\,max} = \sigma_x \cdot \dfrac{k_n \cdot \gamma}{2} \cdot (\gamma + U)$
Zou [12]	$\sigma_{max} = \dfrac{E_a \cdot k_{14} \cdot N \cdot h}{4 \cdot \xi^2 \cdot h_a}$

(*) Hart-Smith [8] proposed their shear stress formula for use only with balanced adherents, while their normal stress formula was developed for balanced and unbalanced adherents.

Table 8 Interlaminar and FRP panel design stresses for E-glass and polyester laminate.

Fibre	f_c [(1)]	Interlaminar design stresses		FRP panel design stresses, S_D [(1)]			
		S_t [(1)] [MPa]	Z_t [(1)] [MPa]	Tensile [MPa]	Flexural [MPa]	Compressive [MPa]	Shear [MPa]
UD [(2)]	60%	15.4	14.6	104.9	94.9	53.5	*28.4*
WR [(2)]	50%	15.25	11.7	62.7	76.7	48.5	25.7
CSM [(2)]	30%	17.95	9.5	29.7	50.2	38.6	20.5

(1) Lloyd´s Register [29]
(2) UD: Uni-directional fibre, WR: Woven Roving, CSM: Chopped strand mat

3.1. The joint length between stiffeners and composite panels

For adhesive joint applications with composite adherents when joining a stiffener or beam to panels, Lloyd's Register [29] recommends the following as the minimum staggered length for an adhesive joint:

$$l = 25 + 15 \cdot (n - 1) \tag{6}$$

And:

$$l \geq 20 \cdot t_f \tag{6a}$$

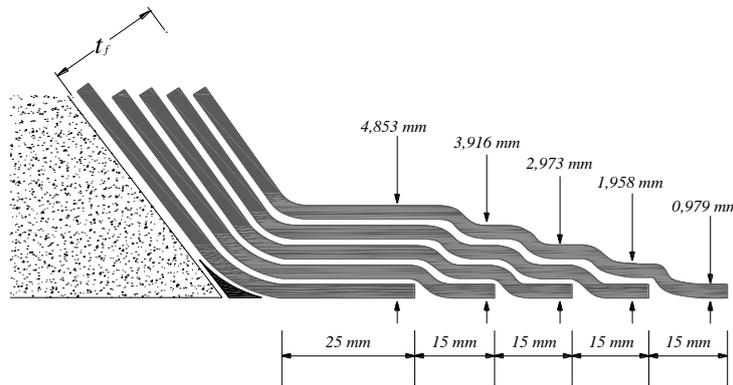Where t_f is the thickness of the upper adherent and n is the number of layers.

Fig. 8: Detail of the staggered adhesive joint length recommended by Lloyd's Register [29].

In any case, the number of layers of t_f depends on the laminate of the stiffener or the girder.

3.2. Minimum joint length for lengthening

The lengthening can be applied to a hull or a superstructure, see Fig. 9. In both cases, it is necessary to determine the forces applied in the joint. These forces can be estimated from the bending stresses and moment, as indicated in the following:

    – Hull lengthening: The bending stress is calculated as a result of the hull girder stress analysis following naval architecture recommendations; it is first necessary to obtain the weight distribution curve to estimate the bending moment in calm water. Then, the wave bending moment is obtained following classification societies' formulas. The total bending moment M_f is the sum of them. Once the lengthening section modulus has been defined, the bending stress $\underline{\sigma_{avg}}$ is obtained using formula (8b).

Fig. 9: Diving yacht: a) Lengthening, +3.55 m, in the midsection; b) final overall length 25.66 m. *Courtesy of Tecnavin S.A.*

- Superstructure lengthening: The bending stress calculation is like that used for hull lengthening, with the difference being that the superstructure section modulus should be used.

Finally, the estimation of the minimum joint length for the lengthening (l) is developed from the formula for a double-lap joint length [23] due to symmetry of the joint at lengthening:

$$l = \left(\frac{\sigma_{avg}}{\tau_a} + \frac{4}{\beta} \right) \cdot \bar{t} \tag{7}$$

Where

$$\tau_a = \frac{\tau_{a-yield}}{10 \cdot f_s} \tag{8a}$$

$$\sigma_{avg} = \frac{M_f}{SM} \tag{8b}$$

Where σ_{avg} is the bending stress; SM is the section modulus, τ_a is the design shear stress of the adhesive, $\tau_{a-yield}$ is the yield shear stress of the adhesive, $\bar{t}$ is the panel thickness and f_s is the safety factor[29].

Once the length of the overlapping joint of the respective lengthening is estimated, it is advisable to use the quadratic criterion of failure of interlaminar forces to validate the design [1].

3.3. Relationship to estimate the normal stress of a non-balanced joint

Once the normal stress of the balanced joint is known (**Table 7**), the following relationship is proposed to estimate the normal stress for a non-balanced joint.

$$\frac{\sigma_{max(NB)}}{\sigma_{max(B)}} = \left[\frac{2}{1 + \frac{k_{b1} \cdot E_1 \cdot (1-\nu_2)^2 \cdot t_1^3}{k_{b2} \cdot E_2 \cdot (1-\nu_1)^2 \cdot t_2^3}} \right]^{1/2} \tag{9}$$

Where adherent 1 corresponds to the reinforcement laminate and adherent 2 is the FRP panel.

3.4. Application of single lap joint

3.4.1. Single lap joint applied to hybrid tubular joint

An application of the adhesive joint is presented in Dominguez [1] to develop a methodology for a hybrid bond with FRP laminated tubular reinforcement that allows the bonding between FRP panels and steel decks.

3.4.2. Yacht hull lengthening

The study of the joint for the elongation of a diving yacht built with hand layup using polyester resin and type E fibreglass is presented below, as shown in Fig. 9. In this application, the adhesive joints will be considered as balanced on the hull bottom, hull side, and deck. The yacht main data is shown in table 9 and laminates properties in table 10.

Table 9 Diving Yacht Main Data

Variable	Symbol	Value	Unit
Length	L	25.66	m
Breadth	B	5.40	m
Depth	D	2.82	m
Draft	H	1.15	m
Displacement	Δ	60.60	ton
Block coefficient	Cb	0.475	
Speed	v	12	knots

$^{(**)}$ *Minimum Cb value for vertical wave bending*
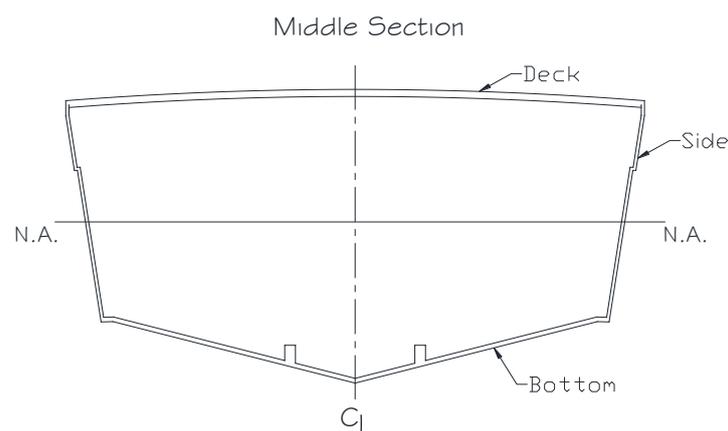*moment calculation 0.6,[29].*

Fig. 10: Diving yacht middle section.

Lloyds Register [29] suggest 13.8 N/mm^2 as yield shear strength $\tau_{a\text{-}yield}$ and a safety factor f_s of 3.

From Fig A.9, the maximum still water bending moment M_S is 1002 kN-m. The maximum vertical wave bending moment M_W of 2571 kN-m, is calculated using formulas of Lloyd´s Register [29], getting a total bending moment of M_f de 3573 kN-m.

On Fig. 10 and Table 11 the amidships section of the diving yacht is shown, where a section modulus SM of 0.502 m^3 for the hull bottom and 0.611 m^3 for the deck are calculated, the smallest modulus will be used in the calculations.

Table 10 Equivalent isotropic properties and thickness of Diving Yacht

Variable	Value	Unit
Side thickness	0.030	m
Bottom thickness	0.044	m
Deck thickness	0.044	m
Resin thickness (adhesive)	0.0025	m
Equivalent elastic modulus of side [**]	2756	MPa
Equivalent elastic modulus of bottom[**]	2338	MPa
Equivalent elastic modulus of deck[**]	2338	MPa
Elastic modulus of the resin	3400	MPa
Shear modulus of the resin	1250	MPa

[**]*For the calculation of equivalent modules, the formulas indicated in Dominguez [1]- Appendix B.1.3 have been used.*

The maximum value between the results of Table 12 and Table 13 is selected, 0.696 m.

The adhesive joint for superstructure lengthening must be estimated with the same procedure used for the hull.

Table 11 Estimation of section modulus and bending stress

Variable	Symbol	Value	Unit
Area	A	0.614	m^2
Distance from bottom to the neutral axis	NA_{bottom}	1.551	m
Distance from deck to the neutral axis	NA_{deck}	1.275	m
Inertia in the neutral axis	I_{NA}	0.778	m^4
Section modulus (bottom)	SM_{bottom}	0.502	m^3
Section modulus (deck)	SM_{deck}	0.611	m^3
Total bending moment	M_f	3573	kN-m
Bending stress (bottom)	$\sigma_{botttom}$	6.626	N/mm^2
Bending stress (deck)	σ_{deck}	5.446	N/mm^2

On Table 12 the estimation of the minimum joint length for hull lengthening is presented using the equation (7):

Table 12 Estimation of minimum joint length

Variable	Symbol	Value	Unit
Section modulus (bottom)	SM	0.502	m^3
Total bending moment	M_f	3573	kN-m
Yield shear stress	$\tau_{a\text{-}yield}$	13800	kN/m^2
Safety factor	f_s	3.00	-
Average thickness	$\bar{t}$	0.044	m
Parameter	β	11.33	-
Bending stress - equation (8b)	σ_{avg}	7118	kN/m^2
Design shear stress - equation (8a)	τ_a	460	kN/m^2
Minimum joint length - equation (7)	l	**0.696**	**m**

On Table 13 the estimation of the minimum joint length for hull lengthening is presented using the formulas of Clark [7] shown in Table 4 :

Table 13 Estimation of minimum joint length, Clark [7]

Variable	Symbol	Value	Unit
Assumed stress fraction	$\tau_{a\text{-}yield} / \tau_{avg}$	21.399	-
Average thickness	$\bar{t}$	0.044	m
Parameter	β	11.33	-
Minimum joint length - Clark [7]	l	**0.196**	**m**

4. Features influencing the single-lap joint on marine applications

The various studies covered in this review propose formulas that show good applicability for use in the industry. However, in designing an adhesive single-lap joint for marine applications, the following aspects should be considered: eccentricity moment, adherent thickness, adhesive length, adherent properties, and adhesive strength.

4.1. Eccentric moment

Hart-Smith presented a formula for the eccentricity factor k that is easy to apply and useful for different thicknesses of adherents. Oplinger proposed a formula for eccentricity factor k that is more accurate compared to FEA calculations; however, it is limited in that it only applies to adherents of the same thickness. Goland and Reissner were the first to introduce the formula for the eccentricity factor k; however, this can only be applied in the context of balanced joints and thin adherent thicknesses. Bigwood and Crocombe and Zou did not directly consider the eccentricity factor k in their calculations.

4.2. Adherent thickness

In marine applications, the variability of the adherent thicknesses is important. To address this variability, the methods proposed by Hart-Smith, Bigwood and Crocombe, and Zou consider the formulas to calculate the shear and normal stresses in the adhesive. The other methodologies were proposed for balanced joints.

4.3. Adhesive length

In marine applications, only the formulations for the elastic behaviour of the adhesive are considered; therefore, the strengths and the direction applied in the lap joint must be identified. In this context, three applications are identified, namely the reinforcement laminate on an FRP panel, the joint between two panels in a hull lengthening, and the joint in a superstructure lengthening.

Regarding the bonding in the reinforcement of an FRP panel, Renton and Vinson made some recommendations for the relationship between joint length and the thickness of the adhesive; however, Lloyd's Register proposes a greater adhesive length in this case. Both authors recommended the bevelling of the adherent ends to decrease the respective stresses. Oplinger provided a formula that allows the calculation of the effective length; its adhesive length values are approximated to the Lloyd's Register formula when the adherent thickness is increased.

Regarding the joint between two panels of a hull lengthening, the formula proposed in section 3.2. is useful to evaluate the adhesive length with equal or unequal adherents' thickness. Alternatively, in cases of equal adherent thickness, the formula proposed by Clark can be used. When these two alternative formulas are used, the required adhesive joint length is to be the greater of the calculated values.

A superstructure lengthening is a case of hull lengthening. The particularity is that the adherents are more likely to be of different thicknesses because it is possible to present the joint between two sandwich panels as well as a combination of a sandwich panel and a single laminate.

From the joint lengths calculated in the application case of diving hull lengthening, it is shown the importance of considering the bending stresses of the hull girder, which is not considered by Clark [7].

A limitation when trying to lengthen a steel hull using FRP adhesive joints, in the middle section, is the difference between the modulus of elasticity of the two materials since it generates different elongations in the hull girder that will cause structural fractures by corrosion. The lengthening with FRP panels can be applied at one end of the hull, provided that the hybrid joint methodology is used taking into account the difference in the mentioned elongations.

4.4. Adherent properties

When joining a reinforcement FRP with an FRP panel, it is important to keep in mind the difference between the equivalent mechanical properties of the adherents because the adhesive stresses obtained with different adherent thicknesses are less efficient than those expected to be obtained with adherents of equal thickness.

4.5. Adhesive stress

The formulas developed by Hart-Smith, Bigwood and Crocombe and Zou yield very similar values for normal stresses. Zou's proposed formulas can calculate the stresses in the adhesive only for equal adherent thicknesses. Hart-Smith's formula allows the normal stresses to be calculated for different adherent thicknesses, whereas shear stresses can be calculated only for balanced joints. Bigwood and Crocombe formula has the advantage that it allows the normal and shear stresses to be calculated for different adherent thicknesses. In all revised formulas, the maximum stresses are shown at the ends of the adhesive joint.

5. Concluding remarks

In the previous sections, the researchers considered the adhesive for a single-lap joint as an isotropic material. Furthermore, thirty per cent of the proposed methodologies considered adherents as orthotropic or anisotropic in their analysis. Due to the mathematical complexity involved, some authors who used anisotropic materials recommended performing a numerical analysis or FEA to complete the solution.

The authors who involved nonlinearity in their analysis concluded that their results are more approximate compared to the results obtained in experimental tests.

The work of Hart-Smith contributed significantly to the utility of such formulas for single-lap joints since this author considered the importance of accounting for the plastic area of the adhesive. This is necessary because single-lap joints are subjected to large deformations, so the shear stresses generated could exceed the elastic limit, in which case a formula would estimate erroneous results.

Another practical conclusion, showed by Hart-Smith and Oterkus, is that an adhesive joint bevel of staggered type has shown to diminish the normal stress on ends. This bevel is recommended by Lloyd's Register to bond stiffeners to an FRP panel and to hull or superstructure lengthening.

The proposed equations (7) and (9), to estimate the minimum joint length and the stresses in the adhesive, respectively, are recommended to preliminary design. The resultant adhesive stresses are useful to estimate the interlaminar stresses of the first adjacent laminate layer; however, to complete FRP panel design, the interlaminar stresses of all laminate layers should be analysed. The last stage is beyond the scope of this study.

To avoid high interlaminar stresses at the ends of the adhesive joint in a lengthening study, it is necessary to make a stepped bevel of $l/t \approx 100$ for each layer. The adhesive joint has been considered to have the same thickness as the hull or superstructure, however, to avoid osmosis effects in the adhesive joint, it is necessary to seal on the opposite side with at least 3 layers using isophthalic NPG resin.

CONFLICTS OF INTEREST: The authors declare that they have no known competing for financial interests or personal relationships that could have appeared to have influenced the work reported in this paper.

NOMENCLATURE

Method with closed-form / Variables	Volkersen	Goland and Reissner	Hart-Smith	Allman	Bigwood and Crocombe	Oplinger	Zou	Zhao
Overlap joint width	b	b						
Overlap joint length	l	l	l			l		l
Non-overlap joint length						l_0		
Elastic length of adhesive			d					
Average length of overlap joint	c		c	a			l	
Adhesive thickness	t_a	t_a	n	t	t	t_b	h_a	t
Adherents thickness	t_1, t_2	t	t, t_1, t_4	h, h_1, h_2	h_1, h_2	t, t_u, t_l	h, h_1, h_2	t_1, t_2
Elastic modulus of the adhesive		E_a	E_a		E_a	E_b	E_a	
Elastic modulus of adherents	E_1, E_2	E	E_1, E_4	E	E_1, E_2	E, E_u, E_l	E_1, E_2	
Shear modulus of the adhesive	G_a	G_a	G_a		G_a	G_b	G_a	
Poisson number of adhesives		ν_a	ν_a		ν_a	ν_b	ν_a	
Poisson number of adherents		ν	ν, ν_1, ν_4	ν	ν_1, ν_2	ν, ν_u, ν_l	ν_1, ν_2	
Flexural stiffness of the adhesive			D_a				$D_{overlap}$	
Flexural stiffness of adherents			D_1, D_4		D_1, D_2	D_u, D_l	D_{11}	D_i
Axial load	P	P, p	P	T		T	N	T
Average shear stress	τ_{avg}		τ_{avg}					
Average normal stress			σ_{avg}			σ_x		

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Appendix: Formulas for the shear and normal stresses of the adhesive

A.1   Explicit methods:
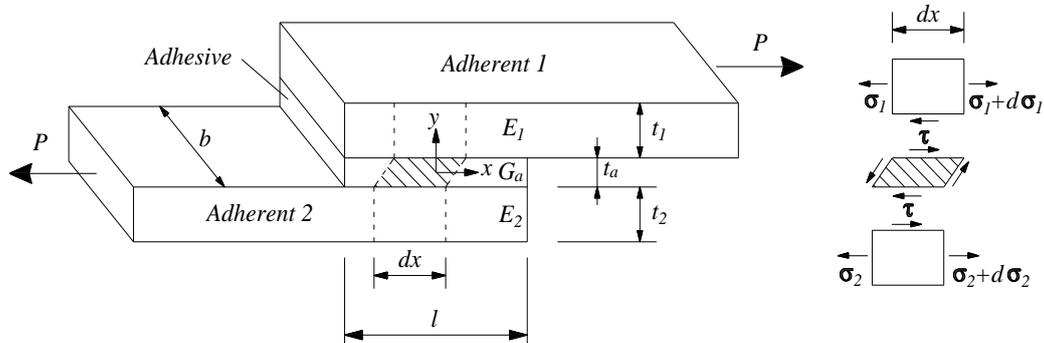
A.1.1 Volkersen method (1938)

Fig. A 1 Single joint geometry; Volkersen [5].

The formulas for shear stress distribution; see Volkersen [5].
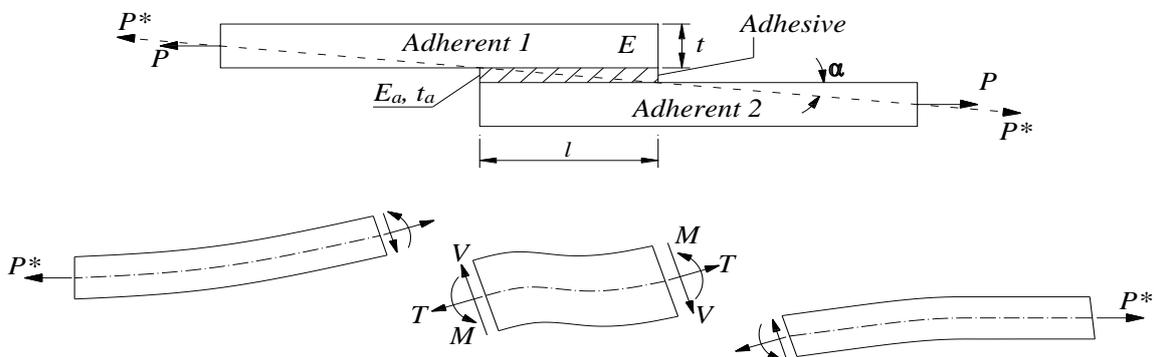
A.1.2 Goland and Reissner method (1944)

Fig. A 2 Single-lap joint geometry; Goland and Reissner [6].

The formulas to estimate the shear and normal stresses; see Goland and Reissner [6].

A.1.3 Hart-Smith method (1973)

Fig A.3 presents the geometry analysed by Hart-Smith [8] for a single balanced or unbalanced adhesive joint and the estimated shear and normal stress distributions for the adhesive.
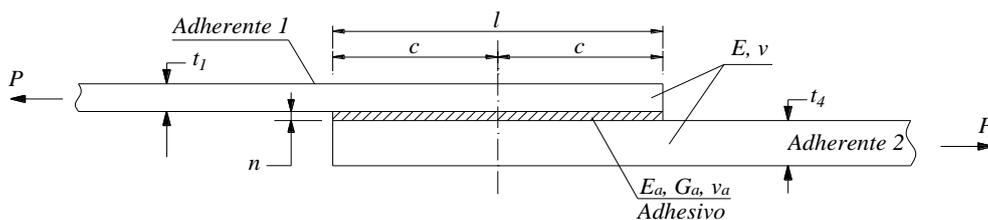
Fig. A 3 Single adhesive joint geometry; Hart-Smith [8].

The moment for a balanced joint generated by the eccentricity of the applied loads is:

$$M_0 = k_1 \cdot P \cdot \frac{t_1}{2} \cdot \left(1 + \frac{n}{t_1}\right)$$

(10)

Being the formula for estimation of k_1 indicated in Table 5.

The parameters λ and λ' for the calculation of the balanced joint shear stress, indicated in Table 6, are as follows:

$$\lambda^2 = \frac{2 \cdot G_a}{E_1 \cdot t_1 \cdot n} \qquad\qquad \lambda'^2 = \left[\frac{1 + 3(1 - v_1^2)/k_b}{4}\right] \cdot \lambda^2$$

(11)

The parameter ξi is used for both balanced and unbalanced adherents:

$$\xi_i^{\,2} = \frac{P}{D_i} = \frac{12 \cdot P \cdot \left(1 - v_i^2\right)}{k_b \cdot E_i \cdot t_i^{\,3}}$$

(12)

The average shear and normal stresses are estimated as follows:

$$\tau_{avg} = \frac{P}{l} \qquad\qquad \sigma_{avg} = \frac{P}{t}$$

(13)

Where t is the weakest adherent thickness, P is the load per unit length, and k_b is the stiffness coefficient of the material, which is 1 for isotropic materials.
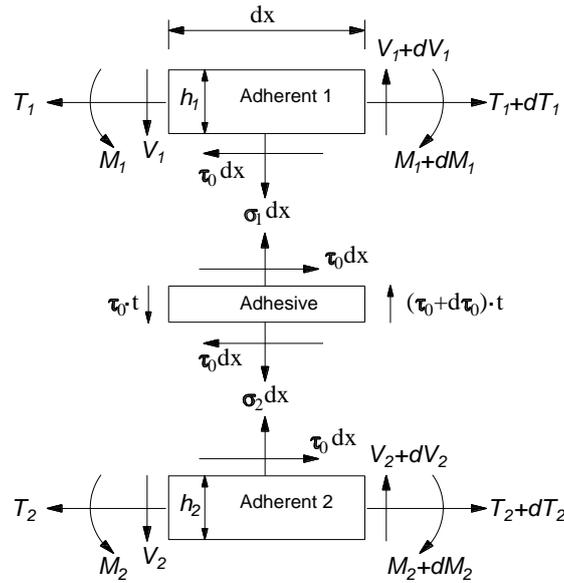
A.1.4 Allman method (1977)
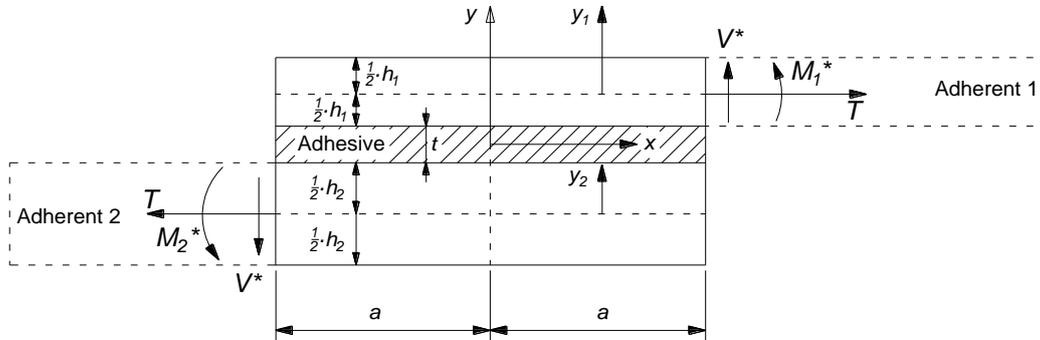
Fig. A 4 Single joint geometry; Allman [9].

The formulas to estimate the shear and normal stress; see Allman [9]

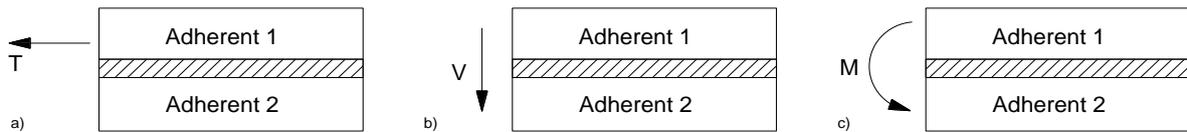A.1.5 Bigwood and Crocombe method (1989)

Fig. A 5 Scheme of loads applied to evaluate the stresses in the adhesive; a) application of tensile stress, b) application of shear stress, c) application of momentum [10].
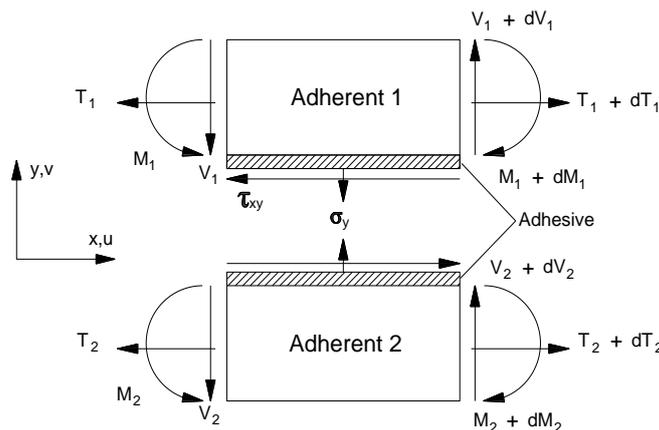

Fig. A6 Diagram of single joining forces; Bigwood [10].

Bigwood [10] simplified the equations of movements based on the consideration that the variations of the shear and normal stresses along the joint are small; the differential equations are:

$$\frac{d^4\sigma_y}{\partial x^4} + 4K_5{}^4\sigma_y = 0$$

(14)

$$\frac{d^3\tau_{xy}}{\partial x^3} - K_6{}^2 \frac{d\tau_{xy}}{\partial x} = 0 \tag{15}$$

Where:

$$K_5{}^4 = \frac{E_a}{4 \cdot t}\left(\frac{1}{D_1} + \frac{1}{D_2}\right) \tag{15a}$$

$$K_6{}^2 = \frac{4 \cdot G_a}{t}\left(\frac{(1-v_1{}^2)}{E_1 \cdot h_1} + \frac{(1-v_2{}^2)}{E_2 \cdot h_2}\right) \tag{15b}$$

Using the following compliance factors for the calculations:

$$\beta_1 = \frac{12 \cdot E_a \cdot \left(1-v_1{}^2\right)}{E_1 \cdot h_1{}^3 \cdot t} \qquad\qquad \beta_2 = \frac{12 \cdot E_a \cdot \left(1-v_2{}^2\right)}{E_2 \cdot h_2{}^3 \cdot t} \tag{16}$$

$$\alpha_1 = \frac{G_a \cdot \left(1-v_1{}^2\right)}{E_1 \cdot h_1 \cdot t} \qquad\qquad \alpha_2 = \frac{G_a \cdot \left(1-v_2{}^2\right)}{E_2 \cdot h_2 \cdot t} \tag{17}$$
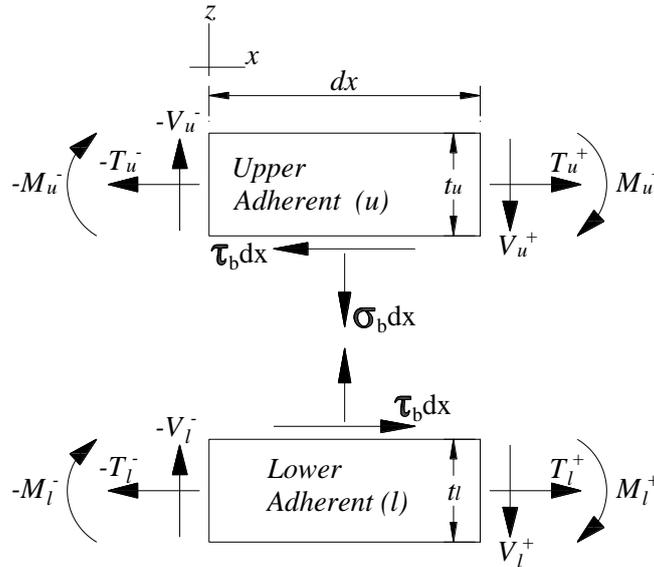
A.1.6 Oplinger method (1991)

Fig. A 7 Single-lap joint geometry; Oplinger [11].

The formulas used to estimate the distribution of the shear stress; see Oplinger [11].

The parameters used to calculate the coefficient k_n, mentioned in Table 5 and enhanced for any thickness of adherents, are:

$$R = \sqrt{\frac{12 \cdot \sigma_x \cdot t_b}{G_b \cdot t}} \qquad\qquad R_1 = \frac{\sqrt{8 \cdot v_1}}{R} \qquad\qquad R_2 = \sqrt{\frac{v_2}{8}} \tag{18a}$$

$$v_1 = a + b \qquad\qquad v_2 = a - b \tag{18b}$$

$$a = 4 \cdot \left(1 + \frac{3}{4} \cdot \rho_t\right) + \frac{R^2}{4} \qquad b = \sqrt{a^2 - R^2}$$

(18c)

$$u_1 = \frac{R_1 \cdot R \cdot \beta}{\sqrt{8}} \qquad u_2 = \sqrt{8} \cdot R_2 \cdot \beta$$

(18d)

$$\beta = \sqrt{\frac{\rho_G}{\rho_t}} \qquad \rho_t = \frac{t_b}{t} \qquad \sigma_x = \frac{T}{t_b}$$

(18e)

with

$$C_1 = \frac{J_1}{64 \cdot J_2} \qquad C_2 = \frac{1}{48 \cdot J_2}$$

(19a)

$$J_1 = \frac{1}{R_2{}^2 - \frac{R^2}{16}} \qquad J_2 = \frac{R_2{}^2}{4R_2{}^2 - 1}$$

(19b)

For adhesive thicknesses much greater than the thickness of the adhesive ($t \gg t_b$), the coefficient k_n can be simplified to:

$$k_n = \frac{1}{1 + \sqrt{8} \cdot \dfrac{T_{h22}}{T_{h1}}}$$

(20)

Where

$$T_{h1} = \tanh(U \cdot \lambda_0) \qquad T_{h21} = \tanh\left(\frac{u_1 \cdot \lambda}{2}\right) \qquad T_{h22} = \tanh\left(\frac{u_2 \cdot \lambda}{2}\right)$$

(21)

The necessary parameters to estimate the effective adhesive length, mentioned in Table 4, are:

$$\Lambda^4 = t_u{}^4 \cdot \frac{K_b}{4} \cdot \left[\frac{1}{D_u} + \frac{1}{D_l}\right]$$

(22a)

$$\lambda^2 = t_u{}^2 \cdot \frac{K_b}{2 \cdot C_v} \cdot \left[\frac{1}{B_u} + \frac{1}{B_l} + \frac{t_u{}^2}{4 \cdot D_u} + \frac{t_l{}^2}{4 \cdot D_l}\right]$$

(22b)

$$K_b = \frac{E_b}{t_b} \qquad C_v = \frac{E_b}{2 \cdot G_b} = 1 + v_b$$

(22c)

$$B_u = \frac{E_u \cdot t_u}{1 - v_u{}^2} \qquad B_l = \frac{E_l \cdot t_l}{1 - v_l{}^2}$$

(22d)

$$D_u = \frac{E_u \cdot t_u^{\,3}}{12 \cdot \left(1 - \nu_u^{\,2}\right)} \qquad\qquad D_l = \frac{E_l \cdot t_l^{\,3}}{12 \cdot \left(1 - \nu_l^{\,2}\right)}$$
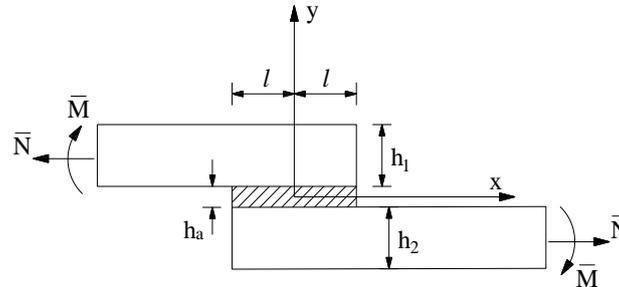
(22e)

A.1.7 Zou method (2004)

Fig. A 8 Single-lap joint diagram proposed by Zou [12]

The formulas used to estimate the shear and normal stress; see Zou [12].

A.2 Implicit Methods

A.2.1 Renton and Vinson method (1973)

Renton and Vinson [13] developed a system of equations that allow the estimation of the adhesive normal shear stresses of a balanced or unbalanced single joint, considering the adherent material as anisotropic composite and the adhesive as isotropic. The methodology is based on the method developed by Goland and Reissner [6], who used the theory of linear behaviour to estimate the loads at the endpoints and solved an ordinary linear differential equation of the eighth order to estimate the shear and normal stresses. In addition, the method performs experimental tests of tension and fatigue to determine the behaviour of the failures in the laminate of the single-lap joint.

Renton and Vinson [13] recommend:
   – To reduce stress peaks, care must be taken to maintain a similar planar stiffness of the adherents.
   – A single joint is more efficient if the elasticity modulus of the adhesive is smaller than the elasticity modulus of the adherents.
   – The adhesive failure is independent of the length of the adhesive and is very little related to the thickness of the adhesive.
   – The strength of the joint can be improved by increasing the thickness of the adhesive at its ends.

A.2.2 Ojalvo method (1977)

Ojalvo [14] focused on analysing the influence of the adhesive thickness on the estimation of the shear and normal stresses. His research was based on the approach of Goland and Reissner, but he modified the differential equation and used three assumptions related to the behaviour of the single joint to define the methodology. Finally, he concluded that the thickness of the adhesive is important in the estimation of the stresses, mainly for the maximum values that are generated at its ends because when the effect of the adhesive thickness is included in the calculations, the shear stress increases and the normal stress decreases.

A.2.3 Delale method (1981)

Delale [15] developed a methodology for single-lap joints of the balanced adherents. This methodology is applied for linear-elastic analyses, considering [15]:

– The adherents are an orthotropic plate material and for their analysis, the transversal shear stresses are used.

– The adhesive is a linear-elastic material.

– The stress variation in the adhesive thickness direction is negligible.

– The deformations in the z-direction of adhesive are zero, and only coplanar deformations are considered.

A.2.4 Adams and Mallick method (1992)

Adams and Mallick [16] analysed a single joint subjected to thermal stress loads. This methodology is applied to non-balance adherents, in which the adhesive is considered as a unidirectional anisotropic material for non-linear analysis. The adherents are analysed as flexural plates, while the adhesive is a series of tension and shear springs. Beginning with the theory of two-dimensional elasticity, these authors developed implicit formulas for calculating tensile and normal stresses in the upper and lower parts of the adhesive. These formulas include the terms for the effects of bending, shear, and hydrothermal deformation in the adherent and adhesive.

A.2.5 Tong method (1996)

Tong [17] assumed in his investigation that the adhesive has a non-linear stress-strain behaviour while the adherents have linear-elastic behaviour. Normal and shear deformations in the adhesive are constant through the adhesive thickness. The adherent-adhesive-adherent sandwich model is used to predict the strength of the joint only for balanced adherents. This author also explains that the product of the deformation energy density and the thickness of the adhesive is equal to the energy release rate for fracture failure modes.

A.2.6 Smeltzer method (2003)

The method proposed by Smeltzer [18] allows an evaluation of the distribution of normal and shear forces along with the adhesive. In its analysis, this method considers the adherent plates as anisotropic and elastic-linear and the adhesive as isotropic nonlinear, elastic and plastic, behaving in a cylindrical form under a condition of flat deformation. This author presented both linear and non-linear examples and compared the results of his method to those of Goland and Reissner [12] and Bigwood and Crocombe [10] [30], obtaining lower maximum normal and shear stresses.

A.3 Hull lengthening study case

Table A 1 Weights, centroids, and location for the calculation of still water moment

		Longitudinal			Transversal	Vertical
Item Name	Weight	Arm	Aft Limit	Fwd Limit	Arm	Arm
	ton	m	m	m	m	m
Lightship	40.577	-2.004	-2.004	-2.004	0	2.609
Passengers + crew	1.35	-2.000	-2.000	-2.000	0	3.08
Fix. Ballast	0	-4.085	-4.085	-4.085	0	-0.024
Tank grey water	1.719	2.418			0	0.531
Tank DO PS	3.131	-4.015			-1.257	1.195
Tank DO SB	3.131	-4.015			1.257	1.195
Tank FW Aft PS-Ballast	1.554	-12.743			-1.391	1.013
Tank FW Aft SB-Ballast	1.554	-12.743			1.391	1.013
Tank Black & Grey empty	0	6.041			0	0.047
Tank FW PS	2.854	-10.529			-1.437	0.968
Tank FW SB	2.854	-10.529			1.437	0.968
Tank black water	1.877	-1.375			0	0.509
TOTAL	**60.601**	**-3.420**			**0**	**2.113**
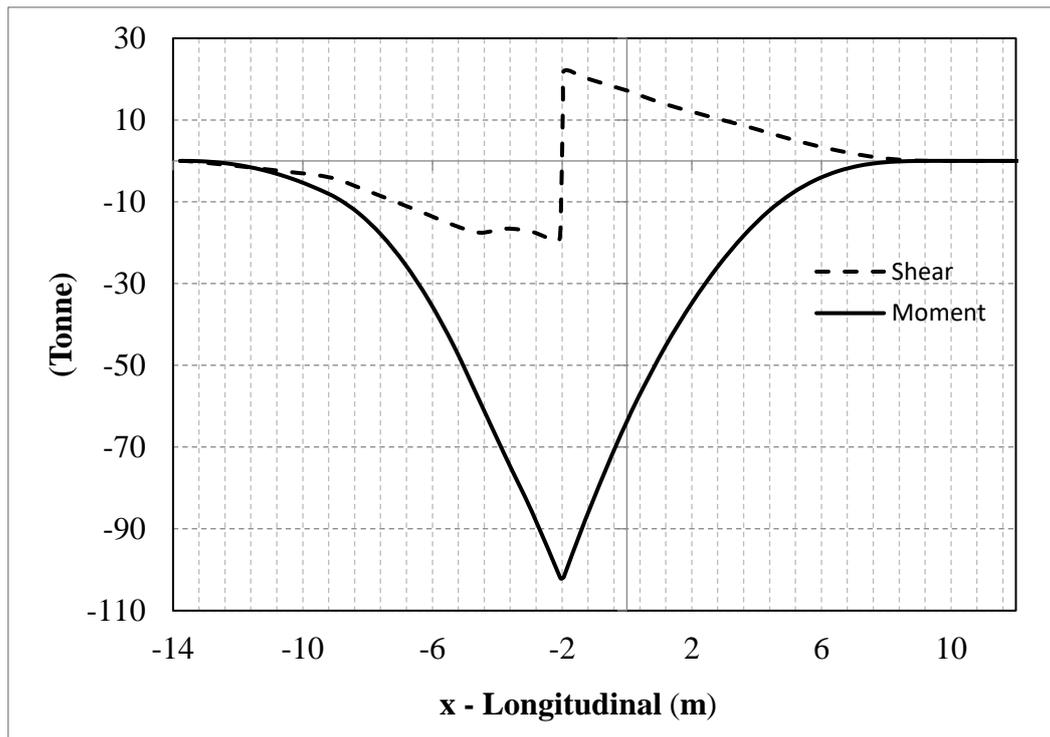
Fig. A 9: Moment and shear stress on still water for the diving ship