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Determining Natural Resonant Frequencies of Large Power Transformer Windings

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Abstract—**Design, assembly process and preparation of windings for large power transformers are complex procedures. It is very important to recognize resonant frequencies of the windings during the design phase in order to avoid increase of vibrations, stresses or noise levels during normal operation of the transformer. Increased vibrations and stresses can occur when the frequency of the excitation is close to the natural resonant frequency of the system. In that case amplitudes will increase, and their values will depend, among other factor, also on the amount of damping present in the system. Therefore, in order to avoid very large amplitude of vibration at resonance, the natural frequency must be known.**

In this work analytical calculation of natural frequencies of high voltage and low voltage windings, is described. Additionally, numerical analysis (FEA) is performed in the software package Ansys. Analysis contains descriptions of geometry simplifications, boundary conditions, mesh validation and applied loads. Finally, results obtained with numerical analysis (FEA) are compared with analytical results.

Index Terms—**Power transformer, natural frequencies, vibrations, finite element analysis**

I. INTRODUCTION

Large power transformer windings are a complex construction comprised of several very different materials. Certain portion of the winding height consists of winding insulation which is, usually, mostly paper and transformerboard. These materials are significantly less stiff than copper, which causes the winding to act in axial direction as a spring, or set of springs, when under stress due to axial forces exerted on the copper conductors.

Axial force per unit of the winding height is usually at its highest value, in absolute form, near the winding ends (bottom and top), with the force near the winding top acting downwards and force near the winding bottom acting upwards. In such cases the accumulated axial force is at its highest value near the middle of the winding. An example of the axial force distribution on the copper conductors and the accumulated axial force along the winding height is given in Figure 1. These forces are periodic, with their base frequency being 100 Hz in 50 Hz network. For this reason, it is important to know the natural resonant frequencies of the transformer windings, in order to be sure that they are not close to excitation frequency or its higher harmonics. In this paper, the object of analysis is to determine the natural resonant frequencies in axial direction of the 500 MVA autotransformer windings using the developed FEM model and analytical model.

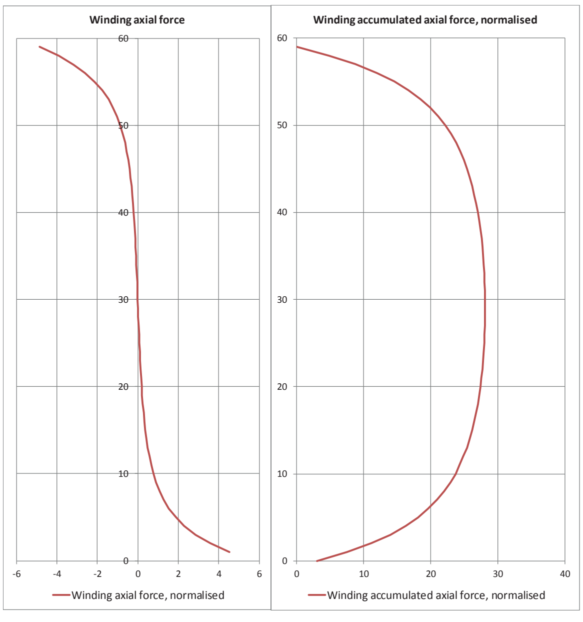

Figure 1 - Example of the winding axial and axial accumulated forces

II. GENERAL

In order to calculate the natural resonant frequencies in axial direction an appropriate model has to be developed. The model has to take into account all important features of the winding geometry and the properties of the materials used, without, in the end, being too complex or time-consuming for simulation and calculation.

For the purpose of this calculation, large power transformers windings model will consist of:

- copper conductors (which can be continuously transposed conductors or CTCs)
- paper insulation
- transformerboard spacers (when the winding has radial cooling channels)
- bottom and top support

For simplification purposes, some geometry details of the actual transformer windings will be omitted from the models such as lead entrances, disc crossovers, transpositions etc. For the same reason, the top and bottom supports, will be modeled as springs with the appropriate characteristics, instead of the full-detailed design of supports, which, depending on the whole winding assembly of one limb, could be complex and not necessarily rotationally-symmetric due to all other aspects such as winding leads, oil paths etc.

Two model variations are developed. In the first one, each disc is modeled as a separate element; while in the other variation whole windings are modeled as a homogeneous column with its own set of properties. The model with separate copper parts and insulation parts as well as a homogenous column model are shown in Figure 2

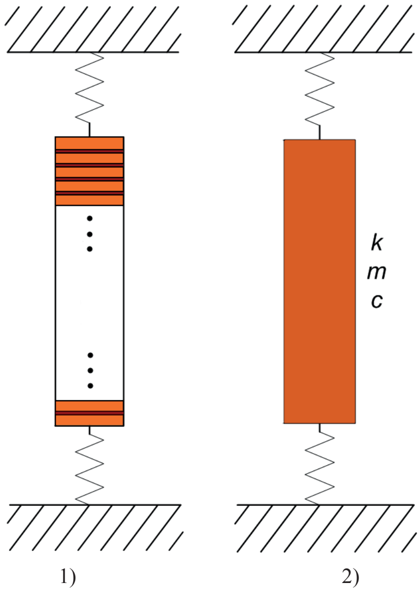

Figure 2 - Model of a transformer winding 1) with copper parts and insulation parts and 2) homogenous column

For the calculation of the natural frequencies, two distinct approaches have been employed.

A. *Finite element method approach*
   for model 1)
B. *Analytical approach*
   for model 2)

The general data of the transformer windings is given in TABLE I.

TABLE I GENERAL DATA OF THE TRANSFORMER WINDINGS

Property	Winding A	Winding B
Number of discs	184	130
Height of copper disc	10,9 mm	12,7
Height of paper insulation	1,12 mm	1,2 mm
Height of transformerboard spacers	N/A	4 mm
Winding nominal height	2220 mm	2286 mm
Winding mass	7435 kg	5052 kg

III. NUMERICAL APPROACH

Numerical simulation, using finite element method, is performed in software package ANSYS 2019. For this type of calculation, where the natural resonant frequencies are in the focus of the analysis, the module “Modal Analysis”, available in the Ansys Workbench setup, is used.
CAD models of windings A and B, used for calculation are given in Figure 3

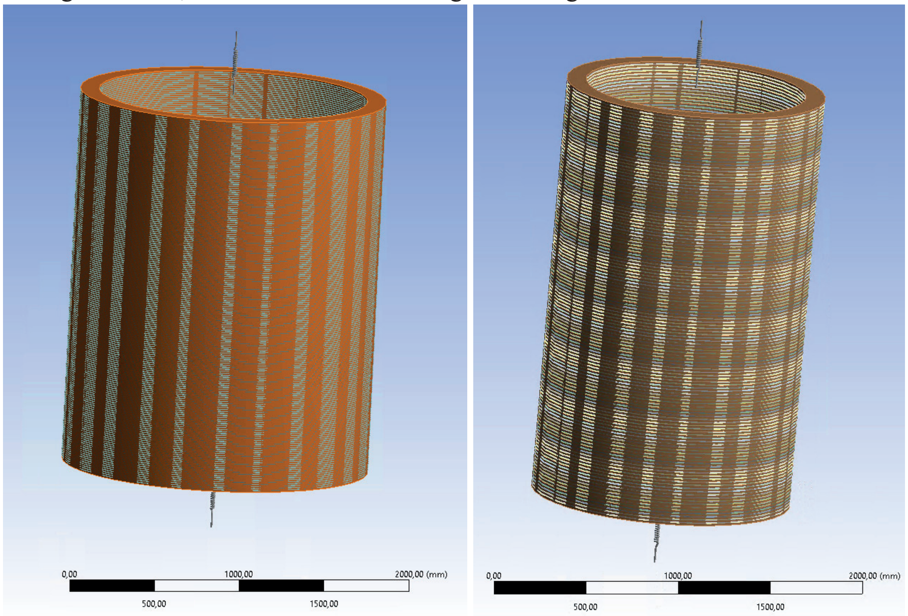

Figure 3: Models of the windings in FEM software

As the focus of this paper are axial natural frequencies, the mesh density in radial direction is of low significance and can be set to 1 mesh element per winding width. However, the mesh density in axial direction is important, and needs to, at the very least, be set at 1 mesh element per winding copper disc and 1 element per insulation between discs. Several cases of calculations were performed, and based on this, in authors’ opinion, adequate mesh, which gives appropriate accuracy for the calculation, was chosen.
In case of winding A, the top paper insulation of a copper disc and bottom paper insulation of the disc immediately above can be modeled together as one part, therefore reducing the number of elements along the height of the winding. A detail, showing layers of copper discs and layer of insulation, of the model is given in Figure 4.

Figure 4 - Layers of copper (brown) and paper insulation (grey) during meshing process

For the winding B, which has transformerboard spacers, the exact sequence would be “copper – paper – spacer – paper – copper disc”. However, without compromising calculation procedure, this can be modeled as “copper – double paper – spacer – copper”. For any winding with spacers, the actual number of spacers can easily reach several thousands, eg. 151 discs with 60 spacers between any two discs would give 9000 spacers. In order to avoid modelling each of the 9000 spacers, the simplification is introduced and all 60 spacer along the winding circumference are modeled as a single disc (as shown in Figure 5) with equivalent parameters, most importantly, the Young’s modulus of elasticity of such a disc is altered, due to the fact that now spacer covered surface is significantly larger.

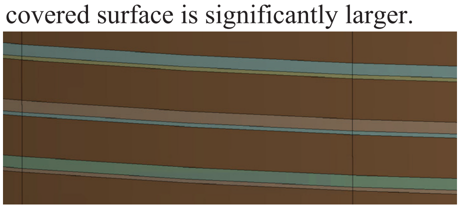

Figure 5 – Layers of copper discs (brown), paper insulation and spacer layer during meshing process

As stated in previous chapter, top and bottom winding support has been modeled using springs that are set between the bottommost disc and ground, as well as, between topmost disc and ground. In order to determine the necessary characteristics of the equivalent spring, several important factors have to be taken into account, notably the construction of the winding top and bottom support including the total surface that can adequately transfer the axial force, the properties of the materials used for their production and relative proportion in the total height of the winding support. Although in the real transformer, depending on the other aspects of design, such as dielectric requests, the height of the top and bottom support, which will affect the equivalent spring characteristic, isn't necessarily the same, for the purpose of the current work the characteristics of the top and bottom spring will be taken to be equal, with values as shown in Table II.

TABLE II EQUIVALENT SPRING CONSTANTS OF THE TOP AND BOTTOM WINDING SUPPORT

No.	Winding A	Winding B
TOP support	6,1 10^5 N/mm	5,0 10^5 N/mm
BOTTOM support	6,1 10^5 N/mm	5,0 10^5 N/mm

The ideal model for such a spring behavior would actually be non-linear, at least in the fact that no tensile force can be transmitted through such a spring. However, compression-only spring is a non-linear element, so it cannot be modeled as such in the Modal analysis. However, with assumption that no separation of elements occurs under appropriate prestress (which holds true for all practical purposes), the modeled spring can be accepted during this calculation procedure.

The results obtained through the numerical simulation, using finite element method in Ansys 19 are given in Table III and Figure 6 (winding A) and Figure 7 (winding B).

TABLE III NATURAL RESONANT FREQUENCIES WINDING A AND B – NUMERICAL APPROACH

No.	Winding A	Winding B
1	56,7 Hz	52,8 Hz
2	137,0 Hz	110,7 Hz
3	236,1 Hz	174,5 Hz
4	340,2 Hz	242,2 Hz
5	446,7 Hz	312,2 Hz

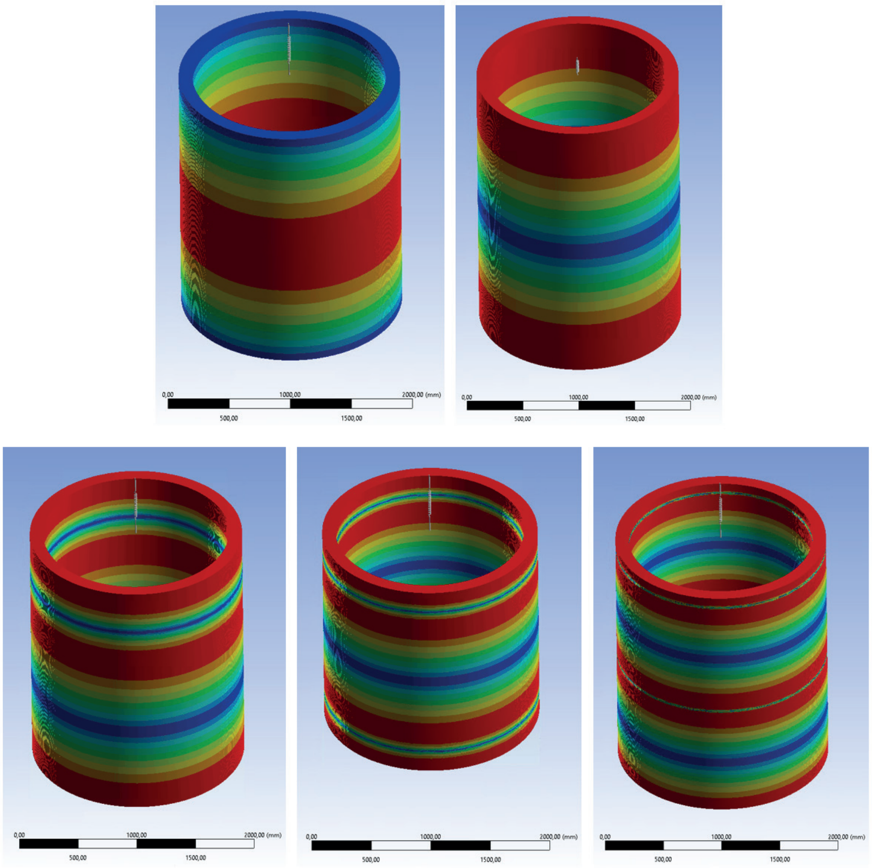

Figure 6 - First, second, third, fourth and fifth mode of axial vibration of winding A

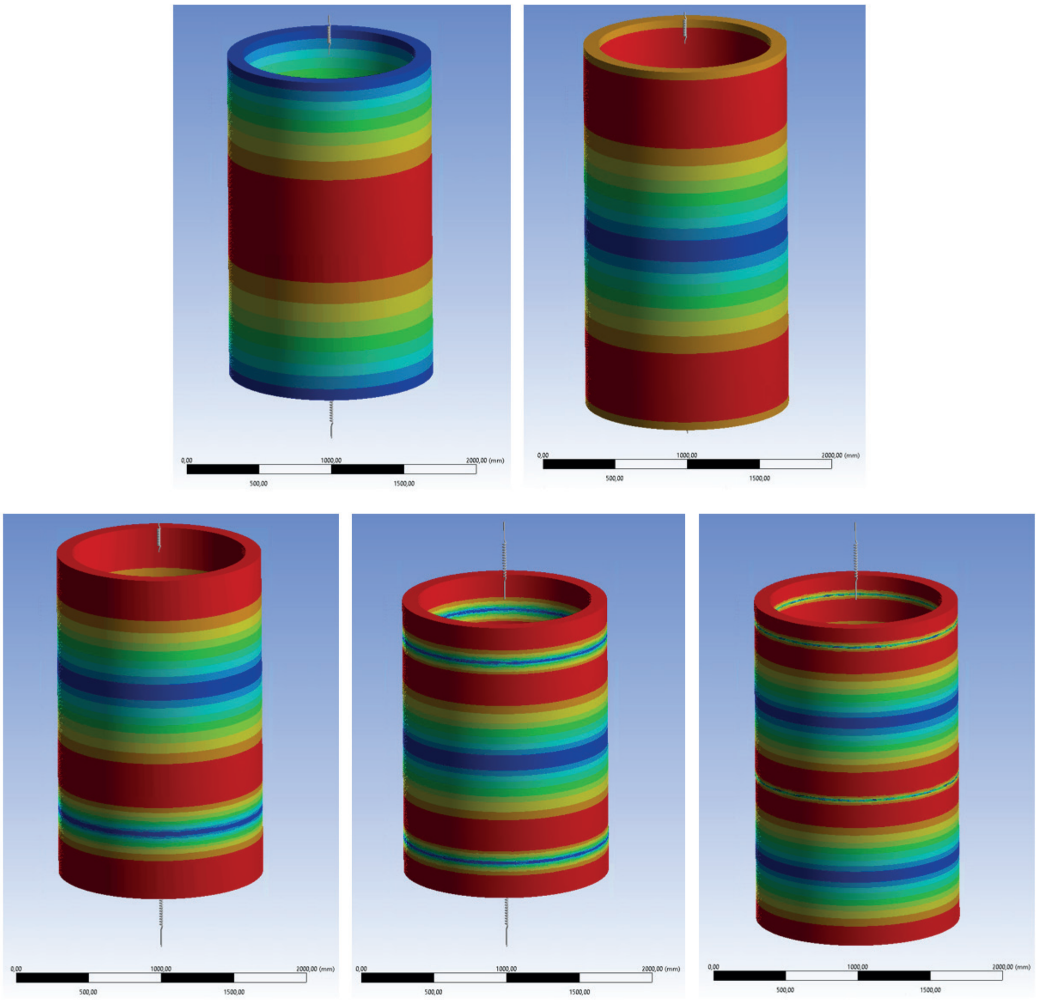

Figure 7 - First, second, third, fourth and fifth mode of axial vibration of winding B

IV. ANALYTICAL APPROACH

As in the case of numerical analysis, similar simplification to the real winding geometry has to be done, in order to derive the differential equation governing the longitudinal dynamic behavior of the winding. The basic schematic for the derivation is given in Figure 8. The winding in this approach is modeled as a homogenous column with its own specific properties, such as mass and stiffness. As several parts of the analytical approach to the task of calculating the winding natural frequencies are explained in [1], detailed step-by-step procedure can here be omitted.

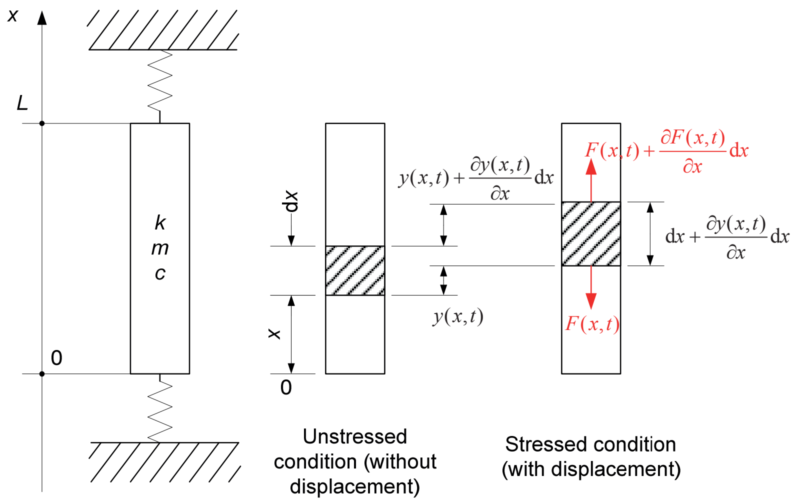

Figure 8 - Basic schematic of a winding and insulation elastic system

The displacement of the differential infinitesimal element of the winding is a function of the coordinate x. If $y(x,t)$ denotes the displacement of the point x in the winding from the rest position, in a strained condition, the element of the length dx changes its length by (∂y/∂x)dx and gets strained by (∂y/∂x). Due to the elastic properties of the winding, the restoring force F_{el} arises:

$$F_{el} = k\frac{\partial y}{\partial x} \tag{1}$$

Let the mass, the spring constant and damping per unit length of the winding be denoted by m, k, c, respectively. The applied force generated per unit length is:

$$P_0(x,t) = P_1(x)G(t) \tag{2}$$

The Newton's law of motion can be applied to the winding element and (3) follows:

$$\begin{aligned}
&m\cdot \mathrm{d}x\cdot\frac{\partial^2 y}{\partial t^2} = (F(x,t)+\frac{\partial F(x,t)}{\partial x}dx - F(x,t)) - c\cdot \mathrm{d}x\cdot v + f(x,t)\cdot \mathrm{d}x \\
&m\cdot \mathrm{d}x\cdot\frac{\partial^2 y}{\partial t^2} = k\cdot\frac{\partial^2 y}{\partial x^2}\mathrm{d}x - c\cdot \mathrm{d}x\cdot\frac{\partial y}{\partial t} + f(x,t)\cdot \mathrm{d}x \\
&m\cdot\frac{\partial^2 y}{\partial t^2} + c\cdot\frac{\partial y}{\partial t} - k\cdot\frac{\partial^2 y}{\partial x^2} = f(x,t)
\end{aligned} \tag{3}$$

This is an inhomogeneous partial differential equation of the second order. To complete the problem definition, the initial and boundary conditions must be applied. The boundary conditions consist of the forces applied to the winding ends:

$$\begin{aligned}
&K_1 y(0,t) - k\frac{\partial y(0,t)}{\partial x} = 0 \\
&K_2 y(L,t) + k\frac{\partial y(L,t)}{\partial x} = 0
\end{aligned} \tag{4}$$

Initial conditions are:

$$\begin{aligned}
&y(x,0) = 0 \\
&\frac{\partial y(x,0)}{\partial t} = 0
\end{aligned} \tag{5}$$

Using the method of separation of variables, the solution to the equation can be sought as a product of the spatial function that depends only on the x coordinate and the time varying part. Therefore, it can be assumed that the solution is of the form:

$$\begin{aligned}
&y(x,t) = S(x)\cdot T(t) \\
&\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}(S(x)\cdot T(t)) = T(t)\frac{\partial}{\partial x}S(x) + S(x)\frac{\partial}{\partial x}T(t) = T(t)\frac{\partial}{\partial x}S(x) = TS' \\
&\frac{\partial^2 y}{\partial x^2} = TS'' \\
&\frac{\partial y}{\partial t} = T'S \\
&\frac{\partial^2 y}{\partial t^2} = T''S
\end{aligned} \tag{6}$$

Plugging the derivatives (6) into the original equation leads to:

$$mT''S + cT'S - kTS'' = f(x,t)\,. \tag{7}$$

First, the homogeneous equation is observed:

$$mT''S + cT'S - kTS'' = 0\,. \tag{8}$$

Rearranging and dividing through by kTS gives:

$$\begin{aligned}
&mT''S + cT'S = kTS'' \\
&\frac{m}{k}\frac{T''}{T} + \frac{c}{k}\frac{T'}{T} = \frac{S''}{S}
\end{aligned}\,. \tag{9}$$

On the left-hand side are functions of time, and on the right-hand side is a function of spatial coordinate. Therefore, for the equation (9) to hold, both sides must be constants. The constant can be chosen arbitrarily. It is convenient to choose

$$\frac{S''}{S} = k = -\mu^2 \tag{10}$$

Now, the spatial differential equation, part of the Sturm-Liouville system as explained in [1], allows us to find eigenvalues of the differential equation with boundary conditions

$$\begin{aligned} &S'' + \mu^2 S = 0 \\ &K_1 S(0) - kS'(0) = 0 \\ &K_2 S(L) + kS'(L) = 0 \end{aligned} \tag{11}$$

First, the homogeneous ODE is solved to obtain

$$S(x) = A\cos(\mu x) + B\sin(\mu x). \tag{12}$$

Now, the boundary conditions are rewritten

$$\begin{aligned} &K_1 S(0) - kS'(0) = 0 \\ &K_2 S(L) + kS'(L) = 0 \\ &S(0) = A \\ &S(L) = A\cos(\mu L) + B\sin(\mu L) \\ &S'(0) = \mu B \\ &S'(L) = -\mu A\sin(\mu L) + \mu B\cos(\mu L) \end{aligned} \tag{13}$$

Further, for nontrivial solution *A* and *B* are not identically zero and

$$\begin{aligned} &K_1 A - k\mu B = 0 \\ &B = \frac{K_1 A}{k\mu} \\ &K_2(A\cos(\mu L) + \frac{K_1 A}{k\mu}\sin(\mu L)) + k(-\mu A\sin(\mu L) + \mu\frac{K_1 A}{k\mu}\cos(\mu L)) = 0 \\ &K_2 A\cos(\mu L) + k\mu\frac{K_1 A}{k\mu}\cos(\mu L) + K_2\frac{K_1 A}{k\mu}\sin(\mu L) - k\mu A\sin(\mu L) = 0 \\ &(K_2 + K_1)A\cos(\mu L) + (\frac{K_1 K_2}{k\mu} - k\mu)A\sin(\mu L) = 0 \\ &K_2 + K_1 + (\frac{K_1 K_2}{k\mu} - k\mu)\tan(\mu L) \\ &\tan(\mu L) = \frac{K_1 + K_2}{k\mu - \frac{K_1 K_2}{k\mu}} \end{aligned} \tag{14}$$

holds. Finally, after some rearranging and substitutions the equation is as in [1]:

$$\begin{aligned} &\mu L \Leftrightarrow \lambda, \bar{K}_1 = K_1 / K_0, \bar{K}_2 = K_2 / K_0, k = K_0 L \\ &\tan(\mu L) = \frac{(K_1 + K_2)K_0 L\mu}{(K_0 L)^2 \mu^2 - K_1 K_2} \\ &\tan(\lambda) = \frac{(K_1 + K_2)K_0\lambda}{K_0^2\lambda^2 - K_1 K_2} \\ &\tan(\lambda) = \frac{(\bar{K}_1 + \bar{K}_2)\lambda}{\lambda^2 - \bar{K}_1\bar{K}_2} \end{aligned} \tag{15}$$

The solution to the spatial part of the differential equation is used in solving the time function part. However, solving the time equation can be avoided in this analysis and the circular frequency of the undamped harmonic oscillation can be written immediately as

$$\omega_n = \lambda_n\sqrt{\frac{K_0}{M_0}}; M_0 = mL. \tag{16}$$

The oscillations of the winding are damped, but the damping coefficient is low and so, due to the (17), (16) can be used to calculate the circular frequencies.

$$\omega_{r,n} = \omega_n \sqrt{1-\varsigma^2} \approx \omega_n \tag{17}$$

The natural frequencies are:

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\lambda_n \sqrt{\frac{K_0}{M_0}} \,. \tag{18}$$

The spring constant of the winding insulation can be calculated using Hooke's law

$$K_0 = \frac{E'_d \, S_{ins}}{H_{ins}} \,, \tag{19}$$

where E'_d is an equivalent dynamic Young's modulus of elasticity, S_{ins} is a cross section area of the insulation perpendicular to the vibration direction, and H_{ins} is the height of the insulation in the winding.
The equivalent dynamic Young's modulus of elasticity depends on several factors, such as the relative content of different materials in the insulation column and for the purpose of this calculation is set at $123{,}8 \cdot 10^8$ Pa for winding A and $165{,}8 \cdot 10^8$ Pa for winding B.

The calculated values for the first five natural frequencies using numerical and analytical approach, are given in Table IV.

TABLE IV COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS

No.	Winding A		Winding B	
No.	Numerical result	Analytical result	Numerical result	Analytical result
1	56,7 Hz	56,4 Hz	52,8 Hz	52,4 Hz
2	137,0 Hz	136,0 Hz	110,7 Hz	110,0 Hz
3	236,1 Hz	232,0 Hz	174,5 Hz	173,6 Hz
4	340,2 Hz	334,1 Hz	242,2 Hz	241,0 Hz
5	446,7 Hz	438,4 Hz	312,2 Hz	310,8 Hz

V. CONCLUSION

Determining the natural resonant frequencies of transformer windings is an important task in evaluating the short-circuit withstand capability of large power transformer. Special attention must be given to this phenomenon during the design and production of large power transformer, in order to ensure that the final product is not affected by it. Otherwise, if such analysis is performed incorrectly or with wrong parameters, the electromagnetic forces may incur overstresses to the transformer windings which can in turn result in the transformer failure.

In this paper two different approaches have been employed in order to determine the natural resonant frequencies of large 500 MVA autotransformer windings. The numerical calculation, using FEM software package ANSYS 2019 and its dedicated module for modal analysis, was performed on the CAD models where over 100 winding discs have been modeled. In the analytical approach the winding was represented by a homogeneous column with equivalent parameters.

The results, for both windings which are of different types, showed that the analytically calculated values of natural frequencies correspond well with the values obtained through the numerical calculation on the developed and more detailed FEM model, which points to the conclusion that analytical calculation is adequate for determination of the windings' natural frequencies, under the condition that the equivalent parameters for both the winding insulation and the top and bottom support are correctly taken into account. In order to take all aspect of winding construction correctly into account when dimensioning transformer windings, one must rely on previous vast experience and gained knowledge.

The dimensioning of large power transformer windings is a very complex procedure which, besides expert knowledge, must be based on extensive successful experience in the design, construction and production of large power transformers and all the design rules and "know-how"-s that come with it.

VI. REFERENCES

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