

StaTips Part VIII: Confidence interval for the sample mean

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ABSTRACT

When conducting research on a given type of patients, it is impossible to examine all the existing subjects of that type (population) to derive the true mean of the parameter of interest. More realistically, by the investigation of a small group of subjects (sample) from the whole population, researchers can estimate an interval into which the true mean of the population lies. In statistics, such interval is referred to as confidence interval (CI). The calculation of the CI from a sample mean is simple and gives important information, not only regarding the true mean of the population, but also on the statistical significance of the difference between groups being compared. For these reasons, the reporting of the CIs is preferred over the p value alone.

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FRAMING OF THE PROBLEM

When conducting research on a given type of patients it is impossible to examine all the existing subjects of that type (population) to derive the true mean of the parameter of interest. Therefore, a small group of subjects (sample) from the whole population is investigated. The more the sample is representative of the population from which is derived, the more the results will be reliable. However, when a mean with the corresponding standard deviation (SD) is derived from a data set (i.e., ANB angle in a sample of patients), one cannot be sure that the true mean of the population from which the sample is derived is exactly that one. More realistically, researchers can estimate an interval into which the true mean of the population lies starting from the analysis of the sample. In statistics that interval is referred to as confidence interval (CI) and it may be applied to all statistical parameters, even though the one related to the sample mean¹ is dealt herein.

CONFIDENCE INTERVAL (AND CONFIDENCE LEVEL)

As stated in the name, the CI is an interval (thus a couple of numbers) that is derived from an inference procedure, in other words the calculation of the CI is a statistical test, while for instance the calculation of the sample mean is a mathematical procedure. That is why CI is bound to a probability that is referred to as confidence level of the CI, which it is usually set at 95% (95% CI), even though 90% and 99% are also used. An understanding of the CI has to deal with the normal distribution of a population. As reported in Figure, in the case of a Gaussian (normal) distribution a total of 95% of the population lies within a range that is equal to the mean $\pm 1.96SD$. The remaining 5% of the population is equally distributed at the sides of the curve (lower and upper tails). The coefficient 1.96 is referred to as z-score. This concept implies that there is a 95% chance that the true population mean lies within the upper and lower scores of the CI. Of note, different z-scores correspond to different percentages of population within the upper and lower sides of the curve, according to which confidence levels can be 90% or 99%, and so on.

The calculation of the CI from a sample mean is pretty simple and made according to the following formula:

$$95\% \text{ CI} = \text{mean} \pm 1.96 \frac{\text{SE}}{\sqrt{n}}$$

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where: **CI**, confidence interval; **mean**, arithmetic mean of the data set; **SE**, standard error of the data set; **n** = sample size. The symbol \pm indicates that the calculation has to be repeated twice deriving the upper (+) and lower (-) scores of the interval. The value 1.96 represent the z-score for a 95% CI, while for other confidence levels, different z-scores are used (Table). Again, the CI is inferred, meaning that calculation is made from a data set of the sample, while the outcome is representative to the population from which the sample is derived. The same concept applies to the standard error of the mean (SE).

WORKED EXAMPLE

As an example, it can be considered orthodontic research recruiting skeletal Class II patients, where the ANB angle for a group of 20 subjects would be 6.0° with a corresponding SE of 1.5° . The 95% CI is then calculated as follows:

Lower score:

$$95\% \text{ CI} = 6.0^\circ - 1.96 \frac{1.5^\circ}{\sqrt{20}} = 6.0^\circ - 0.66^\circ = 5.34^\circ$$

Upper score:

$$95\% \text{ CI} = 6.0^\circ + 1.96 \frac{1.5^\circ}{\sqrt{20}} = 6.0^\circ + 0.66^\circ = 6.66^\circ$$

In this case, the true mean of the population would lie between 5.34° and 6.66° , with a confidence level of 95%. Similarly, the 99%CI for this case would be 5.13° - 6.87° .

OTHER USES OF THE CONFIDENCE INTERVAL

Provided that the parameter under investigation is continuous and assumptions for using parametrical methods exist, the CI is also very useful when applied to the comparison between groups. Indeed, while the p value may denote a difference as statistically significant or not, the CI also gives an estimation of the difference between the true means the populations from which the samples being investigated are derived. In this case, two different CIs are calculated for either group being compared. To assess a statistically significant difference between the groups, the two CIs must not overlap. Coming back to the worked example on the ANB angle, if a group has a 95%CI of 5.34° - 6.66° and the other one has a 95%CI of 4.50° - 5.20° , then it can be concluded that the difference between the groups is statistically significant at $p < 0.05$. In case of 99%CI, when no overlap is seen, then the corresponding p value for the difference between the groups will be < 0.01 , and so on (Table). Of interest, when dealing with longitudinal data (2 time points, T0 and T1, within the same group) the calculation of the CI is not made directly from the data sets belonging to each time point, but rather from a third data set obtained as 'scoreT1-scoreT0' for each subject. Then the corresponding mean and SE derived from this third data set are used for calculation and statistically

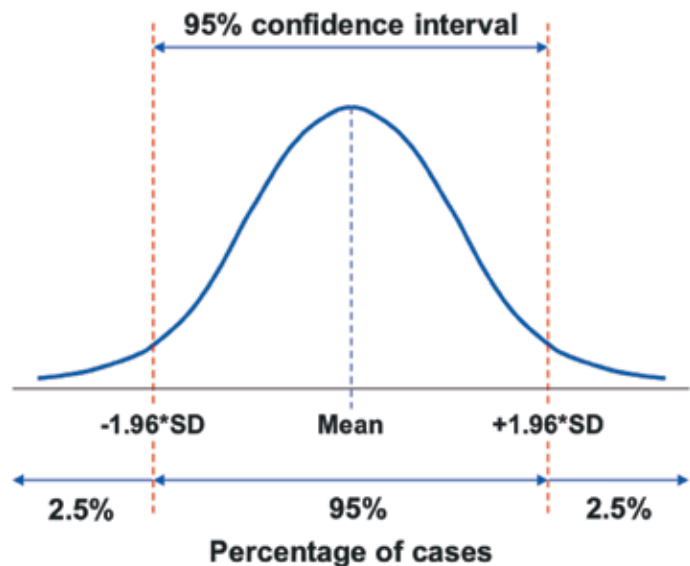
significant difference is assigned when the CI does not include the zero value.

Taking into account all of the considerations above, the reporting of the CIs (or CIs and corresponding p values) would be preferable over the of the p values alone. Finally, the CIs carry more information as compared to the p value, making this piece of data very useful for meta-analyses.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

Figure. Diagram relating confidence intervals with a normal distribution.



Mean, Arithmetic mean; *SD*, standard deviation.

Table. The different z-scores used to calculate the confidence intervals with corresponding confidence levels and p values.

Z-score	Confidence level	P value
1.65	90%	0.10
1.96	95%	0.05
2.58	99%	0.01

REFERENCE

1. Glantz S. Primer of Biostatistics. 7th ed. Columbus: McGraw-Hill Education; 2011.