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A philosophical argument for the beginning of time

JACOBUS ERASMUS
School of Philosophy, Faculty of Arts, North-West University
Private Bag X6001, Potchefstroom 2520, South Africa
je@jacobuserasmus.com

LAUREANO LUNA
IES "Doctor Francisco Marin", Siles, Jaén, Spain
laureanoluna@yahoo.es

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ABSTRACT: A common argument in support of a beginning of the universe used by advocates of the *kalām* cosmological argument (KCA) is the argument against the possibility of an actual infinite, or the "Infinity Argument". However, it turns out that the Infinity Argument loses some of its force when compared with the achievements of set theory and it brings into question the view that God predetermined an endless future. We therefore defend a new formal argument, based on the nature of time (just as geometrical reasoning is based on the nature of space), which addresses more directly the question of beginningless time.

KEY WORDS: *Kalām* cosmological argument, predetermined future, Benardete paradox, Yablo paradox, recursive determination rule, normal time, beginningless time

I

In philosophy, the question whether a beginningless time is possible has received increased attention because of the role it plays in the *kalām* cosmological argument and because of some empirical and mathematical results in cosmology.[1] The *kalām* cosmological argument (KCA) is

[1] We do not mean only the prevalent Big Bang cosmology based on redshift observations but also results like singularity theorems (e.g. Penrose 1965) or the so-called BGV theorem (Borde, Guth and Vilenkin 2003).

a notable theistic argument that tries to prove three claims, namely, (i) that the universe came into being, (ii) that the universe, therefore, has a cause or external reason for its existence, and (iii) that this cause or reason is grounded in a transcendent, personal being (or God). A part of the KCA may, thus, be presented as the following syllogism (see Craig 1999: 737):

1. Whatever comes into being has an external reason for its existence.
2. The universe came into being.
3. Therefore, the universe has an external reason for its existence.

The phrase "the universe came into being" in premise 2 is to be understood as "the time of the universe had a beginning." A distinguishing characteristic of the KCA is that it makes use of the notion of infinity to show that the universe had a beginning. A common argument in support of a beginning of the universe used by advocates of the KCA is the argument against the metaphysical possibility of an actual infinite. Let us refer to this as the "Infinity Argument".[2] According to the Infinity Argument, if the universe had no beginning, then an actually infinite number of past events (such as days) have occurred. However, it is metaphysically impossible for an infinite number of things (such as events) either to exist (i.e. be instantiated in the real world) simultaneously or to come into existence in such a way that there is a time at which all the infinitely many things have already occurred. Thus, since an infinite sequence of events cannot be completed, the sequence of past events must be finite. It follows, then, that the universe had a beginning.

Our intention in this paper is twofold. Firstly, we show that the Infinity Argument is problematic because (i) set theory is consistent with the existence of an actual infinite and (ii) the Infinity Argument brings into question the view that God predetermined an endless future. We do not intend to argue that the Infinity Argument is unsound but, rather, that it faces problems that render it inconclusive. We then, secondly, defend a new formal argument against beginningless time that is compatible with the possibility of both an actual infinite and an endless future. More precisely, we argue that, under certain common-sense assumpti-

[2] The Infinity Argument is not the only philosophical argument for a cosmic beginning offered by the proponents of the KCA. Other arguments include the argument from the impossibility of the formation of an actual infinite by successive addition (Craig and Sinclair 2012: 117-125), the argument based on traversing infinite time (Erasmus 2018: 114-117), the so-called Grim Reaper Paradox (Koons 2014), and Alexander R. Pruss' (2018) arguments for causal finitism. The latter two arguments are similar to the argument we present below.

ons concerning the nature of time, the notion of beginningless time is inconsistent. We will begin by briefly describing the Infinity Argument.

II

As mentioned above, the Infinity Argument attempts to show that the universe had a beginning because an actual infinite cannot exist and beginningless time entails an actual infinite. Accordingly, we may formulate the Infinity Argument as follows (Craig and Sinclair 2012: 103):[3]

(A1) An actual infinite cannot exist.
(A2) A beginningless sequence of events is an actual infinite.
(A3) Therefore, a beginningless sequence of events cannot exist.

In order to understand this argument better, we must distinguish between the actual infinite and the potential infinite. The *actual infinite* is an endless set whose members, nevertheless, exist all at once or at least are such that there is a time at which all of them have already existed. Thus, the actual infinite is a completed totality of infinitely many distinct members. On the other hand, the *potential* infinite is an indefinite process, such as endless addition. Such a process or sequence is dynamic because it increases endlessly towards infinity but at no point does it become actually infinite. Thus, the potential infinite is always finite and never complete. The crucial difference, then, between the actual infinite and the potential infinite is that the latter is not a completed totality whereas the former is.

In defense of (A1), many proponents of the KCA use the Hilbert's Hotel thought experiment to show that certain alleged absurdities would result if an actual infinite were metaphysically possible (Craig and Sinclair 2012: 108-110): Imagine a hotel with an infinite number of rooms, with each room being occupied by a guest. Now, suppose that the guest in room number 1 departs and, thus, the room becomes available. Oddly enough, however, although there is one less guest in the hotel, the number of guests in the hotel has not changed and remains infinite. Thus, infinity minus one equals infinity. Now, suppose that each guest in an odd numbered room departs. In that case, although an infinite

[3] The Infinity Argument dates back to John Philoponus (AD c. 490-c. 570) in the fifth century, and the argument was also refined and defended by numerous Muslim philosophers in the Middle Ages (see Erasmus 2018: 41-64). Hence, there are various versions of the Infinity Argument. Nevertheless, for simplicity, we will briefly focus on the Infinity Argument offered by William Lane Craig and James D. Sinclair (2012).

number of guests have left, the number of guests in the hotel remains the same, namely, infinity. Consequently, infinity minus infinity equals infinity. However, if all the guests in room numbers 5 and above depart, then only four guests would remain – those in room numbers 1, 2, 3 and 4. Thus, infinity minus infinity equals four. This, in turn, apparently leads to three inconsistent situations:

1. Infinity minus one equals infinity ($\aleph_0 - 1 = \aleph_0$)
2. Infinity minus infinity equals infinity ($\aleph_0 - \aleph_0 = \aleph_0$)
3. Infinity minus infinity equals four ($\aleph_0 - \aleph_0 = 4$)

Thus, the Hilbert's Hotel thought experiment apparently illustrates that, if an actually infinite number of objects can exist in the real world, then an equal quantity may be subtracted from an equal quantity and yet produce different results. However, according to the proponents of the Infinity Argument, such a situation is impossible and, thus, an actual infinite cannot exist. Furthermore, as William Lane Craig and James Sinclair (2012: 115) note, (A2) seems obviously true because "if there has been a sequence composed of an infinite number of events stretching back into the past, then the set of all events in the series would be an actually infinite set." Accordingly, the Infinity Argument concludes that an infinite temporal regress of events cannot exist and, thus, the universe had a beginning.

III

Although the Infinity Argument is not necessarily unsound, we believe it faces difficulties. In the first place, finite sets behave very differently to infinite sets. Unlike with finite sets, subtraction with infinite sets is not well defined and, thus, it leads to different answers based on which members of the set are "subtracted" or taken away. However, this is not a logical contradiction. As James East argues:

> If actual infinite collections were to exist, then they would naturally have properties that were not shared by finite collections…The story of Hilbert's Hotel simply highlights … such [a] property that distinguishes actual infinite collections from finite ones: just knowing that an infinite subcollection has been removed from an infinite collection of objects does not allow one to determine how many objects remain. But this property itself does not entail that actual infinite collections are impossible. (East 2013: 433)

Thus, set theory is compatible with the existence of an actual infinite. For this reason, the Infinity Argument depends, not on pure logical or mathematical modality, but on our intuitions about what is metaphysically possible and impossible. However, as Craig and Sinclair (2012: 106) acknowledge, "[a]rguments for metaphysical [...] impossibility typically rely upon intuitions and conceivability arguments, which are obviously much less certain guides than strict logical consistency or inconsistency." Therefore, the Infinity Argument will not convince those who do not share the metaphysical intuitions of its proponents.

Secondly, the Infinity Argument brings into question the view that God predetermined an endless future. The problem is that the proponents of the KCA are inclined to believe that God can predetermine an endless sequence of events but it *appears* that, if God did so, an actual infinite would exist (namely, the set of all future truths), either as a self-subsisting abstract object[4] or as a content of God's mind (see Morriston 2010). Craig's solution to this problem is that abstract objects do not exist and that God's knowledge is non-propositional in nature because God has a simple, undivided intuition of all reality and, thus, God does not know an actually infinite number of propositions (Craig and Smith 1993: 94-96). Jacobus Erasmus and Anné H. Verhoef (2015: 411-427) offer a different solution, namely, that God can actualize an endless future without consciously thinking about each future event in it. However, both these solutions depend on controversial metaphysical positions, since the former requires the rejection of Platonism[5] and the latter defends an unusual account of omniscience. Therefore, these solutions can hardly be said to completely solve the problem posed by the predetermined future objection.

In light of these problems, we believe the Infinity Argument is inconclusive. Nevertheless, since the KCA tries to show that an actually infinite sequence of *past events* is impossible, it is unnecessary for the KCA to deny the possibility of any actual infinite whatsoever. Therefore, we suggest that the advocates of the KCA would profit from supplemen-

[4] Abstract objects are generally said to be those objects that are not persons and are non-spatial, non-temporal, non-physical, and causally inert. However, regardless of how one should define abstract objects, we are merely presupposing that propositions, truths, and mathematical objects (such as sets and numbers) are abstract objects.

[5] Of course, the proponent of the Infinity Argument that already denies Platonism might not view this as a problem. However, if the proponent *is* a Platonist, then this would be a problem for him. Likewise, if Platonists claim to have very powerful arguments in favour of Platonism, then the proponent of the Infinity Argument would have to take on the difficult task of refuting Platonism in order to defend the Infinity Argument.

ting the Infinity Argument with an argument that denies the possibility of beginningless time only. We will now propose one such argument inspired by Laureano Luna (2009a, 2014).

IV

Let us begin by offering an informal version of our philosophical argument for a beginning of the universe. We shall offer a thought experiment, which we shall call the "gong peal paradox", to illustrate the impossible situation that could result from beginningless time. This paradox is based on Luna's (2009a: 304) gong peal paradox, which is based on José A. Benardete's (1964: 259) gong peal thought experiment. The paradox is as follows. Imagine that the universe is beginning-less and past eternal (i.e. past time is infinite). Imagine further that Smith, who is immortal and has been around for the entire past, rings a peal once every day. Since Smith has always been around, he has always been doing this. Furthermore, imagine that the following proposition is true:

(B) The peal is so loud that Smith would be struck deaf permanently upon hearing the peal iff he was not previously deafened by the peal.

(B) entails that Smith can be deaf only as a consequence of hearing the peal. Now, for any past day *x*, Smith must already have been deaf prior to ringing the gong peal on *x*. The reason for this is that Smith's act of ringing the peal on the day immediately prior to *x* ensures that Smith would be deaf on *x*. Hence, Smith has been deaf throughout the entire past. But if Smith were deaf before he rings the peal on any past day, then there is no day on which he became deaf by hearing the peal. However, Smith can become deaf only by hearing the peal. Consequently, we are led to the following inconsistent states:

1. Smith can be deaf only as a consequence of hearing the peal.
2. Smith has never heard the peal.
3. Smith is deaf.

Since these three states are inconsistent, the entire situation is logically impossible. Intuitively, one may see that this entire situation was produced by the beginningless-ness of the temporal regress of past events. If time had a beginning, then the contradiction would vanish because Smith would become deaf on the first day.

One might object that the paradox does not result if time is beginningless and Smith rings the peal on a finite number of days only. In other words, one may argue that it is, in fact, the following two incompatible states that result in the impossible situation above:

(i) Time is beginningless.
(ii) For every day in time, Smith rings a peal that is so loud that Smith is struck deaf permanently upon hearing the peal iff Smith was not previously deafened by the peal.

Hence, the objection is that one may affirm (i) while denying the possibility of (ii) given (i) (note that this does not mean that (ii) is impossible if (i) is true but, rather, that (ii) is false in all possible worlds in which (i) is true). The problem with this objection, however, is that (ii) is obviously possible if time is finite and, therefore, if (i) is true in some possible worlds, then something must prevent (ii) from being true in all those possible worlds. But what could this something be? Stephen Yablo (2000) voices an unusual criticism of Benardete's God-wall paradox (Benardete 1964: 259–260), which is similar to the gong peal paradox. According to Yablo, denying an infinite regress is not the only solution to these Benardete-type paradoxes. Another way out of these paradoxes, claims Yablo, is to argue that logic prevents one of the seemingly possible situations in the thought experiment from happening. Adapted for the gong peal paradox, Yablo's solution is that logic renders (ii) impossible when (i) is true. In other words, if time is beginningless, then, although Smith intends to ring the peal every day at noon, logic prevents him from doing this an actually infinite number of times; logic permits Smith to ring the peal a finite number of times only, thereby causing Smith to become deaf by hearing the peal during his first ring.

Unfortunately, Yablo's solution seems to treat logic as a causal force. However, as Luna (2009b: 95) points out, logic is not a causal force that could step in and stop Smith from ringing the peal on certain days. Indeed, if logic could stop Smith from ringing the peal, on which days would it stop him? There is simply no logical necessity that Smith cannot ring the peal on certain days in all worlds in which (i) is true. Yablo's proposal is more a re-statement in other words of the already known impossibility than an explanation thereof.

Thus, there is no reason to think that something would render (ii) false in all possible worlds in which (i) is true. Therefore, the impossibility of the conjunction of (i) and (ii) does not seem to come from mere incompatibility between those propositions but from an intrinsic impossibility of (i).

It is helpful to note that the preceding argument is akin to the usual intuitive argument against the possibility of time travel: if one could go backward in time, what would prevent one of impeding one's own birth by killing one's own grandfather before he met one's own grandmother? What is obviously impossible is the following conjunction: "one can both travel backward in time and kill one's grandfather, so as to make one's own birth impossible." However, since nothing seems to be able to prevent the latter given the former, one is inclined to believe that the former is impossible on its own.

We are not claiming that the preceding argument is logically compelling. Instead, the argument is simply a heuristic device used to convey the intuition that ungroundedness in causal chains is not just counterintuitive, as traditionally believed, but also inconsistent if aptly tweaked. If one compares the famous Russell set of all non self-membered sets with the set of all self-membered sets, or the Liar with the Truth-Teller, one sees that they turn indeterminateness due to ungroundedness into contradiction.[6] Benardete's and Yablo's paradoxes seem to play a similar role as regards ungrounded temporal chains. Yablo's paradox (Yablo 1993) is as follows: let $S = s_1, s_2, s_3 \ldots$ be an infinite sequence of sentences such that for all n,

$$s_n = \text{“}\forall m{>}n\ [s_m \textit{ is untrue}]\text{”}.$$

Therefore, for each n, s_n is true iff none of the following sentences in S is. Suppose some s_n is true; then for all $m > n$, s_m is untrue; hence for all $p > n + 1$, s_p is untrue, which renders s_{n+1} true, hence s_n false; contradiction; as s_n was arbitrary, all members of S are false but then for all $m > n$, s_m is untrue, which makes s_n true; contradiction again. This is the paradox. Suppose now each s_n was stated at time t_n and reverse the order of the members of S from $s_1, s_2, s_3 \ldots$ to $\ldots s_3, s_2, s_1$ so that t_{n+1} immediately precedes t_n; then you get a structural analogue to Benardete's paradox (as shown in Shackel 2005).

We will now present a more formal argument. Surely, the intuition behind the claim that (ii) has to be valid as an effective rule to determine

[6] Let S be the set of all self-membered sets; we have that $S \in S \leftrightarrow S \in S$, which leaves the issue indeterminate; however, for Russell's R: $R \in R \leftrightarrow R \notin R$, which is a contradiction. Consider the Truth-Teller sentence: "this sentence is true": it is true iff it is true, and we have no other criterion to determine its truth value. Consider the (Strengthened) Liar sentence "this sentence is not true": it is true iff it is not true.

Smith's deafness is that the past is determinate to the effect of determining its future. This seems to follow from the irreversibility of time. Now, if we exploit the irreversibility of time and the determinateness of the past as essential features of time, we can prove theorems about time just as we can prove theorems about space based on its essential traits. This permits to offer a more rigorous version of the preceding argument in the next section: we will rely on the determinateness of the past and will use a rule akin to (B) in order to argue against the possibility of beginningless time.

V

DEFINITION 1

i. By a *strict chain* we understand a pair $S = (Q, <)$, where Q is a set with at least two members and $<$ is a strict total order on Q.

The order among the members of Q can be expressed by supplying its members q_i with indices from a subset I from the adequate set of numbers (natural, integer, real numbers…). Let q_B be the $<$-first member of Q, if it exists (we can take the index B to be always 0; however, $q_0 \neq q_B$ if the chain has no $<$-first member).

ii. A chain is *ungrounded* iff

$$\forall i \in I \, \exists j \in I \, [q_j < q_i].$$

REMARKS ON DEFINITION 1

Chains are usually defined as totally ordered subsets of partially ordered sets: strictness is usually not included. However, for brevity, we will omit the adjective "strict" from now on.

If we wish to concern ourselves with causal-temporal chains of events, time can provide the ordering $<_T$, interpreted as a relation of temporal precedence.

DEFINITION 2

Let $S = (Q, <)$ be a chain. A *recursive determination rule D* on S is a rule that determines the state of the items in Q (except q_B, if it exists) on the basis of the states of $<$-prior items in Q. More precisely, let Σ be the set of all states of the members of Q and, for any $i \in I$, let σ_i be the state of q_i. D is a recursive determination rule iff, when represented as a function

$D : 2^{\Sigma} \rightarrow \Sigma$ (where 2^{Σ} is the power set of Σ), it is such that

$$\forall i \in I / \{B\} \left[\exists X \neq \emptyset \left[X \subseteq \Sigma \,\&\, \forall j \in I \left[\sigma_j \in X \rightarrow q_j < q_i\right] \,\&\, \sigma_i = D(X)\right]\right].$$

As advanced, this means that each state, except the initial one if it exists, is determined by D on the basis of some nonempty set of previous states. If we define X_i as follows: for any $i \in I$, let

$$X_i = \{\sigma_j \in \Sigma \mid q_j <_T q_i\},$$

then a special form of recursive determination rule is the rule D_{Xi} in which X_i is never empty and

$$\forall i \in I \left[\sigma_i = D_{Xi}(X_i)\right],$$

and any such D_{Xi} is a recursive determination rule, since it determines each σ_i based on all σ_j preceding σ_i.

DEFINITION 3

Time is *normal* iff the content of the past is determinate to the effect of determining on its basis the content of subsequent time units. This can also be more formally defined as follows. Let $T = (Q_T, <_T)$; let D_T be a *well-defined* (i.e. univocal, that is to say. able to be represented as a function, as D in Definition 2) recursive determination rule on T; T is normal iff *any* D_T is able to determine σ_i for each $i \in I$, except B (if $B \in I$).

LEMMA 1

If time is beginningless, then a well-defined recursive determination rule exists that fails to make σ_i definite for each $i \in I$.

PROOF

Let $T = (Q_T, <_T)$ be a beginningless chain of time intervals. We define a recursive rule L_R on T. L_R is inspired by Benardete's gong peal paradox (Benardete 1964: 259) and Yablo's Infinite Liars paradox. According to L_R, the state of any member of Q_T is 1 if no state of a previous member is 1; otherwise, it is 0:

$$(L_R) \;\; \forall i \in I \left[\sigma_i = 1 \leftrightarrow \forall j \in I \left[q_j <_T q_i \rightarrow \sigma_j \neq 1\right]\right].$$

Note that L_R has the same form as rule B in the gong peal paradox: for any day, there is something that occurs on that day iff it has never occurred on any previous day. Note also that L_R is both unambiguously defined and a recursive determination rule, since (letting for simplicity

"L_R" denote the corresponding function)[7] for any $i \in I$:

$$\sigma_i = L_R(X_i).$$

We can show by purely logical means that L_R is inconsistent for T. Here is the sketch of a derivation:

1. $\exists i \in I\,[\sigma_i = 1]$	Assumption
2. $\sigma_k = 1$	1,
3. $\forall j \in I\,[q_j <_T q_k \rightarrow \sigma_j \neq 1]$	2, L_R
4. $\forall j \in I\,[q_j <_T q_{k-1} \rightarrow \sigma_j \neq 1]$	3
5. $\sigma_{k-1} = 1$	4, L_R
6. $\exists j \in I\,[q_j <_T q_k\ \&\ \sigma_j = 1]$	5
7. $\sigma_k \neq 1$	6, L_R
8. $\nexists i \in I\,[\sigma_i = 1]$	Contradiction between 2 and 7
9. $\forall i \in I\,[\sigma_i \neq 1]$	8
10. $\forall i,j \in I\,[q_j <_T q_i \rightarrow \sigma_j \neq 1]$	9
11. $\forall i \in I\,[\sigma_i = 1]$	10, L_R
12. $\exists i \in I\,[\sigma_i = 1]$	11

Note the contradiction between 8 and 12.

NOTE ON LEMMA 1

Trivially, if $q_B \in Q_T$, then by L_R, $\sigma_B = 1$ and

$$\forall i \in I\,[B <_T i \rightarrow \sigma_i = 0],$$

which renders L_R successful in providing states for all members of Q_T: inconsistency vanishes as soon as the chain is grounded and an infinite regress is impossible.

THEOREM 1

If time is normal, then it is not beginningless.

PROOF

There is a contradiction between definition 1 of normality of time, according to which each well-defined recursive determination rule succeeds in determining states for the items of Q_T, and Lemma 1.

[7] The definition of L_R as a function would be: $\forall i \in I\,[L_R(X_i) = 1$ if $\forall x \in X_i\,[x \neq 1]$, and $L_R(X_i) = 0$, otherwise].

DETERMINATENESS OF THE PAST AND NORMALITY OF TIME

If we assume that a temporal chain is both beginningless and normal, we can prove that at least the recursive determination rule L_R governing the chain both succeeds and fails, which is the fundament of Theorem 1. It fails because it deals with an ungrounded chain and falls into an infinite regress. It succeeds because time is normal. But do we really need normality of time to show it cannot fail or can the determinateness of the past do the job? Indeed, how could the recursive rule not succeed if it is only determining items in a well-defined way on the basis of already determinate items?

Time, in our intuitive conception, is normal. This intuition relies on the conviction that what has already occurred is determinate and unchangeable: no influence goes backwards in time; time is, in this sense, *irreversible*. Stating that time is irreversible is stating that the past cannot be altered from its future, that is, that the past is *irreversibly determinate*. So far, we have a formal proof that normal time is not beginningless and a strong intuition that the past is irreversibly determinate. *We need an argument to the effect that if the past is irreversibly determinate, as intuition suggests, then it is normal and then, as our Theorem 1 shows, it has a beginning.* This we provide in the following paragraphs.

Consider the following question: how could an absolutely univocal instruction returning just one output for each input fail to determine an output? Intuitively, this can only occur in case the input is not determinate. And one can easily imagine how this could be the case in temporal causal chains; imagine an instruction determining the content of time t in terms of its *future;* as the future of t may depend causally on t, it may fail to be determinate to the effect of determining the content of t: the instruction may lead to a vicious circle or a loop. But precisely this is what cannot be the case for recursive determination rules in temporal chains *if the past is irreversibly determinate.* If the past is so determinate, the input of a rule like L_R is always determinate.

Or consider an agent A, omniscient as regards everything determinate and lying in the past, located at the beginning of time unit t and trying to determine the content of t according to rule L_R. How could A fail? As L_R is absolutely well-defined and univocal, A will be able to decide the content of t as soon as he knows the content of the past of t; as A knows every bit of the past (unless it is indeterminate), A can only fail if the past of t is indeterminate. Thus, if the past is determinate for any t, A can never fail. If A can never fail to determine t on the basis of a recursive determination rule, then no such rule can fail and time is surely normal.

As no loop or any other cause of indeterminateness can arise in a recursive determination rule for a temporal chain if the past is irreversibly determinate, everything works as if the irreversible determinateness of the past grounded the rule so as to prevent it from failing. This is why we have that for a beginningless time, if the past is always irreversibly determinate, L_R must both fail and succeed.

Thus, it seems that, on the intuitive condition that the past is irreversibly determinate, time has to be normal; and if so, by Theorem 1, it must have a beginning. This completes our main argument.[8]

Consider, furthermore, that it is hard to see how causal laws could work if time were not normal, so that past events were not determinate to the effect of determining upcoming events. The fact that causal laws exist and work provides a direct intuition that time is normal. Thus, we believe that the result that normal time cannot be beginningless is itself a strong result because time seems very likely to be normal in the sense defined.

Normality is a condition of time as we conceive it that can be used to establish some facts about it, in the same way as the properties of space permit to prove geometrical theorems. For instance, it can be shown that the notion of *beginningless normal* time is inconsistent.

Appendix. Replies to possible objections

This article has a history of objections by colleagues that range from subtle to weird. This is why we have deemed it convenient to add this appendix.

Objection 1. "Smith could suffer from eternity from permanently deafening otitis."

In response we note that this objection fails to appreciate the nature of a thought experiment. A thought experiment is an imaginary scenario that is used to test which of a few supposedly possible propositions are, in fact, possible. The "possibility" in mind is not merely a practical possibility, but a logical or metaphysical possibility. Thus, the critic cannot simply assert that the thought experiment is too far-fetched but he or she must also show that (ii) is logically impossible. In other words,

[8] Note that determinism in the sense of causal determinism is nowhere invoked in the argument.

objecting to (ii) is like objecting to the definition of a newly introduced object: it makes sense only if the stipulation is contradictory; but surely logic doesn't require in the above scenario the presence of something other than the gong peal to deafen Smith.

Objection 2. "One can only ask of irreversible time to be determinable by any recursive rule that is not contradictory the way we have shown L_R to be."

This objection can be given the following two replies: (a) The point is that if the past is irreversibly determinate, it is unintelligible how a rule (whatever it is) could fail to determine one item if it univocally prescribes its content based only on the content of preceding items, i.e. on the content of the past. The objection is irrelevant to this point. (b) Note that our point could be made using the rule "any item is the same as the immediately preceding one" (call it L_P) instead of L_R. The sole advantage of L_R over L_P is that it can be logically proven by *reductio* to fail to determine any item whereas showing the same for L_P is just showing that it is compatible with more than one situation: all days are 0, all days are 1. Nothing, however, is as trenchant as a contradiction. However, the reader can forget about L_R and use L_P instead, which is by no means contradictory.

Objection 3. "An irreversibly determinate past only guarantees an effective determination capacity for a recursive rule if the past is independently given for the application of the rule, that is, if it is not given through the rule itself; otherwise, the situation could be circular whatever the nature of the past."

The objection is unjustified because, as we explain in the paper, if the agent who is to apply the recursive rule is relying in each occasion only on an irreversibly determinate past, the rule cannot fail in any occasion; the point is that the irreversibility of time has to ground the rule. If there are circumstances (as absence of a first time unit) under which a univocal recursive rule finds the past unavailable as a ground for the determination of the future, then, on the assumption that time is irreversible, those circumstances are ontologically impossible, for they are in contradiction with the inherent nature of time. And such is in essence our argument.

Objection 4. "There are ungrounded recursive rules that succeed even if time is beginningless. For instance, this expression determines a unique content for each day:

(1) $\forall t\,[f(t) \in Z^{+}\ \&\ f(t) = f(t-1)^{2}]$

namely, $\forall t\,[f(t) = 1]$."

This objection attempts to refute the intuitive conjecture that the contradiction arising from the combination of L_R and beginningless time ultimately proceeds from the fact that ungrounded determination chains are unable to determine. Note, however, that as an instruction, (1) is useless to determine the value of any t because it leads to an infinite regress; we suggest that, for the purpose of representing the causal dependence relation that obtains between the items in a causal-temporal chain, recursive rules should be interpreted as instructions to compute, in such a way that in the case of (1) one has to compute $t - 1$ in order to compute t; otherwise, the causal dependence relation gets lost in the mere mathematical formulation. Franzén has remarked the distinction between mere equations and computing instructions:

> In general, however, it is not the case that the function computed by a set of equations [...] read as an algorithm is the unique function satisfying those equations read as an assertion. Consider the following:
>
> (2) $G(x) = 2$ *if* $x = 0$, and otherwise $G(x) = G(x+1) * (G(x+1) + 1)$
>
> There is a unique total function satisfying (2), namely the function G for which $G(n) = 0$ for all $n > 0$, and $G(0) = 2$. However, the function computed by (2) read as an algorithm is undefined for all $x > 0$, since the attempt to compute $G(x)$ by first computing $G(x+1)$ never terminates. (Franzen 2004: 154)

References

Benardete, J. A. 1964. *Infinity: An Essay in Metaphysics* (Oxford: Clarendon Press).

Borde, A., Guth, A. and Vilenkin, A. 2003. "Inflationary spacetimes are incomplete in past directions", *Physical Review Letters* 90(15), 151301.

Craig, W. L. and Smith, Q. 1993. *Theism, Atheism, and Big Bang Cosmology* (Oxford: Clarendon Press).

Craig, W. L. 1999. "The ultimate question of origins: God and the beginning of the universe", *Astrophysics and Space Science* 269-270: 723-740.

Craig, W. L. and J. D. Sinclair. 2012. "The kalam cosmological argument", in: W. L. Craig and J. P. Moreland (eds.), *The Blackwell Companion to Natural Theology* (Oxford: Wiley-Blackwell), 101-201

East, J. 2013. "Infinity minus infinity", *Faith and Philosophy* 30(4): 429-433.

Erasmus, J. and Verhoef, A. H. 2015. "The *kalām* cosmological argument and the infinite god objection", *Sophia* 54(4): 411-427.

Erasmus, J. 2018. *The Kalām Cosmological Argument: A Reassessment* (Cham: Springer Nature).

Franzén, T. 2004. *Inexhaustibility: A Non-exhaustive Treatment* (Wellesley, MA: A K Peters).

Koons, R. C. 2014. "A new kalam argument: Revenge of the Grim Reaper", *Noûs* 48(2): 256-267.

Luna, L. 2009a. "Ungrounded causal chains and beginningless time", *Logic and Logical Philosophy* 18(3-4): 297-307.

Luna, L. 2009b. "Yablo's paradox and beginningless time", *Disputatio* 3(26): 89-96.

Luna, L. 2014. "No successful infinite regress", *Logic and Logical Philosophy* 23(2): 189-201.

Morriston, W. 2010. "Beginningless past, endless future, and the actual infinite", *Faith and Philosophy* 27(4): 439-450.

Penrose, R. 1965. "Gravitational collapse and space-time singularities", *Physical Review Letters* 14(3), 57-59.

Pruss, A. R. 2018. *Infinity, Causation, and Paradox* (Oxford: Oxford University Press).

Shackel, N. 2005. "The form of the Benardete dichotomy", *British Journal for the Philosophy of Science* 56(2): 397-417.

Yablo, S. 1993. "Paradox without self-reference", *Analysis* 53(4): 251-252.