

# An Overview and Efficiency Analysis of Dominant Frequencies Estimation Methods with the Application of DIRECT Global Minimization Method

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**Abstract:** This report discusses the application of the following methods that are used to calculate maximum frequency of a spectrum: parabolic, modified parabolic by Rife and Vincent, the method according to Quinn from 1994 and the method according to Quinn from 1997. After introducing DIRECT global minimization method, the results obtained using previously mentioned methods are compared with the results obtained using DIRECT global minimization of the Lipschitz function. Determining vibration frequency and comparing to previously collected data or knowledge base data is a common procedure. In engineering practice it is often necessary to determine not one, but several so-called dominant frequencies or local maxima; the procedure is sometimes called multi peak detection and here we propose an algorithm for computerised multi peak detection.

**Keywords:** DFT, Discrete Fourier transformation, DIRECT global minimization, dominant frequency, Lipschitz, multi peak detection, spectral analysis, vibration

## 1 INTRODUCTION

Monitoring vibration levels is an effective method of forecasting and detecting mechanical failures in rotating components of mechanical systems. Measurements are carried out periodically using hand held portable devices or continuously by means of stationary systems. Measured vibration level may indicate irregularities caused by incorrect assembly even before the occurrence of failure [1]. Ageing of system mechanical components, appearance of wear and minor damage will increase the level of amplitude of certain frequencies; therefore first step in fault detection is interpretation of the measured amplitude and frequency of vibration. Measured data can be used to determine the condition of a mechanical component in exploitation, predict the occurrence of damage and identify type of fault based on the analysis of measured vibration. In cases where there are no specially adapted stationary systems that monitor vibrations of equipment, maintenance staff often uses a visual method to compare measured data and knowledge base that contains spectrum forms of vibrations caused by various damages, classified by type of rotating components and the type of damage.

During visual comparison it is necessary to note and determine any dominant frequencies and their amplitudes. The measured data in time domain is converted to the frequency domain, using Discrete Fourier transform i.e. frequency spectrum is obtained from amplitudes measured in a given period of time using the discrete Fourier transform.

Values of the dominant frequency, phase and amplitude are obtained from the spectrum using one of the known algorithms; see for example [2, 3].

## 2 INITIAL ASSUMPTIONS

Assume measuring vibration of the electro motor housing, which occurs as a result of its operation. Let  $(t_n, y_n), n = 0, \dots, m$  be measured data, while  $t_n$  is time range, (usually equidistant range of values) on  $y_n$  corresponding measured displacement values of a fixed point on an operating motor housing, directed as one unit vector.

Let  $L > 0, m \in \mathbb{N}$  and  $\Delta t = \frac{L}{m}$ . Assume measured data  $(t_n, y_n), n = 0, \dots, m-1$ , can be written

$$t_n = n\Delta t \tag{1}$$

$$y_n = a \cos(2\pi f n \Delta t + \varphi) + \varepsilon_n \tag{2}$$

$$n = 0, \dots, m-1 \tag{3}$$

while  $a, f > 0, \varphi \in [0, 2\pi)$  are unknown parameters, and  $\varepsilon_n \sim N(0, \sigma^2)$  (normally distributed random error with expectation 0 and variance  $\sigma^2$ ). Additionally assume  $m \gg 2f \cdot L$ . Parameter  $f$  is named **dominant frequency**, parameter  $a$  **amplitude** and  $\varphi$  **phase**.

Let  $A_w : \left[0, \frac{1}{2\Delta t}\right] \rightarrow \mathbb{R}$  be function defined as follows

$$A_w = \frac{1}{m} \left| \sum_{n=0}^{m-1} w_n y_n \varepsilon^{-2\pi i f \Delta t} \right|^2 \tag{4}$$

$$A_w = \frac{1}{m} \left( \left( \sum_{n=0}^{m-1} w_n y_n \cos(2\pi f n \Delta t) \right)^2 + \left( \sum_{n=0}^{m-1} w_n y_n \sin(2\pi f n \Delta t) \right)^2 \right) \tag{5}$$

where  $w_n = w(n\Delta t)$ , and  $w : [0, L] \rightarrow [0, +\infty)$  is window function for which applies  $w(t) > 0, t \in [0, L]$  and  $w(t) = 0, t \notin [0, L]$ . Function  $A_w$  is known in the literature as the periodogram [4] with window function  $w$ , assigned to data  $(t_n, y_n), n = 0, \dots, m-1$ .

We observe the following optimization problem

$$f_w^* = \arg \max A_w(f) \tag{6}$$

Assuming  $m$  is large enough, global maximum  $f_w^*$  of function  $A_w$  on interval  $\left[0, \frac{1}{2\Delta t}\right]$  is used as estimator for

dominant frequency. While amplitude  $a_w^*$  and phase  $\varphi_w^*$  can be determined from the following expressions [5]:

$$a_w^* \cos \varphi_w^* = \frac{2}{m} \sum_{n=0}^{m-1} w_n y_n \cos 2\pi f_w^* n \Delta t, \tag{7}$$

$$a_w^* \sin \varphi_w^* = \frac{2}{m} \sum_{n=0}^{m-1} w_n y_n \sin 2\pi f_w^* n \Delta t. \tag{8}$$

Usually window function is a rectangular window [6] defined as

$$w_R(t) = \begin{cases} 1, & t \in [0, L] \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

in vibration analysis is often used Hanning window [6], defined as

$$w_H(t) = \begin{cases} \frac{1}{2} \left( 1 - \cos \frac{2\pi}{(m-1)\Delta t} t \right), & t \in [0, L] \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

Figure shows given data at  $(t_n, y_n), n = 0, \dots, m$  generated based on parameter  $a = 11, f = 4, \varphi = \frac{\pi}{2}, m = 40, L = \pi$  and  $\sigma = 0.5$ , effect of window function on given data and corresponding periodogram.

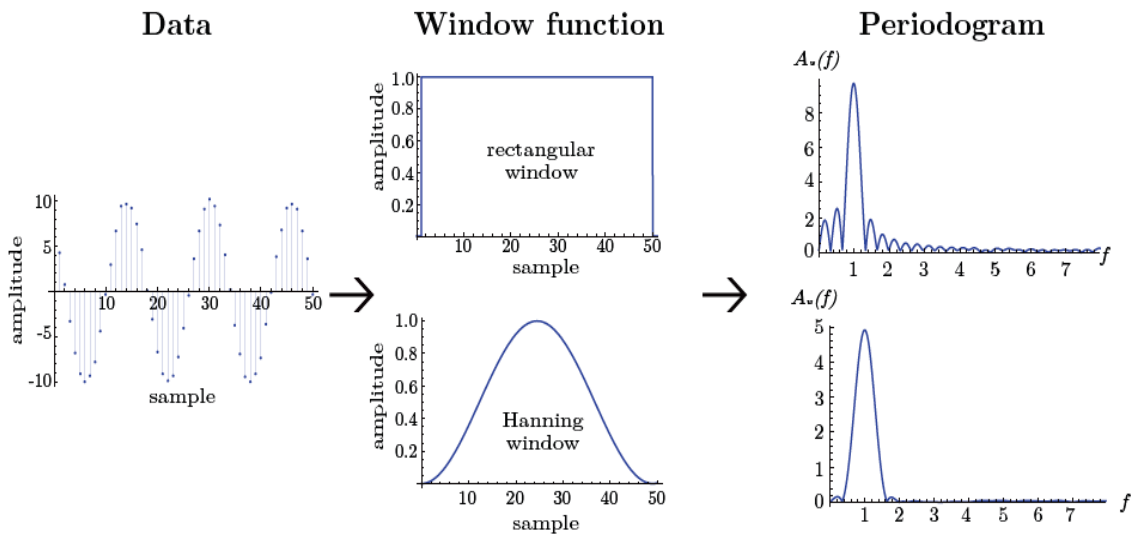


Figure 1 Filtrating data in time domain and transformation of periodogram

From numerical aspect the problem (6) is extremely complex as the function  $A_w$  has a large number of local extrema, and this number is growing by increasing data quantity  $m$ . In the literature, various procedures can be found on local approximation of periodogram  $A_w$  aiming to find a global maximum of the function  $A_w$ . In this article overview of some such functions is given and using numerical experiment their efficiency is examined, Furthermore it is shown that function  $A_w$  is a Lipschitz class function and for the solution of problem (6) can be used DIRECT method for global optimization of Lipschitz functions [7-9]. Such obtained estimator is compared to other methods estimators and its efficiency is determined. Also in order to hasten estimation process DIRECT method is combined with a local method to narrow maximization scope and it will be determined if results can be enhanced comparing the DIRECT method with the mentioned local approximation methods. Finally, based on the numerical experiment findings the method for determining dominant frequencies will be proposed.

### 3 OVERVIEW OF LOCAL APPROXIMATION METHODS FOR THE DETERMINATION OF DOMINANT FREQUENCIES

Within this section an overview will be given of several methods used to determine dominant frequencies based on periodogram approximation.

Assuming given measurement data is  $(t_n, y_n), n = 0, \dots, m - 1$ , window function, corresponding periodogram  $A_w$ , let us define function  $I_w : \left[ 0, \frac{1}{2\Delta t} \right] \rightarrow R$  such that

$$I_w(f) = \sqrt{\frac{1}{m} A_w(f)} \tag{11}$$

Notice

$$f_w^* = \arg \max_{f \in \left[ 0, \frac{1}{2\Delta t} \right]} I_w(f) = \arg \max_{f \in \left[ 0, \frac{1}{2\Delta t} \right]} A_w(f) \tag{12}$$

Function  $I_w$  will be called modified periodogram.

#### 3.1 Discretization of Modified Periodogram

We consider values of modified periodogram  $I_w(f_k)$  on the discrete set of values  $f_k = \frac{k}{m\Delta t}, k = 0, \dots, \left[ \frac{m-1}{2} \right]$ .

Notice:

$$I_w(f_k) = \frac{1}{m} \left| \sum_{n=0}^{m-1} w_n y_n \varepsilon^{-2\pi n i \frac{k}{m\Delta t} \Delta t} \right|, \tag{13}$$

$$I_w(f_k) = \frac{1}{m} \left| \sum_{n=0}^{m-1} w_n y_n \varepsilon^{\frac{-2nk\pi i}{m}} \right|, \tag{14}$$

$$I_w(f_k) = \left| F[w_0 y_0, \dots, w_{m-1} y_{m-1}] \right|, \tag{15}$$

while  $F[w_0 y_0, \dots, w_{m-1} y_{m-1}]$  is discrete Fourier transformation of range  $\{w_0 y_0, \dots, w_{m-1} y_{m-1}\}$  [10].

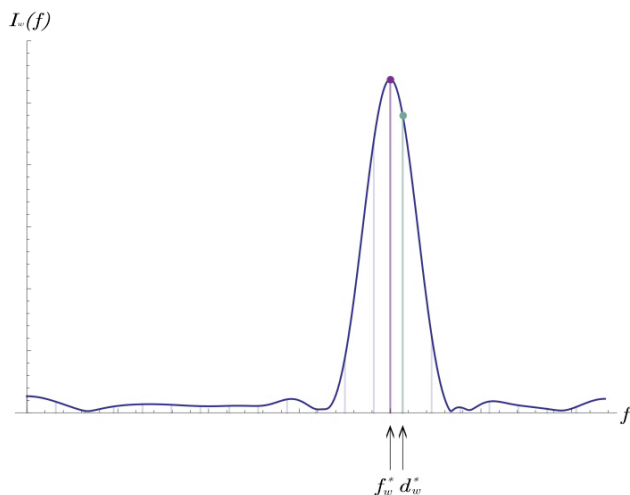


Figure 2 Discretized periodogram

Let  $k^* \in \left\{ 0, \dots, \left[ \frac{m-1}{2} \right] \right\}$  index such that

$$I_w(f_{k^*}) = \max_{n=0, \dots, \left[ \frac{m-1}{2} \right]} I_w(f_k) \tag{16}$$

$$f_K = \frac{k}{m\Delta t}, \quad k = 0, \dots, \left[ \frac{m-1}{2} \right]$$

According to this approach maximization problem of real function of real variable is reduced to maximization problem of discrete set of values and therefore for approximation of global maximum  $f_w^*$  of function, is adopted

$$d_w^* = f_{k^*} = k^* \frac{1}{m\Delta t} \tag{17}$$

Approximation of amplitude and matching phase is obtained from system of Eq. (7) and (8). While it is fair to expect that such an approximation will not be good enough (Fig. 2), point  $d_w^*$  is useful while global maximum  $f_w^*$  of function  $I_w$  can be found in proximity of  $d_w^* = f_{k^*}$  and is therefore meaningful in search of approximation of global maximum point of function  $I_w$  in the form of

$$(k^* + \delta) \frac{1}{m\Delta t}, \quad \delta \in \left\langle -\frac{1}{2}, \frac{1}{2} \right\rangle \tag{18}$$

Whole class of methods is based on this approach, some of them described in the text below.

In Fig. 2 the discreet periodogram is given and values of global maximum  $f_w^*$  and its approximation  $d_w^*$  are shown.

### 3.2 Parabola method

In order to determine dominant frequency, parabola method is proposed in articles [11] and [12], described in this chapter.

Let  $k \in \left\{ 0, \dots, \left[ \frac{m-1}{2} \right] \right\}$  be index for which it applies

(16). Within proximity of point  $f_{k^*} = \frac{k^*}{m\Delta t}$ , function  $I_w$  is

approximated through three points:

$$\left( \frac{k^*-1}{m\Delta t}, I_w(f_{k^*-1}) \right), \left( \frac{k^*}{m\Delta t}, I_w(f_{k^*}) \right), \left( \frac{k^*+1}{m\Delta t}, I_w(f_{k^*+1}) \right),$$

which equation is

$$P(x) = ax^2 + bx + c \tag{19}$$

where it is:

$$a = \frac{1}{2} (I_w(f_{k^*-1}) - 2I_w(f_{k^*}) + I_w(f_{k^*+1})) (m\Delta t)^2$$

$$b = -\frac{1}{2} ((1 + 2k^*)I_w(f_{k^*-1}) - 4k^*I_w(f_{k^*}) - (1 - 2k^*)I_w(f_{k^*+1})) m\Delta t$$

$$c = \frac{1}{2} (k^*(k^*+1)I_w(f_{k^*-1}) - 2((k^*)^2 - 1)I_w(f_{k^*}) + (k^*-1)I_w(f_{k^*+1}))$$

is  $I_w(f_{k^*}) \geq I_w(f_k)$  for each  $k \in \left\{ 0, \dots, \left[ \frac{m-1}{2} \right] \right\}$ ,

$$\text{therefore } I_w(f_{k^*-1}) - 2I_w(f_{k^*}) + I_w(f_{k^*+1}) = (I_w(f_{k^*-1}) - I_w(f_{k^*})) + (I_w(f_{k^*+1}) - I_w(f_{k^*})) < 0$$

namely  $a < 0$ . Thereby quadratic function  $P$  shall reach its maximum in point

$$q_w^* = -\frac{b}{2a} = \left( k^* + \frac{I_w(f_{k^*-1}) - I_w(f_{k^*+1})}{2I_w(f_{k^*-1}) - 4I_w(f_{k^*}) + 2I_w(f_{k^*+1})} \right) \cdot \frac{1}{m\Delta t} \tag{20}$$

and to approximate global maximum point  $f_w^*$  of function  $I_w$ , according to this approach we take point  $q_w^*$ . Aware of parabola method approximation  $q_w^*$ , global maximum of function  $I_w$  is in form (18) while

$$\delta = \frac{I_w(f_{k^*-1}) - I_w(f_{k^*+1})}{2I_w(f_{k^*-1}) - 4I_w(f_{k^*}) + 2I_w(f_{k^*+1})} \in \left\langle -\frac{1}{2}, \frac{1}{2} \right\rangle \tag{21}$$

Approximation of global maximum using parabola method is shown in Fig. 3, parabola was drawn through

three points, the maximum of discretized function and two neighbouring points, and approximation  $q_w^*$  is shown.

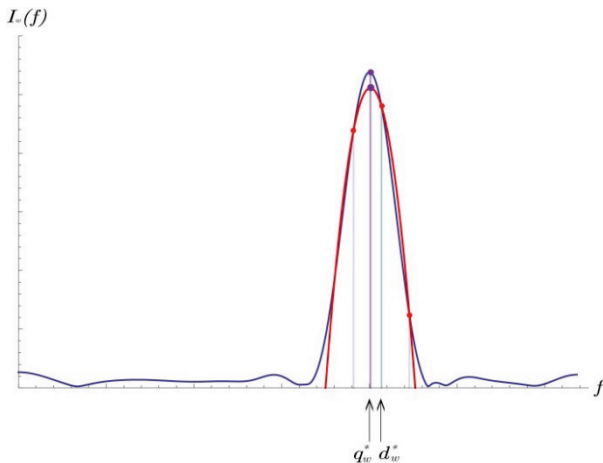


Figure 3 Parabola method

Approximations for amplitude and matching phase shift are obtained from equation system (7) and (8).

### 3.3 Modified Parabola Method

In works [13] and [14] another global maximum approximation method based on approximation of modified periodogram in case when  $m$  is large enough is suggested. Below the method is formulated in analogy to previous chapter.

Let's have  $k \in \left\{0, \dots, \left\lfloor \frac{m-1}{2} \right\rfloor\right\}$  such that it applies (16).

In proximity of point  $f_{k^*} = \frac{k^*}{m\Delta t}$  function  $I_w$  is also approximated by parabola through three points, while detecting two cases:

- If it applies  $I_w(f_{k^*+1}) > I_w(f_{k^*-1})$ , observe parabola  $P(x) = ax^2 + bx + c$  through points

$$\left( \frac{k^*}{m\Delta t}, I_w(f_{k^*}) \right), \left( \frac{k^* + \frac{1}{2}}{m\Delta t}, \frac{3}{4}(I_w(f_{k^*}) + I_w(f_{k^*+1})) \right), \left( \frac{k^* + 1}{m\Delta t}, I_w(f_{k^*+1}) \right) \quad (22)$$

$$\left( \frac{k^* + 1}{m\Delta t}, I_w(f_{k^*+1}) \right)$$

for which applies:

$$a = -(I_w(f_{k^*+1}) + I_w(f_{k^*})) (m\Delta t)^2 \quad (23)$$

$$b = 2(k^* I_w(f_{k^*}) + (1 + k^*) I_w(f_{k^*+1})) m\Delta t \quad (24)$$

$$c = -((k^*)^2 - 1) I_w(f_{k^*}) - k^*(k^* + 2) I_w(f_{k^*+1}) \quad (25)$$

Note that  $a < 0$  and the quadratic function  $P$  reaches maximum at point

$$v_w^* = -\frac{b}{2a} = \left( k^* + \frac{I_w(f_{k^*+1})}{I_w(f_{k^*}) + I_w(f_{k^*+1})} \right) \frac{1}{m\Delta t} \quad (26)$$

$$= (k^* + \delta) \frac{1}{m\Delta t}$$

where it is

$$\delta = \frac{I_w(f_{k^*+1})}{I_w(f_{k^*}) + I_w(f_{k^*+1})} \in \left\langle -\frac{1}{2}, \frac{1}{2} \right\rangle. \quad (27)$$

- If it applies  $I_w(f_{k^*+1}) < I_w(f_{k^*-1})$ , observe parabola  $P(x) = ax^2 + bx + c$  matching points

$$\left( \frac{k^* - 1}{m\Delta t}, I_w(f_{k^*}) \right), \left( \frac{k^* - \frac{1}{2}}{m\Delta t}, \frac{3}{4}(I_w(f_{k^*-1}) + I_w(f_{k^*})) \right), \left( \frac{k^*}{m\Delta t}, I_w(f_{k^*}) \right) \quad (28)$$

$$\left( \frac{k^*}{m\Delta t}, I_w(f_{k^*}) \right)$$

where it applies

$$a = -(I_w(f_{k^*-1}) + I_w(f_{k^*})) (m\Delta t)^2 \quad (29)$$

$$b = 2(k^* I_w(f_{k^*}) + (k^* - 1) I_w(f_{k^*-1})) m\Delta t \quad (30)$$

$$c = -((k^*)^2 - 1) I_w(f_{k^*}) - k^*(k^* - 2) I_w(f_{k^*-1}) \quad (31)$$

$$v_w^* = -\frac{b}{2a} = \left( k^* - \frac{I_w(f_{k^*-1})}{I_w(f_{k^*}) + I_w(f_{k^*-1})} \right) \frac{1}{m\Delta t}$$

$$= (k^* + \delta) \frac{1}{m\Delta t}$$

where it is:

$$\delta = -\frac{I_w(f_{k^*-1})}{I_w(f_{k^*}) + I_w(f_{k^*-1})} \in \left\langle -\frac{1}{2}, \frac{1}{2} \right\rangle$$

Finally for approximation of global maximum point  $f_w^*$  of function  $I_w$ , based on this approach we take point

$$v_w^* = (k^* + \delta) \frac{1}{m\Delta t},$$

where it is

$$\delta = \begin{cases} \frac{I_w(f_{k^*+1})}{I_w(f_{k^*}) + I_w(f_{k^*+1})}, & I_w(f_{k^*+1}) > I_w(f_{k^*-1}) \\ -\frac{I_w(f_{k^*-1})}{I_w(f_{k^*}) + I_w(f_{k^*-1})}, & I_w(f_{k^*+1}) \leq I_w(f_{k^*-1}) \end{cases}$$

$$\delta = \begin{cases} \frac{1}{\frac{I_w(f_{k^*})}{I_w(f_{k^*+1})} + 1}, & \frac{I_w(f_{k^*+1})}{I_w(f_{k^*-1})} > 1 \\ -\frac{1}{\frac{I_w(f_{k^*})}{I_w(f_{k^*-1})} + 1}, & \frac{I_w(f_{k^*+1})}{I_w(f_{k^*-1})} \leq 1 \end{cases}$$

Approximations for amplitude and matching phase shift are obtained from equation system (7) and (8).

### 3.4 Quinn's Period Maximization Method

Two more effective methods for determination of dominant frequencies are suggested in articles [14] and [15]. Without step by step formulation and additional analysis we list these two methods.

$$\delta = \begin{cases} \frac{I_w(f_{k^{*+1}})}{I_w(f_{k^*}) + I_w(f_{k^{*+1}})} \\ \frac{\text{sign}\left(\text{Re}\left(Y_w(f_{k^{*-1}})\overline{Y_w(f_{k^*})}\right)\right)I_w(f_{k^{*-1}})}{I_w(f_{k^*}) - \text{sign}\left(\text{Re}\left(Y_w(f_{k^{*-1}})\overline{Y_w(f_{k^*})}\right)\right)I_w(f_{k^{*-1}})} \end{cases}$$

**Method Quinn 1994** [15]: dominant frequency approximation is

$$\beta_w^* = (k^* + \delta) \frac{1}{m\Delta t} \tag{33}$$

where it applies

$$\delta = \begin{cases} \frac{I_w(f_{k^{*+1}})}{I_w(f_{k^{*+1}}) - I_w(f_{k^*})} > 0 \& \frac{I_w(f_{k^{*+1}})}{I_w(f_{k^{*+1}}) - I_w(f_{k^*})} > 0 \& \\ \frac{I_w(f_{k^{*-1}})}{I_w(f_{k^*}) - I_w(f_{k^{*-1}})} > 0 \\ \text{otherwise} \end{cases} \tag{34}$$

Approximations for amplitude and matching phase shift are obtained from equation system (7) and (8).

## 4 GLOBAL METHOD FOR DOMINANT FREQUENCY ESTIMATION

Function  $A_w$  maximization problem is equal to the following function minimization problem

$$\min_{f \in \left[0, \frac{1}{2\Delta t}\right]} -A_w(f) \tag{35}$$

It is shown that the function  $-A_w$  is Lipschitz function at  $\mathbb{R}$ , namely there is constant  $L > 0$  such that for  $\alpha, \beta \in \mathbb{R}$  applies

$$|A_w(\beta) - A_w(\alpha)| \leq L|\beta - \alpha| \tag{36}$$

**Proposition.** Function  $-A_w : \left\langle 0, \frac{1}{2\Delta t} \right\rangle \rightarrow \mathbb{R}$  is Lipschitz at  $\mathbb{R}$ .

**Proof.** Since  $-A_w$  is derivable at  $\mathbb{R}$  it is sufficient to show

Let it be  $Y_w(f) = \frac{1}{m} \sum_{n=0}^{m-1} w_n y_n e^{-2n\pi i f \Delta t}$ , notice that it is

$$I_w(f) = |Y_w(f)|.$$

**Method Quinn 1997** [14]: dominant frequency approximation is

$$\beta_w^* = (k^* + \delta) \frac{1}{m\Delta t}$$

where it applies

$$\begin{aligned} &\text{sign}\left(\text{Re}\left(Y_w(f_{k^{*+1}})\overline{Y_w(f_{k^*})}\right)\right) = -1 \& \\ &\text{sign}\left(\text{Re}\left(Y_w(f_{k^{*-1}})\overline{Y_w(f_{k^*})}\right)\right) = 1 \\ &\text{otherwise} \end{aligned} \tag{32}$$

$$\begin{aligned} |A'_w(f)| &= \frac{1}{m} \left| -4\pi\Delta t \sum_{n=0}^{m-1} w_n y_n \cos(2\pi f n \Delta t) \right. \\ &\quad \left. \sum_{n=0}^{m-1} n w_n y_n \sin(2\pi f n \Delta t) + \right. \\ &\quad \left. + 4\pi\Delta t \sum_{n=0}^{m-1} w_n y_n \sin(2\pi f n \Delta t) \sum_{n=0}^{m-1} n w_n y_n \cos(2\pi f n \Delta t) \right| \\ &\leq \frac{8\pi\Delta t}{m} \sum_{n=0}^{m-1} m w_n |y_n| \sum_{n=0}^{m-1} w_n |y_n| =: L \end{aligned} \tag{37}$$

it follows  $\alpha, \beta \in \mathbb{R}$  exists  $\zeta \in \langle \alpha, \beta \rangle$  such that

$$|A_w(\beta) - A_w(\alpha)| = |A_w(\zeta)| |\beta - \alpha| \leq L |\beta - \alpha| \tag{38}$$

namely the function  $-A_w$  is Lipschitz function.

As function  $-A_w$  is Lipschitz class function at  $\mathbb{R}$ , it

follows that it's Lipschitz at  $\left[0, \frac{1}{2\Delta t}\right]$  and accordingly

problem (35) can be solved applying DIRECT method for global minimization of Lipschitz function [7, 8, 9, 16].

Instead of minimization of function  $-A_w$  at whole interval,

and applying DIRECT method function at function  $-A_w$ ,

it is meaningful to minimize function only in proximity of

point  $f_{k^*} = \frac{k^*}{m\Delta t}$ , where it is  $k^* \in \left\{0, \dots, \left\lceil \frac{m-1}{2} \right\rceil\right\}$  such

that it applies (16). Motivated by (18) function  $-A_w$  is

minimized on interval  $\left[f_{k^*} - \frac{1}{2m\Delta t}, f_{k^*} + \frac{1}{2m\Delta t}\right]$ , namely

instead of (35) we solve minimization problem on smaller domain.

$$\min_{f \in \left[f_{k^*} - \frac{1}{2m\Delta t}, f_{k^*} + \frac{1}{2m\Delta t}\right]} -A_w(f) \tag{39}$$

### 5 DETERMINATION OF REMAINING DOMINANT FREQUENCIES - MULTYPEAK DETECTION

To determine remaining subsequent dominant frequencies by filtering global maximum in this article is used inverse Fourier transform, can also be used digital filter as shown in [17].

Several methods are described in above sections which can be used to estimate dominant frequency or global maximum, and afterwards using equation system (7) and (8) to obtain amplitude and phase shift.

There are several methods for multiple peak detection, depending on the signal origin and physical meaning eg. [18-20]. Also online can be found many solutions for multiple peak detection; some examples are given in [21, 22].

Here the authors propose the following algorithm to estimate remaining dominant frequencies and their matching amplitudes:

#### 5.1 Dominant frequencies determination algorithm

1. Assign

i)  $L > 0, m \in \mathbb{N}, \Delta t = \frac{L}{m}$ , data  $(t_n, y_n)$ ,  $n = 0, \dots, m-1$

ii) window function  $w: [0, L] \rightarrow [0, \infty)$

iii) number of obvious dominant frequencies

2. Determine dominant frequency  $f_w^*(1)$  using one of the methods described in sections 3 and 4 and matching amplitude  $a_w^*(1)$  and phase shift  $\varphi_w^*(1)$  based on system of Eq. (7) and (8).

3. for  $i = 2$  to  $\kappa$

Redefine data  $(t_n, x_n), n = 0, \dots, m-1$  where it applies

$$y_n = y_n - a_w^*(i-1) \cos(2\pi f_w^*(i-1)n\Delta t + \varphi_w^*(i-1))$$

end

4. Write

i) dominant frequencies  $f_w^*(j), j = 1, \dots, \kappa$

ii) amplitudes:  $a_w^*(j), j = 1, \dots, \kappa$

iii) phase shifts:  $\varphi_w^*(j), j = 1, \dots, \kappa$

### 6 NUMERICAL EXPERIMENT

In following sections are given results of two numerical experiment examples. Experiment was done using numerical procedures.

#### 6.1 Example 1

Let us state  $L > 0, m \in \mathbb{N}, \Delta t = \frac{L}{m}$ , data  $(t_n, y_n)$ ,  $n = 0, \dots, m-1$  is generated such that it applies

$$t_n = n\Delta t, y_n = 11 \cos(2\pi \cdot 1n\Delta t + 1.1) + \varepsilon_n$$

where it is  $\varepsilon_n \sim N(0, \sigma^2)$ .

Let it be  $m = 50$  and  $\sigma \in \{0.1, 0.5, 0.9, 1.7, 2.1, 2.5\}$  for some random set of input data using the above described methods: Discrete Fourier transformation – discretization of periodogram (DFT), Parabola method 1 (P1), Modified parabola method (P2), Method Quinn 1997 (Q97), Method Quinn 1994 (Q94), Global method (GM) we estimate frequency  $f_j$  as well as matching amplitude  $a_j$  and phase shift  $\varphi_j$ .

For all methods reconstructed signal will be compared to the original signal. Results of frequency, amplitude and phase are given in Tabs. 1, 2 and 3 for rectangular window function and Tabs. 4, 5, 6 for Hanning window function.

Table 1 Results of frequencies for rectangular window function

	discreet	parabola	parabola 2	Quinn 97	Quinn 94	Lipschitz
Sigma = 0.1	0.00203134	0.00127251	0.0000288497	0.0000257985	0.0000288497	0.0000005725
Sigma = 0.5	0.00203134	0.00126982	0.0000359473	0.0000318505	0.0000359473	0.0000024904
Sigma = 0.9	0.240226	0.00127108	0.0000501181	0.0000436677	0.0000501181	0.00000699837
Sigma = 1.3	0.764254	0.00127682	0.0000713641	0.000061164	0.0000713641	0.0000141129
Sigma = 1.7	1.05009	0.00128762	0.000180505	0.0000842825	0.0000996327	0.0000238564
Sigma = 2.1	1.193	0.00130371	0.000551392	0.000133457	0.000162137	0.0000362577
Sigma = 2.5	1.24064	0.00132435	0.00100511	0.000194253	0.00026484	0.000051352

Table 2 Results of amplitudes for rectangular window function

	discreet	parabola	parabola 2	Quinn 97	Quinn 94	Lipschitz
Sigma = 0.1	0.398648	0.24326	0.142554	0.141606	0.142554	0.130874
Sigma = 0.5	0.400135	0.246319	0.147337	0.146143	0.147337	0.13522
Sigma = 0.9	0.403327	0.26292	0.164232	0.162515	0.164232	0.151555
Sigma = 1.3	0.400706	0.293156	0.193195	0.190721	0.193195	0.179891
Sigma = 1.7	0.390318	0.337149	0.257849	0.230771	0.234196	0.220247
Sigma = 2.1	0.382874	0.395006	0.386669	0.282555	0.287539	0.272642
Sigma = 2.5	0.388103	0.466728	0.567863	0.349037	0.362886	0.337097

Table 3 Results of phase for rectangular window function

	discreet	parabola	parabola 2	Quinn 97	Quinn 94	Lipshitz
Sigma = 0.1	0.312287	0.15167	0.00480191	0.00441043	0.00480191	0.000148262
Sigma = 0.5	0.312864	0.151927	0.00560693	0.00510285	0.00560693	0.0004413
Sigma = 0.9	0.331657	0.152792	0.00727609	0.0065192	0.00727609	0.00113917
Sigma = 1.3	0.370176	0.154322	0.00980576	0.0086512	0.00980576	0.00224395
Sigma = 1.7	0.393201	0.156578	0.0210787	0.0114934	0.0131873	0.00375796
Sigma = 2.1	0.403089	0.159581	0.059402	0.01707	0.0200951	0.00568383
Sigma = 2.5	0.408133	0.163253	0.10395	0.0239329	0.0313072	0.00802452

**Table 4** Results of frequencies for Hanning window function

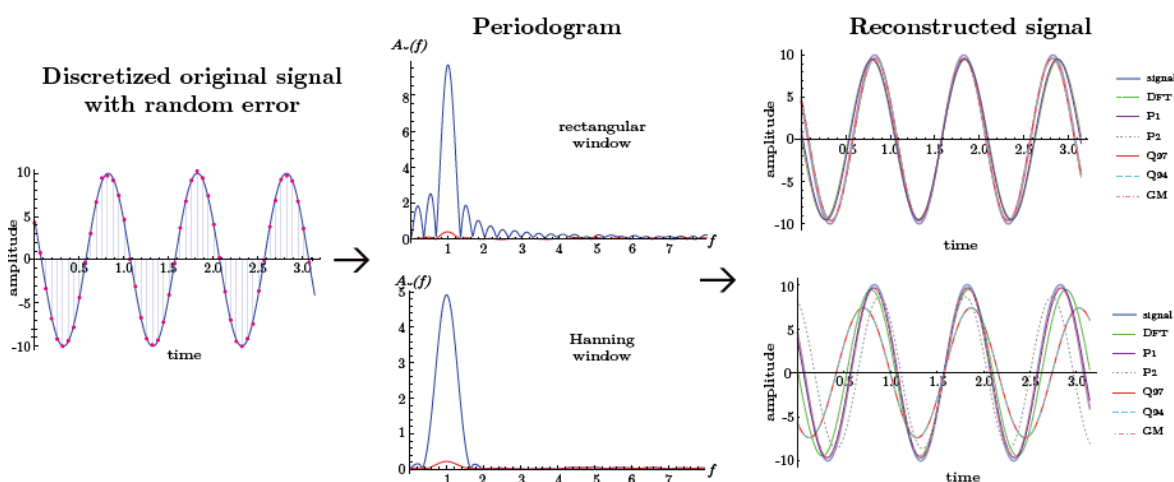
	discreet	parabola	parabola 2	Quinn 97	Qiunn 94	Lipshitz
Sigma = 0.5	0.00203134	0.00010653	0.00603202	4.22485	4.22217	0.00000024904
Sigma = 0.9	0.00203134	0.000119886	0.0060449	4.22496	4.22214	0.00000699837
Sigma = 1.3	0.00203134	0.000142451	0.00605948	4.05723	4.09624	0.0000141129
Sigma = 1.7	0.00203134	0.000174348	0.00607577	3.97363	3.97057	0.0000238564
Sigma = 2.1	0.00203134	0.000215749	0.00630649	3.72112	3.71798	0.0000362577
Sigma = 2.5	0.0844243	0.000266876	0.0063421	3.468	3.46469	0.000051352

**Table 5** Results of amplitudes for Hanning window function

	discreet	parabola	parabola 2	Quinn 97	Qiunn 94	Lipshitz
Sigma = 0.1	0.398648	0.118049	2.06786	16.4745	16.503	0.130874
Sigma = 0.5	0.400135	0.123353	2.05865	16.4914	16.529	0.13522
Sigma = 0.9	0.414848	0.14168	2.05674	16.5193	16.5763	0.151555
Sigma = 1.3	0.442788	0.173054	2.06224	16.2386	16.3771	0.179891
Sigma = 1.7	0.483952	0.217533	2.07527	16.1683	16.2876	0.220247
Sigma = 2.1	0.538329	0.275203	2.16779	15.8613	16.0274	0.272642
Sigma = 2.5	0.605403	0.346182	2.2008	15.5771	15.806	0.337097

**Table 6** Results of phase for Hanning window function

	discreet	parabola	parabola 2	Quinn 97	Qiunn 94	Lipshitz
Sigma = 0.1	0.312287	0.00746186	0.616335	0.179689	0.185053	0.000148262
Sigma = 0.5	0.312864	0.00800005	0.616567	0.180251	0.185757	0.0004413
Sigma = 0.9	0.313583	0.00943011	0.617129	0.18089	0.18668	0.00113917
Sigma = 1.3	0.314443	0.0117661	0.618018	0.267009	0.250307	0.00224395
Sigma = 1.7	0.315442	0.0150281	0.619226	0.312917	0.319446	0.00375796
Sigma = 2.1	0.316581	0.0192429	0.643179	0.440763	0.448251	0.00568383
Sigma = 2.5	0.370351	0.024445	0.647252	0.552666	0.56261	0.00802452



**Figure 4** Signal reconstruction

**6.2 Example 2**

Let it be  $L > 0, m \in \mathbb{N}, \Delta t = \frac{L}{m}$ , data  $(t_n, y_n)$ ,

$n = 0, \dots, m-1$  is generated so that  $t_n = n\Delta t$ ,

$$y_n = 11 \cos\left(2\pi \cdot 4n\Delta t + \frac{\pi}{2}\right) + 8 \cos(2\pi \cdot 2n\Delta t + \pi) + \varepsilon_n$$

where  $\varepsilon_n \sim N(0, \sigma^2)$ . Let it be  $m = 50$ . For each

$\sigma \in \{0.1, 0.5, 0.9, 1.7, 2.1, 2.5\}$  the experiment is repeated 100 times. Applying above mentioned methods: Discreet Fourier transformation - discretization of periodogram (DFT), Parabola method 1 (P1), Modified parabola method (P2), Method Quinn 1997 (Q97), Method Quinn 1994 (Q94), Global method (GM) we estimate frequency  $f_j$  as well as matching amplitude  $a_j$  and phase shift  $\phi_j$ . For each method average error is calculated:

- for frequency  $E_f = \frac{1}{100} \sum_{j=1}^{100} \left| 4 - \hat{f}_j \right|$
- for amplitude  $E_a = \frac{1}{100} \sum_{j=1}^{100} \left| 4 - \hat{a}_j \right|$
- for phase  $E_\phi = \frac{1}{100} \sum_{j=1}^{100} \left| 4 - \hat{\phi}_j \right|$

Experiment was conducted with Hanning and rectangular window function, in Fig. 5 are shown the results. After isolating first dominant frequency it is obvious that frequency spectrum is smoothed and additionally emphasized using Hanning window function.

Results obtained while conducting numerical experiment are shown in Figs. 6 and 7. Most precise is DIRECT global method, with slightly less accuracy at calculating second dominant frequency.

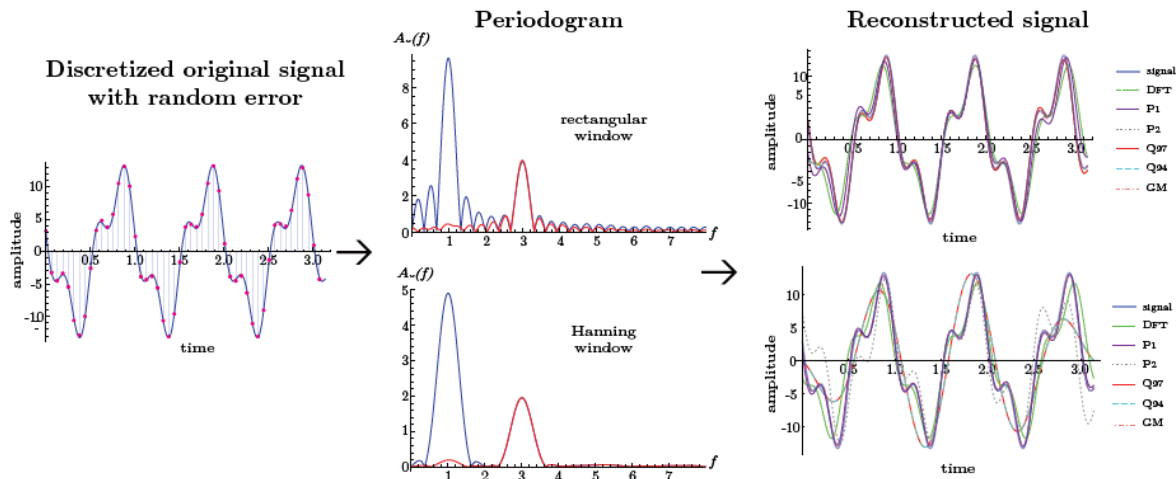
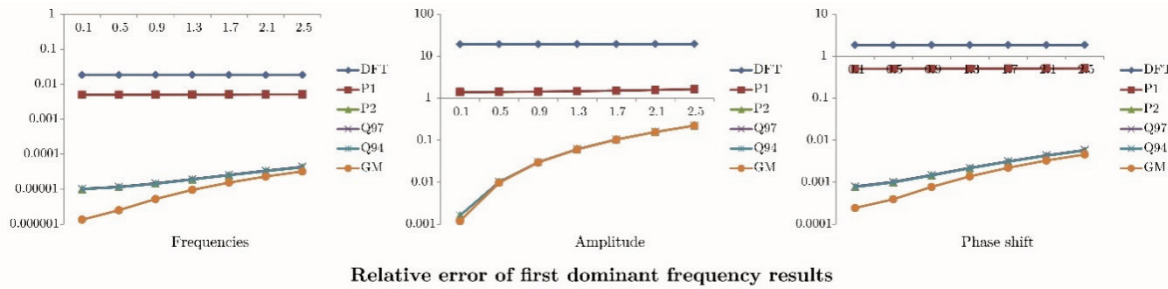
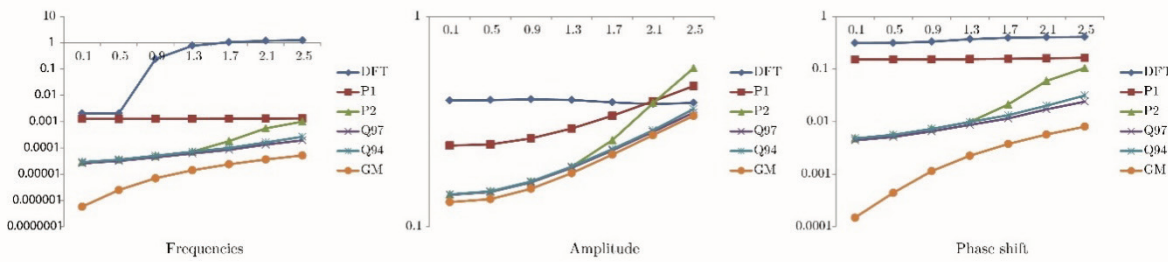


Figure 5 Signal reconstruction after obtaining two dominant frequencies

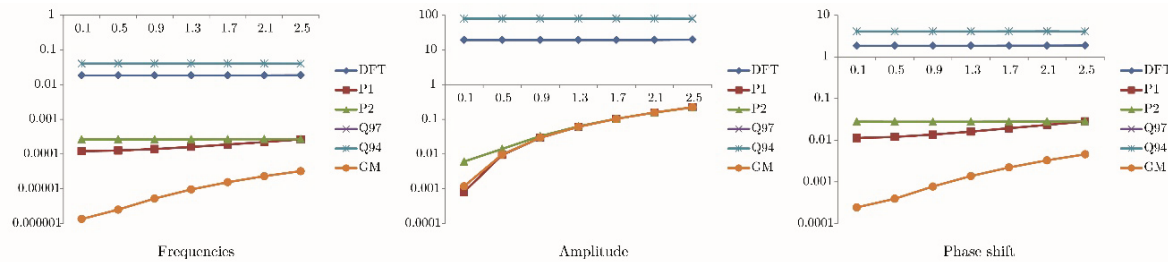


Relative error of first dominant frequency results

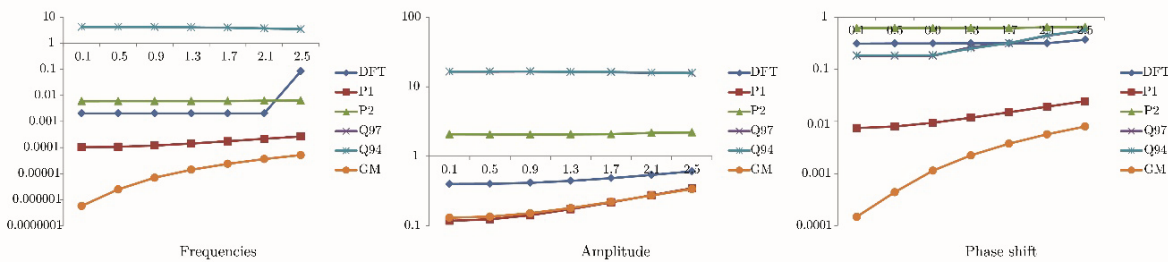


Relative error of second dominant frequency results

Figure 6 Comparison of results obtained using rectangular window function



Relative error of first dominant frequency results



Relative error of second dominant frequency results

Figure 7 Comparison of results obtained using Hanning window function

Comparison of results shows that more accurate results are obtained using Hanning window function at larger data dissipation, but for more accurate assessment algorithm

should be tested on real measurements data or more realistic data should be obtained such as calculated using finite element method.



Testing input functions has shown that in case when dominant frequencies are sufficiently distant, after isolating dominant frequency from periodogram, algorithm can be effectively used to obtain subsequent dominant frequency. If dominant frequencies are in vicinity then inverse transformation of reduced periodogram gives slightly different result which has larger deviation from real value of maximum.

## 7 CONCLUSION

Comparing results of testing methods for vibration dominant frequencies determination, conclusion can be drawn that global method is most accurate in determination of frequencies, amplitude and phase shift in the experiment where rectangular window was used. While using Hanning window DIRECT global method and parabola method give similar results for amplitude but for frequency and phase shift DIRECT global method gives the most accurate results. So overall conclusion that this efficiency analysis shows can be that the global method is more accurate than some previously commonly used methods for frequency estimation.

Method efficiency varies with using different window functions. Further testing shows that algorithm efficiency is questionable at some ratios of amplitude against frequency (when frequency peaks are in vicinity), in some cases Hanning window function causes smoothing of spectra and making frequency isolation more difficult. This problem can be solved by changing sampling ratio and applying rectangular window function.

Proposed algorithm for multi peak detection could be used as part of intelligent fault detection system for rotary machines. In further research algorithm could be used to build more complex computer software and after numerical simulation, results should be tested on experimental rig and real measurement data.

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